Social Laws for Multi-Agent Systems: Logic and Games

Lecture 6: Reasoning about Social Laws

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Introduction

Topic of the day:

- Expressing properties of systems using formal logic
- In particular involving *quantification over coalitions*
- And in particular properties of social laws, involving *compliance*, such as robustness and power properties

Contents

Background

Cooperation logics have received much attention in the multi-agent system literature in recent years

- Idea: modalities saying what a group of agents, or a single agent, has the ability to enforce. More or less independent approaches:
- Bonanno: $\diamond_i \varphi$: agent *i* can unilaterally bring about a state where φ holds
- van Benthem on "forcing": extension to groups
- Pauly's Coalition Logic: $[G]$
- Alur et al.'s Alternating-time Temporal Logic: add temporals
- Seeing-To-It-That (STIT) logics

Coalition Operators

- Coalition: a set of agents
- Coalition operator: $[G]$ where G is a coalition
- Formula

$$[G]\varphi$$

  means that:
  - coalition G can make φ come about
  - there is a strategy for each member of G such that no matter what the agents outside G do, we will end up in a state where φ holds
- Marc Pauly's Coalition Logic:

$$\varphi ::= p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \wedge [G]\varphi$$

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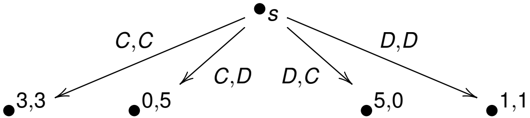
Example

M:



$$M, s \models [Ann]jail_B$$

Coalition Logic

- Equivalent to the next-time fragment of Alternating-time Temporal Logic
- Pauly has shown that CL can be used to express properties of social mechanisms, but for some purposes it is not expressive/succinct enough
- Many extensions have been developed:
  - Temporal
  - Epistemic
  - Quantification
  - Deontic

Contents

Lack of Succinctness in CL

Take the property:

agent 1 is necessary to achieve φ

Its expression in CL is exponentially long in the number of agents in the system. If $Ag = \{1, 2, 3, 4\}$:

$$\neg[\{\}]\varphi \wedge \neg[\{2\}]\varphi \wedge \neg[\{3\}]\varphi \wedge \neg[\{4\}]\varphi \wedge \neg[\{2,3\}]\varphi \wedge \neg[\{3,4\}]\varphi \wedge \neg[\{2,4\}]\varphi \wedge \neg[\{2,3,4\}]\varphi$$

- Ideally, we would like to write something like this:

$$\forall C([C]\varphi \rightarrow 1 \in C)$$

- But we must be careful with complexity
- We introduced Quantified Coaliton Logic to deal with quantification in a tractable way

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Lack of succinctness

- Note that this particular example assumes no *coalition montonicity* (which many variants of coalition logic have).
- It is easy to think of other examples: “every two-agent coalition can achieve φ”, etc.

Quantified Coalition Logic

Collection of unary modal operators indexed by a coalition predicate P:

$\langle P \rangle \varphi$: there exists some coalition satisfying P which can achieve φ

$[P]\varphi$: every coalition satisfying P can achieve φ

Examples of predicates (C' a coalition, n a number):

- $supseteq(C')$: satisfied by C iff $C \supseteq C'$
- $geq(n)$: satisfied by C iff $|C| \geq n$
- $gt(n)$: satisfied by C iff $|C| > n$
- $maj(n) \equiv geq(\lceil (n+1)/2 \rceil)$
- Boolean combinations

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QLC: formally

$$\varphi ::= p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \wedge \langle P \rangle \varphi \mid [P]\varphi$$

$$P ::= subseteq(C) \mid supseteq(C) \mid geq(n) \mid \neg P \mid P \vee P$$

$C' \models_{cp} subseteq(C)$ iff $C' \subseteq C$
$C' \models_{cp} supseteq(C)$ iff $C' \supseteq C$
$C' \models_{cp} \neg P$ iff not $C' \models_{cp} P$
$C' \models_{cp} P_1 \vee P_2$ iff $C' \models_{cp} P_1$ or $C' \models_{cp} P_2$

$K, s \models p$ iff $p \in \pi(s)$ (where $p \in \Phi_0$)
$K, s \models \neg\varphi$ iff $K, s \not\models \varphi$
$K, s \models \varphi \vee \psi$ iff $K, s \models \varphi$ or $K, s \models \psi$
$K, s \models \langle P \rangle \varphi$ iff $\exists C \subseteq Ag$: $C \models_{cp} P$ and $K, s \models [C]\varphi$.
$K, s \models [P]\varphi$ iff $\forall C \subseteq Ag$: $C \models_{cp} P$ implies $K, s \models [C]\varphi$.

Example

agent 1 is necessary to achieve φ

$$\neg\langle\neg supseteq\{1\}\rangle\varphi$$

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Coalition Predicates

We have that

$$subseteq(C) \equiv \bigwedge_{i \in Ag \setminus C} \neg supseteq(\{i\})$$

and

$$supseteq(C) \equiv \bigwedge_{C' \subseteq Ag, C \not\subseteq C'} \neg subseteq(C').$$

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Derived predicates

$$
\begin{aligned}
eq(C) &\triangleq subseteq(C) \wedge supseteq(C)\\
subset(C) &\triangleq subseteq(C) \wedge \neg eq(C)\\
supset(C) &\triangleq supseteq(C) \wedge \neg eq(C)\\
incl(i) &\triangleq supseteq(\{i\})\\
excl(i) &\triangleq \neg incl(i)\\
any &\triangleq supseteq(\emptyset)\\
nei(C) &\triangleq \bigvee_{i \in C} incl(i)\\
ei(C) &\triangleq \neg nei(C)
\end{aligned}
$$

Example: voting

An electorate of n voters wishes to select one of two outcomes ω_1 and ω_2. They want to use a simple majority voting protocol, so that outcome ω_i will be selected iff a majority of the n voters state a preference for it. No coalition of less than majority size should be able to select an outcome, and any majority should be able to choose the outcome (i.e., the selection procedure is not influenced by the "names" of the agents in a coalition).

$$([maj(n)]\omega_1) \wedge ([maj(n)]\omega_2)$$

$$(\neg\langle\neg maj(n)\rangle\omega_1) \wedge (\neg\langle\neg maj(n)\rangle\omega_2)$$

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QCL: Some Results

Expressive Power

Quantified Coalition Logic is no more expressive than Coalition Logic

Succinctness

Quantified Coalition Logic is exponentially more succinct than Coalition Logic

Axiomatisation

We have a sound and complete axiomatisation

QCL: Some Results: Complexity

Model checking

The model checking problem can be solved in polynomial time – assuming an explicit representation of models

Model checking with succinct model representations

The model checking problem assuming an RML representation of models is PSPACE-complete.

QCL Some Results: Satisfiability

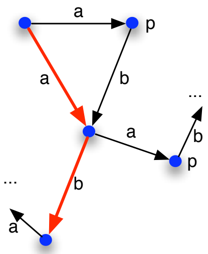
The satisfiability problem is PSCPACE-complete.

Contents

Recap: Social Laws

A social law is simply a labelling of some of the transitions as undesirable or illegal

- It is typically the case that if none of the illegal transitions are used, the system will behave in a desirable way
- Fundamental assumption: agents choose whether or not to comply



Motivation

- Expressing properties of social laws:

$$K, \eta \models \varphi$$

means that the social law η in the context of the system K has the property described by the formula φ then we could use tools from artificial intelligence and computer science to
  - formally reason about the logical principles of the mechanism
  - specify and verify properties of the mechanism
  - synthesise mechanisms
- We have already looked at one such language, *Normative Temporal Logic (NTL)*, allowing expressions such as

$$\mathsf{P}_{Tokyo} \Box eatnoodles \wedge \mathsf{O}_{Tokyo} \Diamond paynoodles$$

- But we are interested in *more expressive* languages, in particular in order to *formally reason about compliance*

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Norm Compliance CTL (NCCTL)

Language: extend CTL with

$$[P]\varphi$$

where P is a coalition predicate, meaning compliance of any coalition satisfying P will ensure that φ is true

Norm Compliance CTL: formally

$$P ::= subseteq(C) \mid supseteq(C) \mid geq(n) \mid \neg P \mid P \vee P$$

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{E}\bigcirc\varphi \mid \mathsf{E}(\varphi\,\mathcal{U}\,\varphi) \mid \mathsf{A}\bigcirc\varphi \mid \mathsf{A}(\varphi\,\mathcal{U}\,\varphi) \mid [P]\varphi$$

$$K, \eta, s \models [P]\varphi \Leftrightarrow \forall C \subseteq Ag\ (C \models_{cp} P \Rightarrow K \dagger (\eta \upharpoonright C), \eta, s \models \varphi)$$

Example

$$K, \eta, s \models [P]\varphi \Leftrightarrow \forall C \subseteq Ag \ (C \models_{cp} P \Rightarrow K \dagger (\eta \restriction C), \eta, s \models \varphi)$$

$\neg[eq(Ag)]\neg\varphi$: the social law is *effective*

Example

$$K, \eta, s \models [P]\varphi \Leftrightarrow \forall C \subseteq Ag \ (C \models_{cp} P \Rightarrow K \dagger (\eta \restriction C), \eta, s \models \varphi)$$

$\neg[\top]\neg\varphi$: there is *some* coalition whose compliance will ensure φ

Example

$$K, \eta, s \models [P]\varphi \Leftrightarrow \forall C \subseteq Ag \ (C \models_{cp} P \Rightarrow K \dagger (\eta \upharpoonright C), \eta, s \models \varphi)$$

$\neg[eq(Ag)]\neg\varphi \wedge [subset(Ag)]\neg\varphi$: the social law is effective but vulnerable

Examples: robustness

- $[supseteq(C)]\varphi$: C are *sufficient* for the social law in the context of the goal φ
- $[\neg supseteq(C)]\varphi$: C are *necessary* for the social law in the context of the goal φ
- $[geq(k)]\varphi$: the social law is *k-sufficient* wrt. the goal φ
- $[geq(n-k)]\varphi \wedge [ceq(n-k-1)]\neg\varphi$: the *resilience* of the social law is k
- $[\neg geq(k)]\neg\varphi$: the social law is *k-necessary* wrt. the goal φ

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- $[eq(Ag)]\varphi$: there *exists some sufficient coalition*
- $[any][any]\varphi$: exercise for the audience!
- $[P][any]\varphi$: there exists *some sufficient coalition satisfying P*
- $\neg \bigwedge_{i \in Ag}[\neg supseteq(i)]\varphi$: there exists non-empty sufficient coalitions

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General Robustness

We can also use coalition predicates to describe more general forms of robustness.

Example

The system will not overheat as long as at least one sensor works as it should and either one of the relief valves is working as it should or the automatic shutdown is working as it should

P characterises the robustness of η w.r.t. K and φ iff:

$$[P]\varphi \wedge [\neg P]\neg\varphi$$

iff

$$\forall C \subseteq A : \quad (C \models_{cp} P) \quad \Leftrightarrow \quad ((K \dagger (\eta \restriction C)) \models \varphi)$$

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Example cont.

$$P = nei(S) \wedge (nei(R) \vee incl(a))$$

characterises robustness in the example, where S is the set of sensors, R the set of relief valves and a the automatic shutdown system

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General Robustness

Theorem

Deciding P-characterisation is co-NP-complete

Examples: power

(Write $[C]$ for $[eq(C)]$)

- $SWING(C, i, \varphi) \equiv [C \cup \{i\}]\varphi \wedge \neg[C]\varphi$: i is *swing* for C when the goal is φ
- $MINBANZHAV(i, k, \varphi) \equiv \bigvee_{C_1, \cdots, C_k \subseteq A \setminus \{i\}, C_i \neq C_j} \bigwedge_{1 \leq j \leq k} SWING(C_j, i, \varphi)$: the *Banzhav score* for i is *at least* k, when the goal is φ
- $MAXBANZHAV(i, k, \varphi) \equiv \neg MINBANZHAV(i, k+1, \varphi)$: the *Banzhav score* for i is *at most* k, when the goal is φ
- $BANZHAV(i, k, \varphi) \equiv MINBANZHAV(i, k, \varphi) \wedge MAXBANZHAV(i, k, \varphi)$: the *Banzhav score* for i is *exactly* k, when the goal is φ
- $POS(\varphi) \equiv \bigvee_{i \in A} MINBANZHAV(i, 1, \varphi)$: there exists a player with a positive Banzhav score

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- $MINBANZHAV(i, k, \varphi) \equiv \bigvee_{C_1,\cdots,C_k \subseteq A \setminus \{i\}, C_i \neq C_j} \bigwedge_{1 \leq j \leq k} SWING(C_j, i, \varphi)$: the *Banzhav score* for i is *at least* k, when the goal is φ
- $MAXBANZHAV(i, k, \varphi) \equiv \neg MINBANZHAV(i, k+1, \varphi)$: the *Banzhav score* for i is *at most* k, when the goal is φ
- $BANZHAV(i, k, \varphi) \equiv MINBANZHAV(i, k, \varphi) \wedge MAXBANZHAV(i, k, \varphi)$: the *Banzhav score* for i is *exactly* k, when the goal is φ
- $POS(\varphi) \equiv \bigvee_{i \in A} MINBANZHAV(i, 1, \varphi)$: there exists a player with a positive Banzhav score

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2. $\langle C\rangle\alpha \leftrightarrow \alpha$ — α an objective formula
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Epistemic Logic

Modalities for expressing properties about agents' knowledge or beliefs

- $K_i\varphi$: agent i knows φ
- $E_G\varphi$: every agent in $G \subseteq Ag$ knows φ
- $C_G\varphi$: φ is common knowledge in $G \subseteq Ag$
- $D_G\varphi$: φ is distributed knowledge in $G \subseteq Ag$

Problem with Succinctness: example

At least two agents know that at most three agents know φ, from an overall set of agents $\{1, 2, 3, 4\}$.

$$E_{\{1,2\}}\psi \vee E_{\{1,3\}}\psi \vee E_{\{1,4\}}\psi \vee E_{\{2,3\}}\psi \vee E_{\{2,4\}}\psi \vee E_{\{3,4\}}\psi \vee E_{\{1,2,3\}}\psi \vee E_{\{1,2,4\}}\psi \vee E_{\{1,3,4\}}\psi \vee E_{\{2,3,4\}}\psi \vee E_{\{1,2,3,4\}}\psi$$

$$\psi = (\neg K_1\varphi \vee \neg K_2\varphi \vee \neg K_3\varphi \vee \neg K_4\varphi)$$

(exponential in the number of agents in the system)

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Epistemic Logic with Quantification over Coalitions (ELQC)

Idea: use coalition predicates for quantification, in the same way as in QCL

- $\langle P \rangle_C \varphi$: there exists some coalition satisfying P which have common knowledge of φ
- $[P]_C \varphi$: every coalition satisfying P have common knowledge of φ
- $\langle P \rangle_E \varphi$: there exists some coalition satisfying P in everybody knows φ
- $[P]_E \varphi$: in every coalition satisfying P everybody knows φ
- $\langle P \rangle_D \varphi$: there exists some coalition satisfying P which have distributed knowledge of φ
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At least two agents know that at most three agents know φ, from an overall set of agents $\{1, 2, 3, 4\}$.

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Example: knowledge dynamics of voting protocols

A committee consisting of Ann, Bill, Cath and Dave, vote for who should be the leader of the committee (it is possible to vote for oneself). The winner is decided by majority voting (majority means at least three votes, if there is no majority there is no winner).

Let una_a mean that *Ann* wins unanimously, and so on.

$$\neg ann\text{-}wins \rightarrow \langle geq(2) \rangle_D \neg \langle geq(3) \rangle_E (\neg una_b \land \neg una_c \land \neg una_d)$$

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Some results

Expressive Power

ELQC is no more expressive than $S5_n^{C,D}$

Succinctness

ELQC is exponentially more succinct than $S5_n^{C,D}$

Axiomatisation

We have a sound and complete axiomatisation

Model checking

Model checking for ELQC is Δ_2^p-complete

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Summary

- Introduced *Quantified Coalition Logic* to improve the *succinctness* of coalition logic
- Particular useful to reason about *compliance properties* of social laws
- Can also be used to quantify over coalitions in epistemic logic

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