Computational approaches to analyze complex dynamic systems: model-checking and its applications.

Part 1: Model-checking of discrete event systems

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NII - *Inoue Laboratory*
École Centrale de Nantes - IRCCyN - *MeForBio team*

Lecture Series - Lecture 1 - 2013/03/21

Overview

Vitæ

2004	Engineer diploma in "Computer Science"
2004	Master thesis in "Automatics and Applied Computer Science"
2007	PhD thesis in "Automatics and Applied Computer Science"École Centrale de Nantes / Université de Nantes $\Rightarrow$ **model-checking**, **dynamic systems**, **expressivity between models**
Since 2008:	Associate professor in Computer Science at École Centrale de Nantes / MeForBio research team at IRCCyN $\Rightarrow$ **bioinformatics**, **large-scale analysis**, **parameters inference**
Since 2010:	E-Learning Project Manager at École Centrale de Nantes

A few words about ICT for Education

Information and Communication Technologies Applied to Education

- An emerging **cross-disciplinary** research topic in our institution
- Several papers and projects about:
  - Skills **assessment** (e.g. Script Concordance Tests)
  - Integration and impact of mobiles devices in the learning environment (adaptation, motivation, ...)
  - **Massive Online Open Courses** (1st French MOOC, ITyPA: 1.400 students)
- **International** collaborations:
  - 2008-2010: HP
  - 2008-2011: OpenOffice.org Education project
  - Since 2009: University of Toronto (co-supervision with Karen Reid of the MarkUs software)
- Co-advisor of 1 PhD Student: Augmented learning in science and engineering

A few words about ICT for Education

E-learning group (ECN)

Morgan MAGNIN
Ass. prof. (CS) & project manager

Francisco CHINESTA
Professor (mechanics)

Guillaume MOREAU
Professor (CS)

Christine EVAIN
Associate professor (English)

Anne-Celine GROLLEAU
Research engineer

Simon CAROLAN
1st year PhD student

Main research activities: Bio-informatics

MeForBio (IRCCyN team, ECN): Formal Methods for Bioinformatics

Research axes

- Models: automata, Petri nets, process algebra ($\rightarrow$ process hitting)
- Extended with: time (**chronometry** vs **chronology**) and/or parameters
- Analysis techniques: model-checking, control, abstraction, parameters inference
- Applied (and/or designed) to biology, e.g. biological regulatory networks

Main research activities: Bio-Informatics

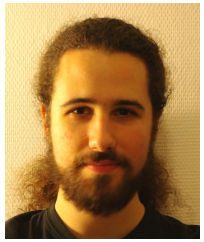
MeForBio (IRCCyN team, ECN): Formal Methods for Bioinformatics

Olivier ROUX
Professor & team leader

Morgan MAGNIN
Associate professor

Carito GUZIOLOWSKI
Associate professor

Julien GRAS
Research engineer



Maxime FOLSCHETTE
2nd year PhD student

Courtney CHANCELLOR
1st year PhD student

Overview

Introduction (1/2)

Real-time system

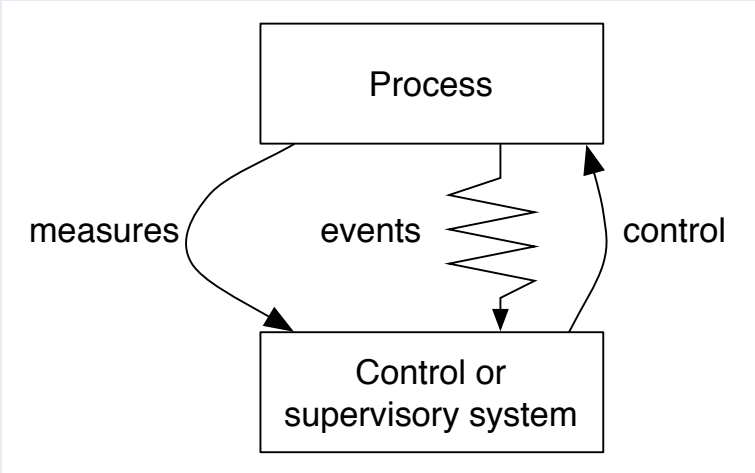


Figure: Analyzed system: { process + control system }

Introduction (1/2)

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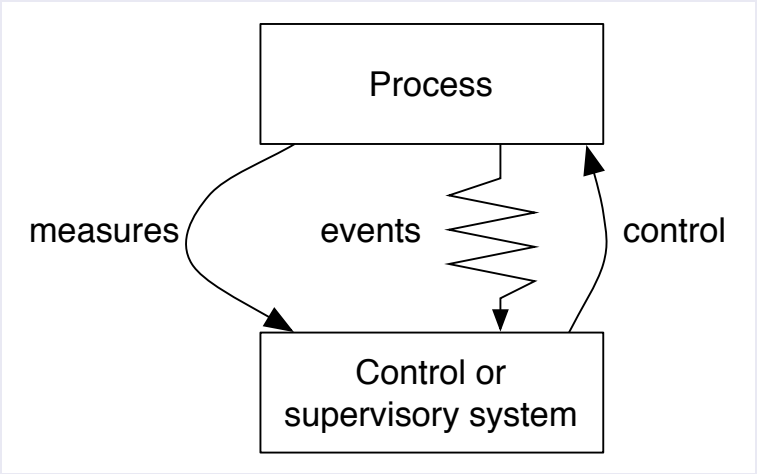

Figure: Analyzed system: { process + control system }

Formal verification of *real-time* systems

- Embedded systems (cars, planes, ...)
- Logistics (systems engineering)
- **Systems biology** (gene regulatory networks)

But still, **why** do we want to perform formal verification?

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Figure: Ariane 5 - Flight 501

But still, **why** do we want to perform formal verification?

The two most well-known computer bugs *(at least, for the model-checking community)*

- Crash of the european Ariane 5 rocket in 1996
  - **Exploded 39 seconds** after takeoff
  - Cost 370 millions US$
  - **Overflow** in a conversion of a 64 bit floating point number into a 16 bit integer data conversion from a 64-bit floating
  - Software from Ariane 4 was re-used for Ariane 5 **without re-testing**
  - `http://en.wikipedia.org/wiki/Ariane_5_Flight_501`
- Pentium FDIV bug, 1994.
  - Flaw in the **division algorithm**
  - Cost an average 500 million US$ and image loss for Intel
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But still, **why** do we want to perform formal verification?

Emergence of **model-checking**

- To automatically find **bugs** and prove **correctness** of *models*
- Edmund Clarke, E. Allen Emerson and Joseph Sifakis: Türing Award in 2007

Motivations

Objective: formal verification of properties

- Model the system S :
  - Discrete models: finite state automata, Petri nets, ... $\Rightarrow$ **Lecture 1**
  - Timed models:
    - timed extensions of finite state automata: timed/hybrid automata $\Rightarrow$ Lecture 2
    - timed extensions of Petri nets: time/stopwatch Petri nets $\Rightarrow$ Lecture 3
- Formalize the specification φ :
  - Observers
  - Temporal logics: LTL, CTL, ITL, ... $\Rightarrow$ **Lecture 1**
  - Timed extensions of temporals logics: TCTL, ... $\Rightarrow$ Lectures 2 & 3
- Does $S \models \varphi$?

Model-checking algorithms

$\Rightarrow$ State space exploration

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Model-checking algorithms

$\Rightarrow$ State space exploration

Some major issues

Need for modeling tasks with suspending/resuming features

Expressivity/**Decidability** compromise to discuss ⇒ Lectures 2 & 3

State space combinatorial explosion

- Need for symbolic approaches ⇒ Lectures 2 & 3
- Need for new models and abstracted algorithms ⇒ Lecture 4

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Overview

How to compare (and choose between) different models?

Theoretical criteria

- **Decidability** results w.r.t. common problems (e.g. state reachability, liveness), *but* ...
- Respective **expressivity**

Pragmatical criteria

- Ease to **understand** and **build** models
- What are the **key features** (e.g. modularity, concurrence, chronology) the model has to capture?
- Existence of **software tools** to design and analyze these models
- **History**

Why Petri nets?

Motivation

- Mathematical **and** graphic formalism
- Easy representation **concurrence and parallelism**
- **Structural** properties (P-invariants, T-invariants, ...)
- **Dynamical** properties (liveness, boundedness, reachability, ...)
- Mature tools : Snoopy, ginSIM, Roméo, etc.

Petri nets: a wide range of models

- Discrete [SBSW07]
- Continuous [KH08]
- Hybrid [MDNM00]
- Stochastic [GP98]: the firing of a transition relates to a probability function, which corresponds to the chemical sensitization depending on the concentration
- Colored [GKP10] : tokens are differentiated
- Time and stopwatches: [MMR09]

Petri net - Introduction

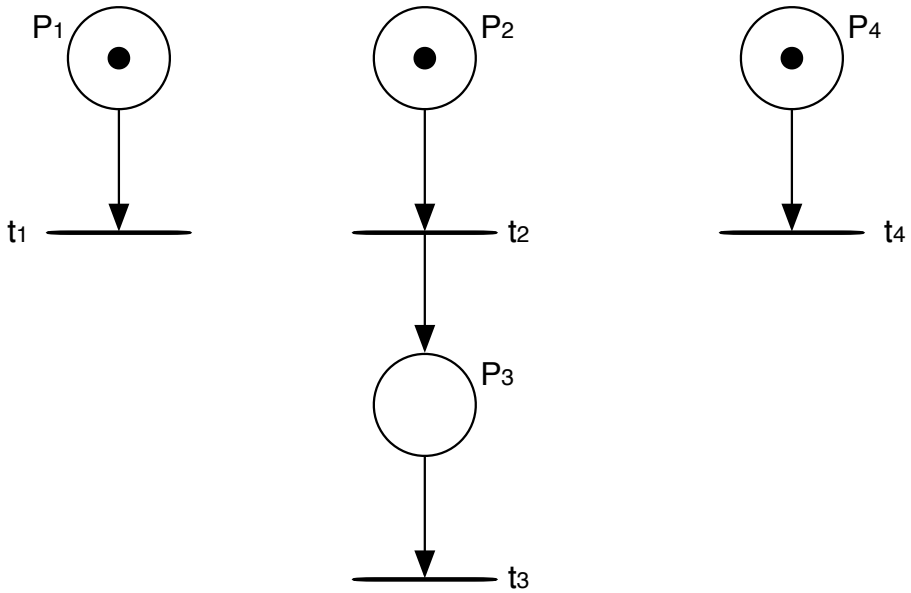
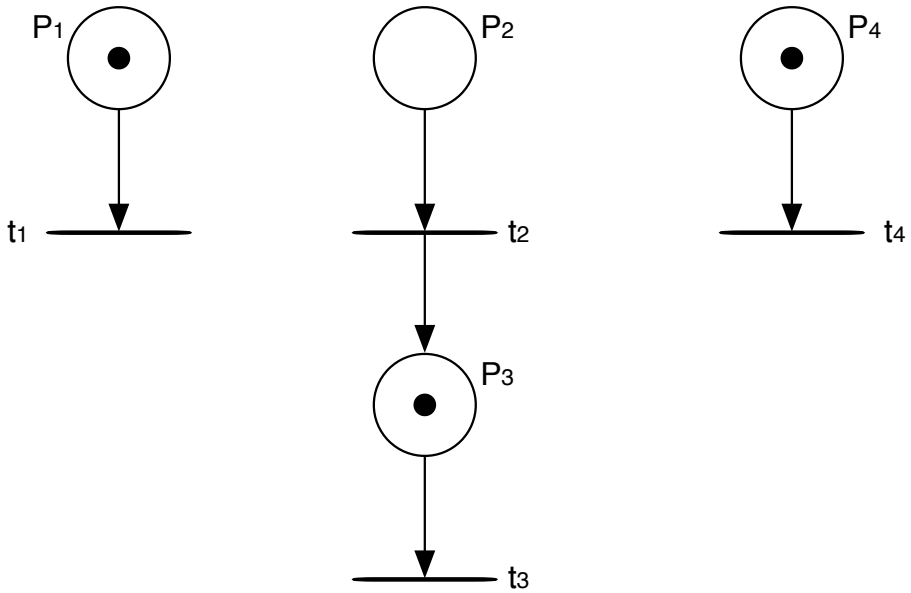


Figure: A Petri net

$$\{P_1, P_2, P_4\}$$

Petri net - Introduction



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$$\{P_1, P_2, P_4\} \xrightarrow{t_2} \{P_1, P_3, P_4\} \xrightarrow{t_1} \dots$$

Petri nets - Introduction

Definition

- A set of places
- A set of transitions
- A backward incidence function
- A forward incidence function,
- *An initial state*

Petri nets - Introduction

Basics

- Non-deterministic execution
- Multiple tokens make PN be suitable to modeling the concurrent behavior of distributed systems

Petri nets - Properties

Some structural properties

- **T-invariant** : sequence of transitions which returns in the same state / marking
- **P-invariant** : marking invariant (e.g. $q_i M(p_i) + q_j M(p_j) + q_k M(p_k) = c$ for any state of the net)

Petri nets - Introduction

Some dynamic properties

- **Liveness**: in every state, every transition will eventually be fired
- **Deadlock**: state in which no more transition can be fired
- Marking **reachability** (given an initial state)

Petri net - Liveness

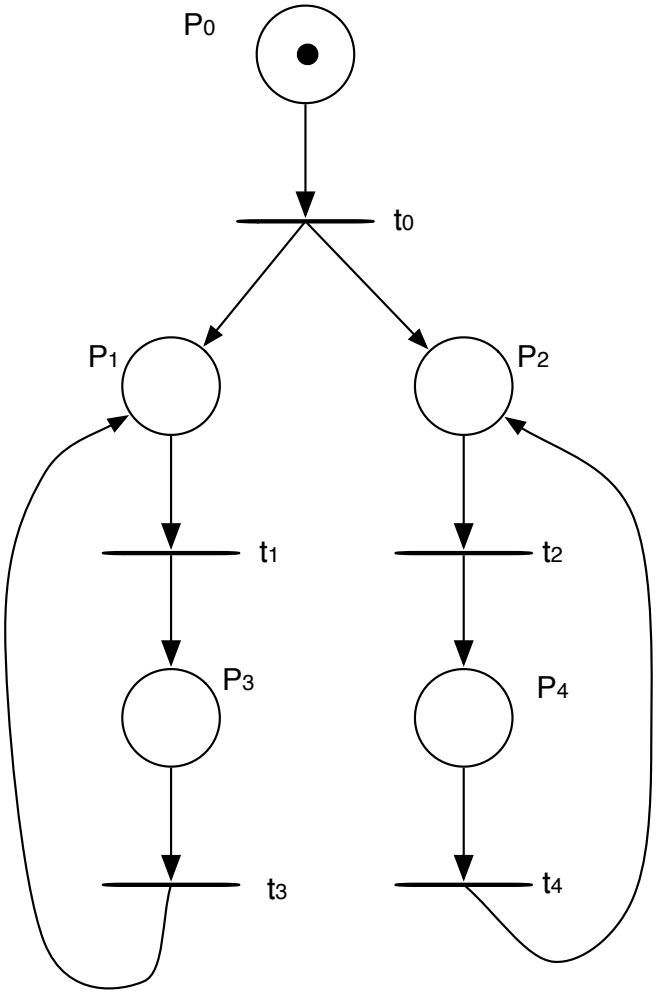
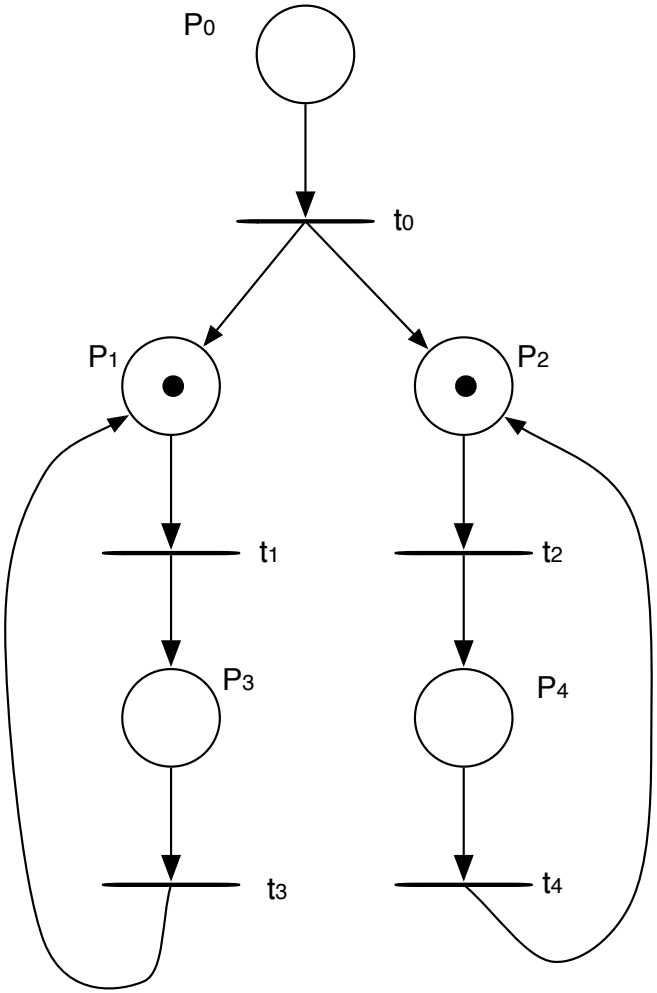


Figure: Is it a live Petri net?

Petri net - Liveness



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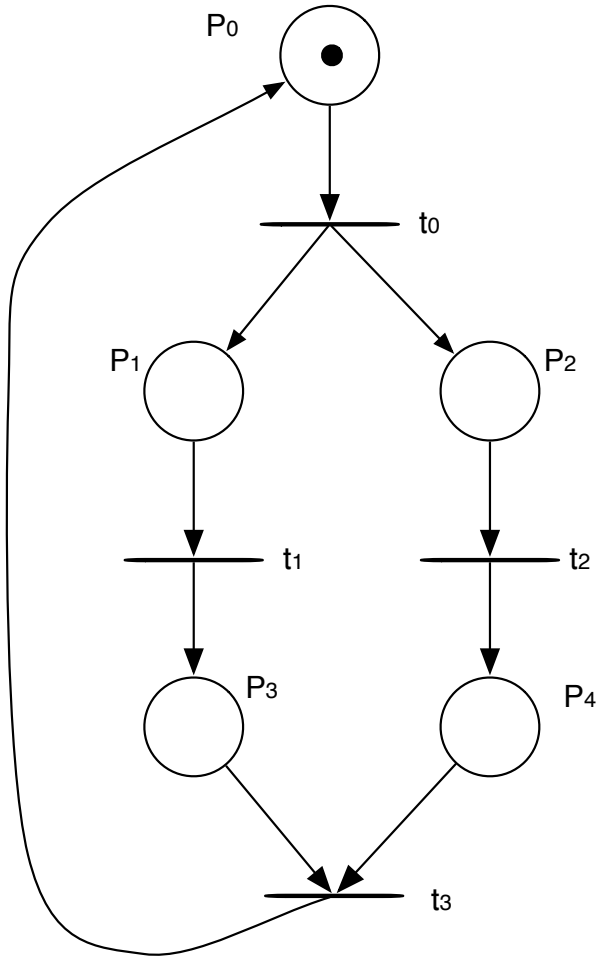

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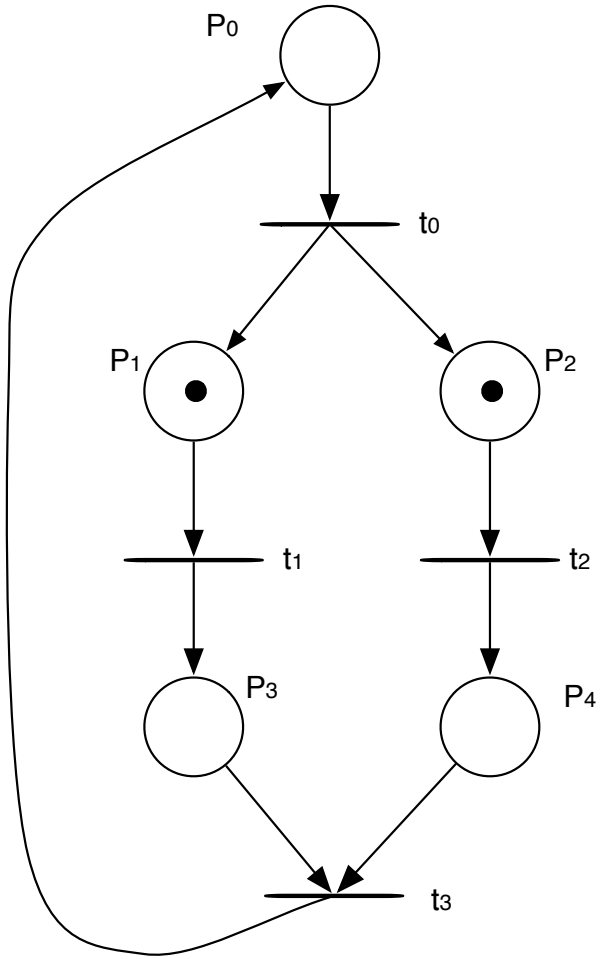

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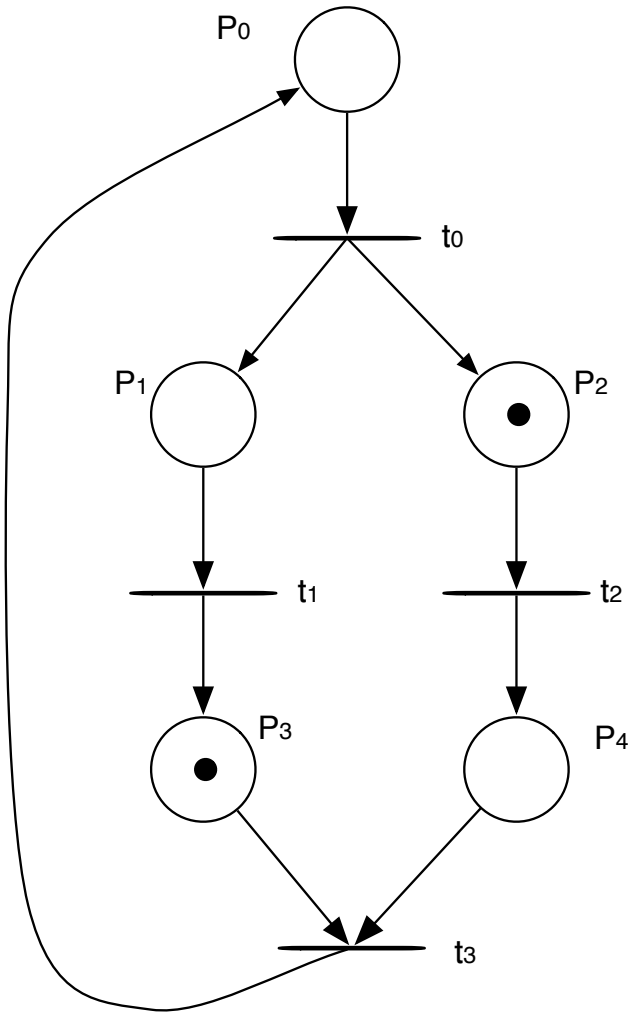

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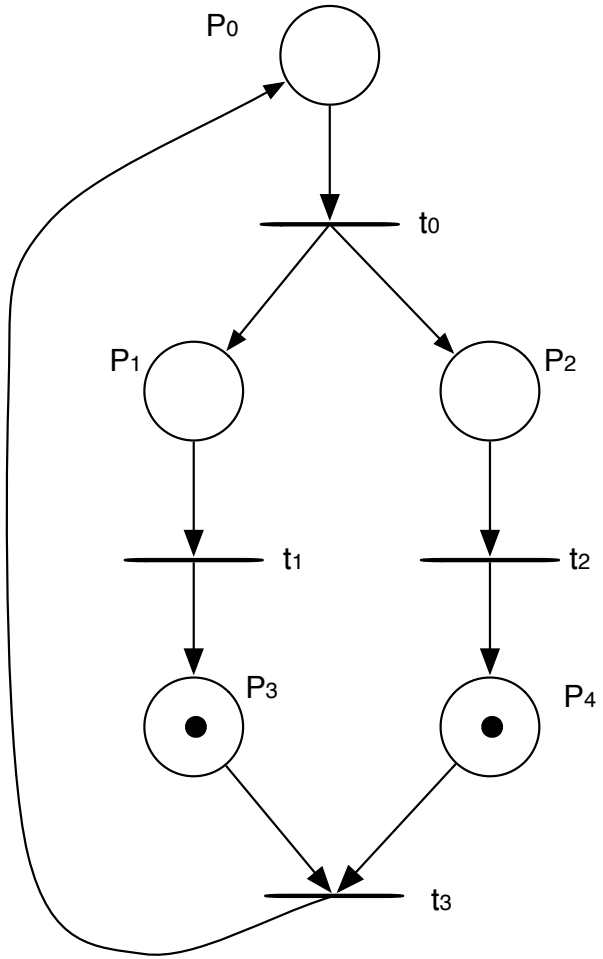

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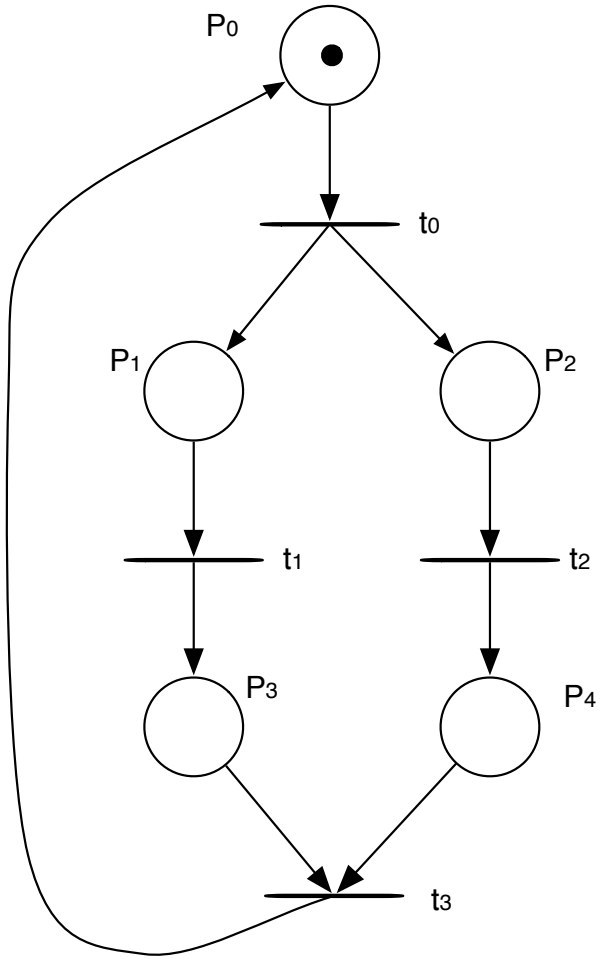

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Petri net - Deadlock

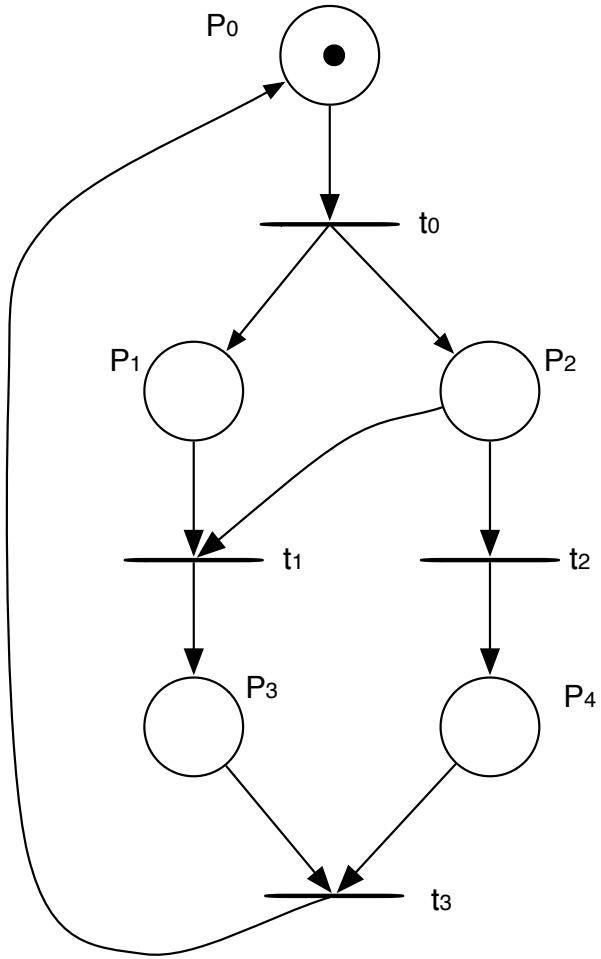
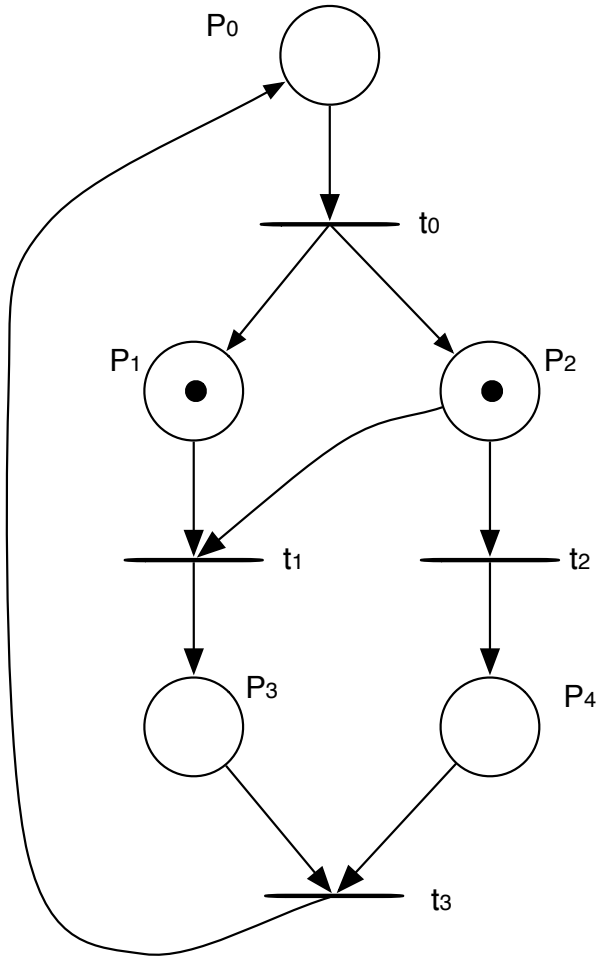


Figure: Will this Petri net lead to a deadlock?

Petri net - Deadlock



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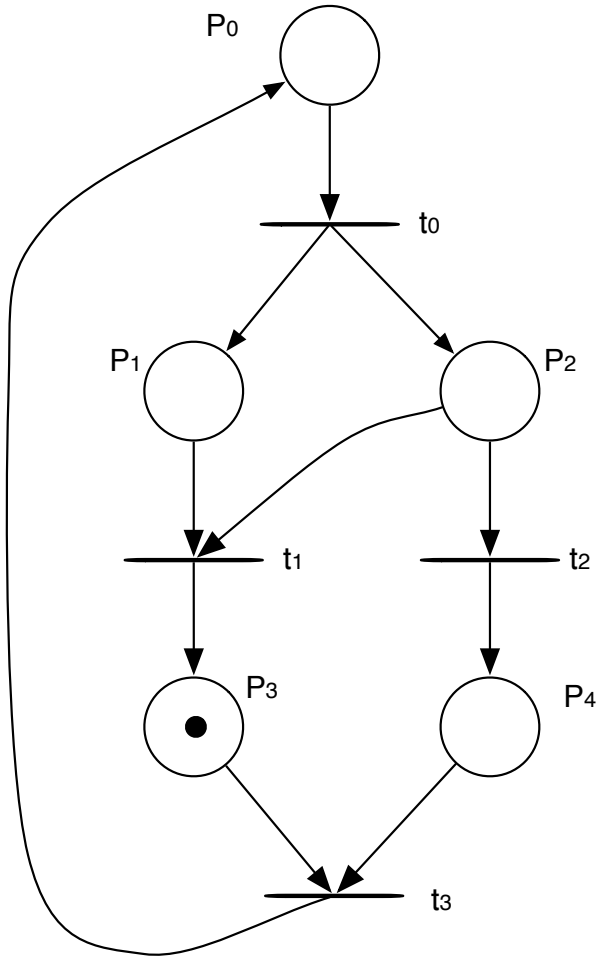

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Petri nets - Applications

- Production systems (factories)
- Logistic deployment systems
- Embedded systems
- Video games (A.I. modeling)
- And, of course, biology!

Overview

Short insight on the meaning of Petri net components

Intuitive meaning

- Marking of a place: **presence/absence** or **quantity** of a component
- Arc : **precedence** or **succession**
- Transition : **event** andor **transformation**
- Weight : necessary **quantity**, consumed and/or produced

Petri nets for modeling biochemical networks

Principle of qualitative modeling

- Places : reactants, products, enzymes
- Transitions : reactions, catalysis
- Weight on the arcs: stoichiometry

Application to the modeling of biochemical reactions

$$2NAD^+ + 2H_2O \rightarrow 2NADH + 2H^+ + O_2$$

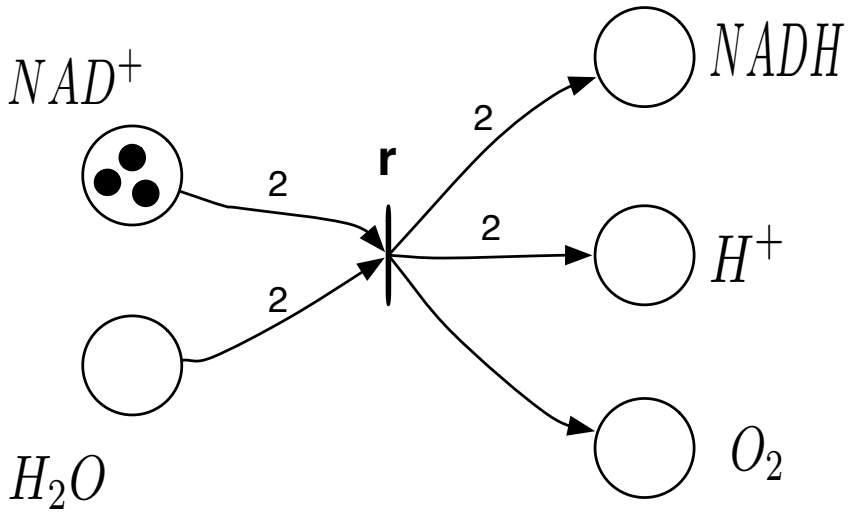

Figure: An example of a very simple translation from a biochemical reaction

Petri nets properties for the modeling of biochemical networks

Structural properties

- Incidence matrix: stoichiometry matrix
- P-invariants: conservation relations
- T-invariants: elementary flux modes

Dynamic properties

- Liveness: components are sufficient to trigger reactions
- Deadlock: stable state

Additional modeling features

Critical issues

- How to **test** the number of tokens in a place (e.g. concentration level of a protein) without decrementing it?
- How to model an action that takes place only **below** a given threshold (e.g. concentration level of a protein)?
- How to reset the number of tokens in a place (e.g. reset of a system) whatever was its previous concentration?

$\rightarrow$ Introduction of new arcs

Petri net - Enriching the expressivity of the model

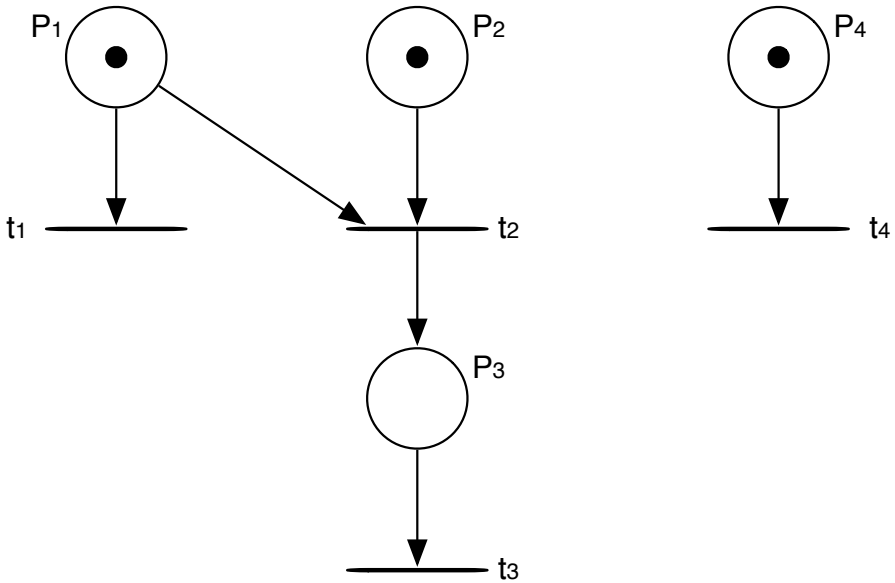

Figure: An other Petri net

$$\{P_1, P_2, P_4\}$$

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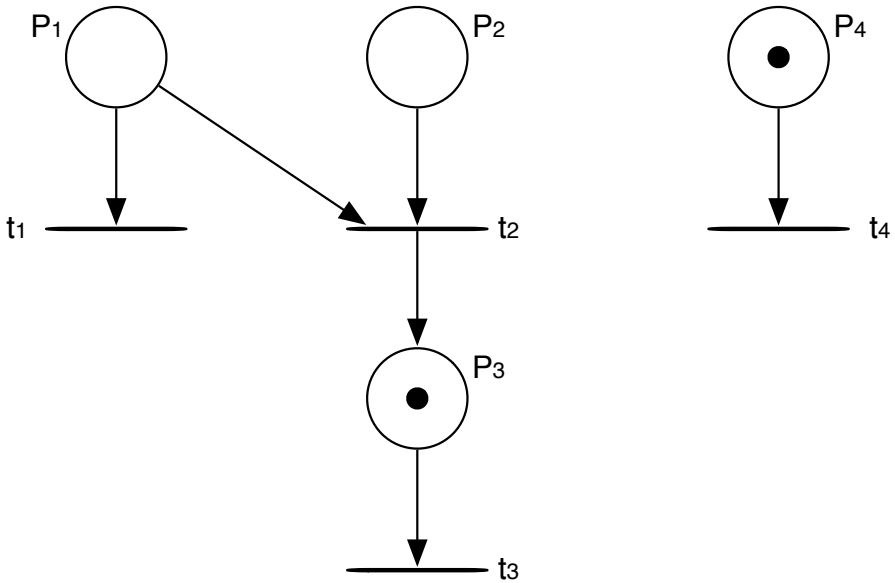

Figure: An other Petri net

$$\{P_1, P_2, P_4\} \xrightarrow{t_2} \{P_3, P_4\} \dots$$

Petri net with reset arcs - Introduction

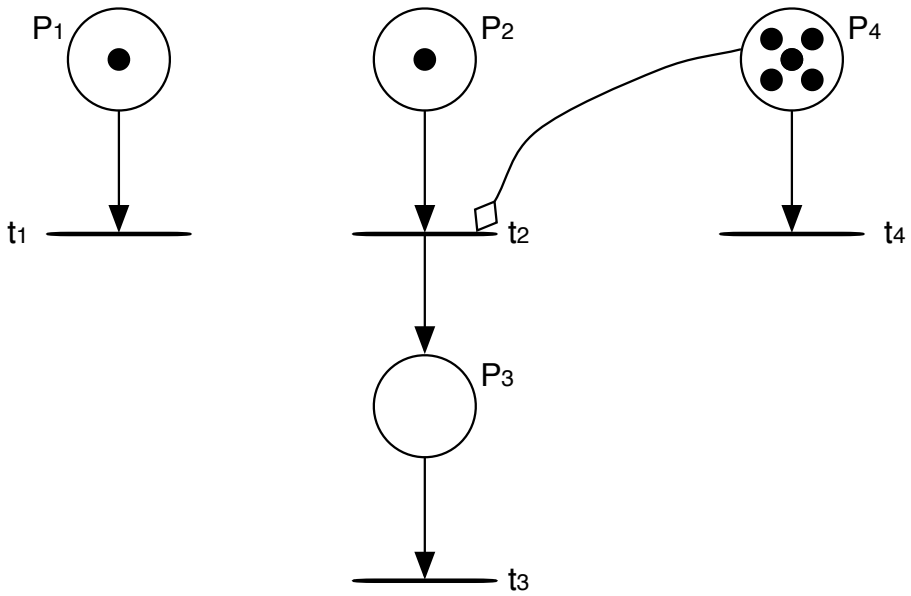

Figure: A Petri net with *reset* arcs

$$\{P_1, P_2, 5 \times P_4\} \xrightarrow{t_2} \dots$$

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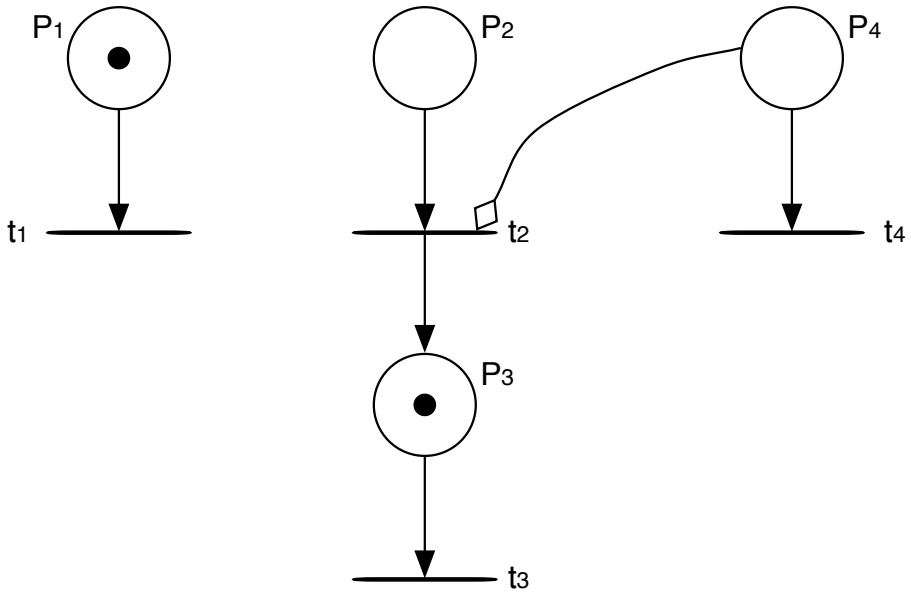

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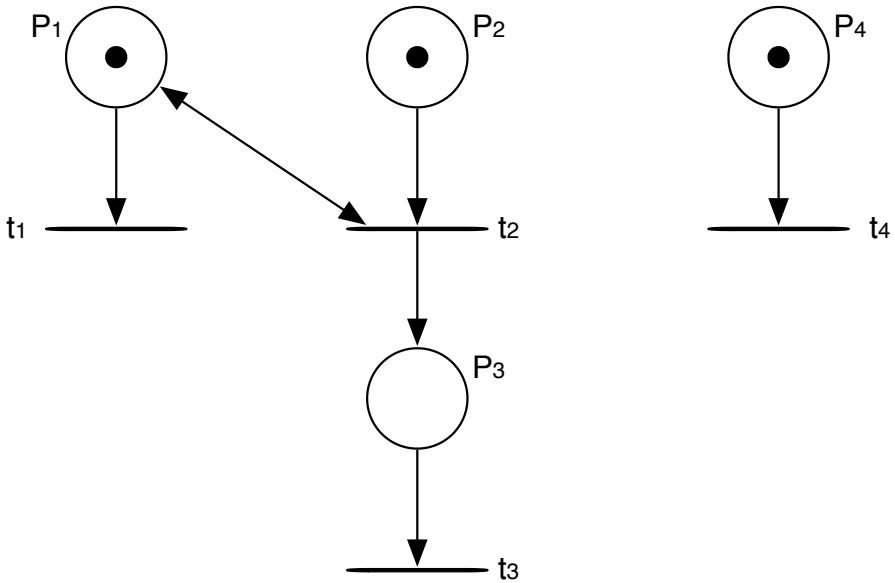
Petri nets with read arcs



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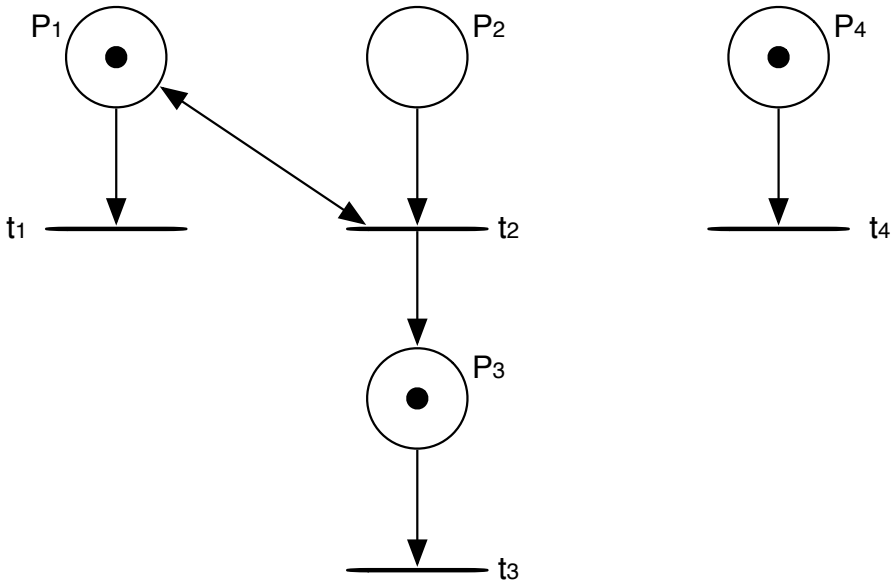

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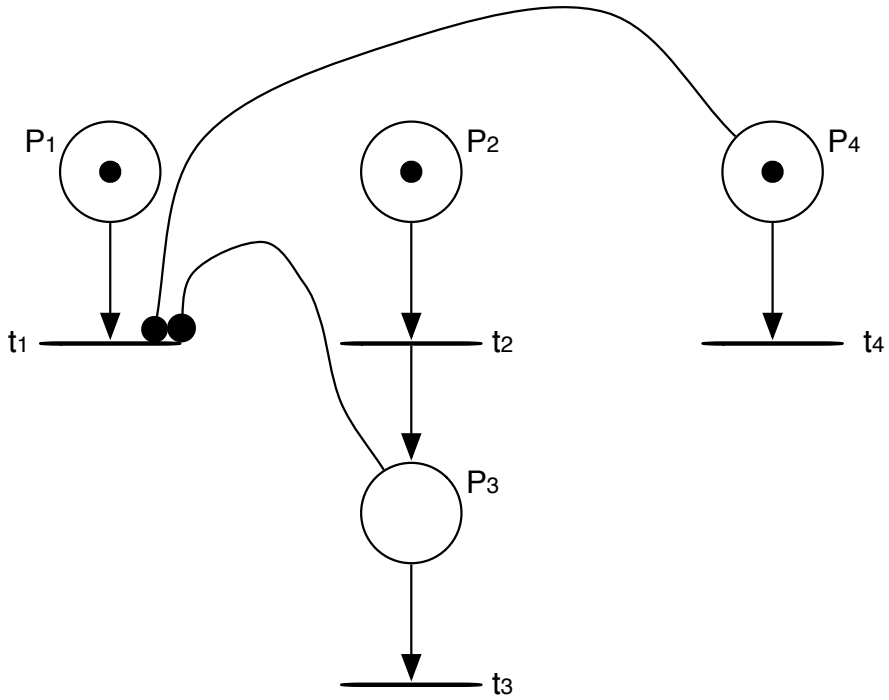
(Logic) Inhibitor Arc Petri nets: OR inhibition



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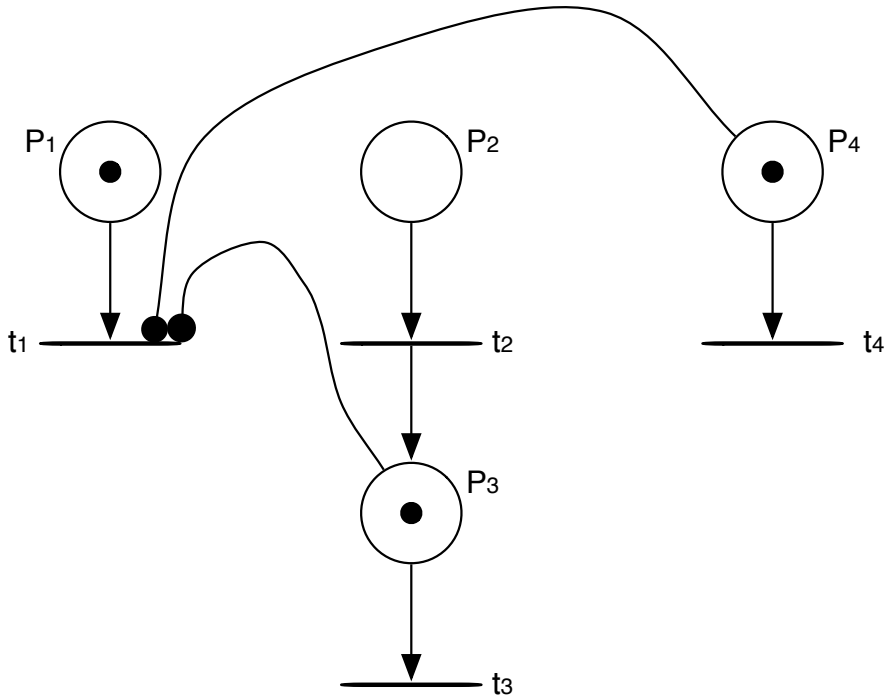

Figure: An inhibitor arc Petri net : t_1 inhibited iff $(M(P_3) \geq 1$ **or** $M(P_4) \geq 1)$

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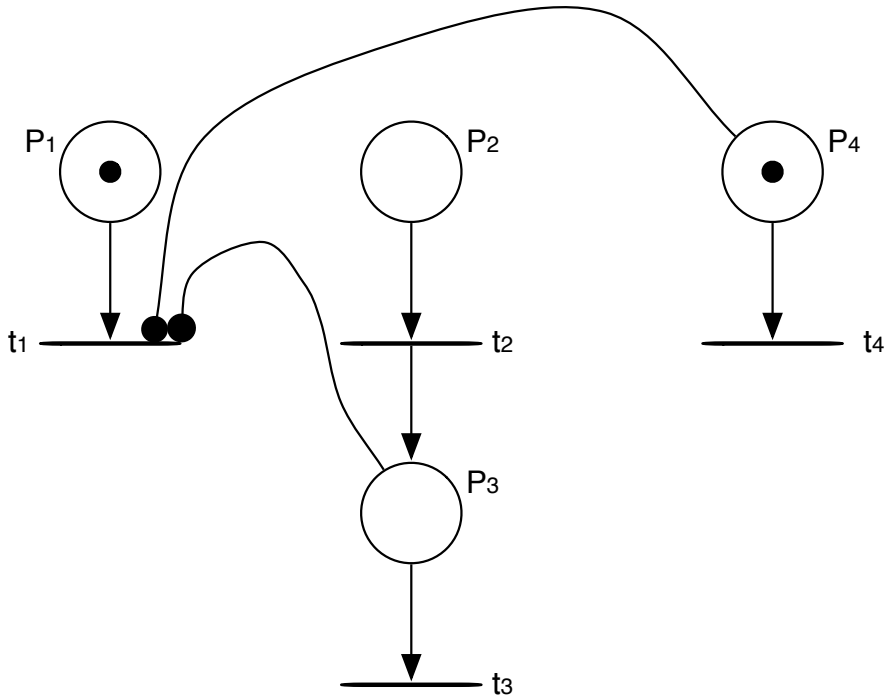

Figure: An inhibitor hyperarc Petri net

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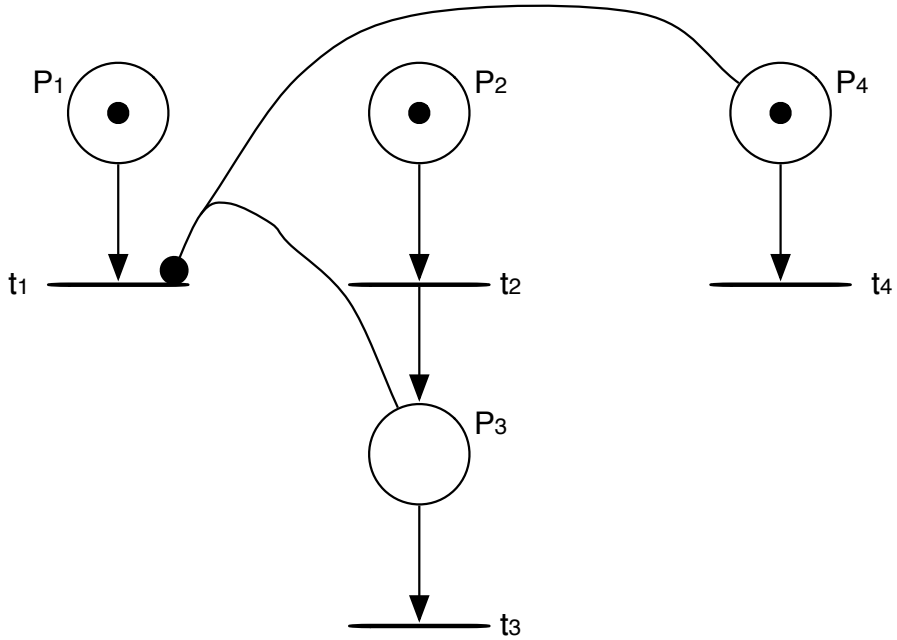
(Logic) Inhibitor Hyperarc Petri nets: AND inhibition



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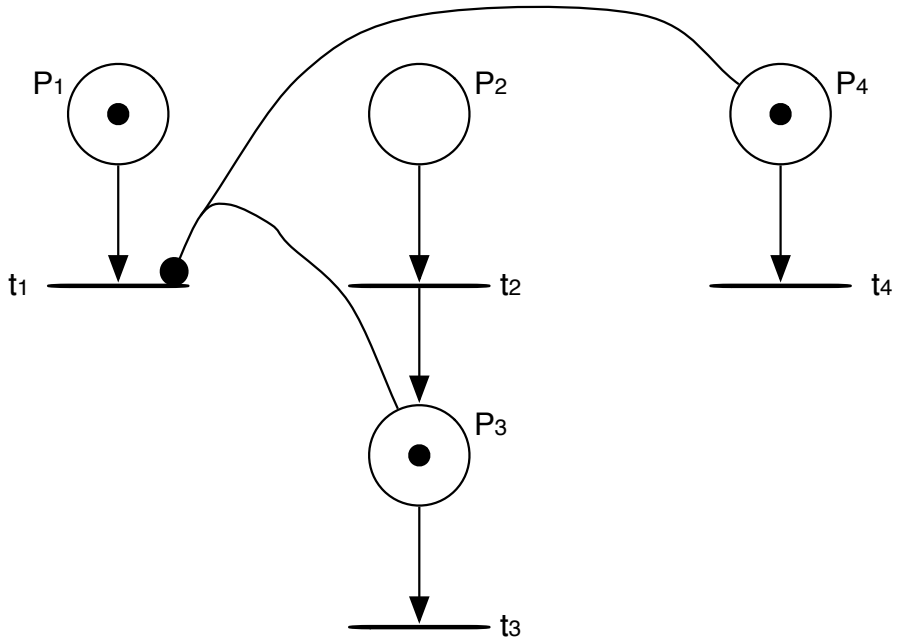

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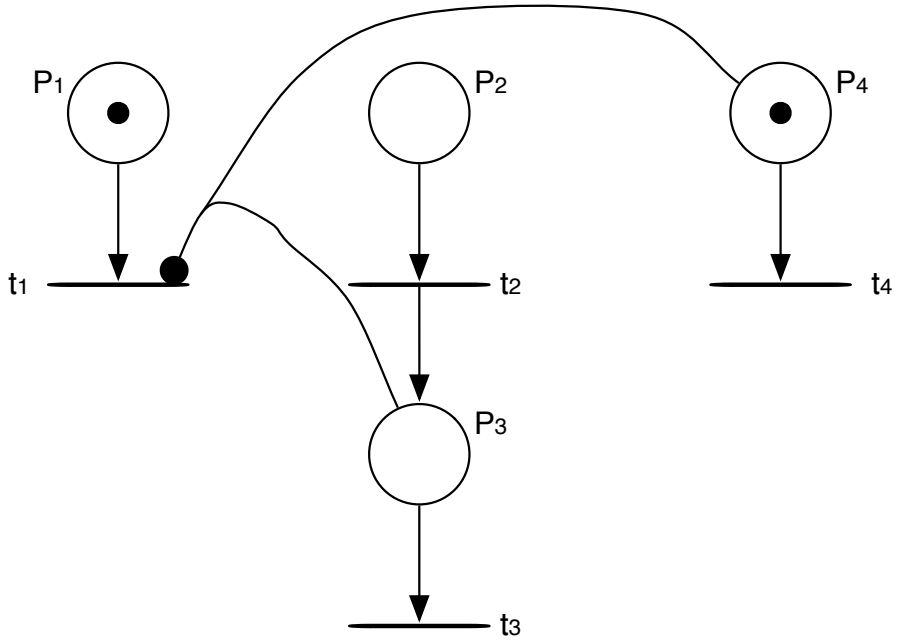

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Implementation

How to automatically compute state space of PNs?

Define **efficient data-structures** designed for:

- Storage of states
- Modeling incidence functions

$\rightarrow$ Get inspiration from *Binary Decision Diagrams* (BDDs)

A word about Binary Decision Diagrams (BDDs)

How to get a compact representation of a boolean function?

- **Compressed** representation of sets
- **Directed acyclic graph**
- To make it **canonical** would allow to make equivalence-checking easier

x1	x2	x3	*f*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Figure: Truth table for $f(x_1, x_2, x_3) = \bar{x_1}\bar{x_2}\bar{x_3} + x_1x_2 + x_2x_3$ (from WIKIPEDIA)

A word about Binary Decision Diagrams (BDDs)

How to get a compact representation of a boolean function?

- One root node
- Terminal nodes: 0, 1
- Each non-terminal node has two children and is labeled by a variable
- No variable appears more than once along a path

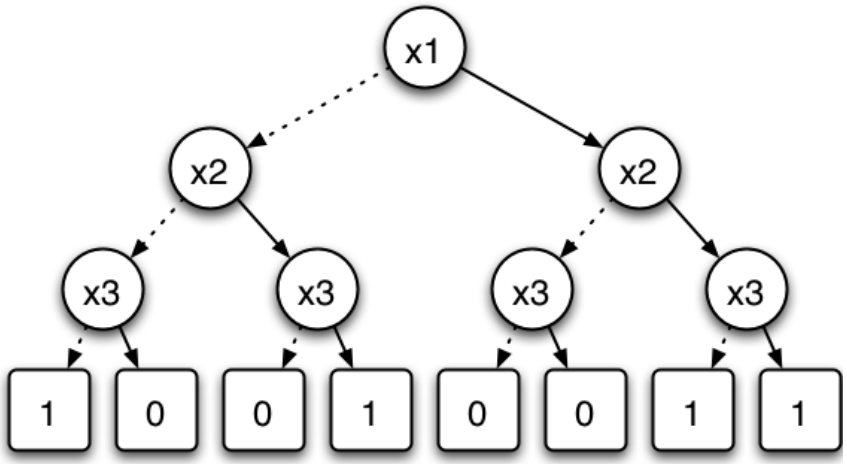


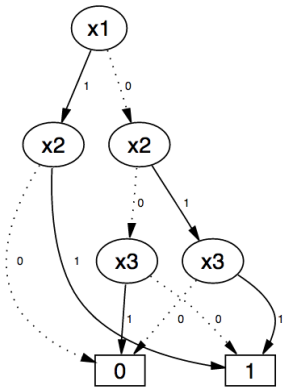
Figure: Binary tree for $f(x_1, x_2, x_3) = \bar{x_1}\bar{x_2}\bar{x_3} + x_1x_2 + x_2x_3$ (from WIKIPEDIA)

A word about Binary Decision Diagrams (BDDs)

How to get a compact and **canonical** representation of a boolean function?

Key ideas formalized by Briant [Bry86] in 1986:

- Variables **ordering**: in all paths, variables appear in the same order
- Reduction :
  - Merge any isomorphic subgraphs
  - Eliminate any node whose two children are isomorphic
- Each non-terminal node has two children and is labeled by a variable



A word about Binary Decision Diagrams (BDDs)

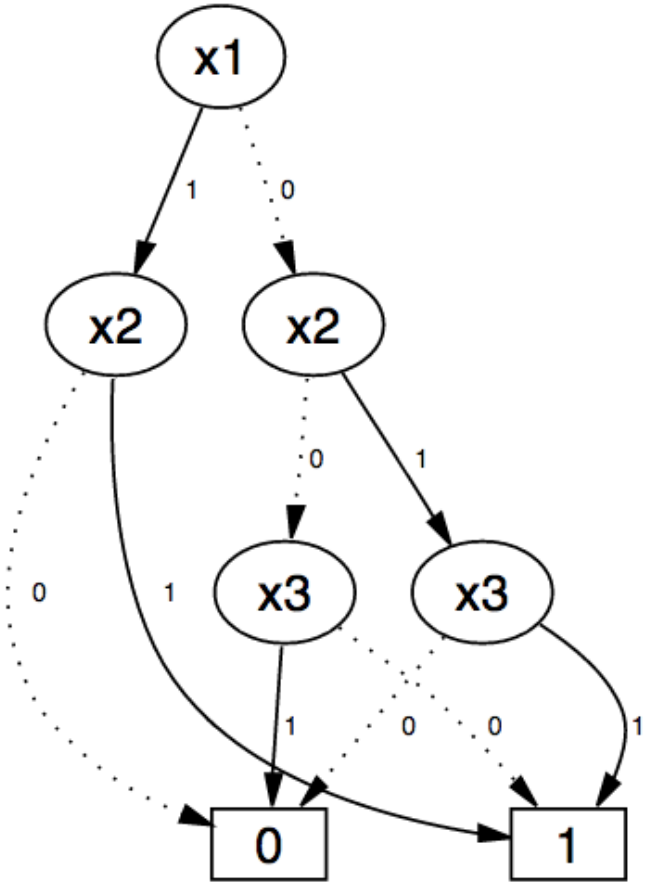


Figure: BDD for $f(x_1, x_2, x_3) = \bar{x_1}\bar{x_2}\bar{x_3} + x_1x_2 + x_2x_3$ (from WIKIPEDIA)

A word about Binary Decision Diagrams (BDDs)

Challenges w.r.t. BDDs

- Size of a BDD **depends** critically on variable ordering
- Finding the *best* variable ordering is NP-hard
- Some efficient **heuristics** still exist!

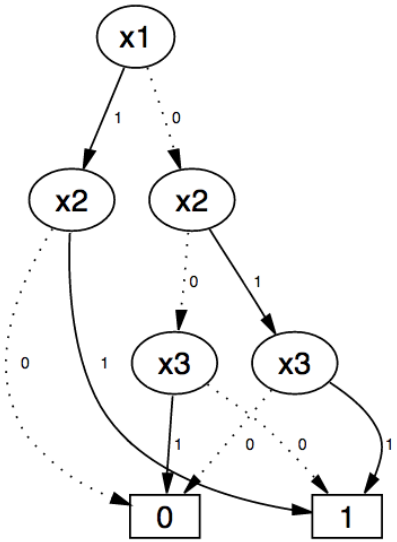


Figure: BDD for $f(x_1, x_2, x_3) = \bar{x_1}\bar{x_2}\bar{x_3} + x_1x_2 + x_2x_3$ (from WIKIPEDIA)

Application of BDD to Petri nets

Illustration of the implementation in the Markg software

- Computes the state space of untimed Petri nets (with **read hyperarcs** and **marking values affectation**)
- Based on data-structures inspired by the *Binary Decision Diagrams* (BDDs) :
  - Vector Decision Diagrams (VDDs) : storage of states;
  - Arithmetic Expression Decision Diagrams (AEDDs) : model incidence functions;
- Deals with potentially infinite markings
- Tackles state space with more than 10^{14} states and 10^{15} transitions
- Download: `http://markg.rts-software.org`

Storage of states: Vector decision diagrams

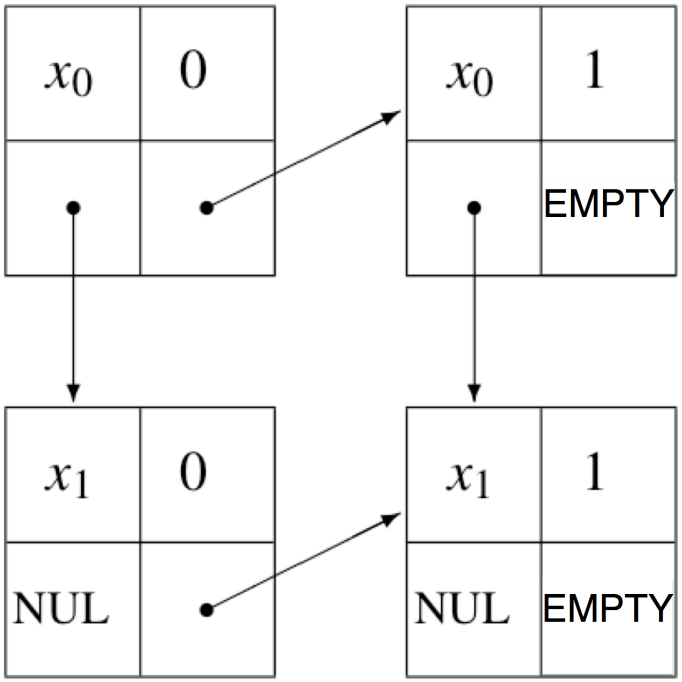


Figure: VDD representing states $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$

Modeling incidence functions: Arithmetic Expression Decision Diagrams

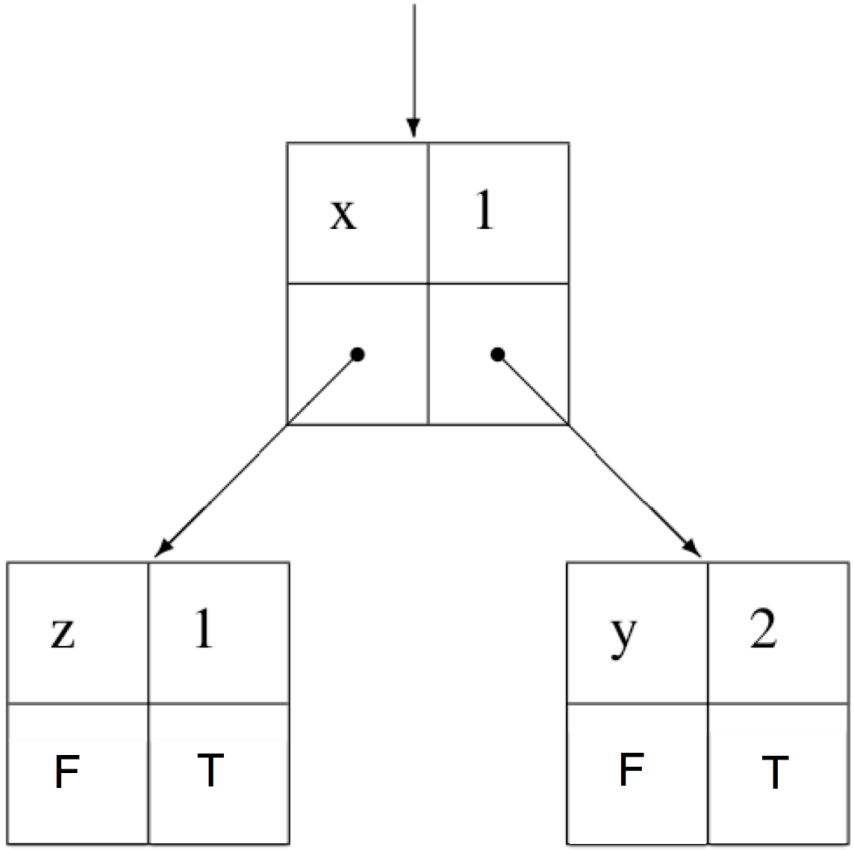


Figure: AEDD modeling $f(x, y, z) = ((x \geq 1) \wedge (y \geq 2) \cup ((x < 1) \wedge (z \geq 1))$

Application of BDD to Petri nets

Up-to-date state of the art in this field

libDDD developed at LIP6, Paris [Mieg et al.]

- C++ library for manipulation of decision diagrams
- Support for Petri nets and some of their extensions
- Available at `http://move.lip6.fr/software/DDD/`

Models classification

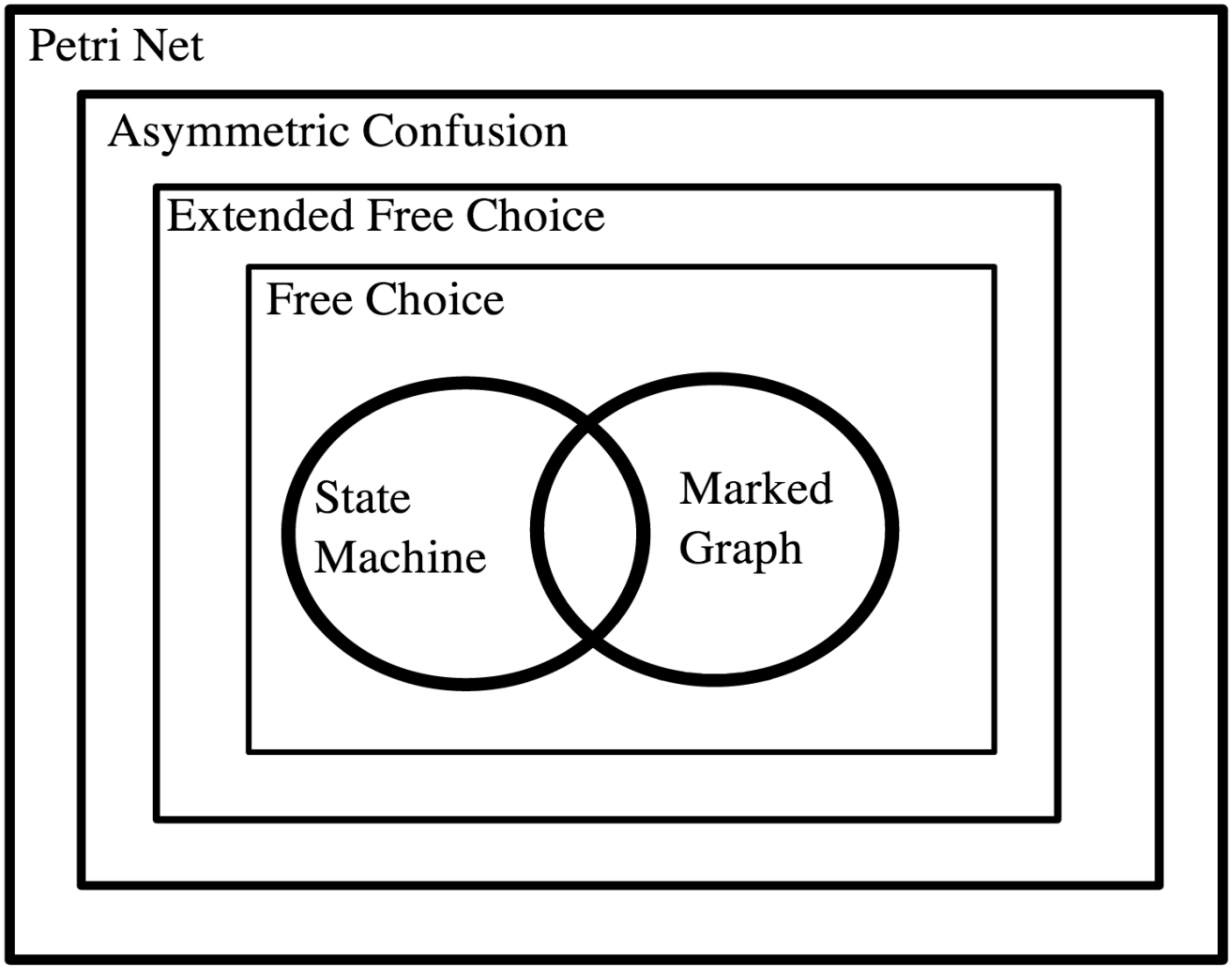


Figure: Various classes of models

Overview

Finite state machine - Introduction

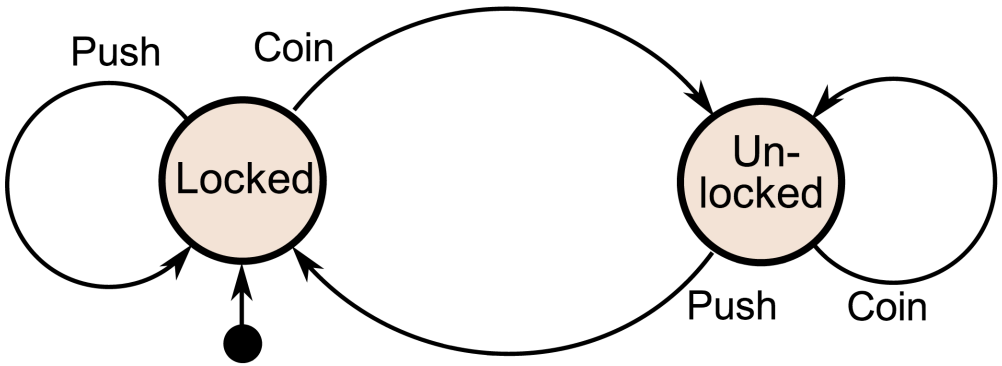


Figure: Example of a FSM modeling a turnstile (taken from `Wikipedia`)

Finite state machine - Introduction

Definition

- A finite number of states
- A set of transitions triggered by events or conditions
- *An initial state*

Finite state machine - Introduction

Basics

- The machine is in only one state at a time
- FSM are just a part of **automata theory**.
- **Powerset construction**: any nondeterministic finite automaton (NFA) can be turned into a deterministic finite automaton (DFA) which recognizes the same formal language.

Automata: a wide family

Variations

- **Input**
  - Finite input: accepts only finite words
  - Infinite input: accepts infinite words
- **States**
  - Finite states automaton
  - Infinite states automaton
  - Pushdown automaton
- **Transition function**
  - Deterministic
  - Nondeterministic

Automata: classification

Classification

Automaton = finite representation of a formal language that may be an infinite set

Automaton	**Recognizable language**
Nondeterministic/Deterministic Finite state machine (FSM)	regular languages
Pushdown automaton (PDA)	context-free languages
Turing machine	recursively enumerable languages
Deterministic Büchi automaton	ω-limit languages
Nondeterministic Büchi automaton	ω-regular languages

Overview

Computation paths vs computation tree

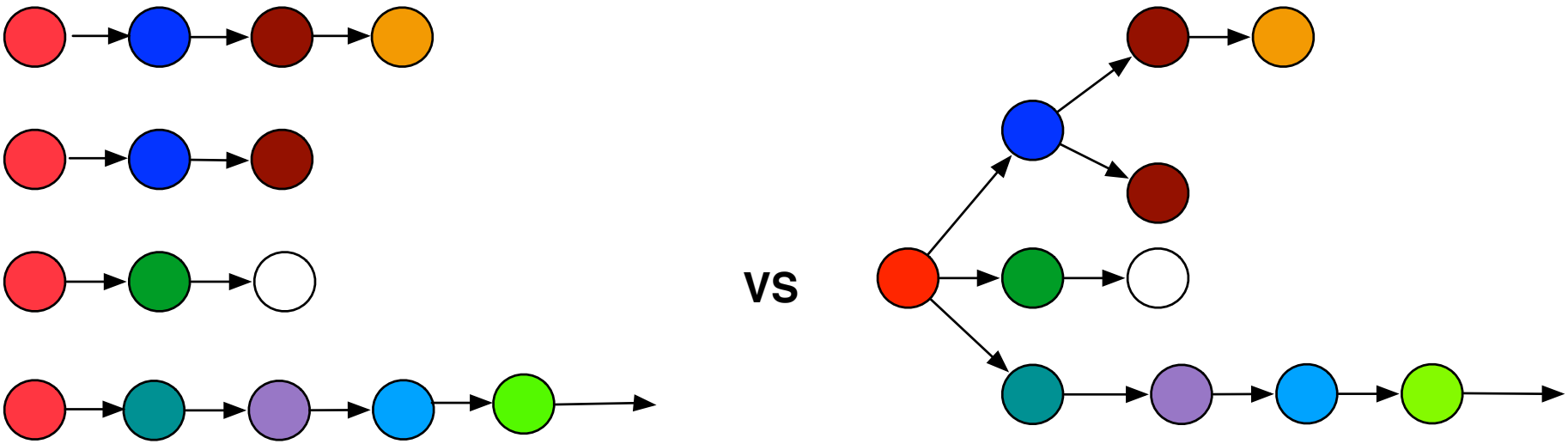


Figure: Execution can be seen as a set of execution paths or as an execution tree

Linear Temporal Logic

Principle

- First proposed for the **formal verification of computer programs** by A. Pnueli in 1977
- Sometimes called propositional temporal logic (PTL)
- LTL is the set of formulae φ_s s.t.:

$$\varphi_1, \varphi_2 := \textbf{true} \mid ap \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid X\varphi_2 \mid \varphi_1 U \varphi_2$$

  with ap an atomic proposition.
- The above definition allows the definition of G and F operators, $\Longrightarrow$ operator, ...:
  - $F\varphi \Leftrightarrow \textbf{true} U \varphi$
  - $G\varphi \Leftrightarrow \neg F \neg \varphi$

Model-checking of LTL properties

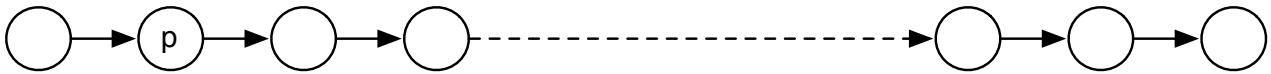


Figure: Xp

Model-checking of LTL properties

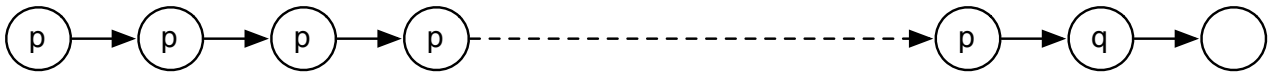


Figure: pUq

Model-checking of LTL properties

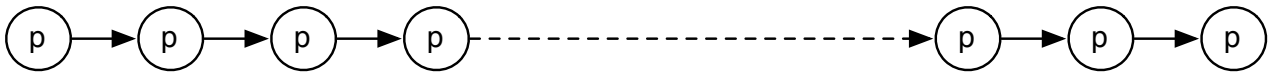


Figure: Gp

Model-checking of LTL properties

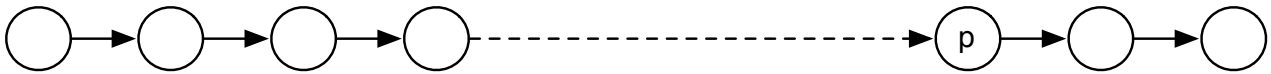
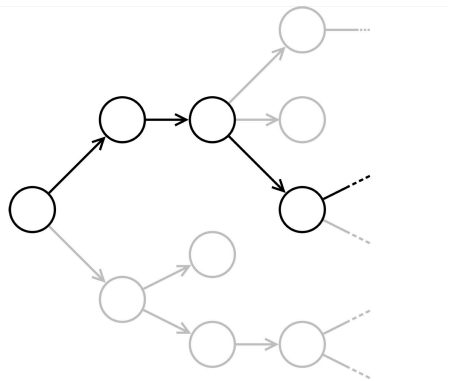


Figure: Fp

Model-checking of LTL properties

Principle

- Express desired properties using LTL operators and actually check if the model satisfies these properties
- Well adapted to:
  - **safety** properties **on a path** (i.e. something bad never happens, stated through $\mathsf{G}(\neg\varphi)$ operator)
  - **liveness** properties **on a path** (i.e. something good keeps happening, stated through $\mathsf{GF}\psi$ or $\mathsf{G}(\varphi \implies \mathsf{F}\psi)$
- But LTL does not allow to express properties on every path



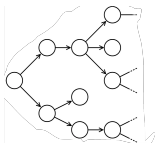
Computation Tree Logic

Principle

- **Branching-time logic**
- Defined by E. M. Clarke and E. A. Emerson in 1981
- CTL is the set of formulae s.t.:

$$\varphi_1, \varphi_2 := \textbf{true} \mid \neg\varphi_1 \mid \varphi_1 \wedge \varphi_2 \mid EX\varphi_1 \mid AX\varphi_1 \mid \varphi_1 EU \varphi_2 \mid \varphi_1 AU \varphi_2$$

  with *ap* is an atomic proposition
- Note that E stands for "*along at least one path*"(there Exists one path)
- The above formal definition allows the definition of additional operations, like: AG, AF, ... (A stands for "along All paths")



Model-checking of CTL properties

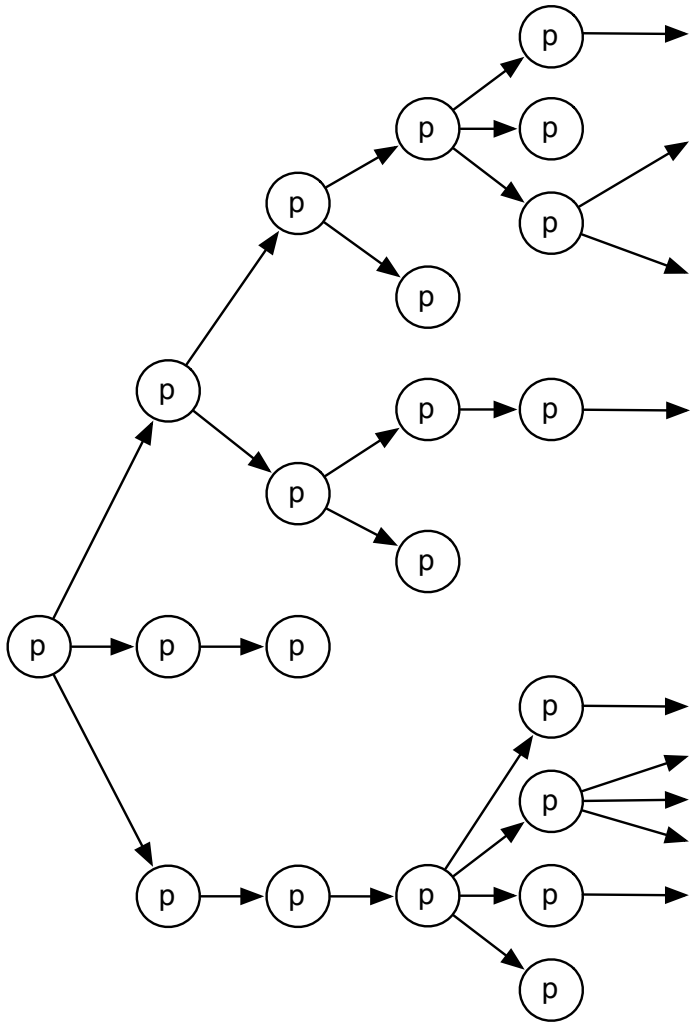


Figure: AGp

Model-checking of CTL properties

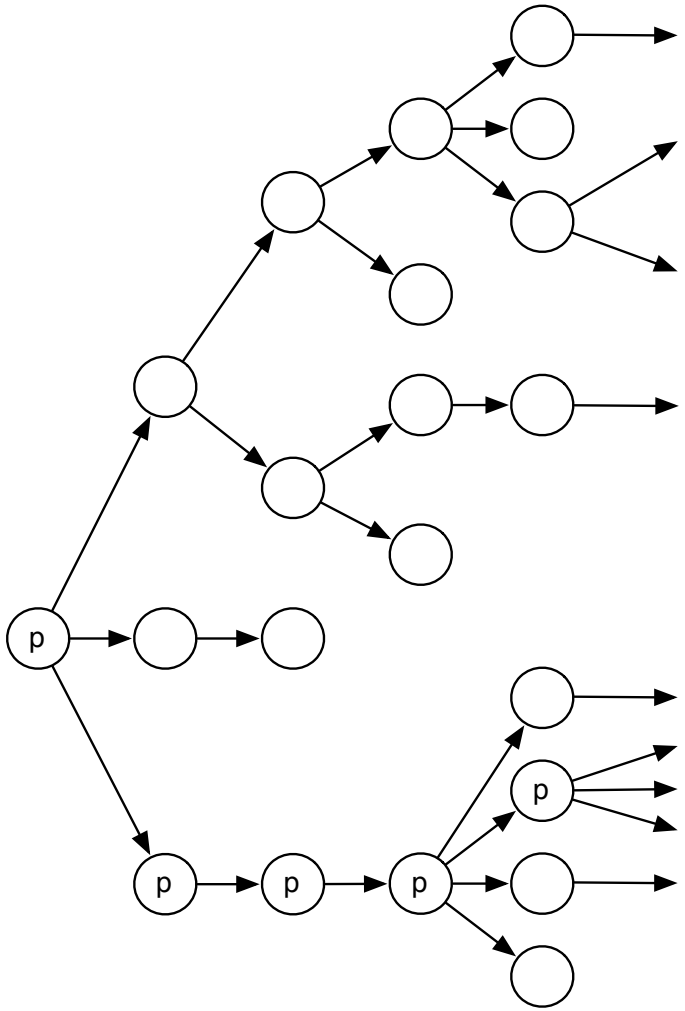


Figure: EGp

Model-checking of CTL properties

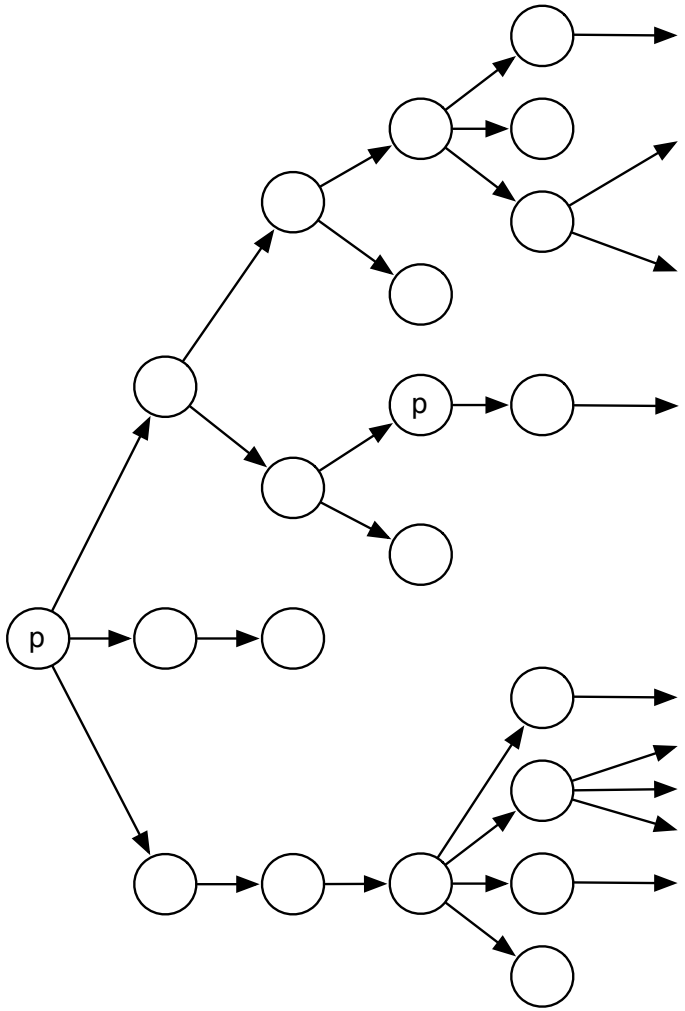


Figure: $EF\,p$

Model-checking of CTL properties

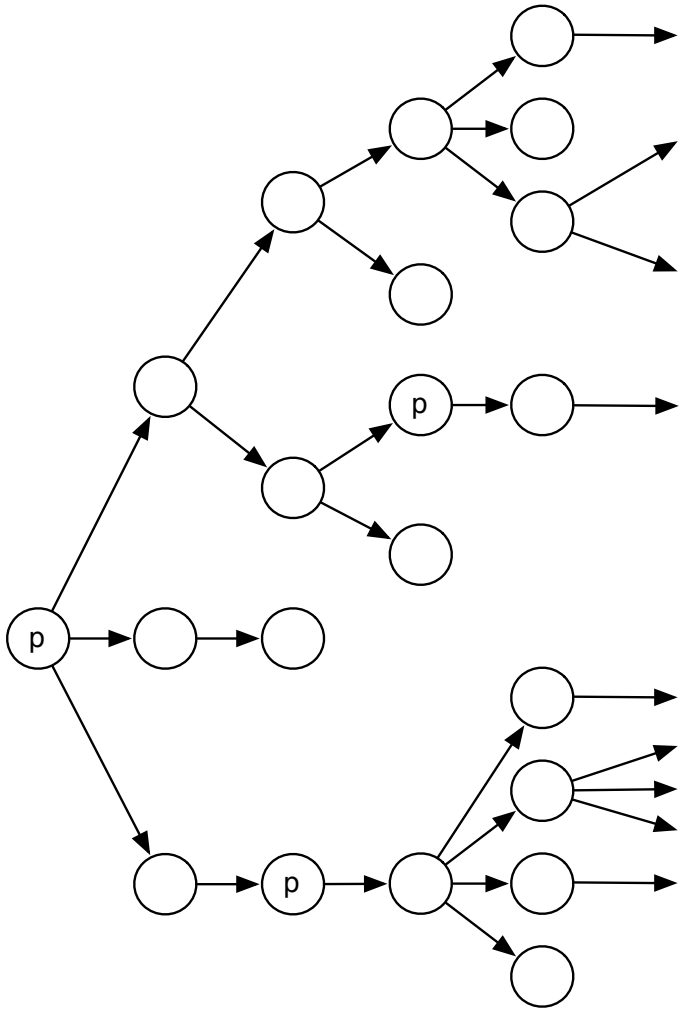


Figure: $EF\,p$

Model-checking of CTL properties

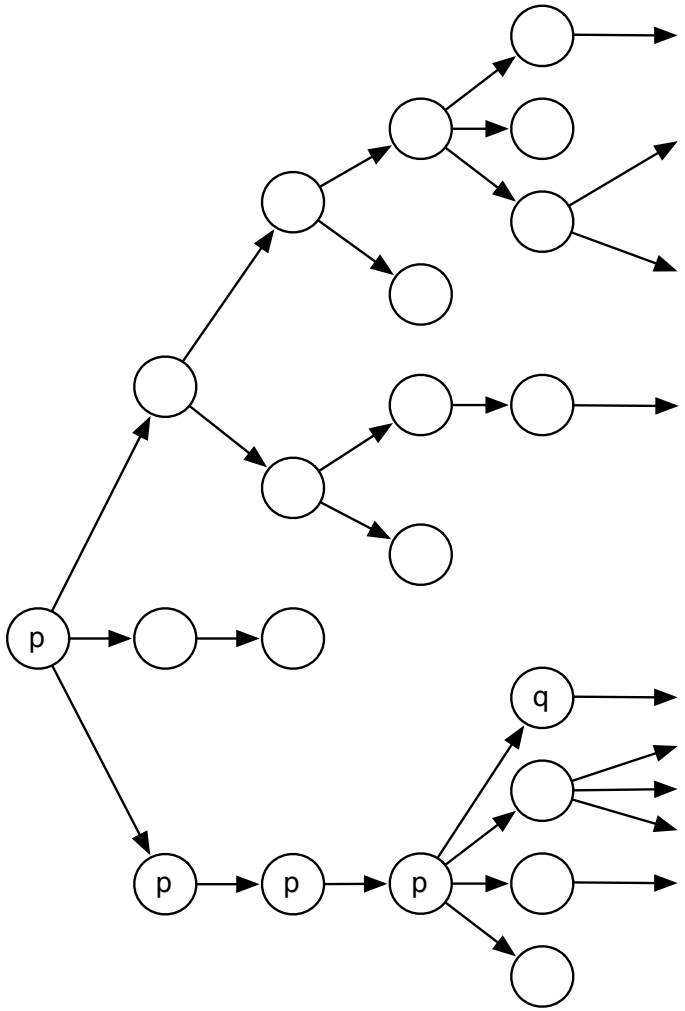


Figure: p*EU*q

Model-checking of CTL properties

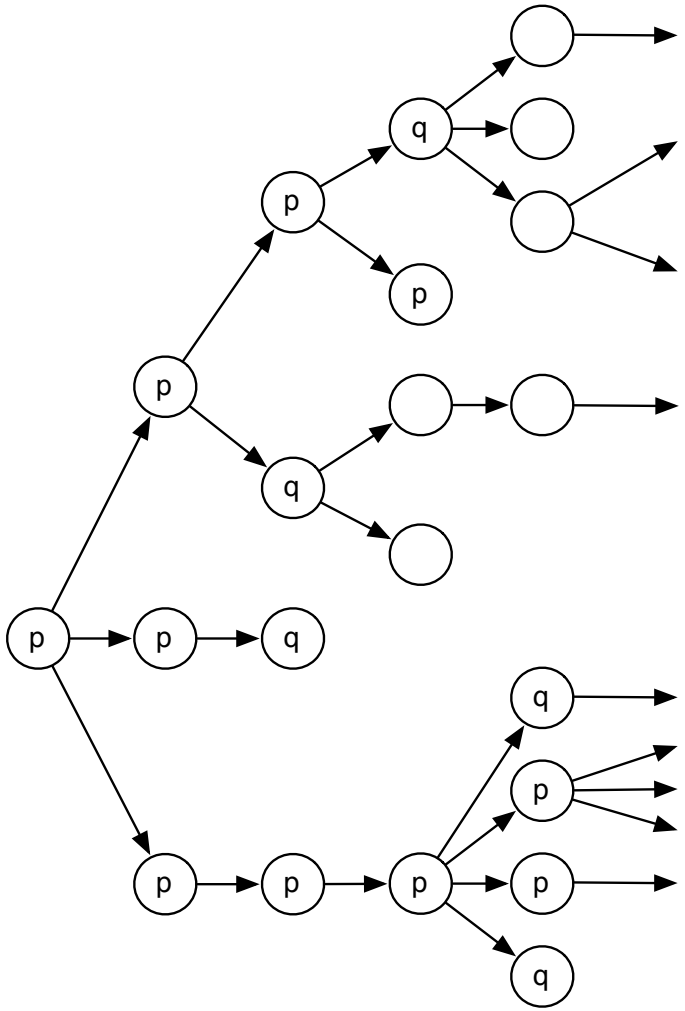


Figure: pAUq

Model-checking of CTL properties

Principle

- Express desired properties using CTL operators and actually check if the model satisfies these properties
- Well adapted to:
  - **safety** properties (e.g. when some initial condition is satisfied, then all possible executions of a program avoid some undesirable condition)
  - **liveness** properties (e.g. when some initial condition is satisfied, then some desirable property keeps happening)

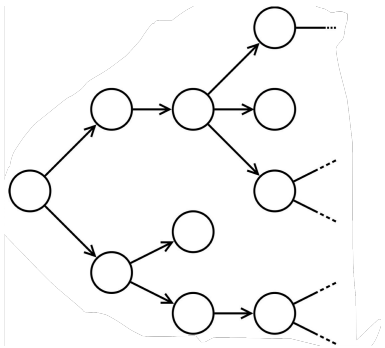


Figure: CTL defines properties on the different paths

LTL vs CTL

Comparison

- Linear time (paths) VS branching time (trees)
- **Incomparable expressive power**:
  - FGp cannot be expressed in CTL
  - $AGEFp$ cannot be expressed in LTL
- LTL may be more intuitive, but seems **too weak**

Computation Tree Logic CTL*

Principle

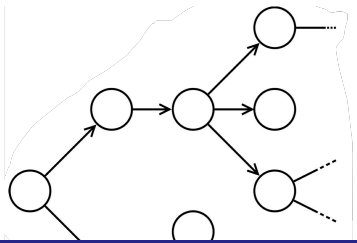
- **Superset** of CTL and LTL
- Combines **path quantifiers** and **temporal operators**
- Defined by E. A. Emerson and J.Y. Halpern in 1986
- CTL* is the set of φ_s formulae s.t.:

$$\varphi_s, \psi_s := \textbf{true} \mid ap \mid \neg\varphi_s \mid \varphi_s \wedge \psi_s \mid A\varphi_p \mid E\varphi_p$$

with

$$\varphi_p, \psi_p := \textbf{true} \mid \varphi_s \mid \varphi_p \wedge \psi_p \mid X\varphi_p \mid \varphi_p U \psi_p$$

- *AGFp* is a CTL* property, not a CTL property.



Computation Tree Logic CTL*

A path quantifier can prefix an assertion composed of arbitrary combinations of the usual linear-time operators.

Path quantifiers

- A : "for every path"
- E : "there exists a path"

Linear-time operators

- Xp : "p holds true next time"
- Fp : "p holds true sometime in the future"
- Gp : "p holds true globally in the future"
- pUq : "p holds true until q holds true"

Model-checking of CTL* properties

Principle

- Express desired properties using CTL* operators and actually check if the model satisfies these properties
- Well adapted to **complex properties**, like "whatever is the state of the system, if an alarm is received, then a signal must be emitted": $AG(alarm \Rightarrow AFsignal)$

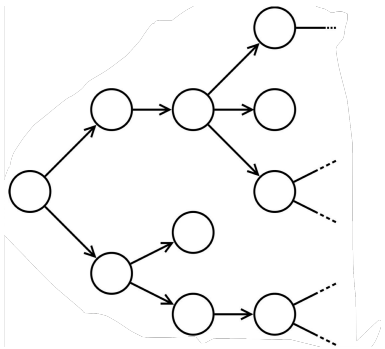


Figure: CTL* defines properties on the different paths

Tips for model-checking CTL* properties

Principles (as summarized by G. Bernot)

- Idea 1: work on the **state graph** instead of the path trees
- Idea 2: check first the **atoms** of φ and then check the connectives of φ with a bottom-up computation strategy
- Idea 3: (computational optimization) group some cases together using **BDD-inspired** techniques

But...

Since the late 90s, progress in BDDs has stalled while SAT solvers performances have increased $\Rightarrow$ some recent papers focus on checking:

- LTL formulae **through SAT solvers** (Biere et al.,1999, 2006)
- CTL formulae **through SAT solvers** (Heymans, 2005)

Model-checking

Summary

- Powerful methodology to:
  - **Verify** a wide range of given properties
  - **Validate** a model
- Application to discrete models, like FSM, Petri nets, ...
- Available automatic tools

Model-checking

Summary

- Powerful methodology to **verify** and/or **validate**
- Application to discrete models, like FSM, Petri nets, ...
- Available automatic tools

How to circumvent state space explosion?

- **Symbolic algorithms** to avoid building the whole state space and regroup states (e.g. BDD-based techniques)
- **Bounded model-checking**: consider only counterexamples of a bounded length $\Rightarrow$ whereas completeness, termination is guaranteed.
- **Abstraction** (e.g. consider only boolean variables)
- **Partial order reduction** (e.g. given a property, if the order between two sequences σ_1 and σ_2 is not meaningful, then it is unnecessary to consider both interleavings $\sigma_1.\sigma_2$ and $\sigma_2.\sigma_1$)

Model-checking

Summary

- Powerful methodology to **verify** and/or **validate**
- Application to discrete models, like FSM, Petri nets, ...
- Available automatic tools

Further work

- Progressive introduction of **time delays**
- Extend the logics expressiveness to tackle properties with **quantitative timing information** $\Rightarrow$ Lecture 2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