離散2次元陽曲線の連結性について

On connectivity of discretized 2D explicit curve

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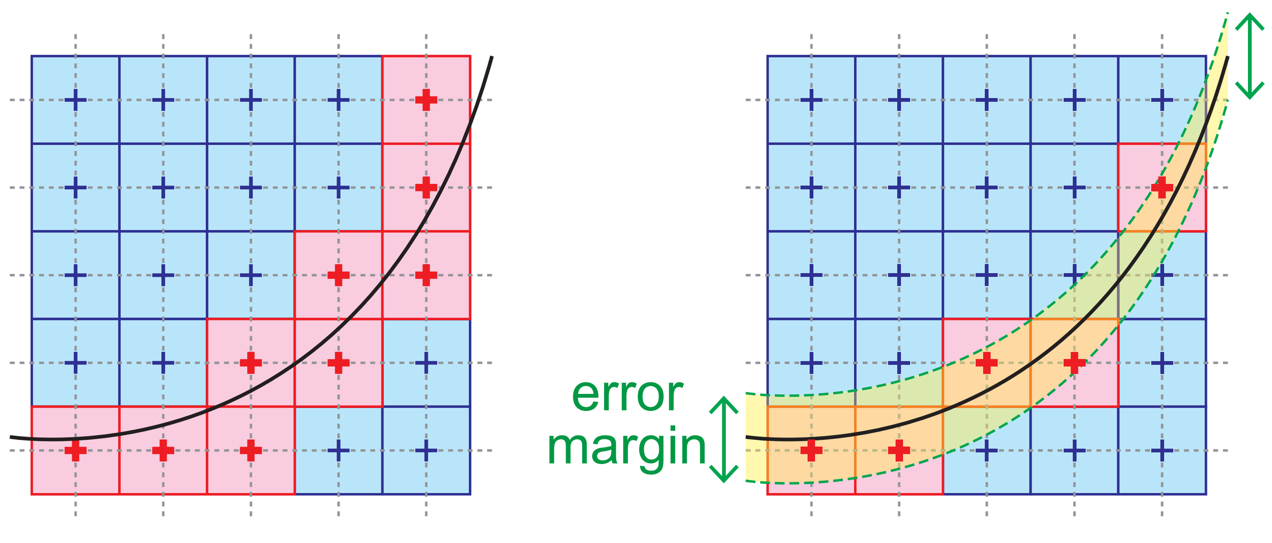
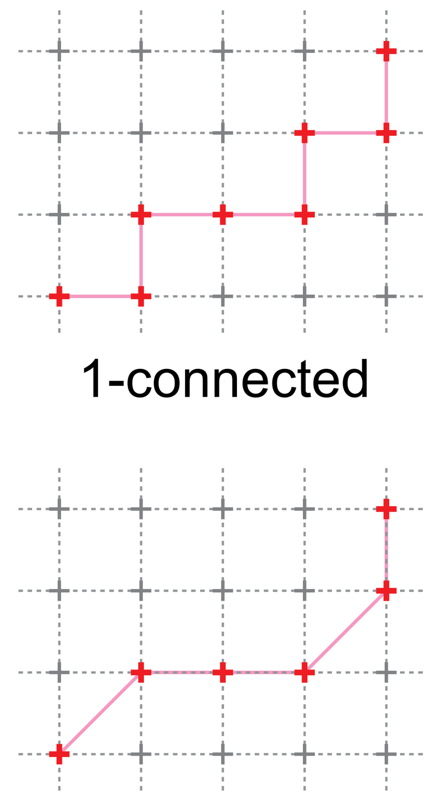
Shape representation in computer

Curves must be discretized in the computer, where preserving connectivity is important.



Connectivity depends on discretization method.

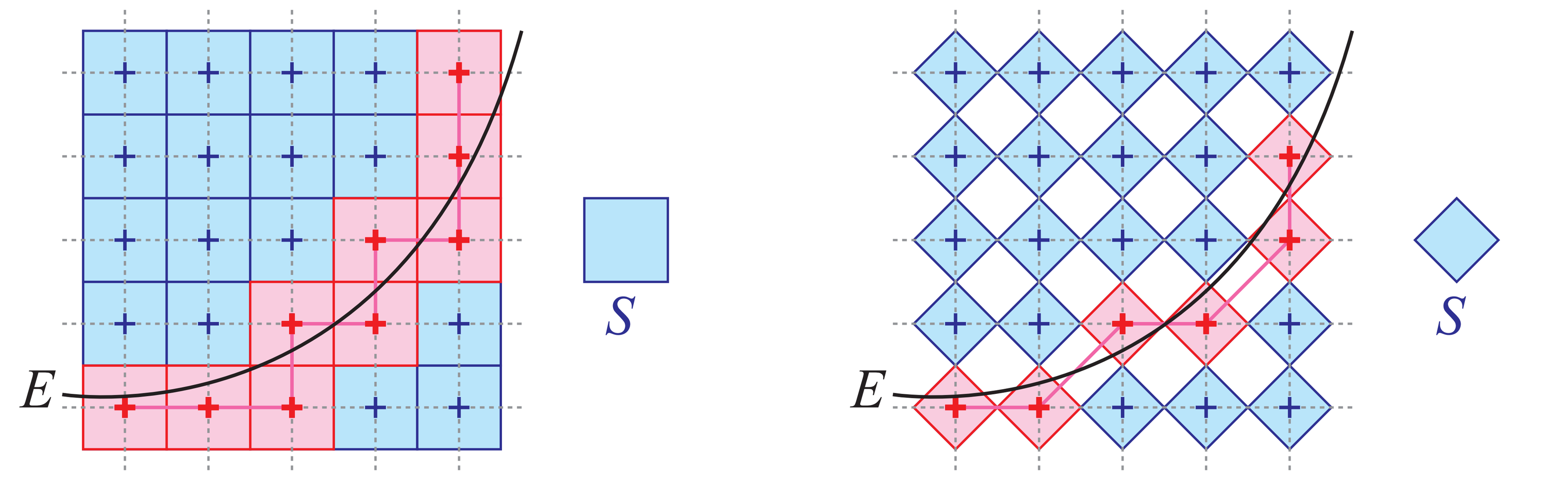


Morphological discretization (M.D.) [Heijmans et al., 1991]

$$D_S(E) = \{\boldsymbol{v} \in \mathbb{Z}^2 : (\{\boldsymbol{v}\} \oplus S) \cap E \neq \emptyset\}$$

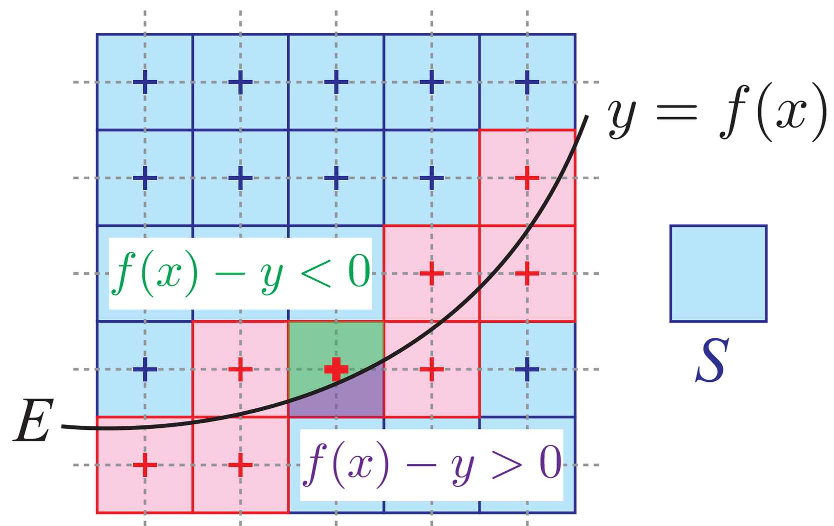
$E \subset \mathbb{R}^2$: original continuous curve

$S \subset \mathbb{R}^2$: structuring element → proper choice preserves connectivity



M.D. of 2D explicit curve

$$D_S(E) = \left\{ (x_{\text{int}}, y_{\text{int}}) \in \mathbb{Z}^2 : \begin{array}{l} y_{\text{int}} \geq \min_{(s_x,s_y)\in S} (f(x_{\text{int}} + s_x) - s_y) \\ y_{\text{int}} \leq \max_{(s_x,s_y)\in S} (f(x_{\text{int}} + s_x) - s_y) \end{array} \right\}$$

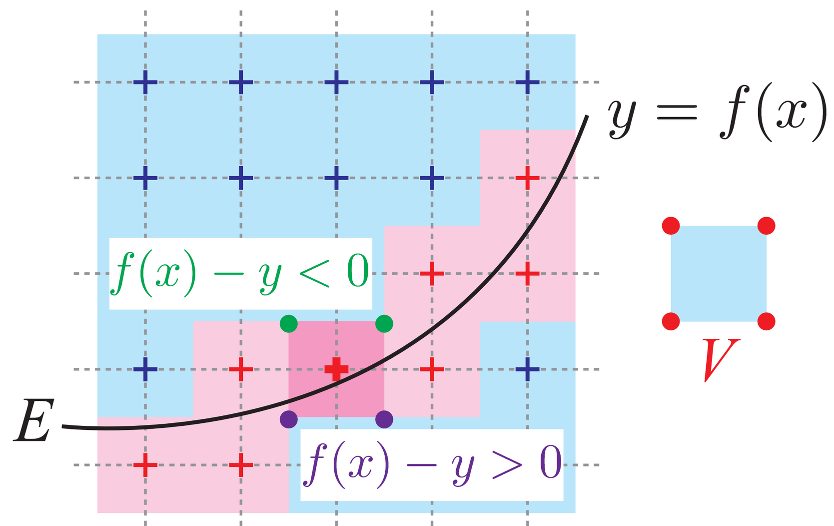


for E represented in $y = f(x)$ ($f : \mathbb{R} \to \mathbb{R}$ is continuous)

Problem:
Evaluating the min and max is computationally expensive since S has infinite elements.

Analytical approximation (A.A.) [Toutant et al., 2014]

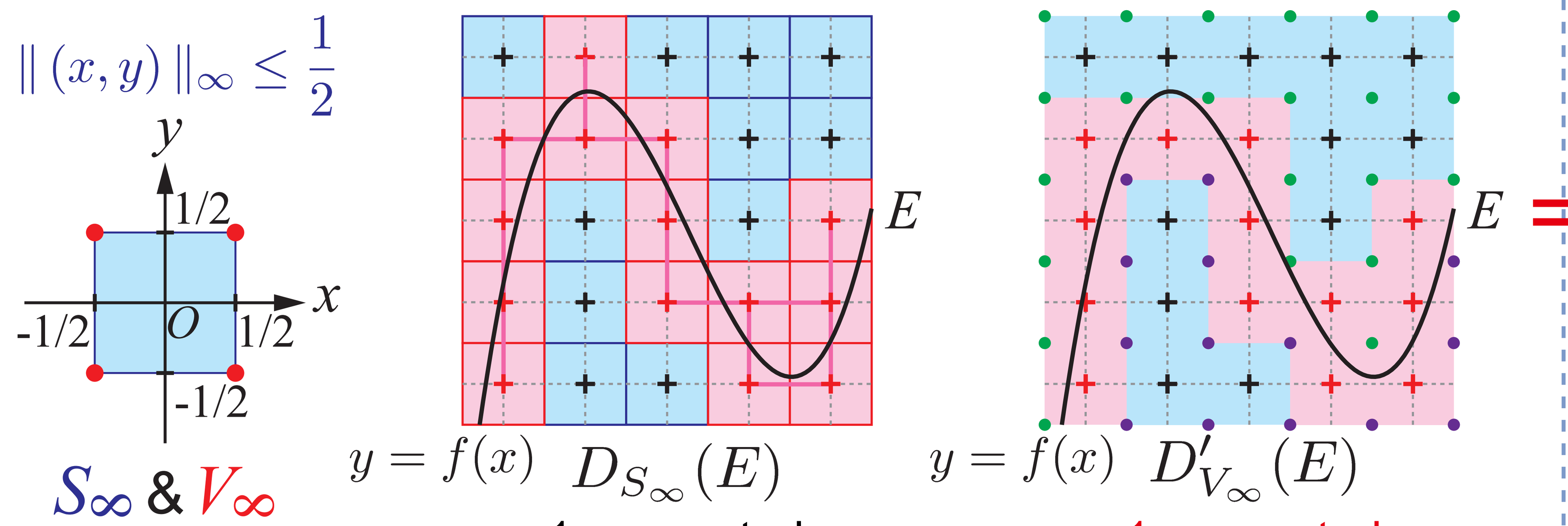
$$D'_V(E) = \left\{ (x_{\text{int}}, y_{\text{int}}) \in \mathbb{Z}^2 : \begin{array}{l} y_{\text{int}} \geq \min_{(s_x,s_y)\in V} (f(x_{\text{int}} + s_x) - s_y) \\ y_{\text{int}} \leq \max_{(s_x,s_y)\in V} (f(x_{\text{int}} + s_x) - s_y) \end{array} \right\}$$



Approximates M.D. by using only a finite subset of S.
=> computationally inexpensive

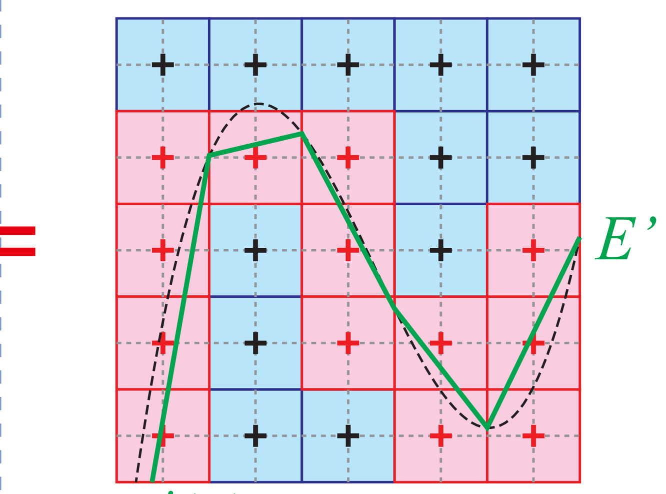
Question:
What becomes of connectivity?

Relationship on connectivity between M.D. and A.A.



Theorem: $D_{S_\infty}(E)$ and $D'_{V_\infty}(E)$ have the same connectivity.

Proof sketch



A.A. can be regarded as M.D. of a piecewise linear approximation to E.

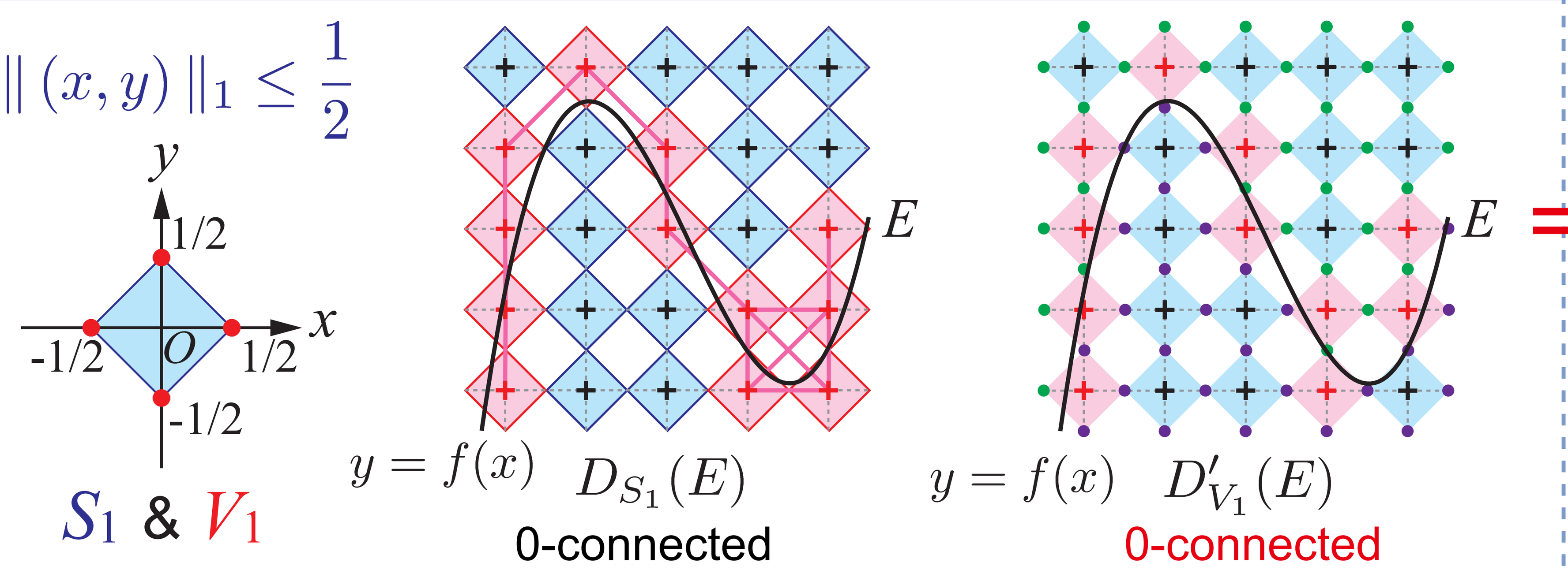
$D_{S_\infty}(E')$: $\min_{(s_x,s_y)\in S_\infty} (f'(x_{\text{int}} + s_x) - s_y) \leq y_{\text{int}} \leq$ (abbrv.)

⇕

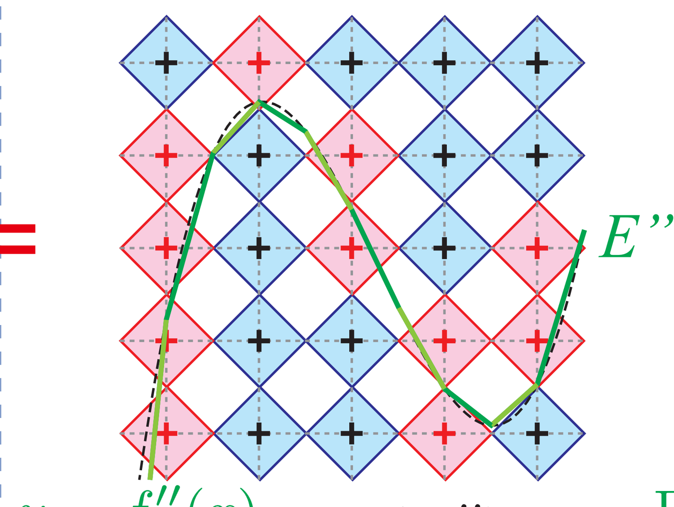
$D'_{V_\infty}(E)$: $\min_{(s_x,s_y)\in V_\infty} (f(x_{\text{int}} + s_x) - s_y) \leq y_{\text{int}} \leq$ (abbrv.)

Properties of f' : for $\forall x_{\text{int}} \in \mathbb{Z}$,

1. Linear within $\left[x_{\text{int}} - \frac{1}{2}, x_{\text{int}} + \frac{1}{2}\right]$
2. $f'\left(x_{\text{int}} + \frac{1}{2}\right) = f\left(x_{\text{int}} + \frac{1}{2}\right)$



Theorem: $D_{S_1}(E)$ and $D'_{V_1}(E)$ have the same connectivity.



$D_{S_1}(E'')$: $\min_{(s_x,s_y)\in S_1} (f''(x_{\text{int}} + s_x) - s_y) \leq y_{\text{int}} \leq$ (abbrv.)

⇕

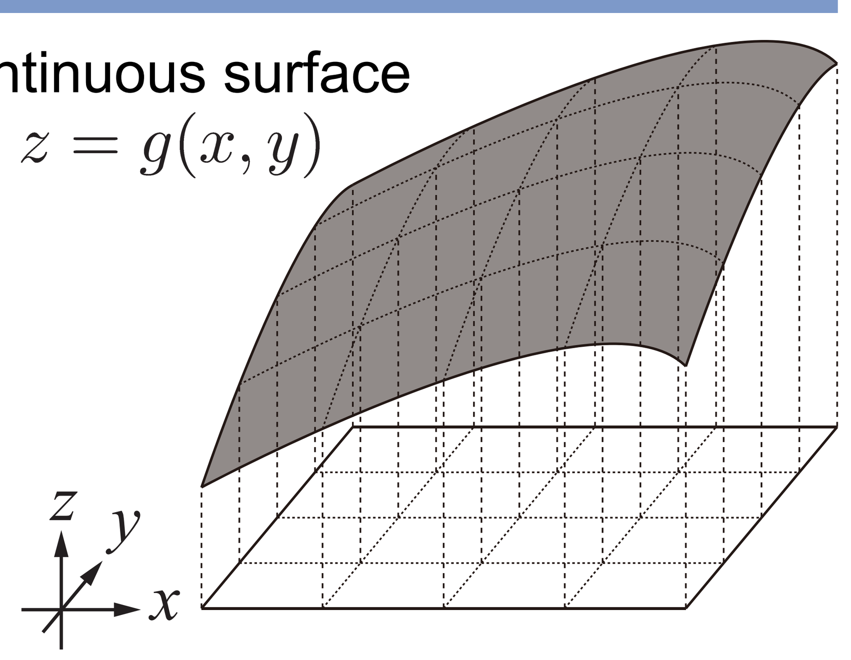
$D'_{V_1}(E)$: $\min_{(s_x,s_y)\in V_1} (f(x_{\text{int}} + s_x) - s_y) \leq y_{\text{int}} \leq$ (abbrv.)

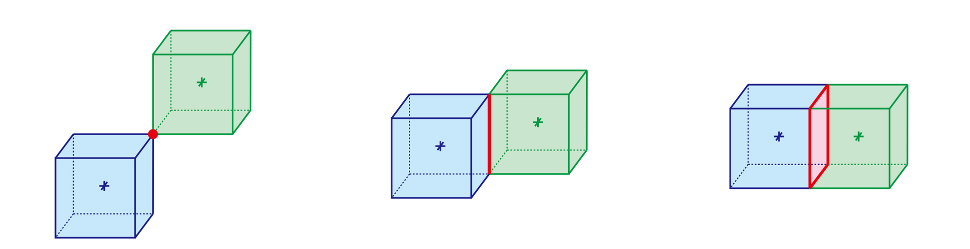
Properties of f'' : for $\forall x_{\text{int}} \in \mathbb{Z}$,

1. Linear within $\left[x_{\text{int}} - \frac{1}{2}, x_{\text{int}}\right]$ and $\left[x_{\text{int}}, x_{\text{int}} + \frac{1}{2}\right]$
2. $f''\left(\frac{x_{\text{int}}}{2}\right) = f\left(\frac{x_{\text{int}}}{2}\right)$

Extension to 3D explicit surface

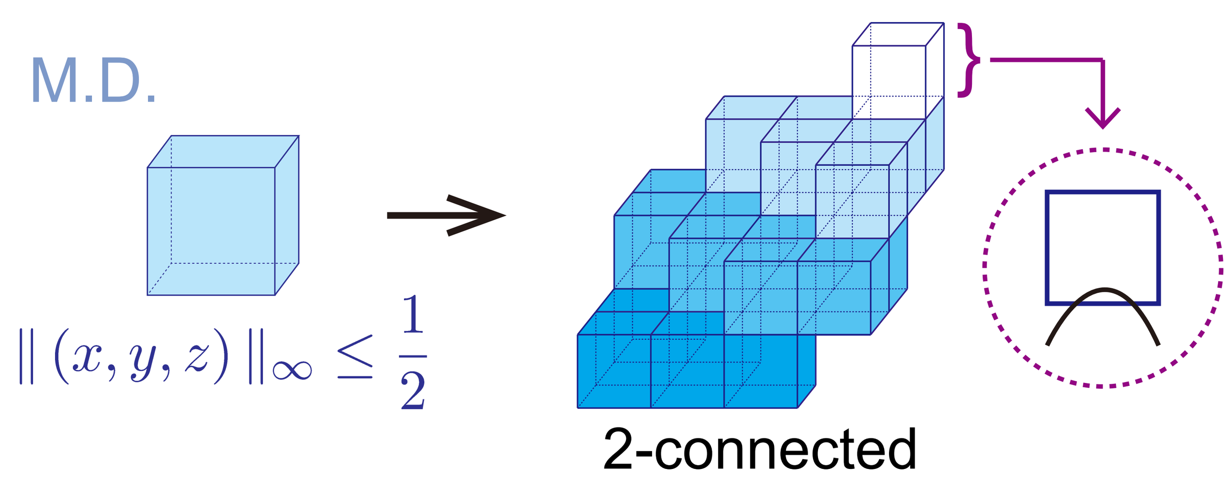
continuous surface $z = g(x, y)$



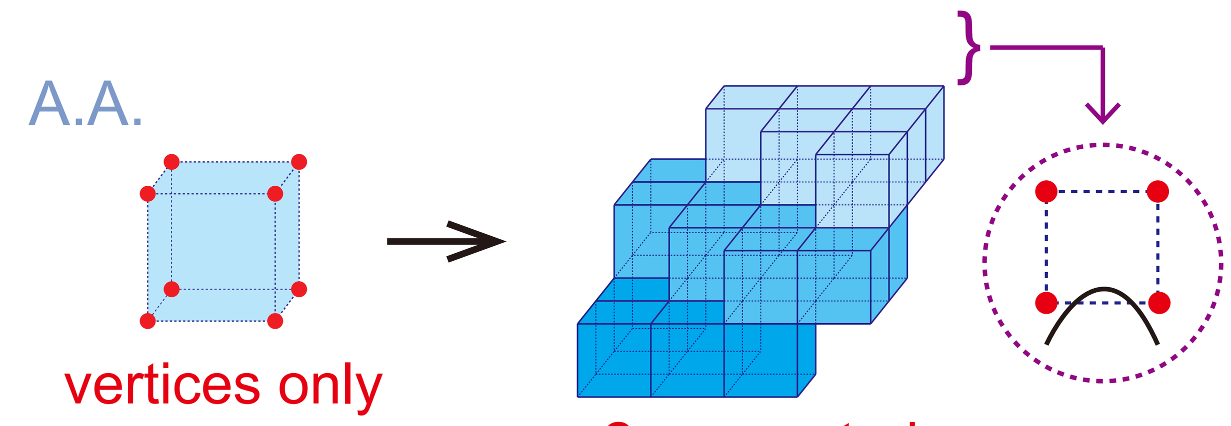


Connectivity relation between M.D. and A.A

M.D.



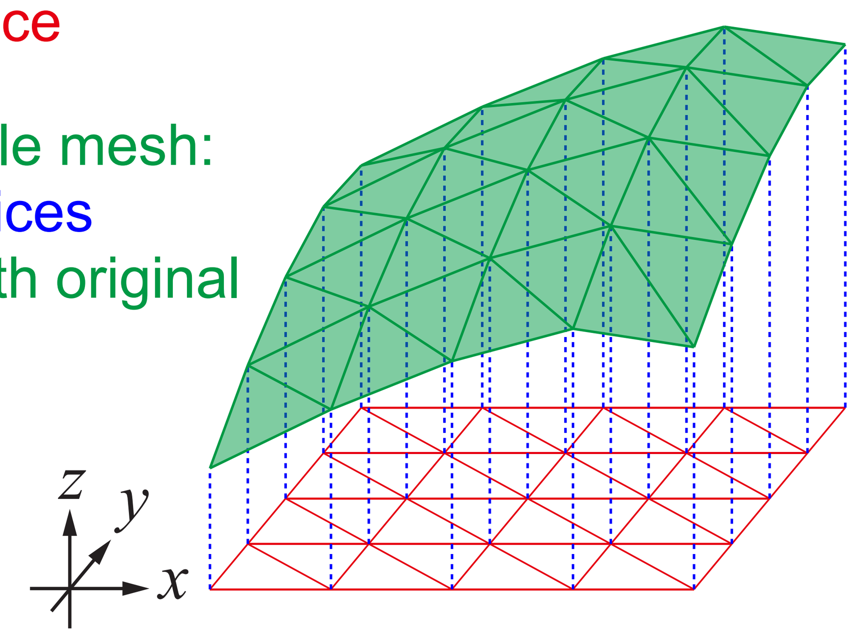
A.A.

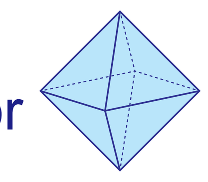


Proof sketch

A.A. can be regarded as M.D. of a piecewise linear approximation (triangle mesh) to the original surface

Properties of triangle mesh:
1. extrema at vertices
2. same height with original at vertices



Same logic holds for  corr. to 1-connectivity

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