School US-Japan seminar 2013/4/4 @Nara

Topological quantum computation

-from topological order to fault-tolerant quantum computation-

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Outline

(1) Introduction: what is topological order?

(2) Majorana fermions & 2D Kitaev model

(3) Thermal instability of topological order

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(4) Error correction on (Kitaev's toric code) surface code

(5) Topological quantum computation defect qubits/ braiding /magic state distillation/ implementations

condensed matter physics

quantum information processing

What is topological order?

Topological order is..........

-a new kind of order in zero-temperature phase of matter.

-cannot be described by Landau's symmetry breaking argument.

-ground states are degenerated and it exhibits long-range quantum entanglement.

-the degenerated ground states cannot be distinguished by local operations.

-topologically ordered states are robust against local perturbations.

-related to quantum spin liquids, fractional quantum Hall effect, fault-tolerant quantum computation.

Landau's symmetry breaking argument

Ising model (e.g. two dimension):

Pauli operators:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Computational basis (qubit):

$$\{|0\rangle, |1\rangle\} \rightarrow Z|0\rangle = |0\rangle,\ Z|1\rangle = -|1\rangle$$

$$\{|+\rangle, |-\rangle\} \rightarrow X|+\rangle = |+\rangle,\ X|-\rangle = -|-\rangle$$

Multi-qubit system $\{|0\rangle, |1\rangle\}^{\otimes N}$:

$$A_i = I_1 \otimes ... \otimes I_{i-1} \otimes A_i \otimes I_{i+1} \otimes ... I_N$$

$$(A = X, Y, Z)$$

temperature T

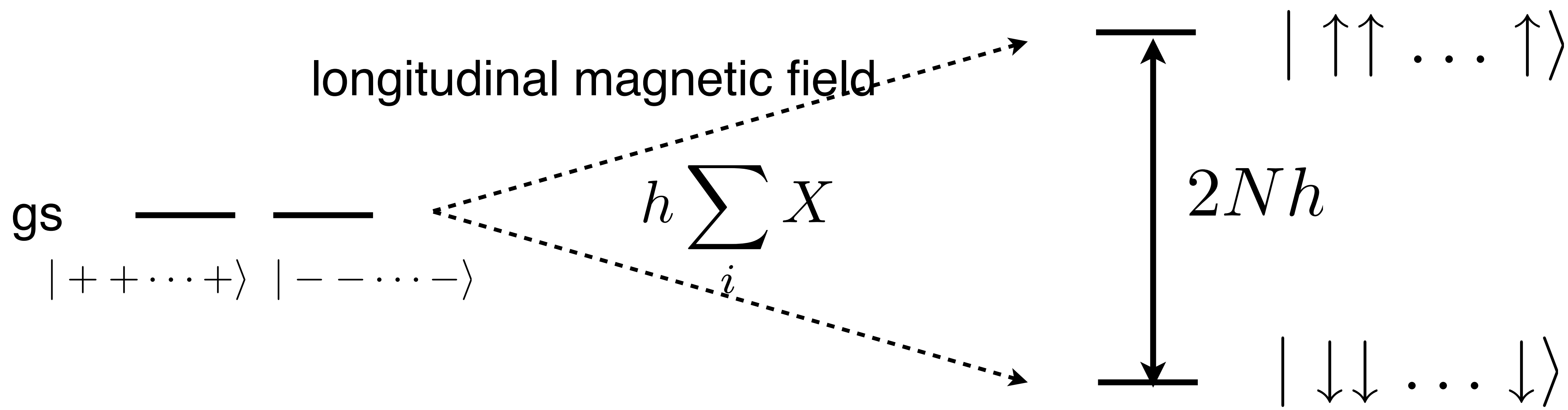
Landau's symmetry breaking argument

Ising model (e.g. two dimension):

$$H = -J \sum_{\langle ij \rangle} X_i X_j$$

The Ising Hamiltonian is invariant under spin flipping *Z* w.r.t. *X*-basis.

The ground states: $\{|++\cdots+\rangle, |--\cdots-\rangle\}$ (Z_2 symmetry)



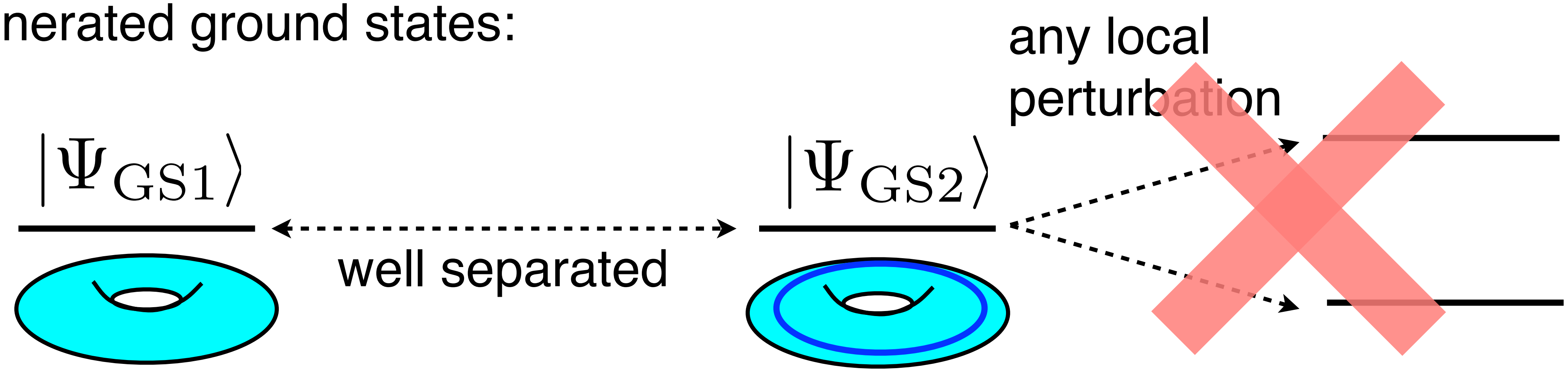
→ground state degeneracy is lifted by longitudinal magnetic field.

→ground state degeneracy is not robust against local perturbation.

Topologically ordered states

In topologically ordered system....

degenerated ground states:



Only nonlocal operators (high weight operator $A^{\otimes O(L)}$) can exchange the ground states.

→ The ground state degeneracy cannot be lifted by any local operations (energy shift is ~ $e^{-\alpha L}$).

e.g) $|\Psi_1\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2}$

$$|\Psi_2\rangle = (|0011\rangle + |1100\rangle)/\sqrt{2}$$

two orthogonal state cannot be distinguished by measuring a single qubit in any basis → second order perturbation first lifts the degeneracy

Topologically ordered states

How can we describe topologically ordered states efficiently?

→ Theory of quantum error correction is a very useful tool to describe topologically ordered system.

quantum error correction codes	topologically ordered system
code subspace	ground state degeneracy
correctability against errors (*k*-error correction code)	robustness against local perturbation (robust up to (2k+1)-th order perturbation)
stabilizer codes (D. Gottesman PhD thesis 97)	stabilizer Hamiltonian

locality and translation invariance

Toric code (surface code)	Kitaev model
classical repetition code (can correct either X or Z errors)	Ising model (non-topological-ordered)

→ thermal stability/ information capacity of discrete systems/ exotic topologically ordered state (fractal quantum liquid)

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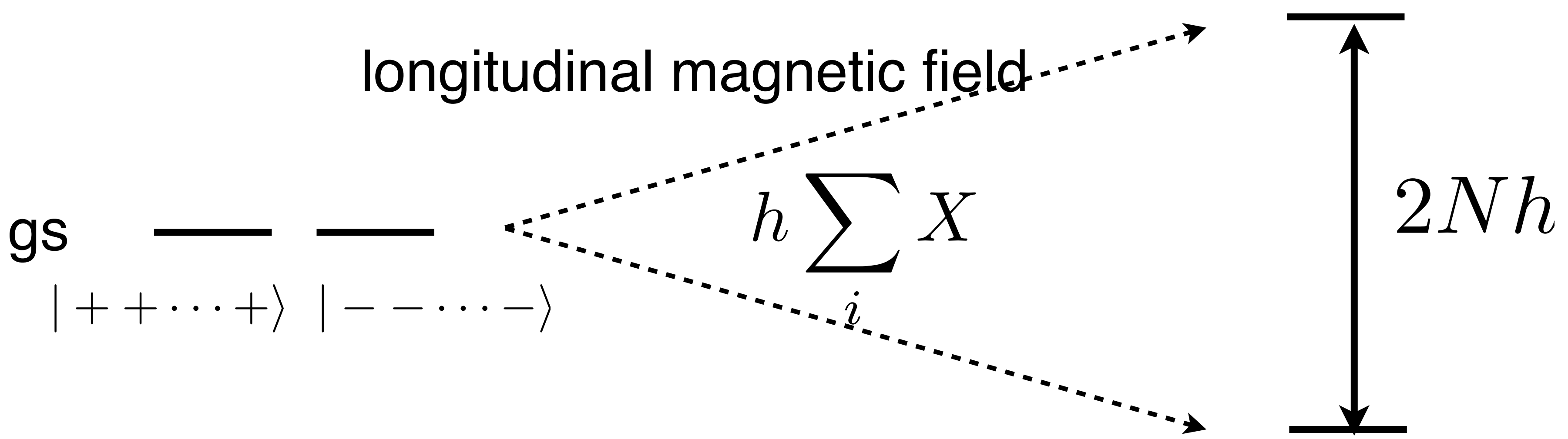
Majorana fermions

Ising model in one dimension (Ising chain):

$$H = -J \sum_{\langle ij \rangle} X_i X_j$$

the Ising Hamiltonian is invariant spin flipping *Z* w.r.t. *X*-basis.

The ground states: $\{|++\cdots+\rangle, |--\cdots-\rangle\}$ (Z_2 symmetry)



a repetition code $\langle X_i X_{i+1} \rangle$ cannot correct any *X* error,
since any single bit-flip error X_i changes the code space non-trivially.

Majorana fermions

Let us consider a mathematically equivalent but physically different system.

Ising chain: $$H_{\text{Ising}} = -J \sum_{i=1}^{N-1} X_i X_{i+1}$$

Jordan-Wigner transformation
(spin $\rightleftarrows$ fermion)

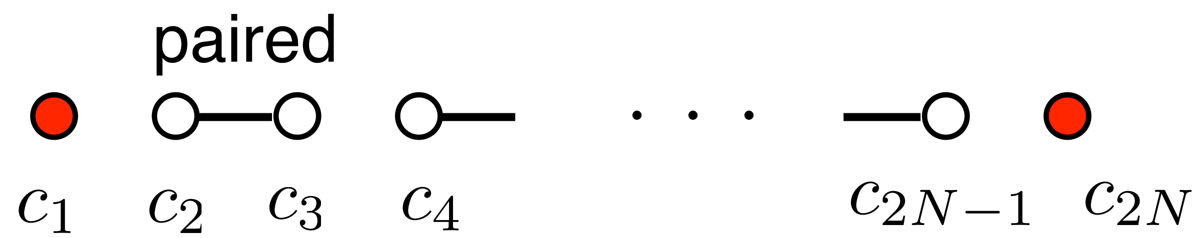
$$c_{2i-1} = Z_1 \ldots Z_{i-1} X_i$$
$$c_{2i} = Z_1 \ldots Z_{i-1} Y_i$$
$$\{c_i, c_j\} = \delta_{ij} I, c_i^\dagger = c_i$$
(Majorana fermion operator)

2*N* spinless fermions: $$H_{\text{Maj}} = -J \sum_{2}^{N-1} (-i) c_{2i} c_{2i+1}$$

superconductor, topological insulator, semiconducting heterostructure
(see A. Kitaev and C. Laumann, arXiv:0904.2771 for review)

Majorana fermions

$$H_{\mathrm{Maj}} = -J \sum_{2}^{N-1} (-i) c_{2i} c_{2i+1}$$



ground states: $(-i)c_{2i}c_{2i+1}|\Psi\rangle = |\Psi\rangle$ for all *i*.

unpaired Majorana fermions ● at the edges of the chain

→ "zero-energy Majorana boundary mode" $\{|\bar{0}\rangle, |\bar{1}\rangle\}$

$$(-i)c_1 c_{2N}|\bar{0}\rangle = |\bar{0}\rangle, \quad (-i)c_1 c_{2N}|\bar{1}\rangle = -|\bar{1}\rangle, \quad c_1|\bar{0}\rangle = |\bar{1}\rangle$$

↓ $Y_1 Z_2 \ldots Z_{N-1} Y_N$ (Z2 symmetry)
If unpaired Majonara fermions are well separated, this operator would not act.

↓ X_1
(act on the ground subspace nontrivially)

But! c_1 or c_{2N} (odd weight fermionic operators) require coherent creation/annihilation of a single fermion, which is prohibited by superselection rule.

→ *X* errors are naturally prohibited by the fermionic superselection rule.

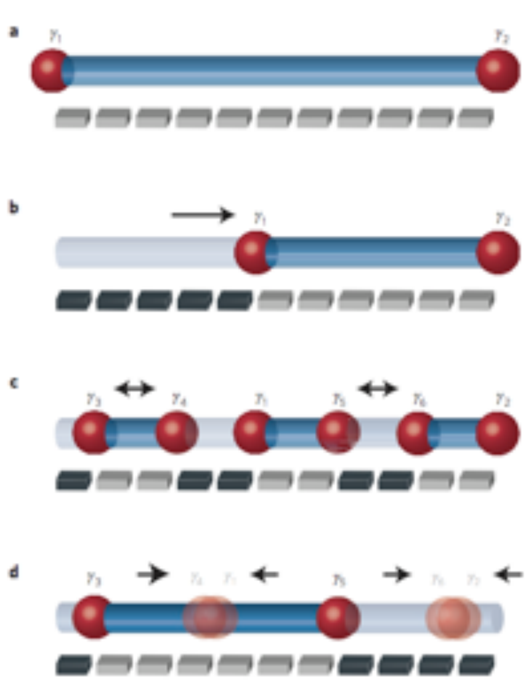
→ Unpaired Majorana fermion is robust against any "physical" perturbation.

Topological quantum computation

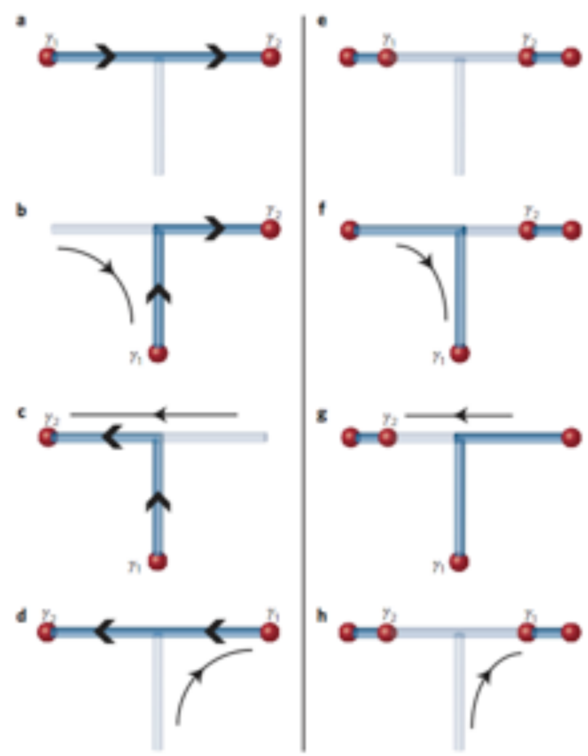
ARTICLES
PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI: 10.1038/NPHYS1915
nature physics

Non-Abelian statistics and topological quantum information processing in 1D wire networks

Jason Alicea[1]*, Yuval Oreg[2], Gil Refael[3], Felix von Oppen[4] and Matthew P. A. Fisher[3,5]



pair creation of Majorana fermions



exchanging Majorana fermions via T-junction

Majorana fermions are non-Abelian, but do not allow universal quantum computation.

Topologically protected gates

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Magic state distillation
[Bravyi-Kitaev PRA **71**, 022316 (2005)]

=

Universal quantum computation

Topological quantum computation

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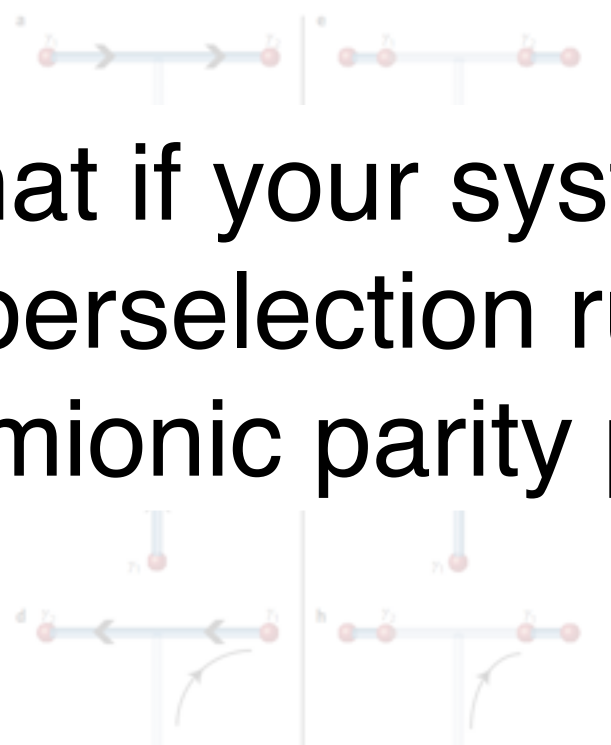


exchanging Majorana fermions via T-junction

...otected gates

...illation

, 022316 (2005)]

Universal quantum computation

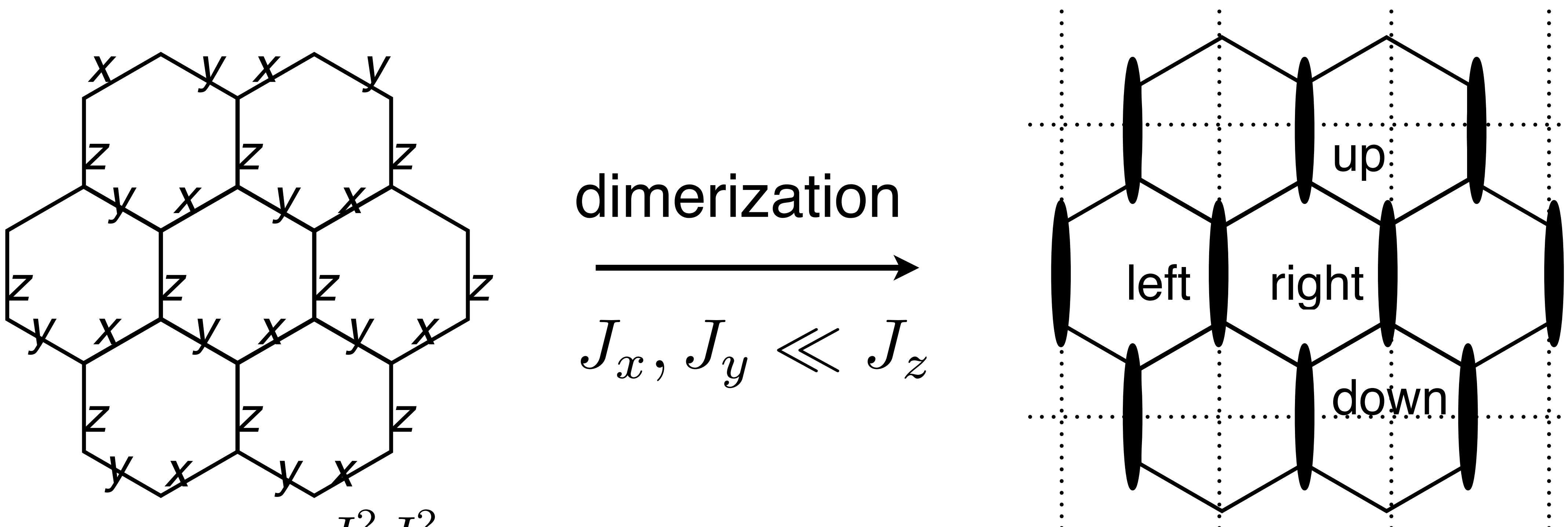
What if your system has no superselection rule, such as the fermionic parity preservation?

Kitaev's honeycomb model

A. Kitaev, Ann. Phys. 321, 2 (2006)

Honeycomb model:

$$H_{\rm hc} = -J_x \sum_{x\text{-link}} X_i X_j - J_y \sum_{y\text{-link}} Y_i Y_j - J_z \sum_{z\text{-link}} Z_i Z_j$$



$$H_{\rm eff} = -\frac{J_x^2 J_y^2}{16|J_z|^3} \sum_p Y_{{\rm left}(p)} Y_{{\rm right}(p)} X_{{\rm up}(p)} X_{{\rm down}(p)}$$

local unitary transformation

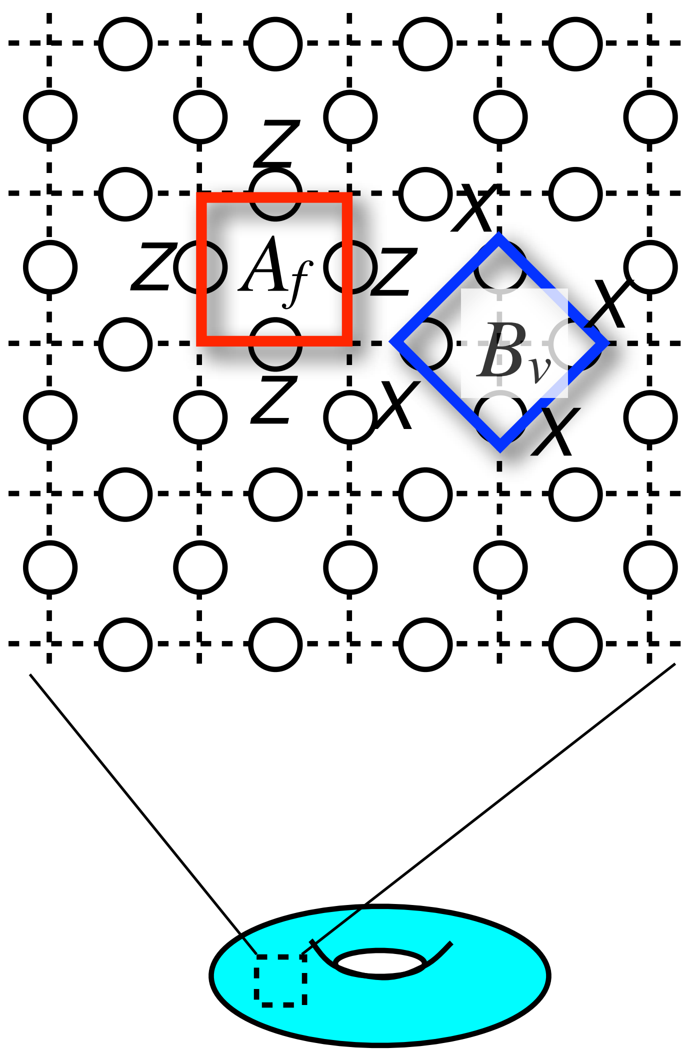
Toric code Hamiltonian:

$$H_{\rm TC} = -J \sum_f Z_{l(f)} Z_{r(f)} Z_{d(f)} Z_{u(f)} - J \sum_v X_{l(v)} X_{r(v)} X_{d(v)} X_{u(v)}$$

A. Kitaev, Ann. Phys. 303, 2 (2003)

Kitaev's toric code model

Kitaev's toric code model is a representative example of topologically ordered system.



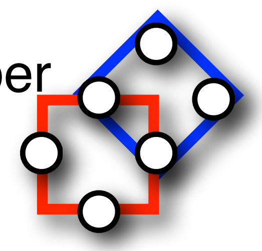
face operator: $A_f = \prod_{i \in \text{ face } f} Z_i$

vertex operator: $B_v = \prod_{i \in \text{ vertex } v} X_i$

Toric code Hamiltonian: $H = -J \sum_f A_f - J \sum_v B_v$

Note that these operators are commutale:

even number crossover



anti-commute × anti-commute = commute

The ground states are given by simultaneous eigenstate of all face & vertex operators (gapped and frustration-free):

$$A_f|\Psi\rangle = |\Psi\rangle, \quad B_v|\Psi\rangle = |\Psi\rangle$$

Kitaev toric code model

Kitaev's toric code model is a representative example of

★Short note on the stabilizer formalism

- n-qubit Pauli group: $\{\pm 1, \pm i\} \times \{I, X, Y, Z\}^{\otimes n}$
- stabilizer group: $\mathcal{S} = \{S_i\}, \text{where } [S_i, S_j] = 0 \text{ and } S_i = S_i^\dagger$
  (commutative) (hermitian)
- stabilizer generators: minimum independent set of stabilizer elements
- stabilizer state: $S_i|\Psi\rangle = |\Psi\rangle$ for all stabilizer generators S_i
- example: $\langle X_1 X_2, Z_1 Z_2 \rangle \rightarrow (|00\rangle + |11\rangle)/\sqrt{2}$
- dimension of the stabilizer subspace: 2^(# qubits - # generators)

B_v

ute

$$A_f|\Psi\rangle = |\Psi\rangle, \quad B_v|\Psi\rangle = |\Psi\rangle$$

Kitaev toric code model

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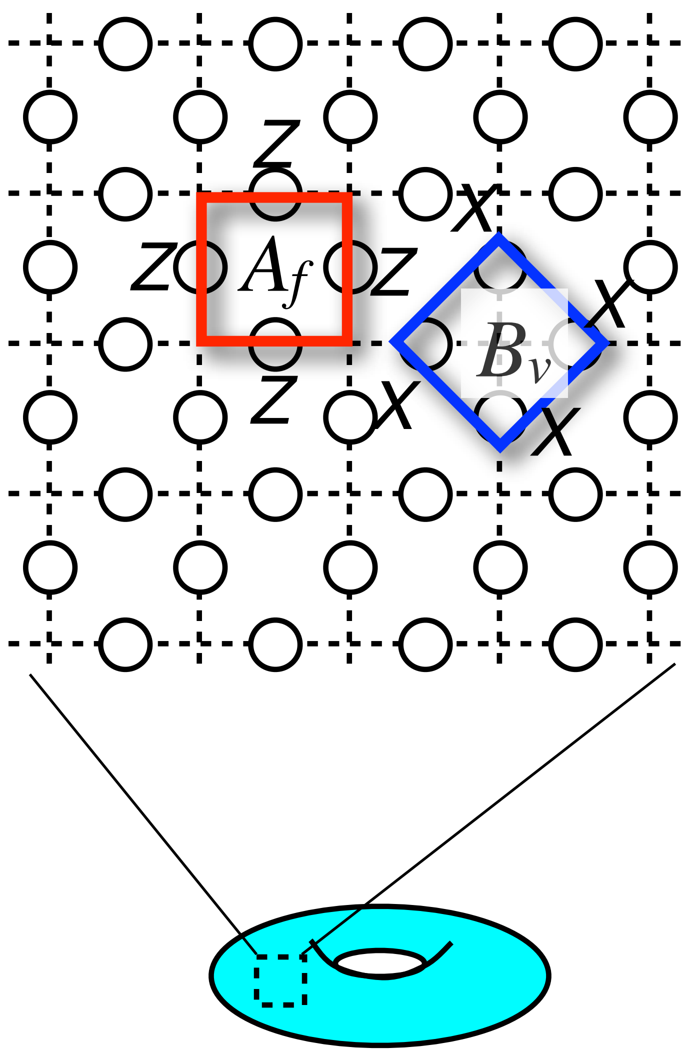

face operator: $A_f = \prod_{i \in \text{ face } f} Z_i$

→stabilizer generators

vertex operator: $B_v = \prod_{i \in \text{ vertex } v} X_i$

Toric code Hamiltonian: $H = -J \sum_f A_f - J \sum_v B_v$

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Note that these operators are commutale:

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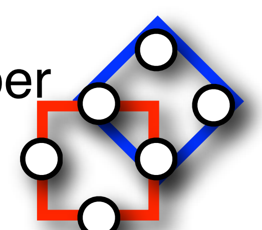

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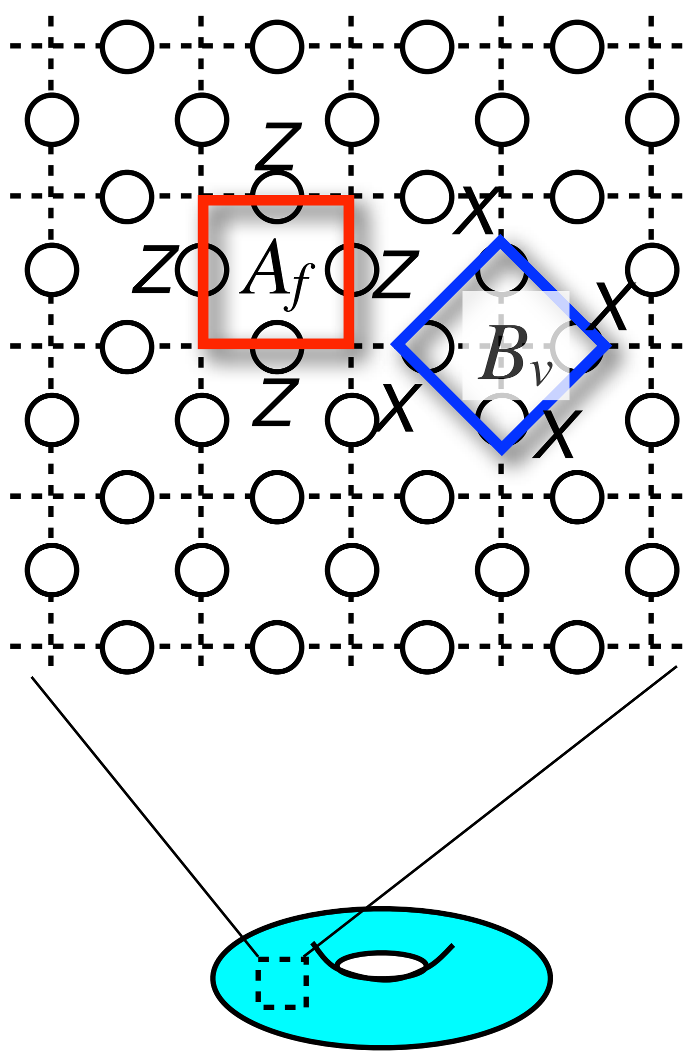
The ground states are given by simultaneous eigenstate of all face & vertex operators (gapped and frustration-free):

→stabilizer subspace $A_f|\Psi\rangle = |\Psi\rangle, \quad B_v|\Psi\rangle = |\Psi\rangle$

Structure of grand states



Degeneracy of the ground subspace:

[Torus]

qubits: (edges) on $N \times N$ torus = $2N^2$

stabilizer generators:
(faces + vertexes - 2) = $2N^2$ - 2

dimension of ground subspace: $2^2=4$

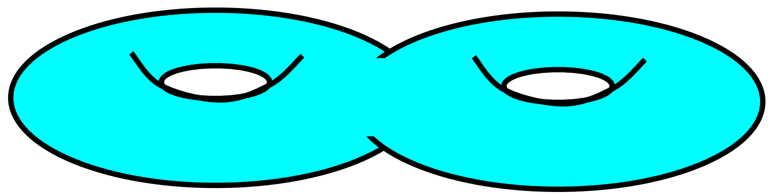
logical qubits → 2 (two logical qubits)

[General surface]

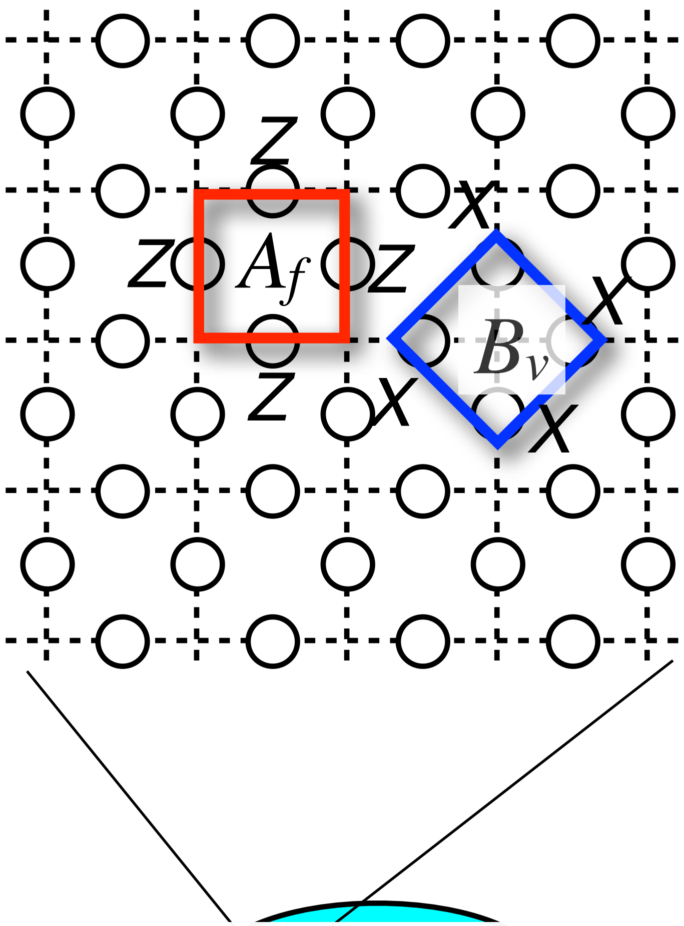
(face)+(vertex)-(edge)=2-2g $\quad$ g = genus

Euler characteristic

logical qubits → (edge)-[(face)+(vertex)-2]=2g



Structure of ground states



Degeneracy of the ground subspace:

[Torus]

\# qubits: (edges) in $N \times N$ torus = $2N^2$

\# stabilizer generators:
(faces + vertexes - 2) = $2N^2 - 2$

\# dimension of ground subspace: $2^2=4$

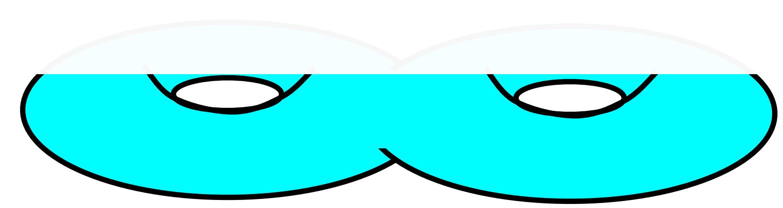
\# logical qubits → 2 (two logical qubits)

[General surface]

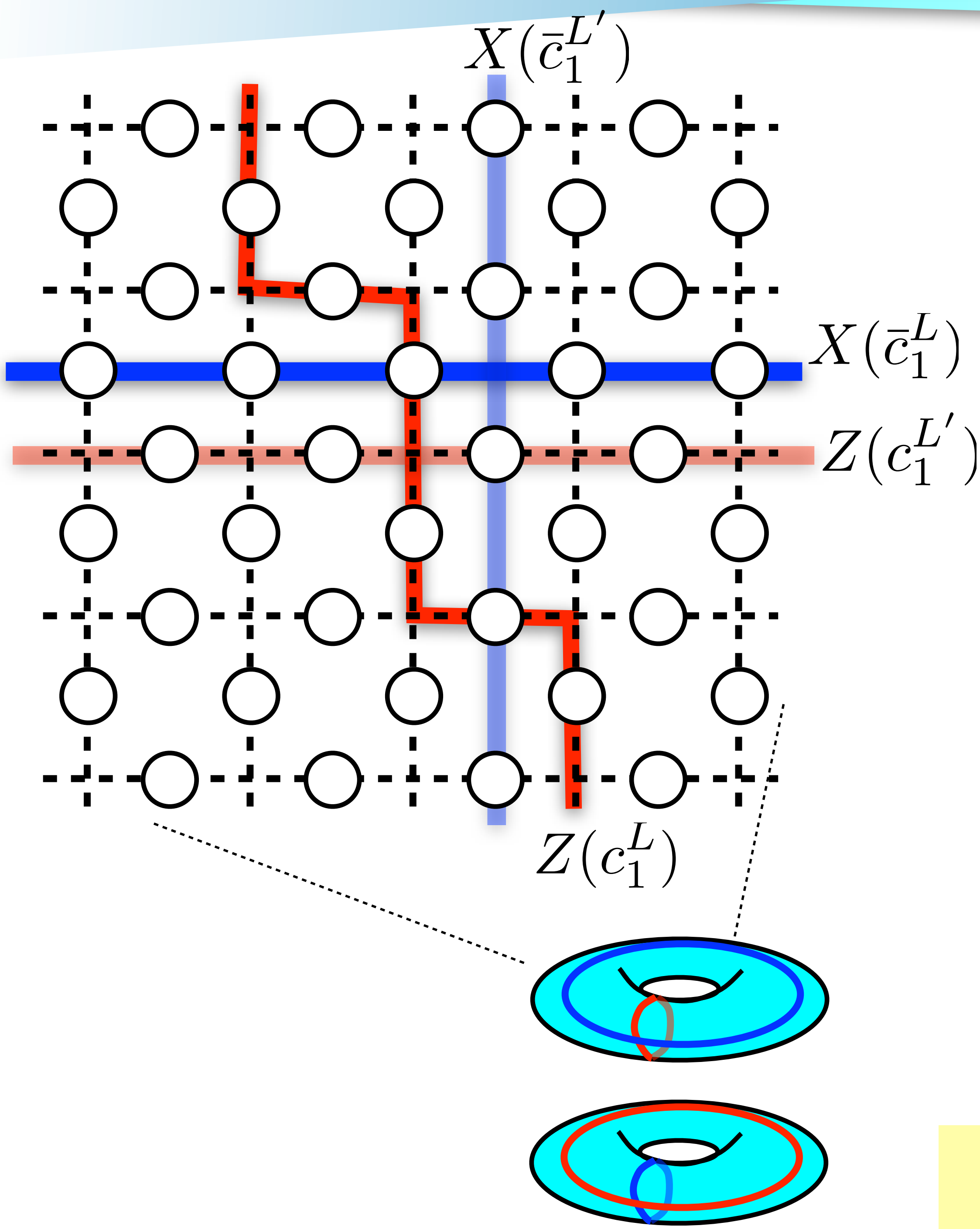
How is the ground state degeneracy described?

→ Find a good quantum number! The operator that acts on the ground subspace nontrivially, "*logical operator*".

$2g$



Non-trivial cycle: Logical operators



The operators on non-trivial cycles $Z(c_1^L), X(\bar{c}_1^L)$ are commutable with all face and vertex operators, but cannot given by a product of them.

$$\{Z(c_1^L), X(\bar{c}_1^L)\} = 0$$

→ logical Pauli operators.

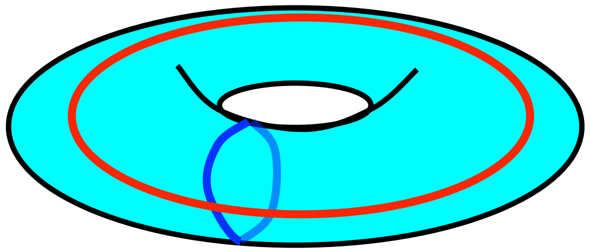
g=1 → # of logical qubit = 2:

$$\{Z(c_1^L), X(\bar{c}_1^L)\}, \{Z(c_1^{L'}), X(\bar{c}_1^{L'})\}$$

(The action of logical operators depend only on the homology class of the cycle.)

The logical operators have weight *N.*
→ *N*-th order perturbation shifts the ground energy.

Stability against local perturbations



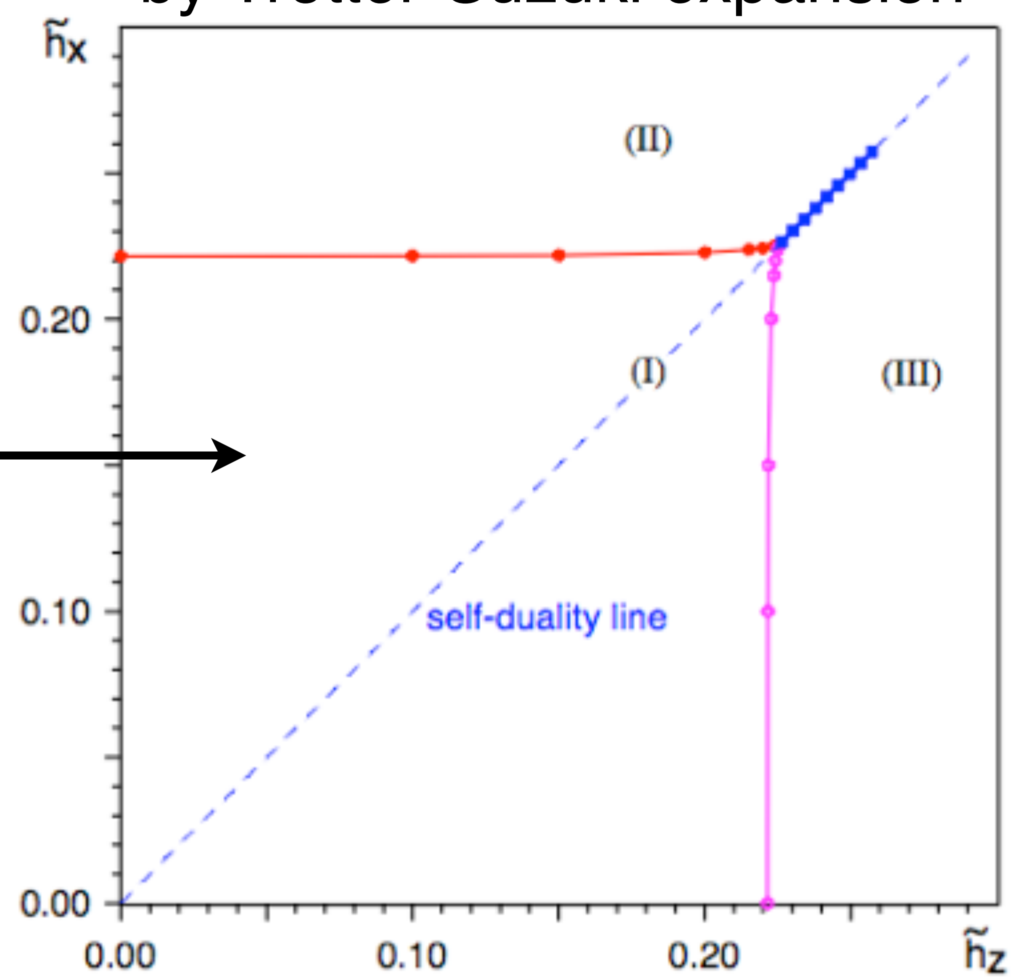
local field terms

$$H = H_{\mathrm{TC}} + h_x \sum_i X_i + h_z \sum_i Z_i$$

quantum/classical mapping
by Trotter-Suzuki expansion

Z2 Ising gauge model
(dual of 3D Ising model)

topologically ordered
(Higgs phase)



Tupitsyn *et al.*, PRB **82**, 085114 (2010)

Stability against local perturbations

local field terms

$$H = H_{\mathrm{TC}} + h_x \sum_i X_i + h_z \sum_i Z_i$$

quantum/classical mapping → Z2 Ising gauge model (dual of 3D Ising model)

Is stability against perturbations enough for fault-tolerance?

No. Stability against thermal fluctuation is also important!

topologically orde
(Higgs phase)

Tupitsyn *et al.*, PRB **82**, 085114 (2010)

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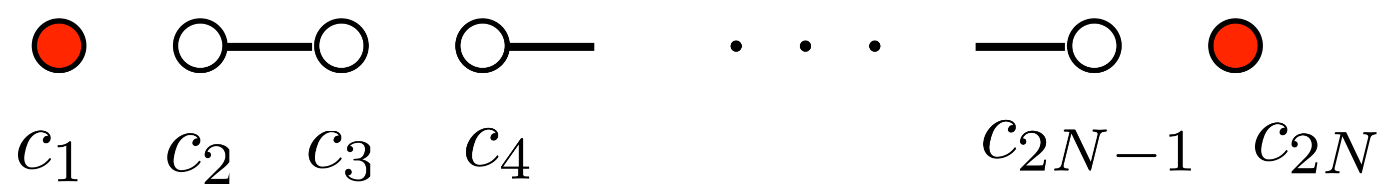
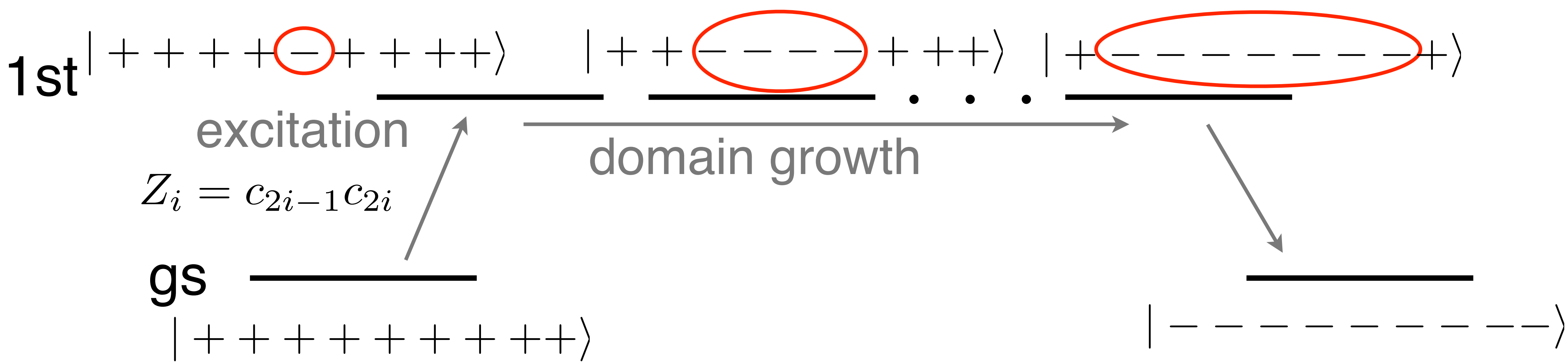
Thermal instability of topological order

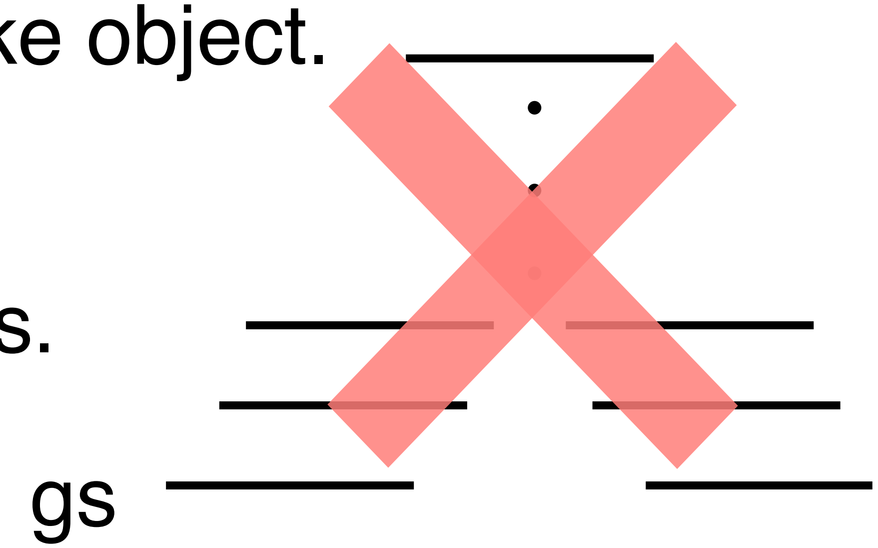
Majorana fermion:





Excitation (domain-wall) is a point-like object.

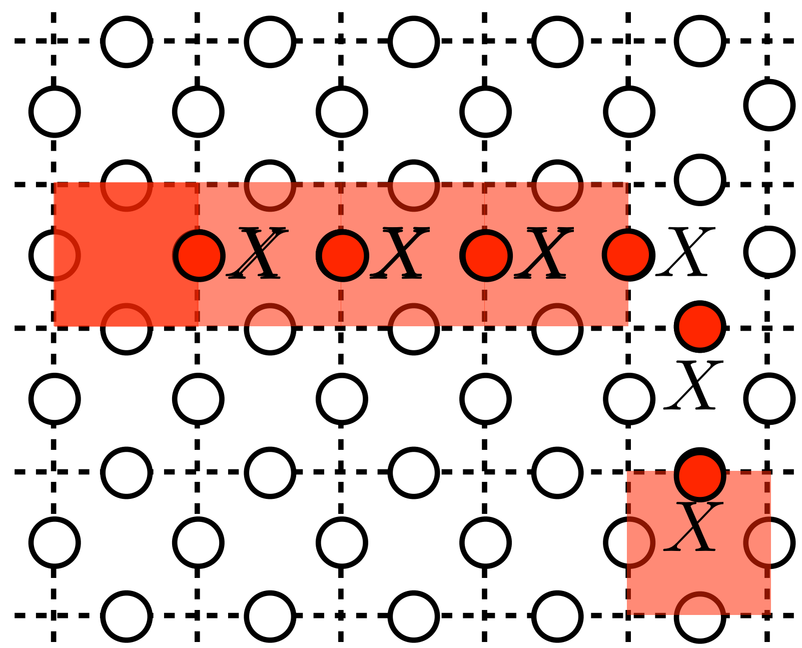
→There is no large energy barrier between the degenerated ground states.



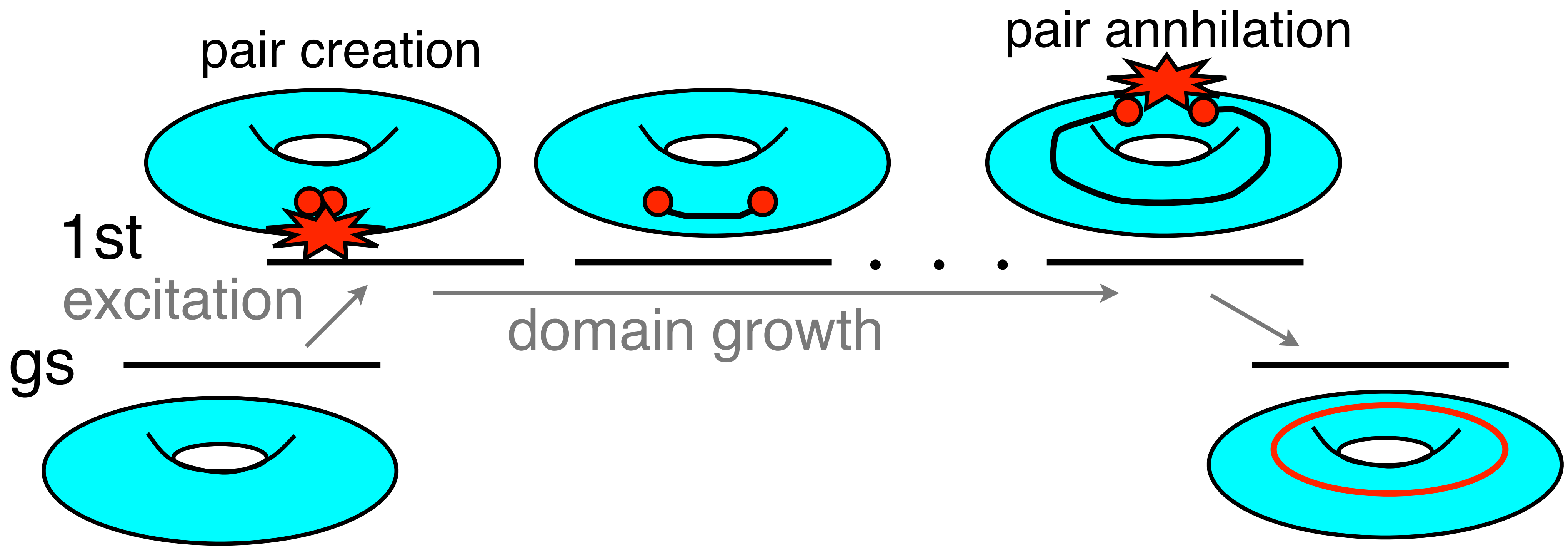
Thermal instability of topological order

Kitaev’s toric code model:

anyonic excitation (Abelian)
→excitation is a point-like object.



Anyon can move freely without any energetic penalty.



Thermal instability of topological order

More generally...

Topological order in any local and translation invariant stabilizer Hamiltonian systems in 2D and 3D do not have thermal stability.

2D: S. Bravyi and B. Terhal, New J. Phys. **11**, 043029 (2009).
3D: B. Yoshida, Ann. Phys. **326**, 2566 (2011).

← quantum error correction code theory

Thermally stable topological order (self-correcting quantum memory) in 4D
by E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J.Math.Phys. **43,** 4452 (2002).

(Excitation has to be two-dimensional object for each non-commuting errors, X and Z. →4D)

Existence/non-existence of thermally stable topological order (= self-correcting quantum memory) in 3 or lower dimensions is one of the open problems in physics!
(see list of unsolved problem in physics in wiki)

Non-equilibrium condition (feedback operations) is necessary to observe long-live topological order (many-body quantum coherence) at finite temperature.

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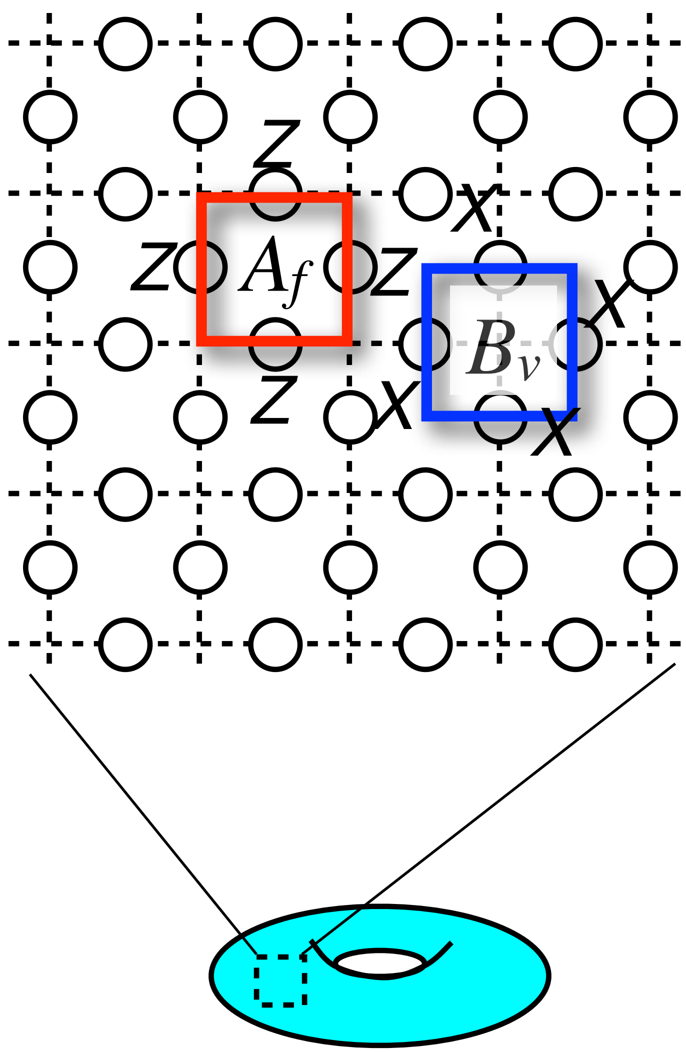
Topological error correction



face stabilizer: $A_f = \prod_{i \in \text{ face } f} Z_i$

vertex stabilizer: $B_v = \prod_{i \in \text{ vertex } v} X_i$

Toric code Hamiltonian: $H = -J \sum_f A_f - J \sum_v B_v$

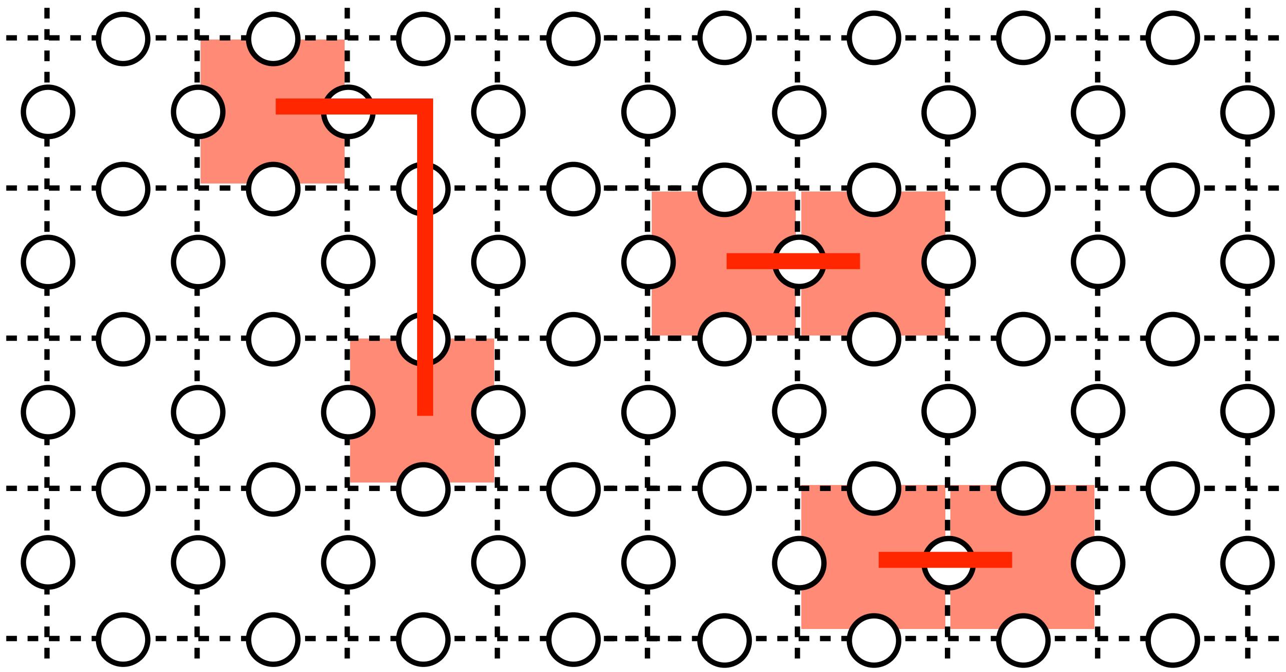
The code state is defied by

$$A_f |\Psi\rangle = |\Psi\rangle, \quad B_v |\Psi\rangle = |\Psi\rangle$$

for all face and vertex stabilizers.

Errors on the surface code

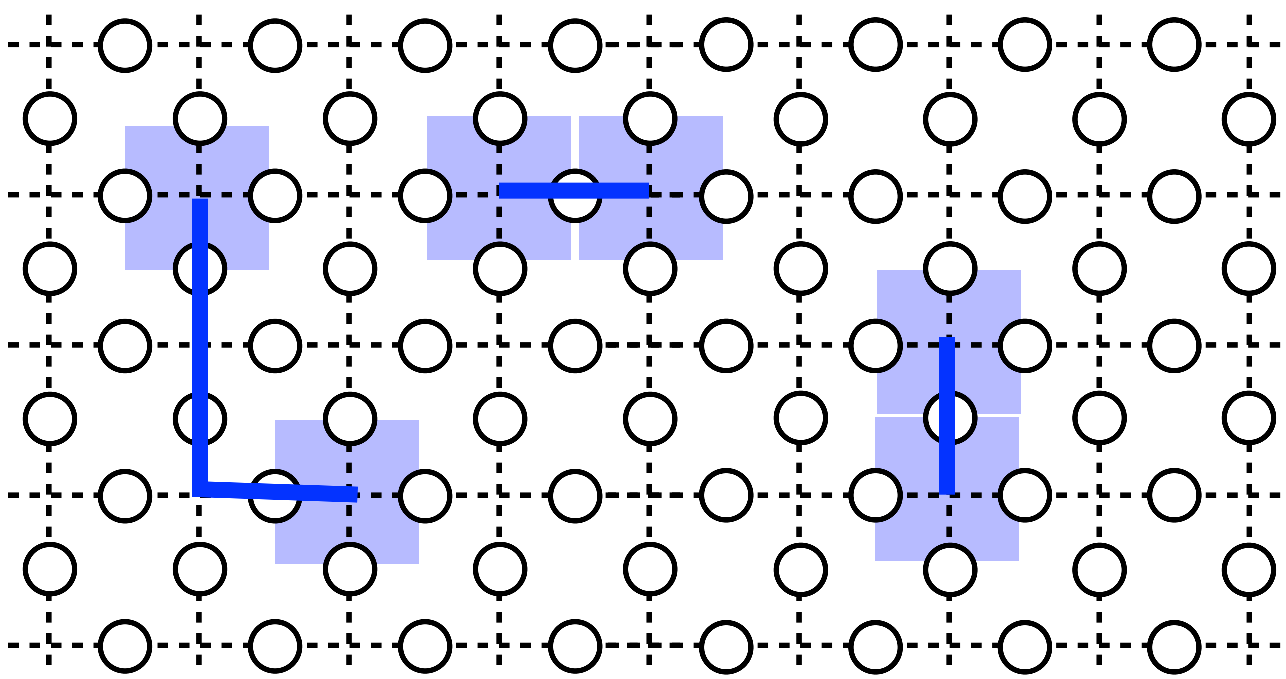
If a chain of *X* (bit-flip) errors occurs, the eigenvalues of the face stabilizers become -1 at the boundary of the error chain.



(In the toric code Hamiltonian, they correspond to the anyonic excitations)

Errors on the surface code

Similarly if a *Z* (phase-flip) error chain occurs, the eigenvalues of the vertex stabilizers become -1 at boundary of the error chian.

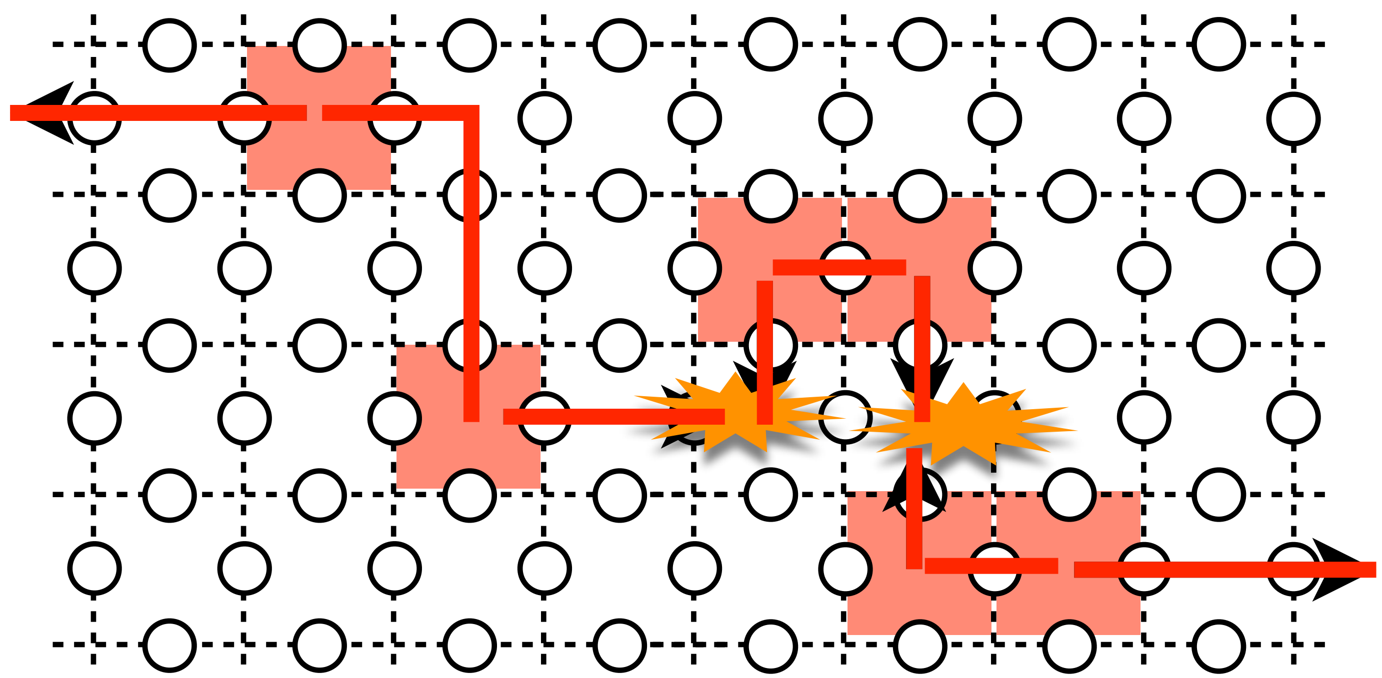


(that is, toric code model have two types of anyonic excitations)

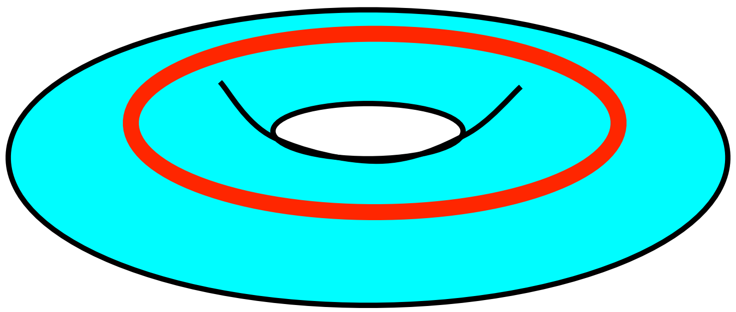
For simplicity, we only consider *X* errors correction below.

Errors on the surface code

If a chain of X (bit-flip) errors occurs, the eigenvalues of the face stabilizers become -1 at the boundary of the error chain.



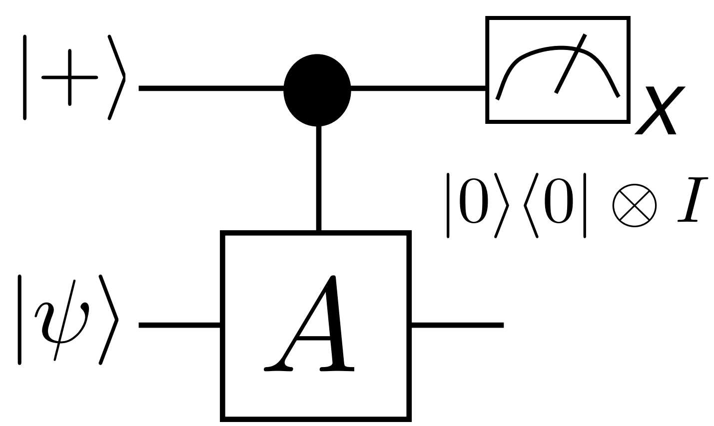
(In the toric code Hamiltonian, they correspond to the anyonic excitations)



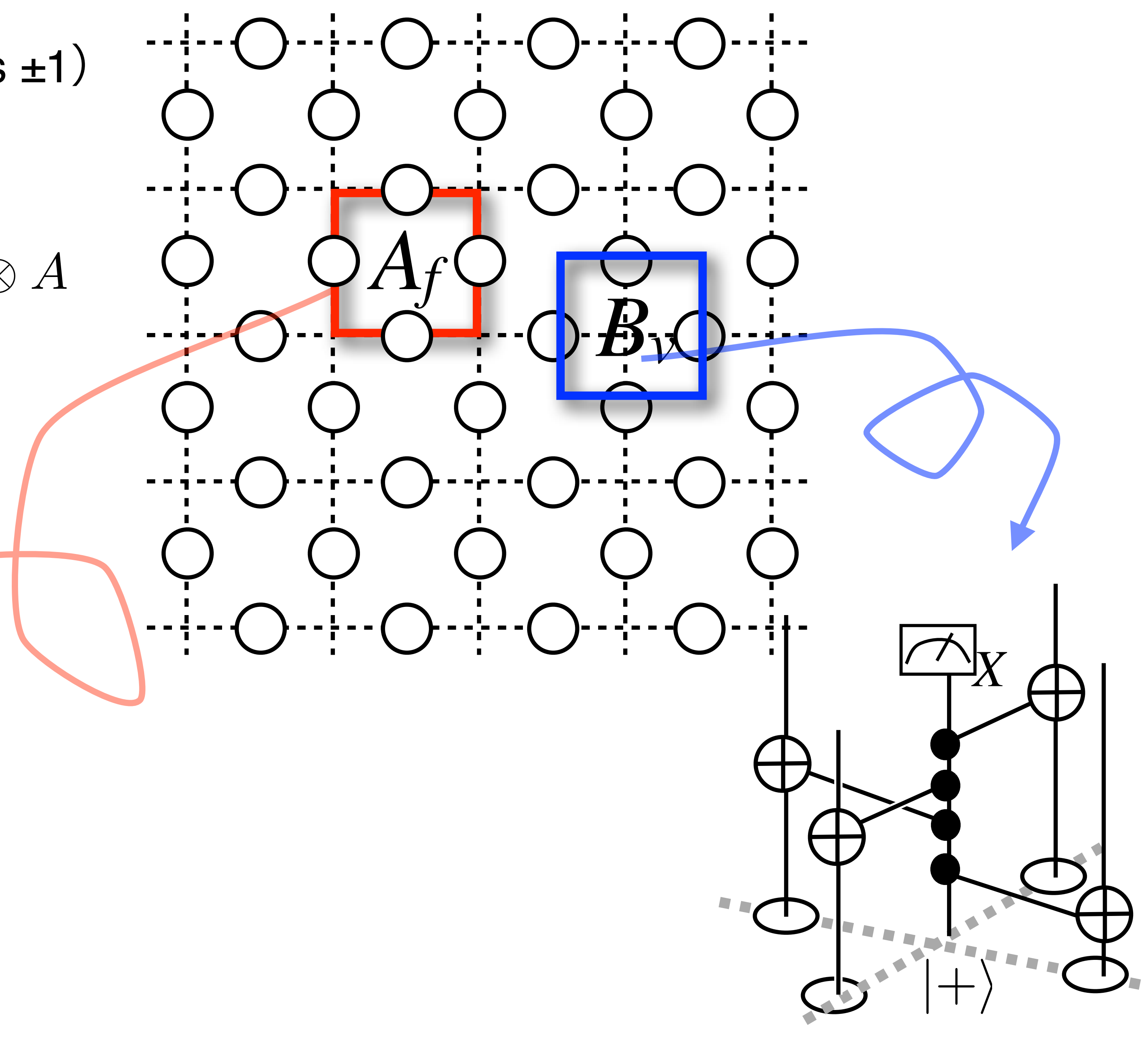
Syndrome measurements

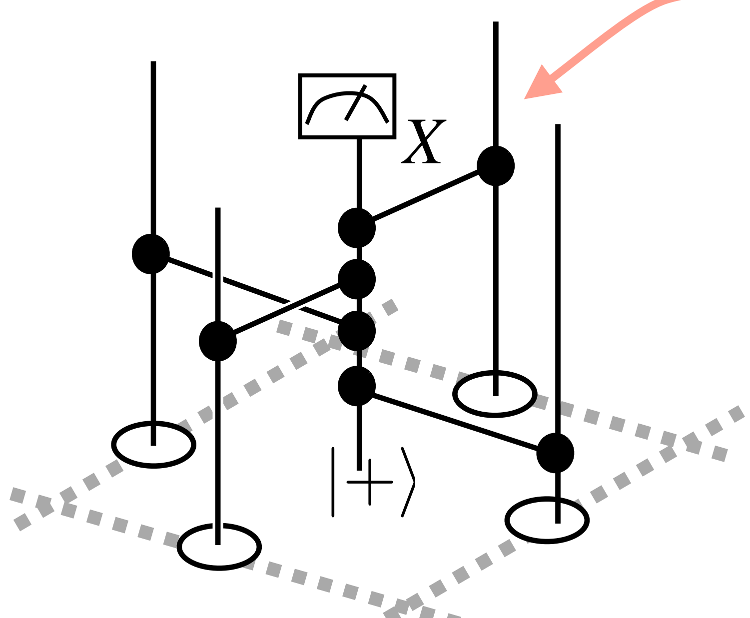
Measure the eigenvalues of the stabilizer operators.

Projective measurement for
an operator *A* (hermitian & eigenvalues ±1)



$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes A$$



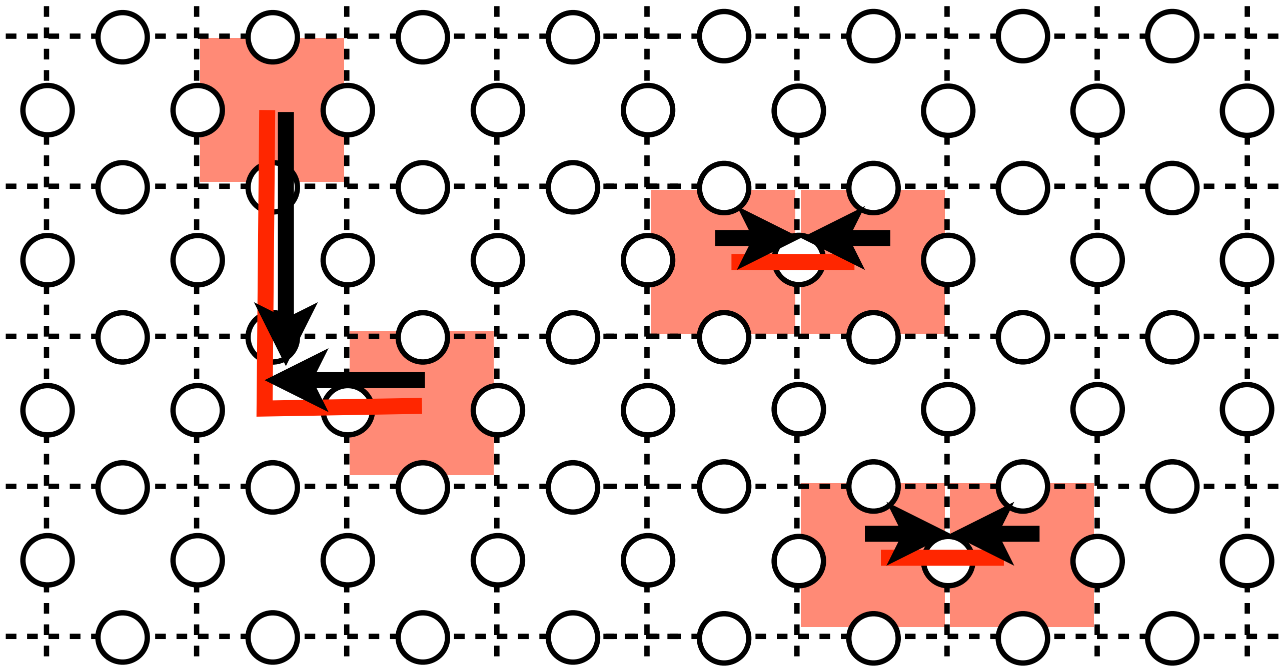


(In the toric code Hamiltonian, the syndrome measurements correspond to measurements of the local energy.)

Topological error correction

The syndrome measurements do not tell us the actual location of errors, but boundaries of them.

(It tells location of excitations, but does not tell the trajectory of the excitaitons)



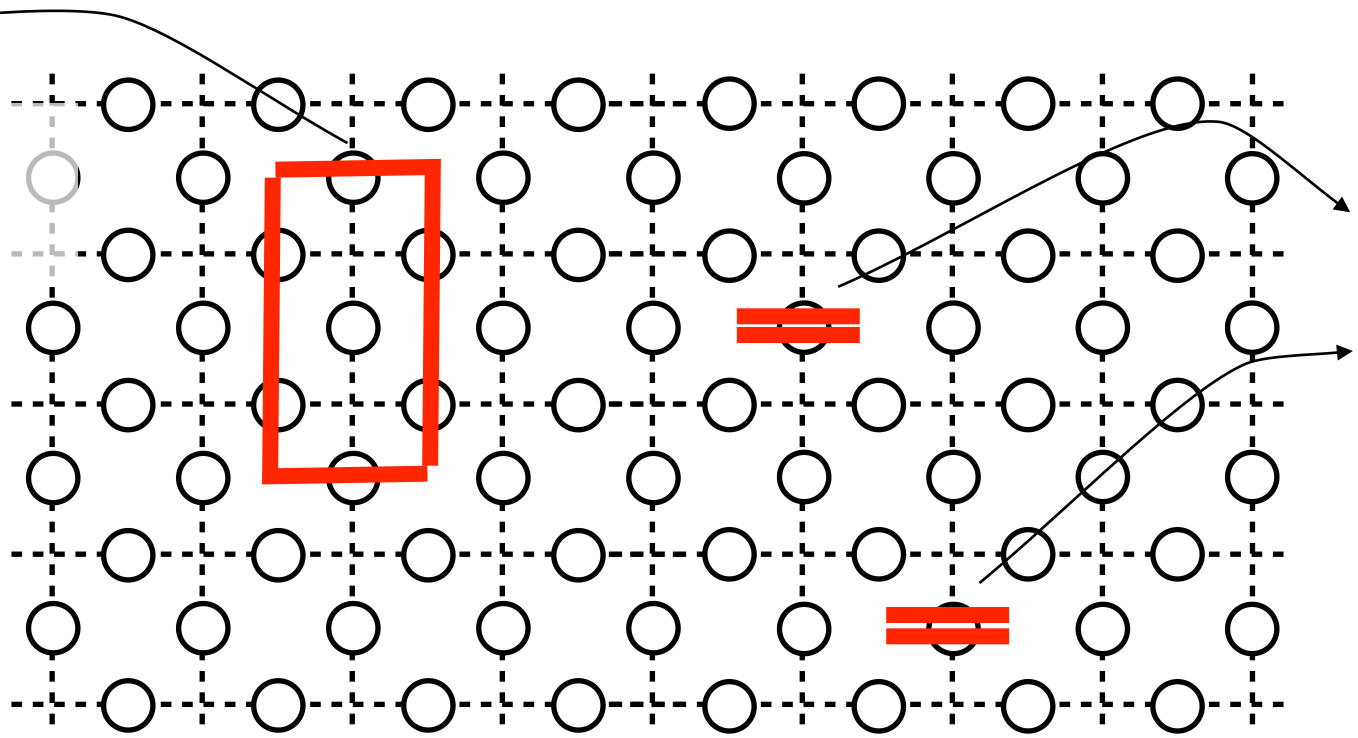
Then we have to infer a recovery chain, to recover from errors.

In the toric code Hamiltonian, this can be viewed as finding an appropriate way to annihilate pairs of anyones.

Topological error correction

If error and recovery chains result in a trivial cycle, the error correction succeeds.

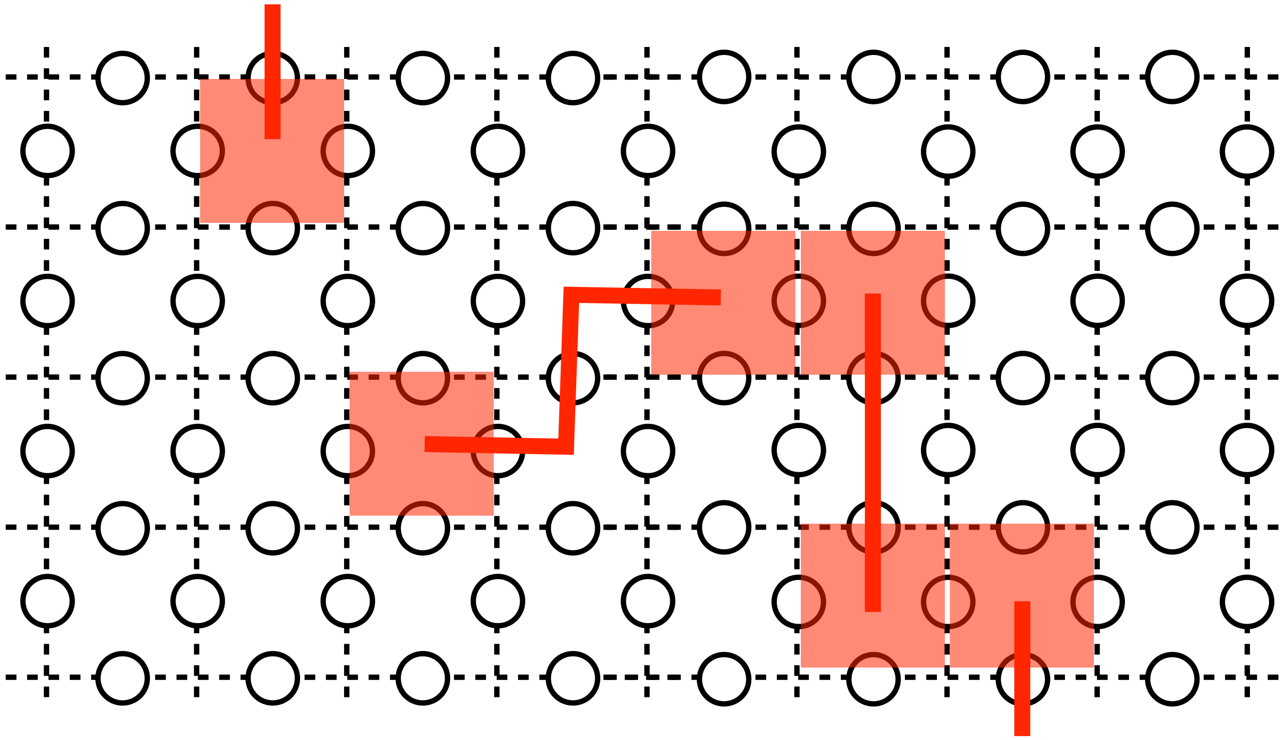
Trivial cycle = stabilizer element



Actual and estimated error locations are the same.

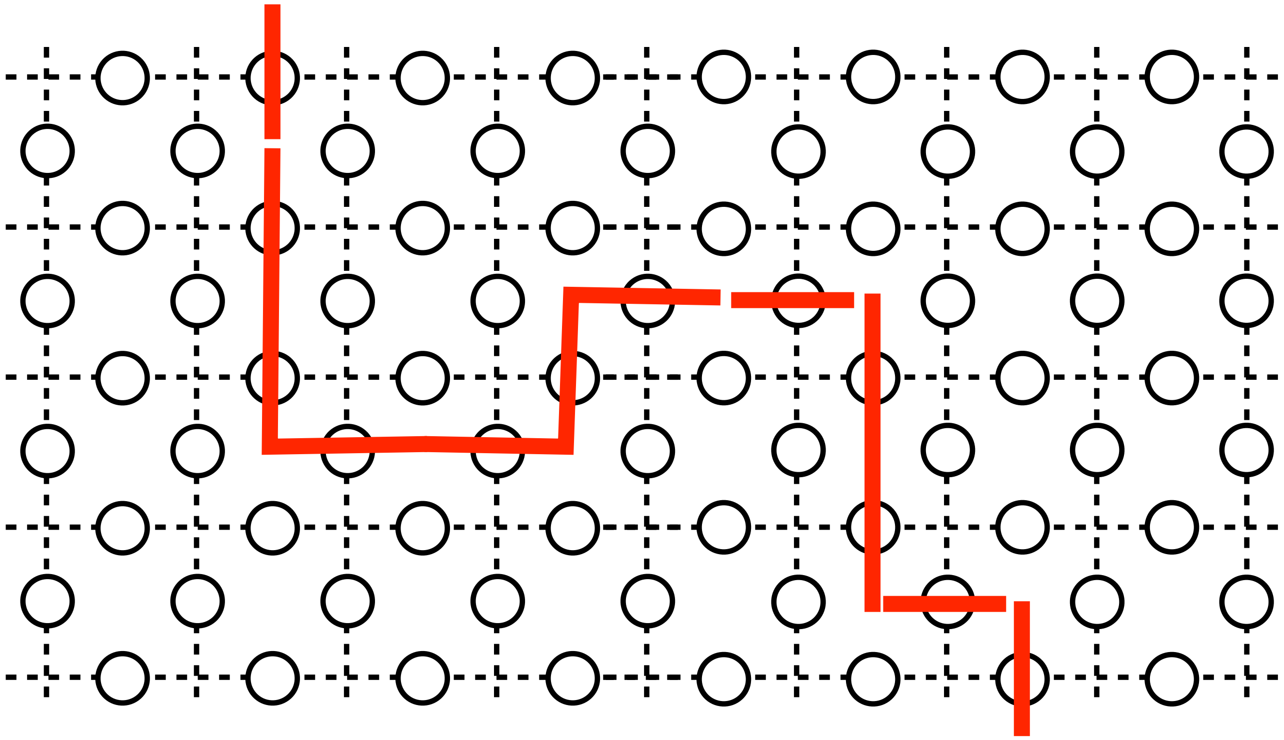
Topological error correction

If the estimation of the recovery chain is bad ....

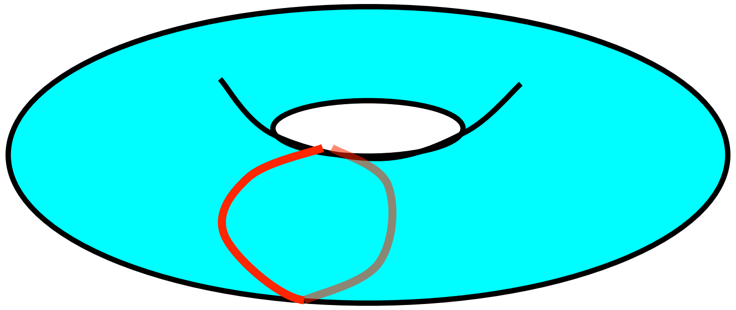


Topological error correction

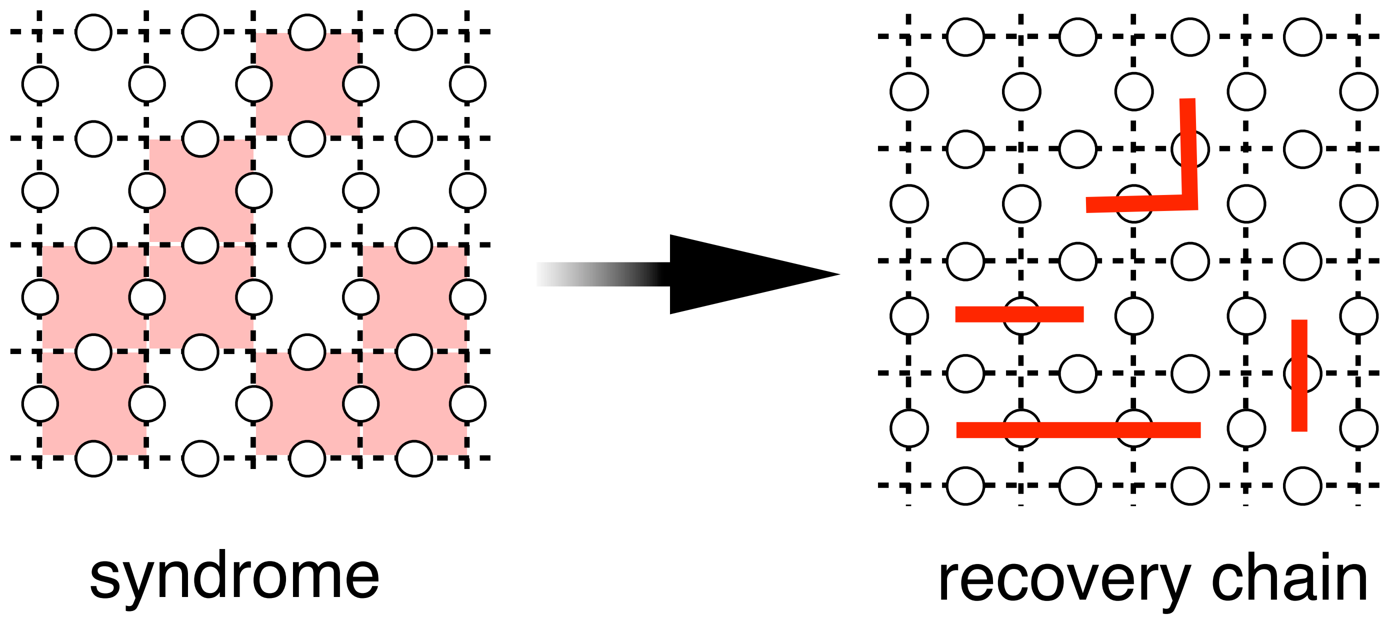
If the estimation of the recovery chain is bad ....



The error and recovery chains result in a non-trivial cycle, which change the code state.



Algorithms for error correction



→ The error chain which has the highest probability conditioned on the error syndrome.

→ minimum-weight-perfect-match (MWPM) algorithm (polynomial algorithm)

Blossom 5 by V. Kolmogorov, Math. Prog. Comp. **1**, 43 (2009).

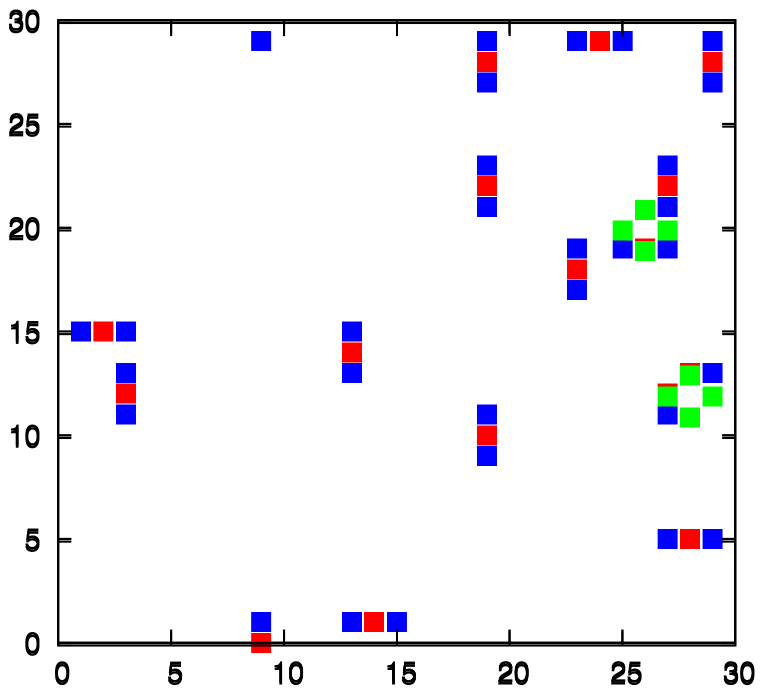
[Improved algorithms]

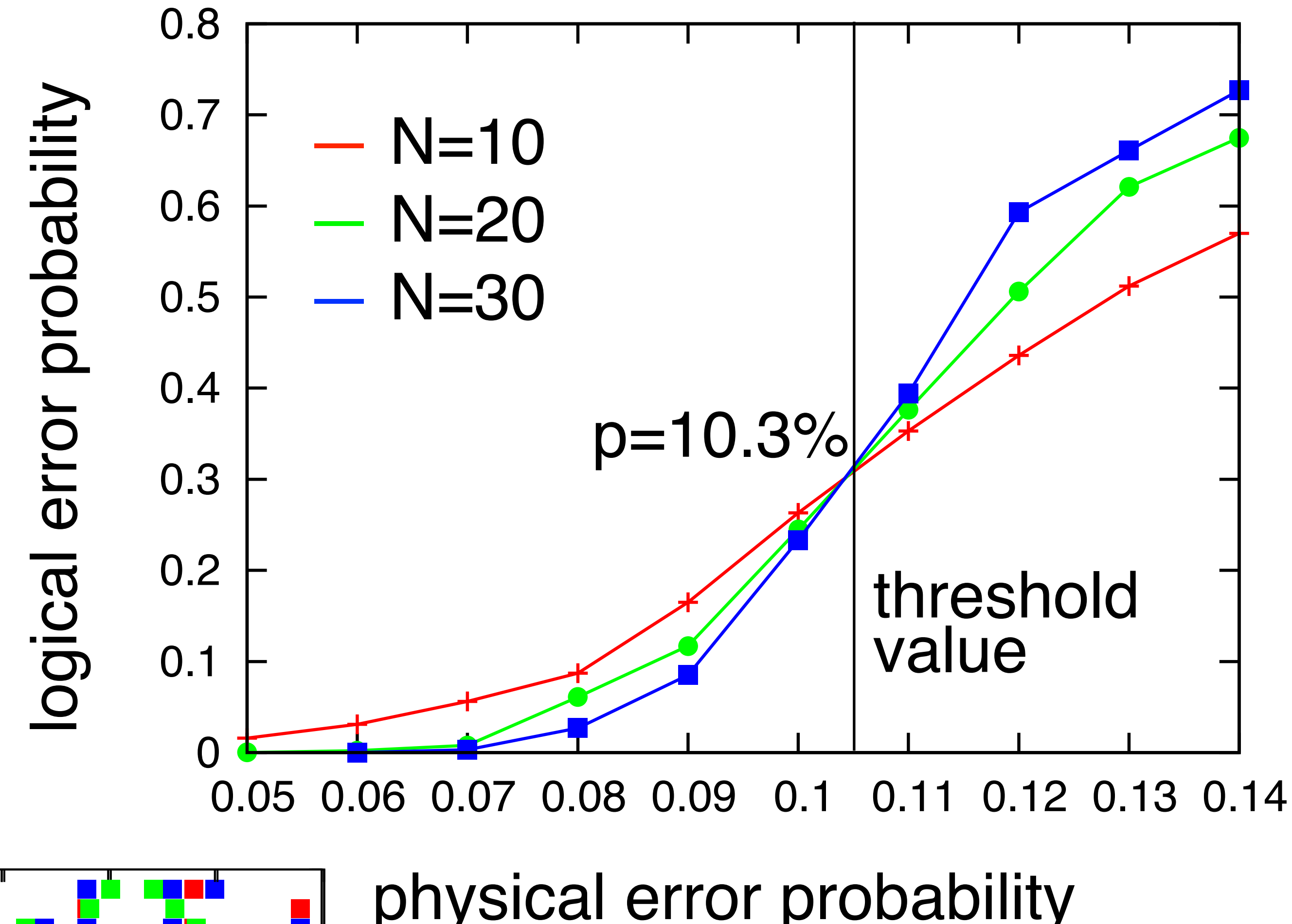
by Duclos-Cianci & Poulin Phys. Rev. Lett. **104**, 050504 (2010).
by Fowler *et al*., Phys. Rev. Lett. **108**, 180501 (2012).
by Wootton & Loss, Phys. Rev. Lett. **109**, 160503 (2012).

E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J.Math. Phys. **43**, 4452 (2002).

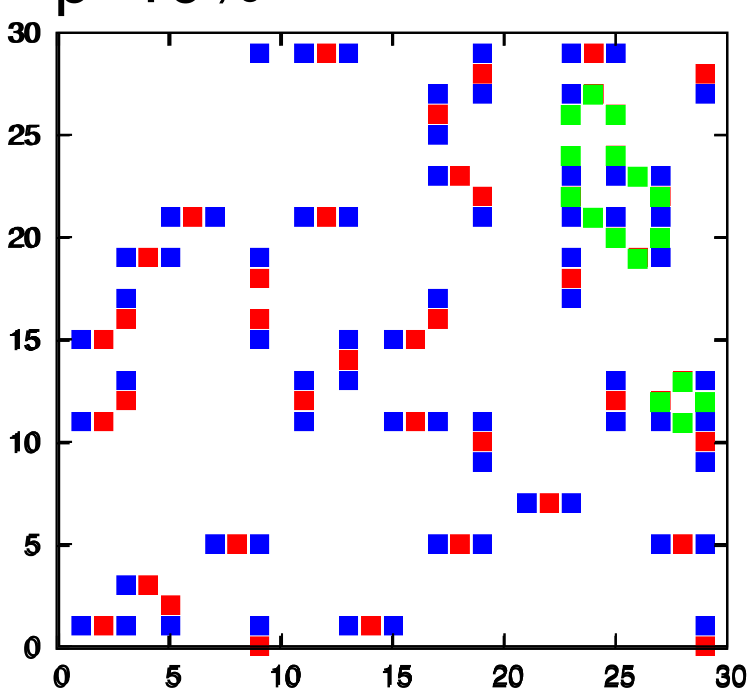
Algorithm for error correction

p=3%

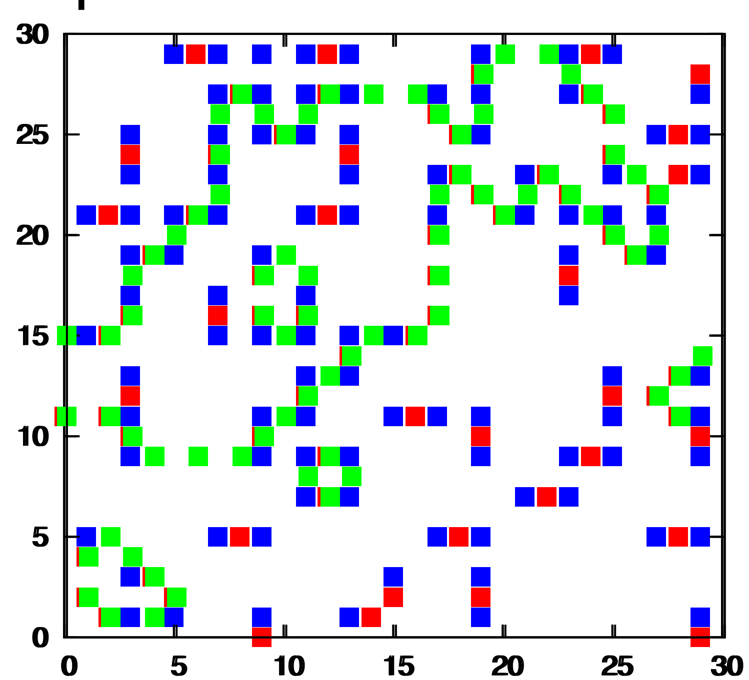




p=10%



p=15%

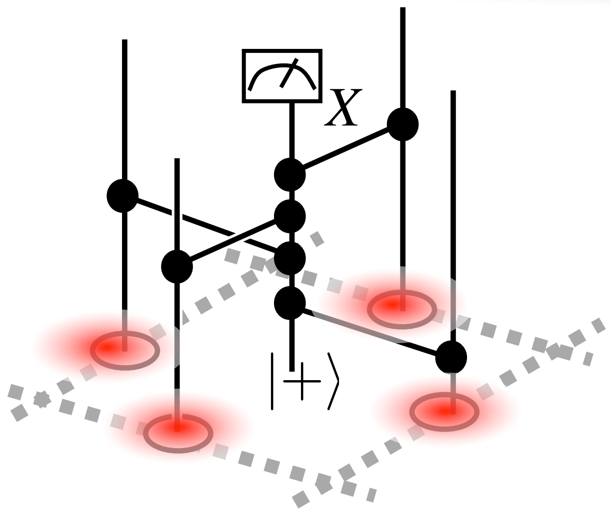


The inference problem can be mapped to a ferro-para phase transition of random-bond Ising model.

E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J.Math. Phys. **43**, 4452 (2002).

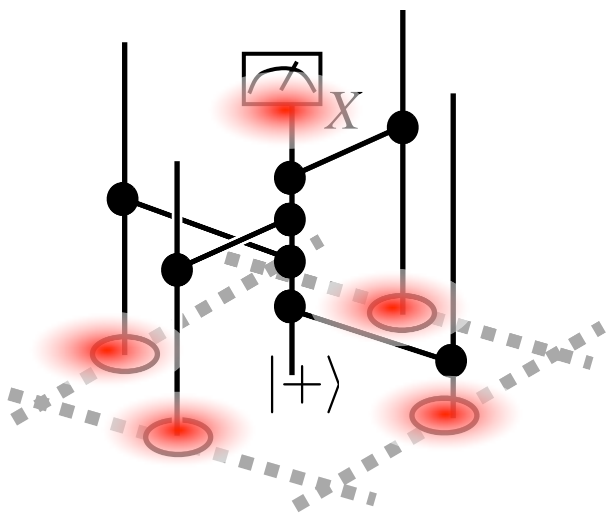
Noise model and threshold values

Code performance:
Independent X and Z errors with perfect syndrome measurements.
[10.3-10.9%]



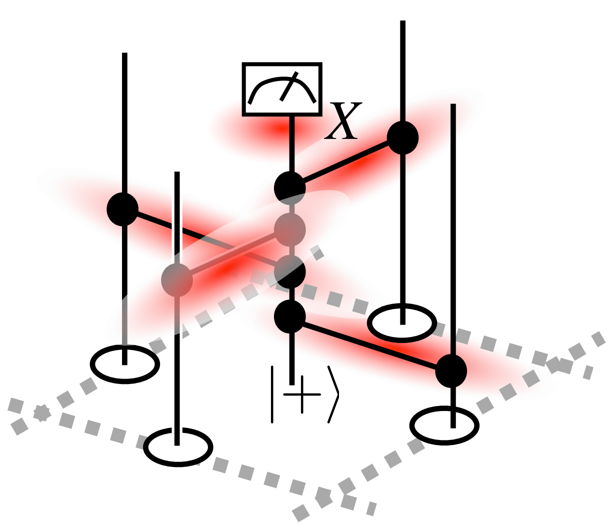
Dennis *et al*., J. Math. Phys. **49**, 4452 (2002).
M. Ohzeki, Phys. Rev. E **79** 021129 (2009).

Phenomenological noise model:
Independent X and Z errors with noisy syndrome measurements.
[2.9-3.3%]



Wang-Harrington-Preskill, Ann. Phys. **303**, 31 (2003).
Ohno *et al*., Nuc. Phys. B **697**, 462 (2004).

Circuit noise model:
Errors are introduced by each elementary gate.
[0.75%]



Raussendorf-Harrington-Goyal, NJP **9**, 199 (2007).
Raussendorf-Harrington-Goyal, Ann. Phys. **321**, 2242 (2006).

Outline

(1) Introduction: what is topological order?

(2) Majorana fermions & 2D Kitaev model

(3) Thermal instability of topological order

---

(4) Error correction on (Kitaev's toric code) surface code

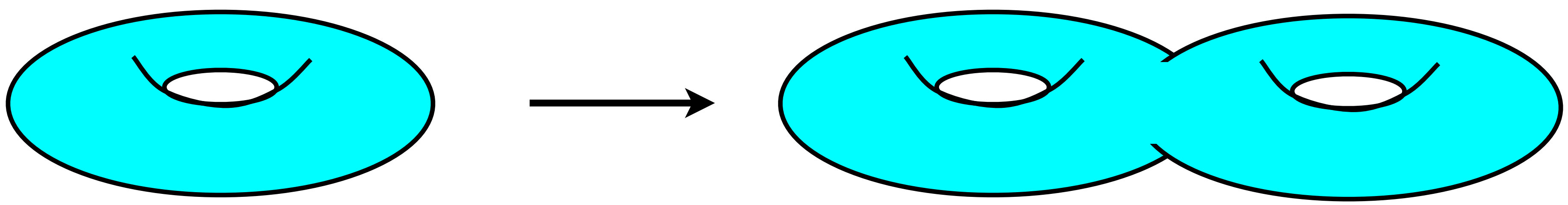
(5) Topological quantum computation defect qubits/ braiding /magic state distillation/ implementations

condensed matter physics

quantum information processing

p- and d-type defects

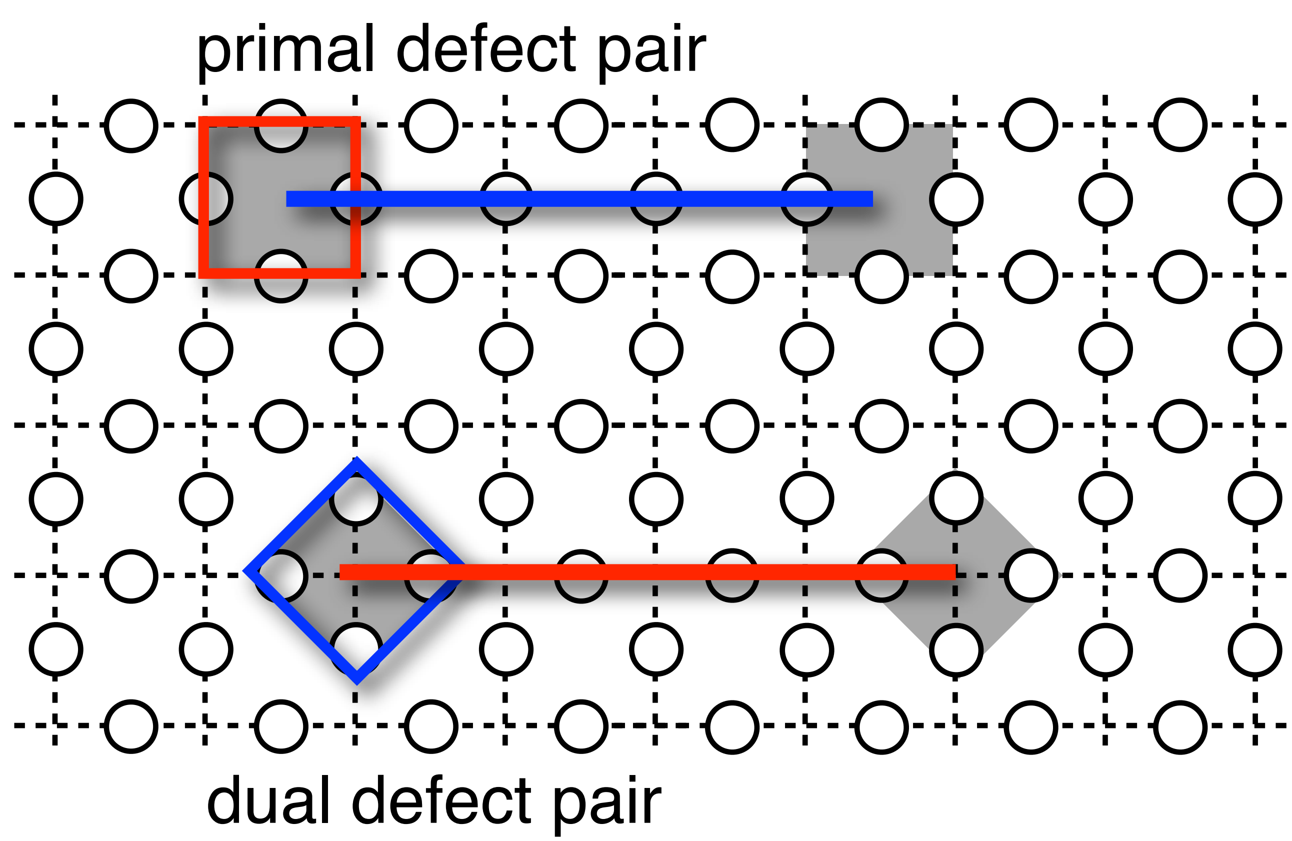
too complex....



too complex....

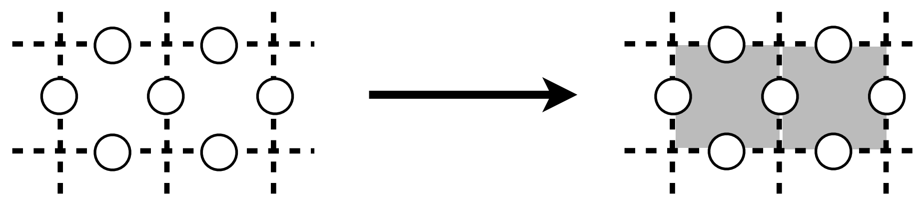
introduce “defects” on the planer surface code

(defect = removal of the stabilizer operator from the stabilizer group, which introduce a degree of freedom)

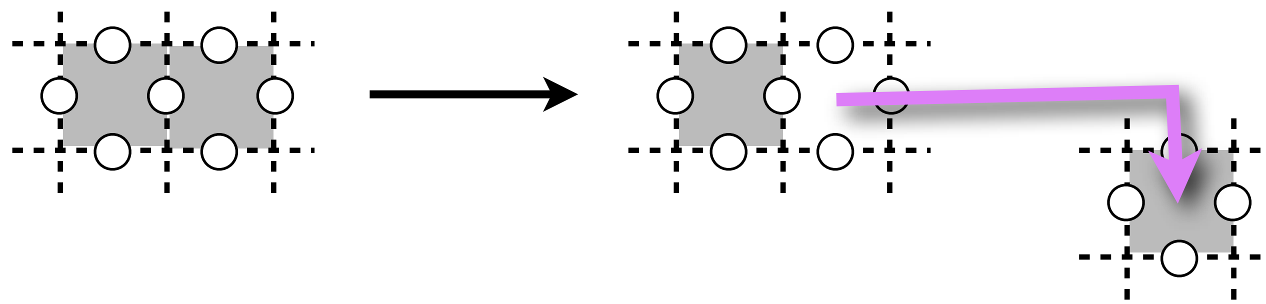


dynamics of defects

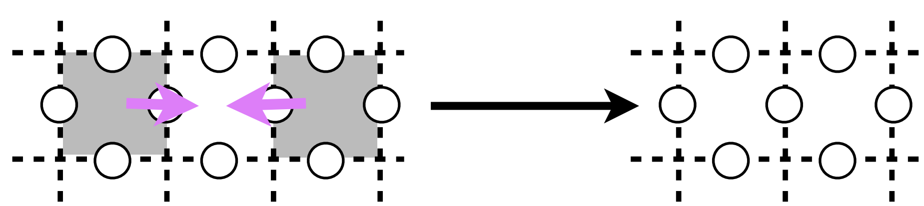
- preparation of logical qubit
→creation of defect pair



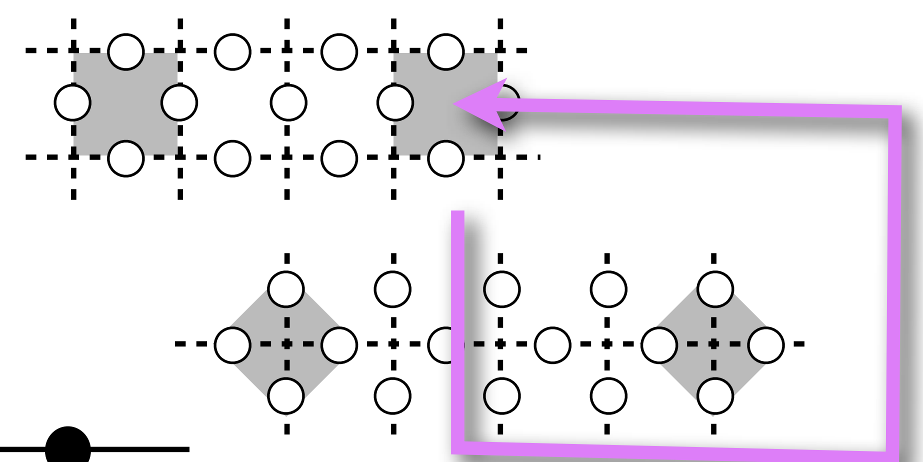
- moving the defect



- measurement of logical qubit
pair annihilation of defects



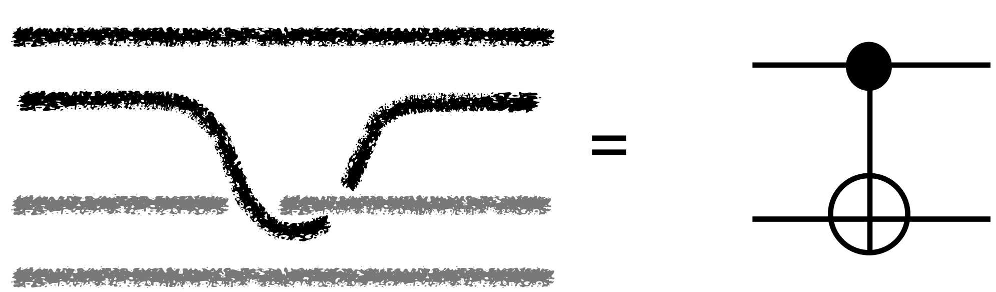
- braiding p-defect around d-defect



→Controlled-Not gate between p-type (control) and d-type (target) qubits.

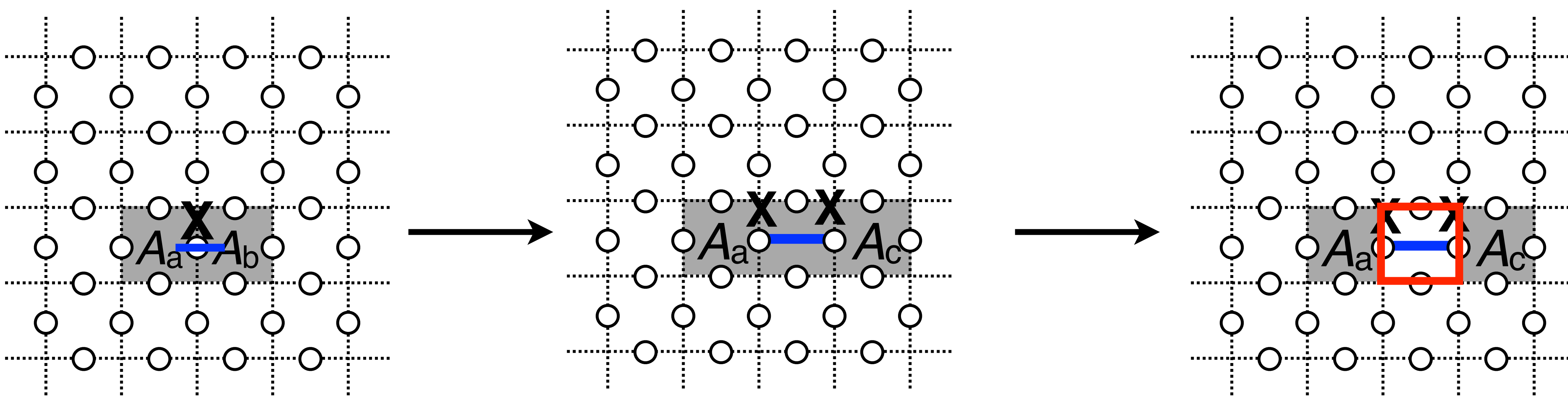
p-type qubit

d-type qubit



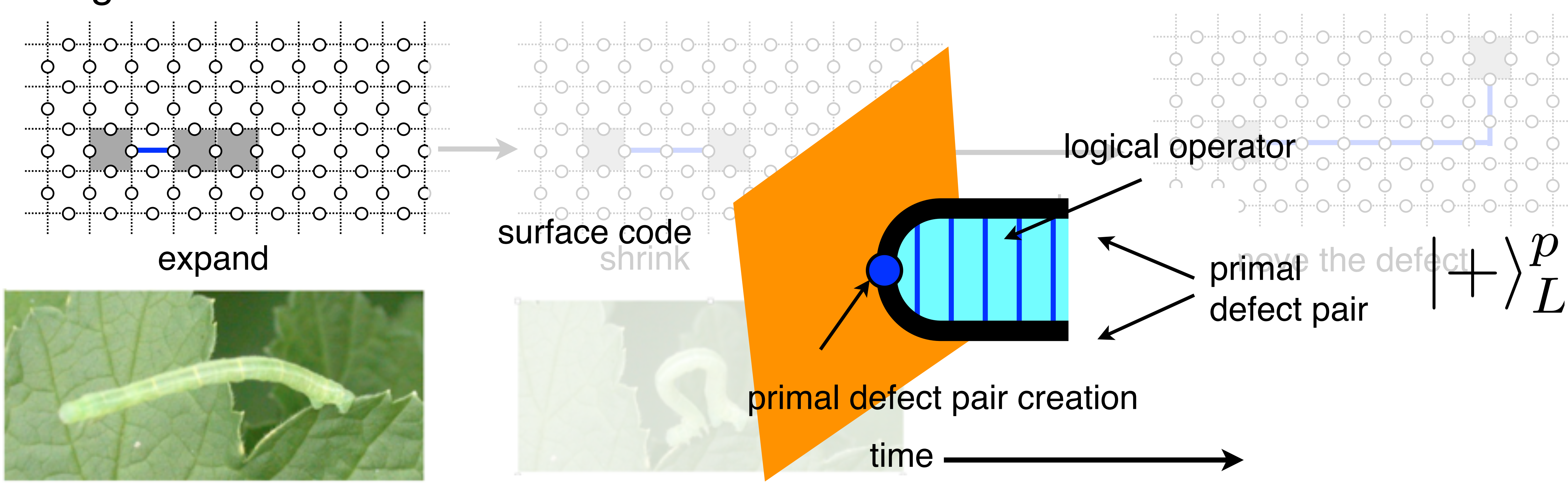
Prepare & move the defect

Preparation of eigenstate $|+\rangle_L^p$ of L_X^p:



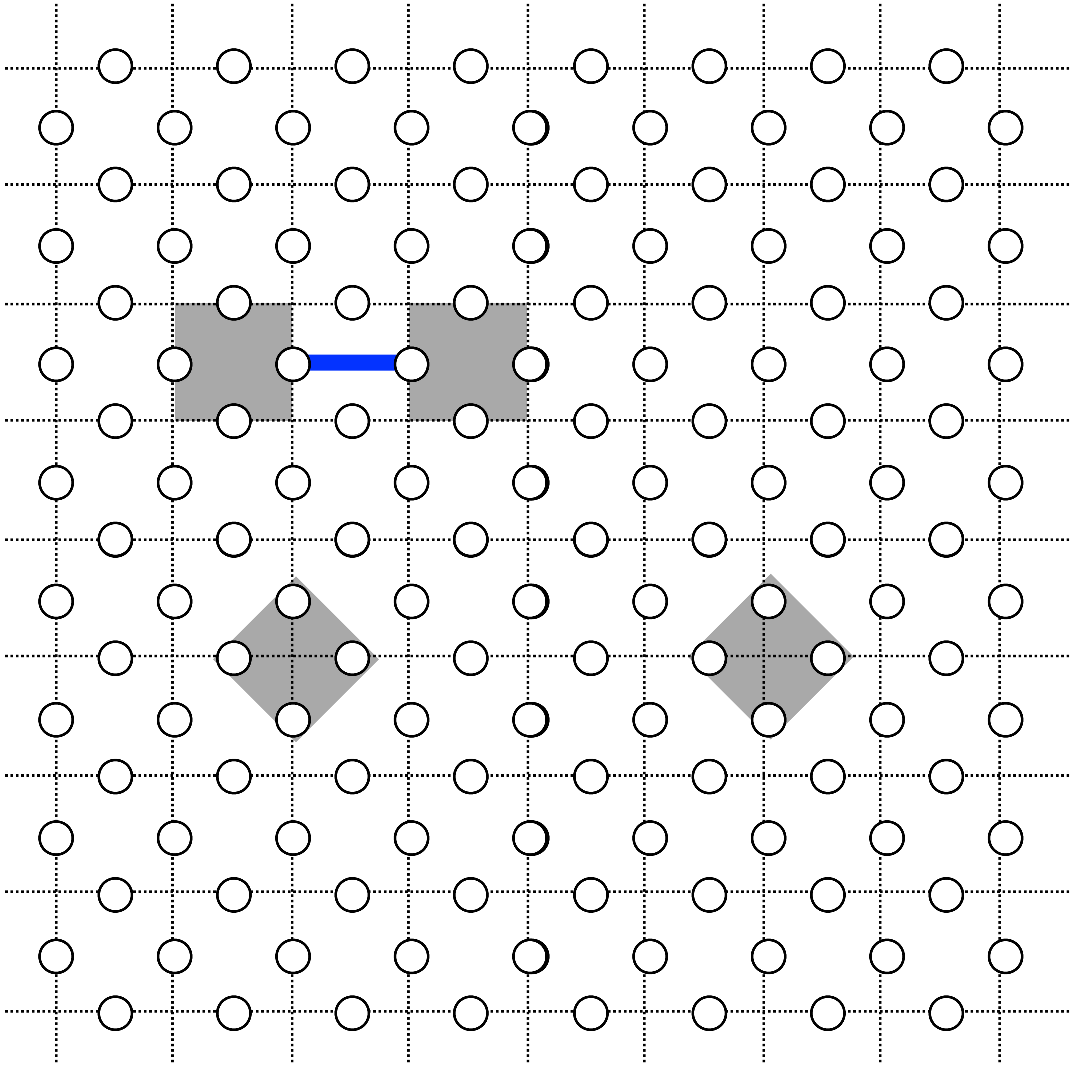
X basis measurement X basis measurement stabilizer measurement

Moving the defect :



CNOT gate by braiding

Observe the time evolution of the logical operator
under the braiding operation of the primal defect around the dual defect.



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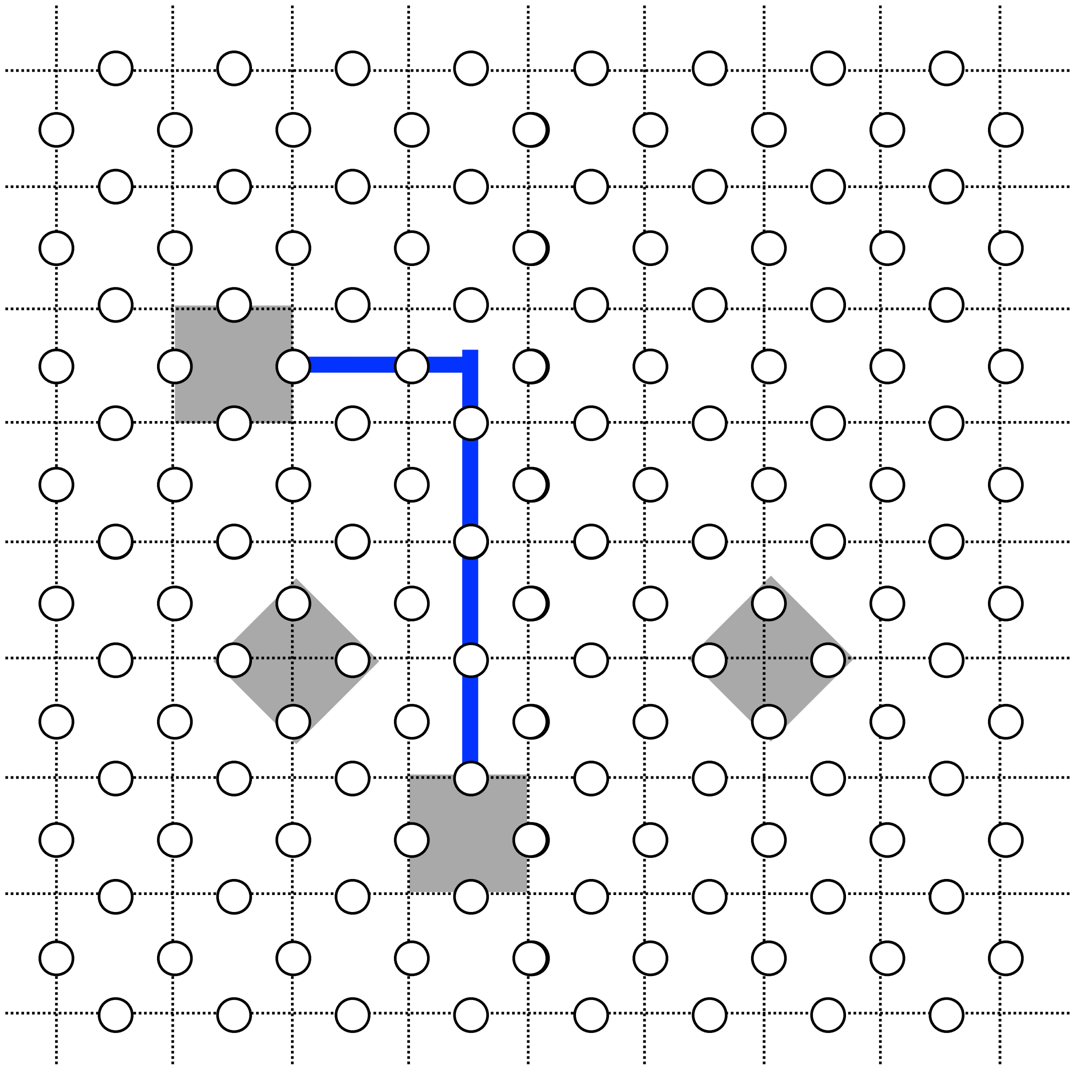

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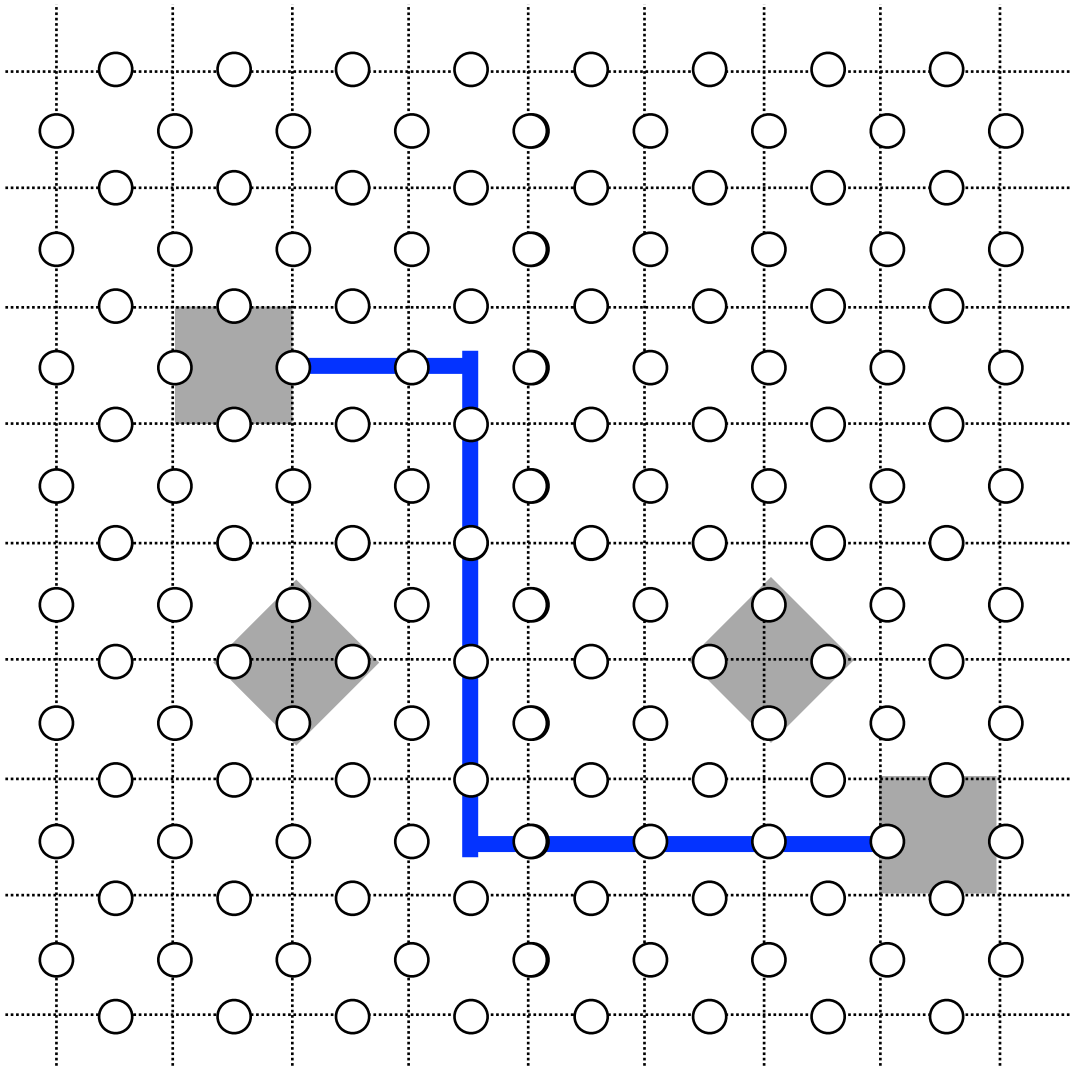

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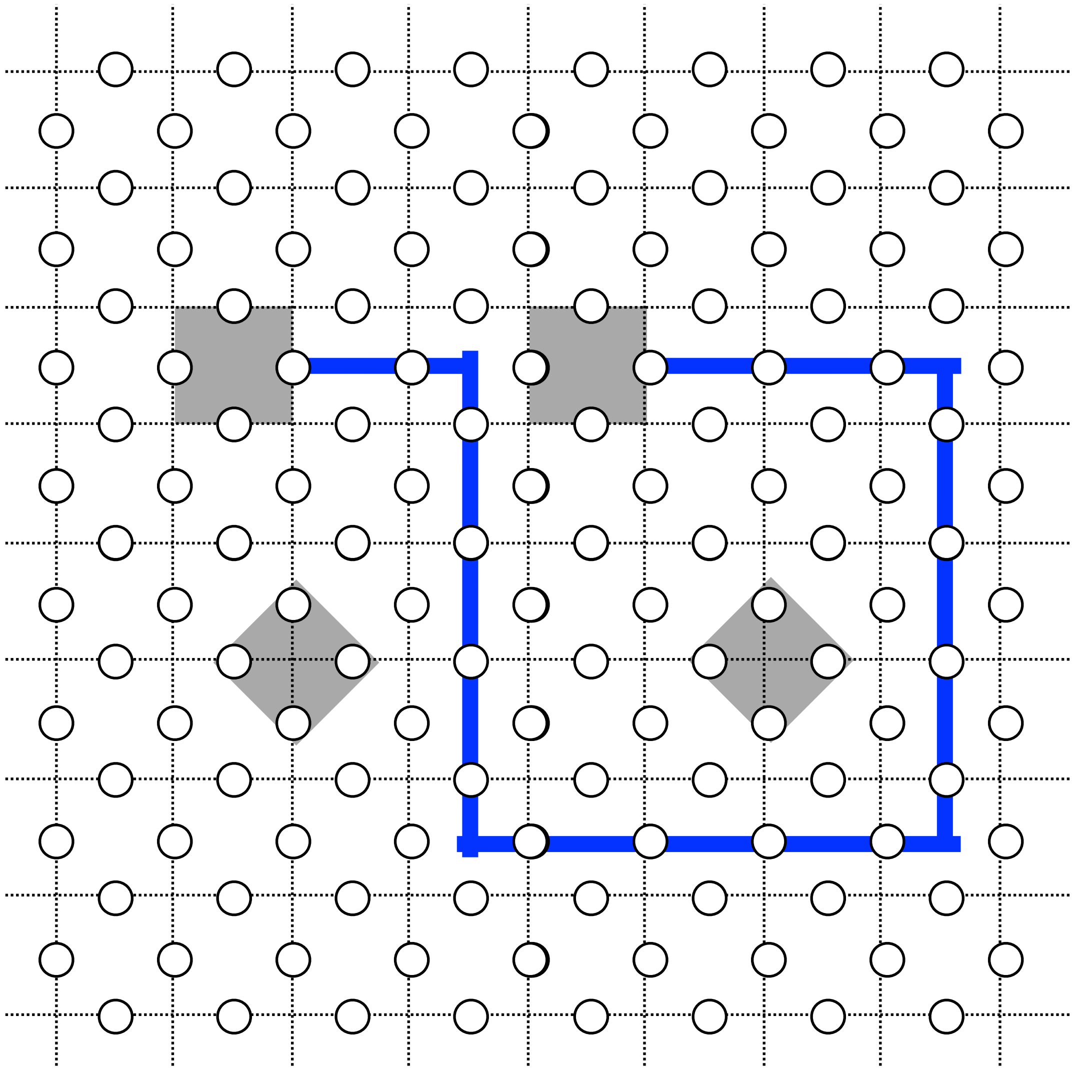

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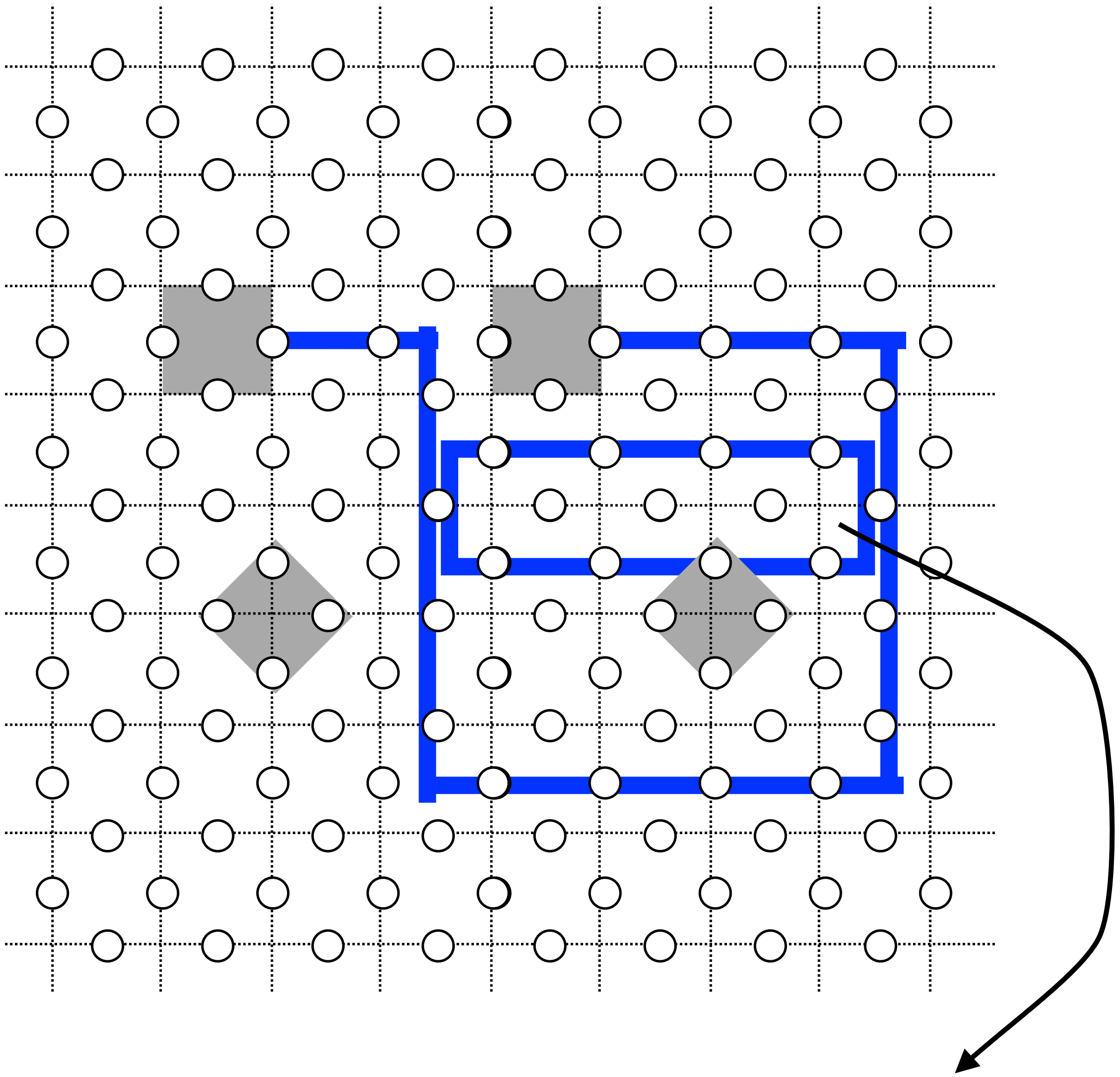
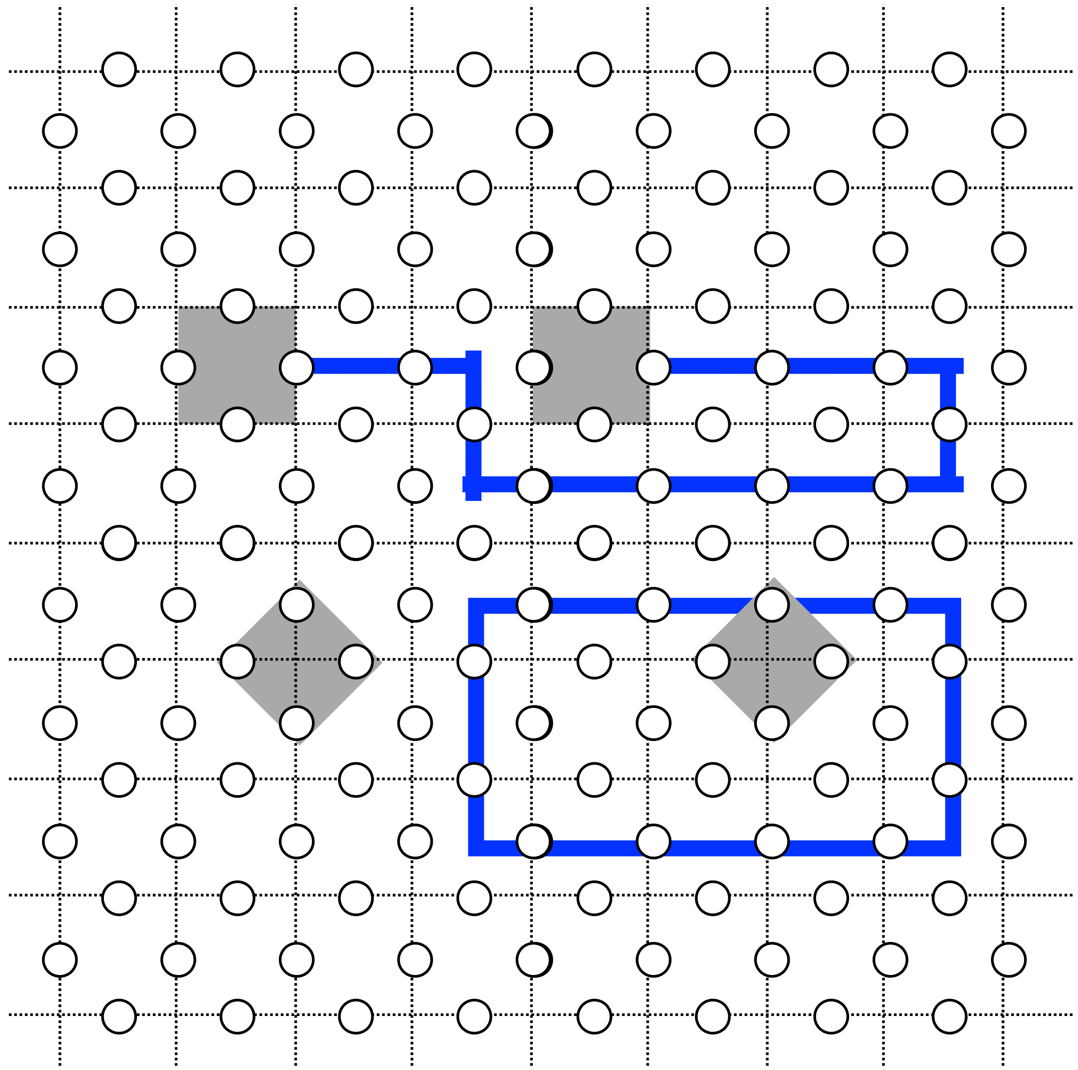


Trivial cycle is a stabilizer operator, and hence acts trivially on the code space.

CNOT gate by braiding

Observe the time evolution of the logical operator
under the braiding operation of the primal defect around the dual defect.



CNOT gate by braiding

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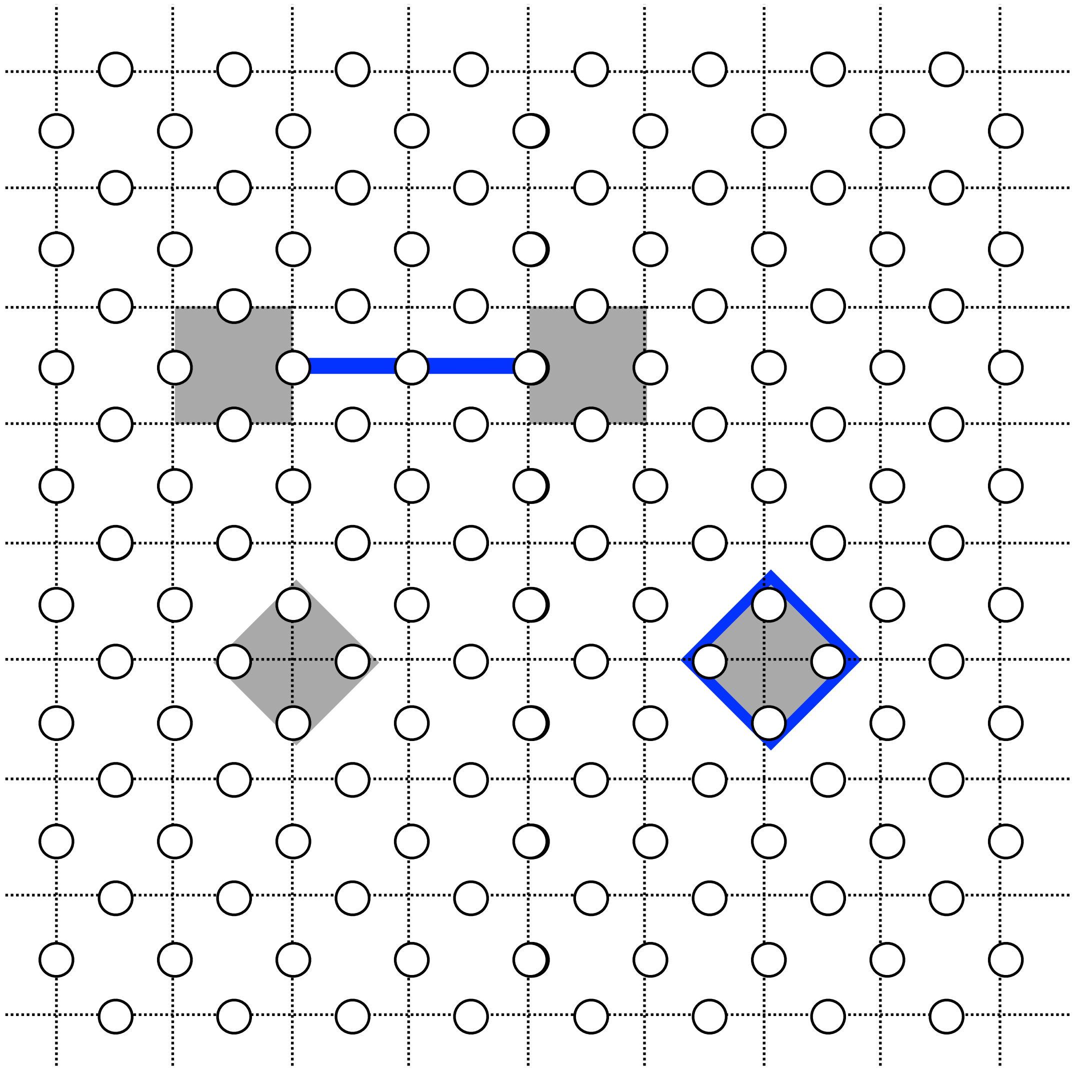
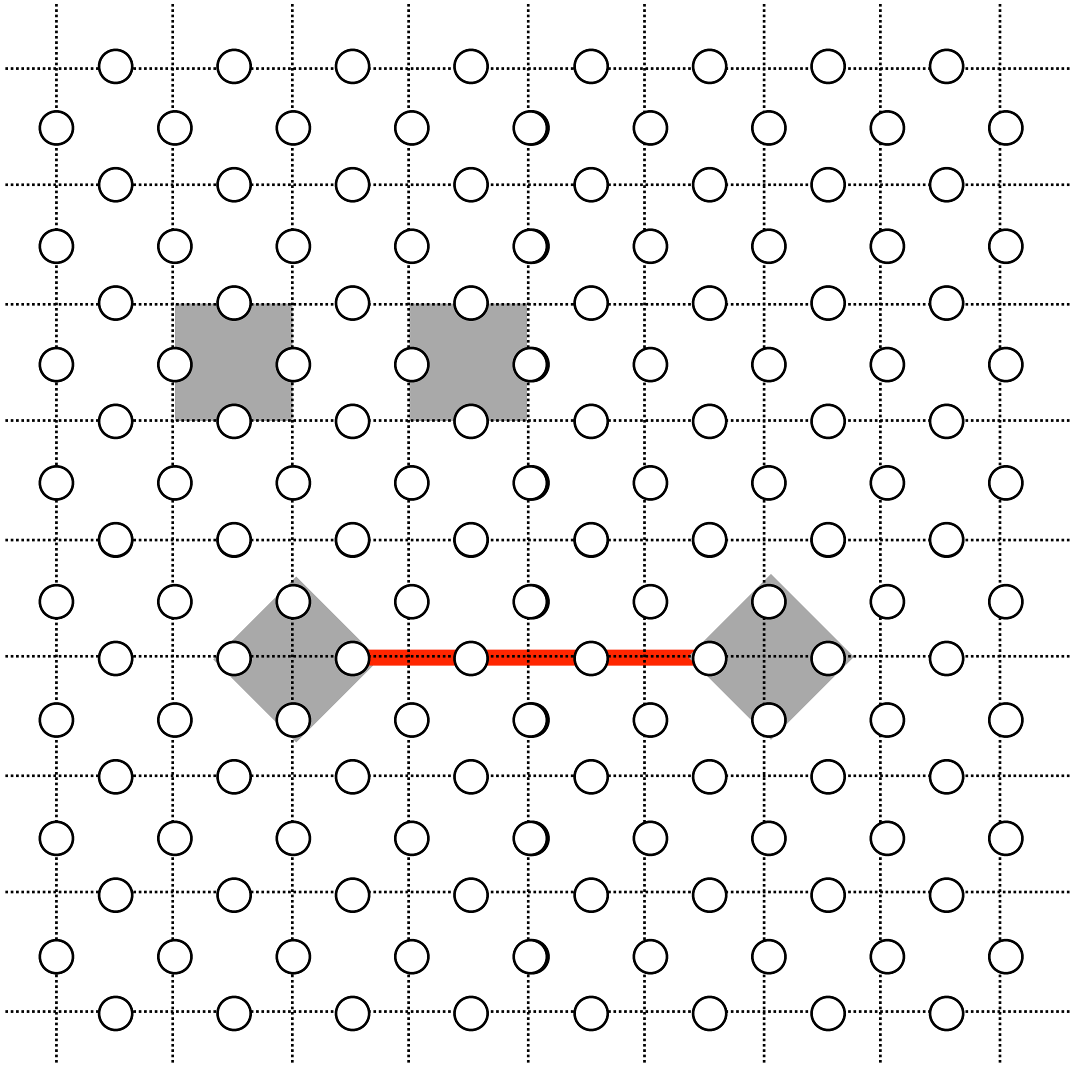


Contraction does not change topology!

$L_X^p \otimes I^d$ is transformed into $L_X^p \otimes L_X^d$!

CNOT gate by braiding

Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.



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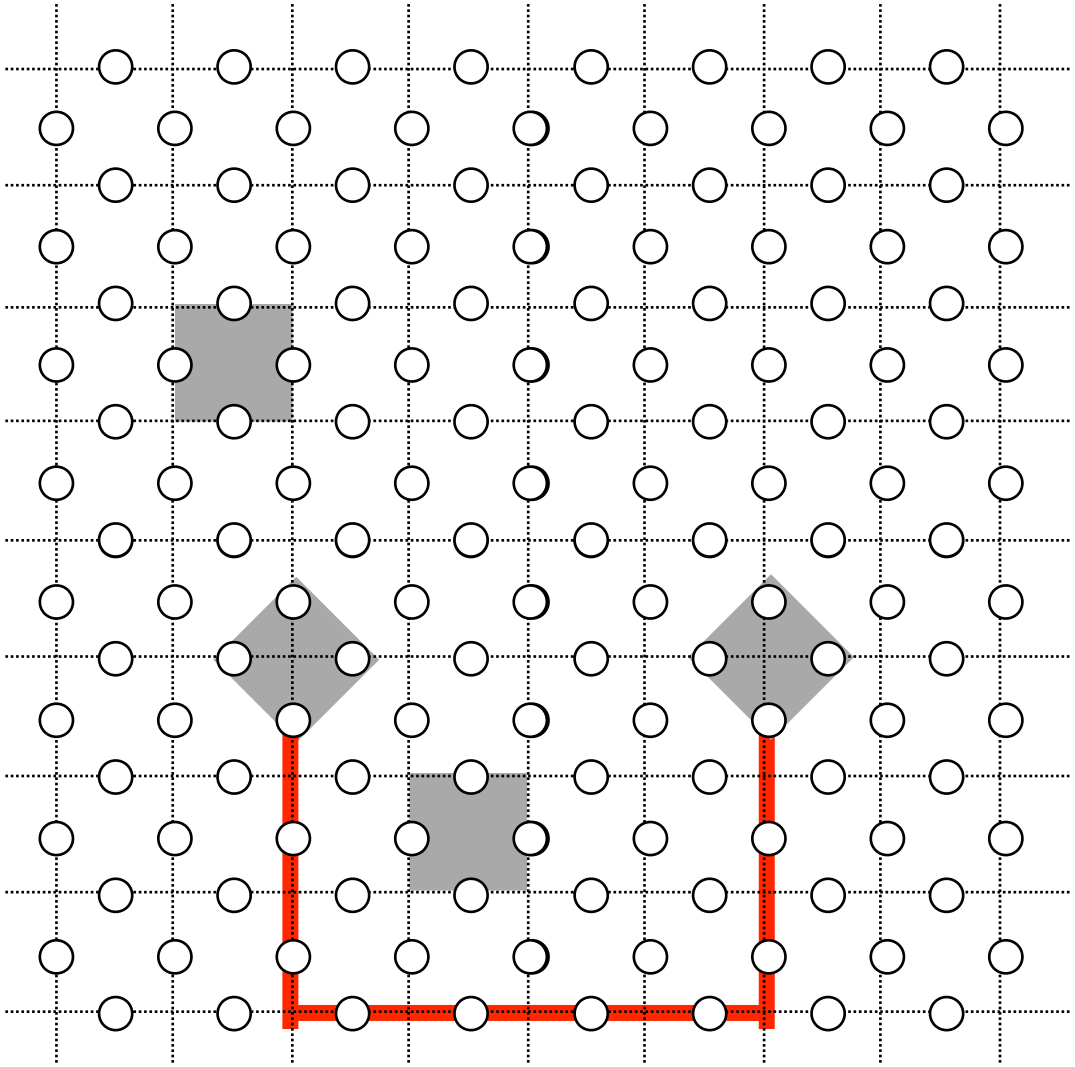

CNOT gate by braiding

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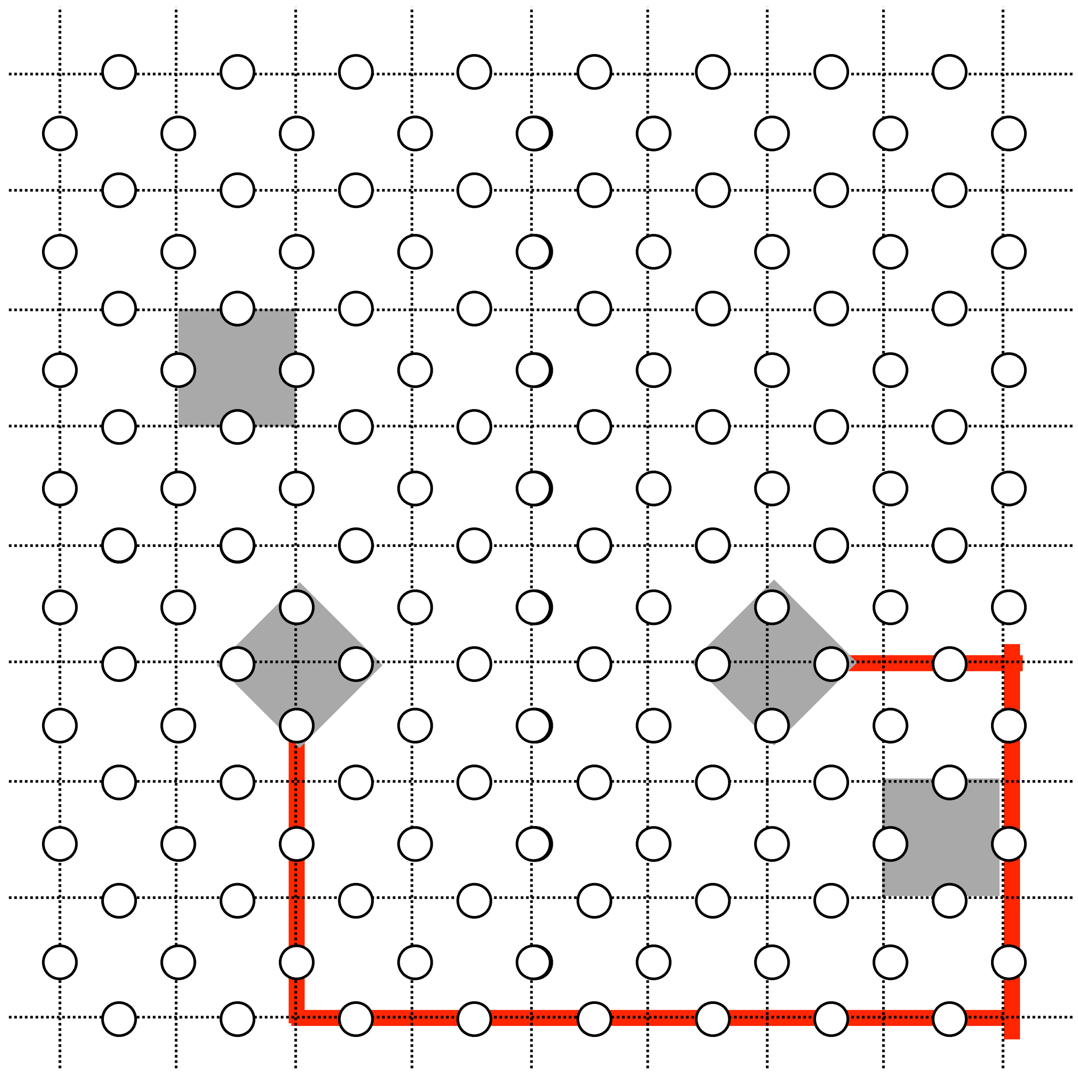
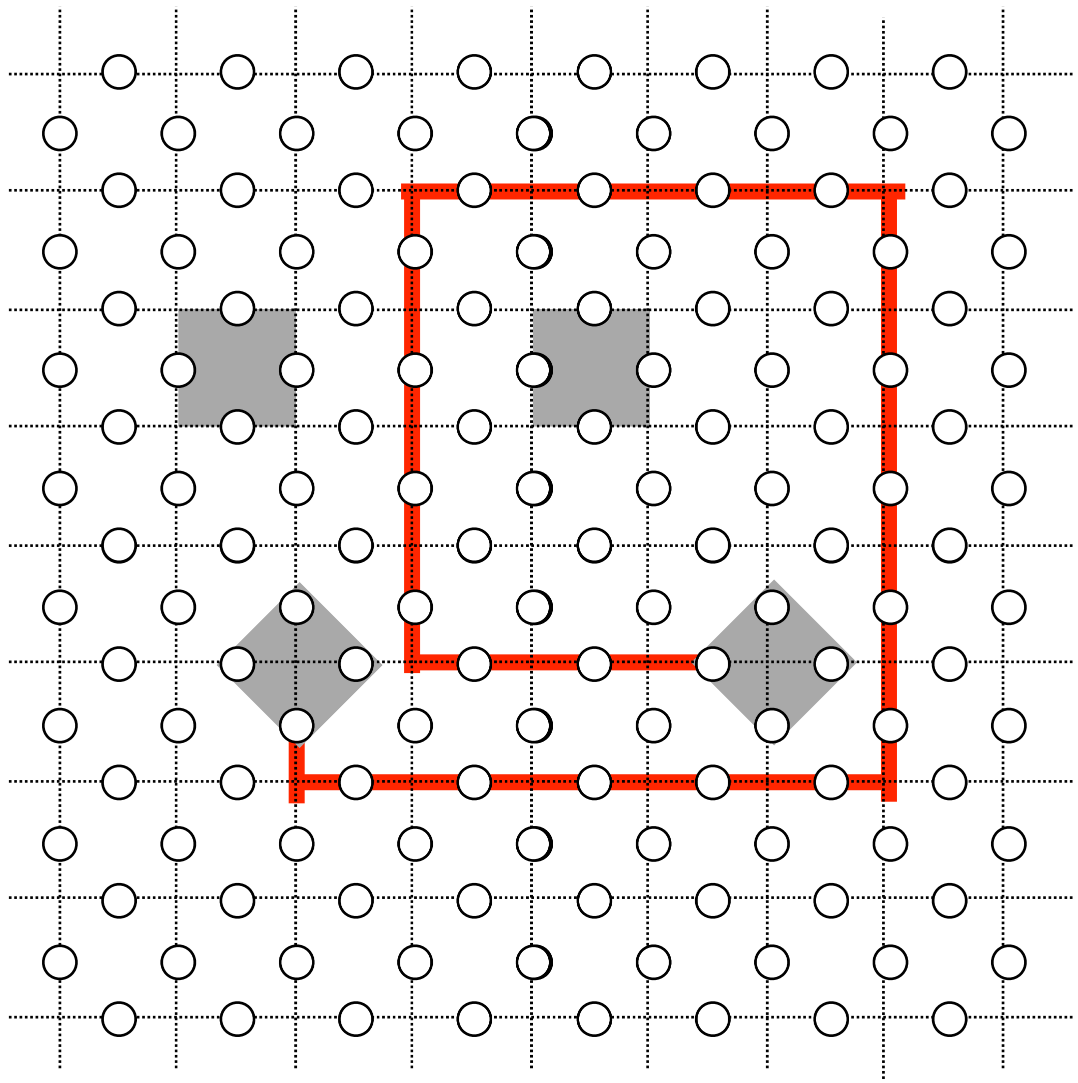


CNOT gate by braiding

Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.



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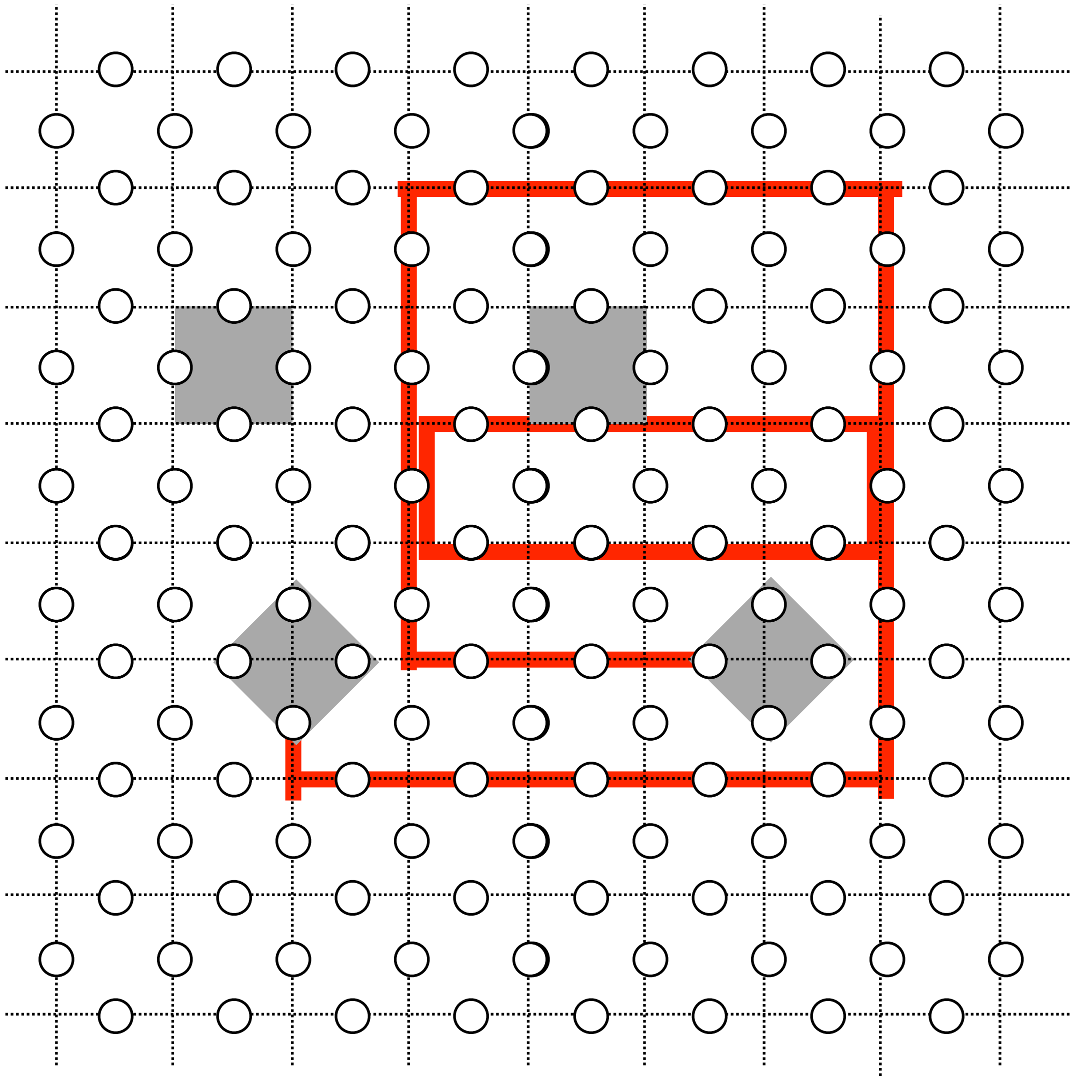

Trivial cycle is a stabilizer operator, and hence acts trivially on the code space.

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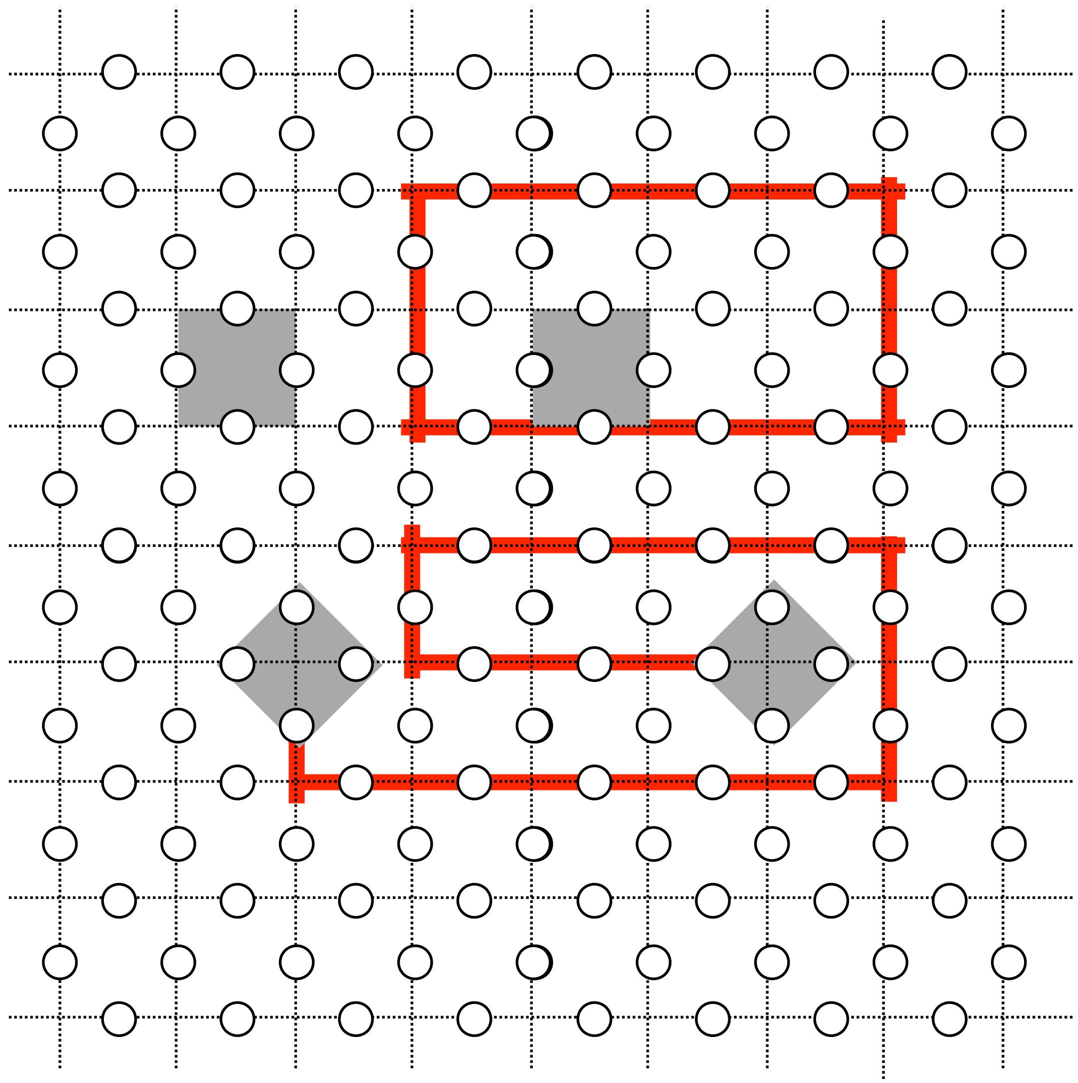

CNOT gate by braiding

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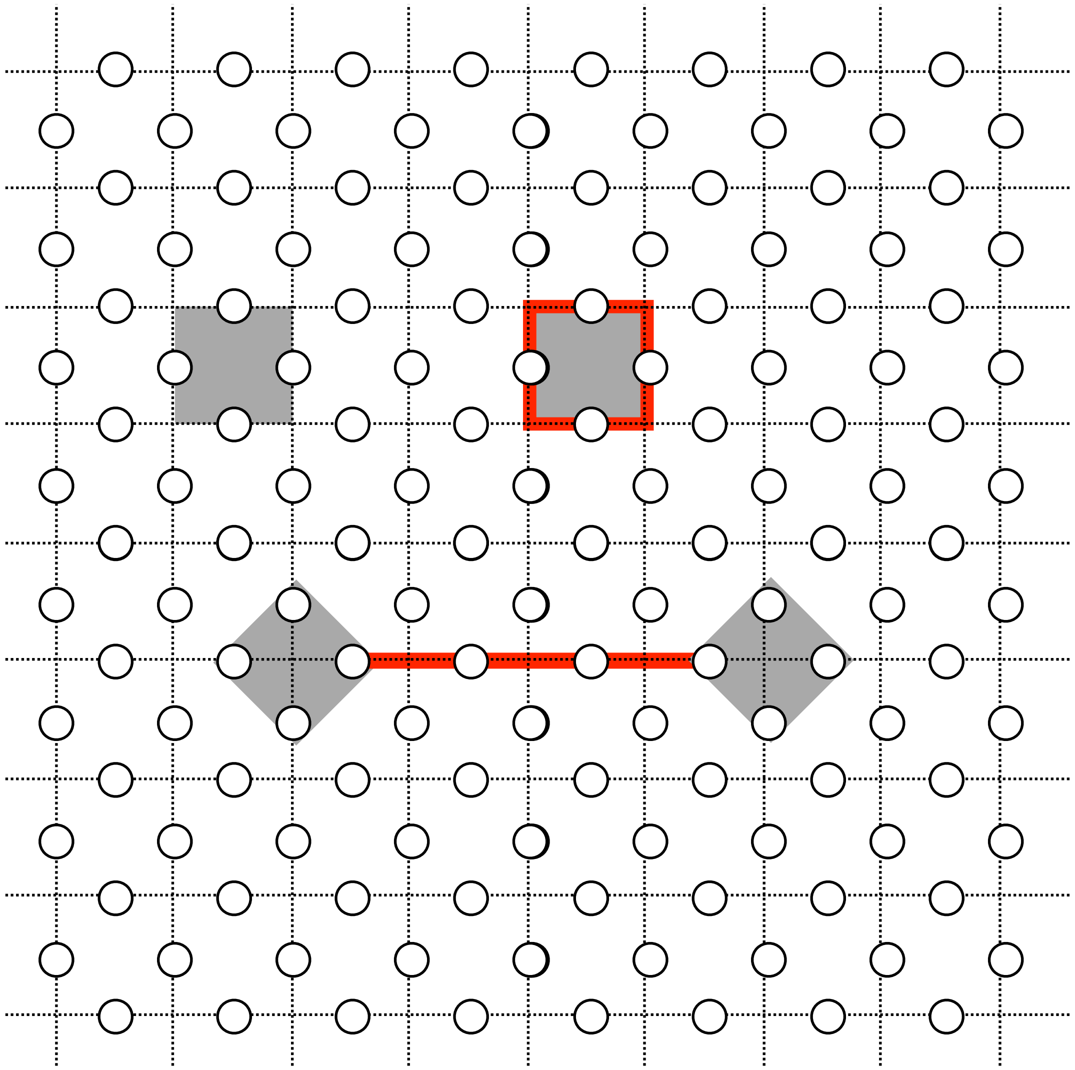
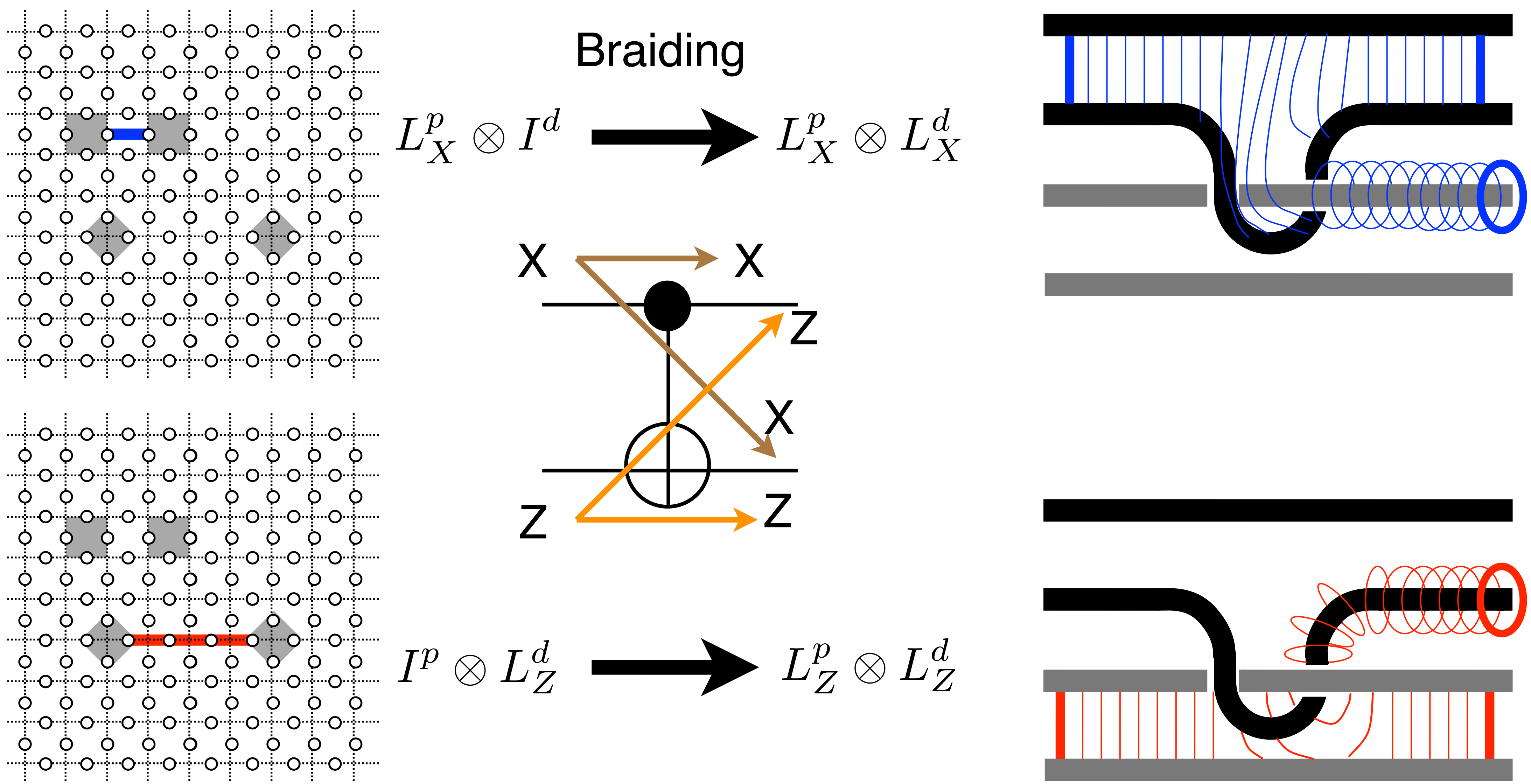


$I^p \otimes L_Z^d$ is transformed into $L_Z^p \otimes L_Z^d$!

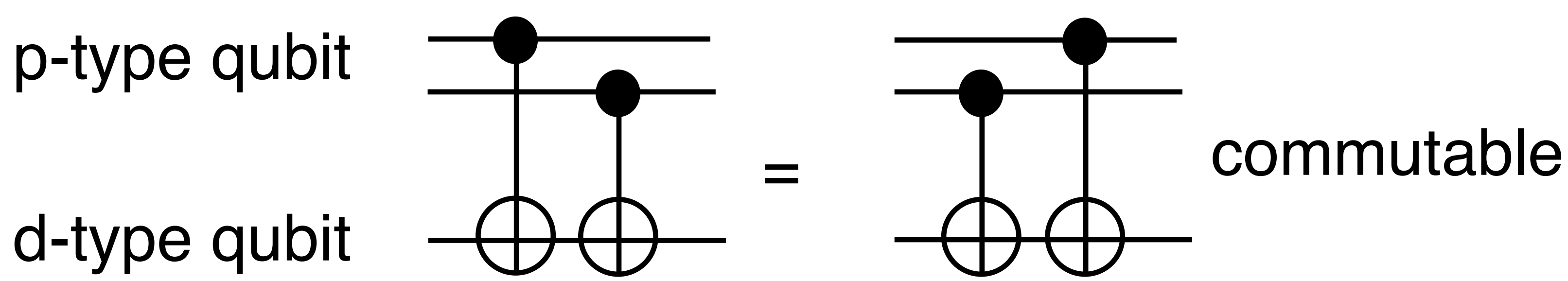
CNOT gate by braiding



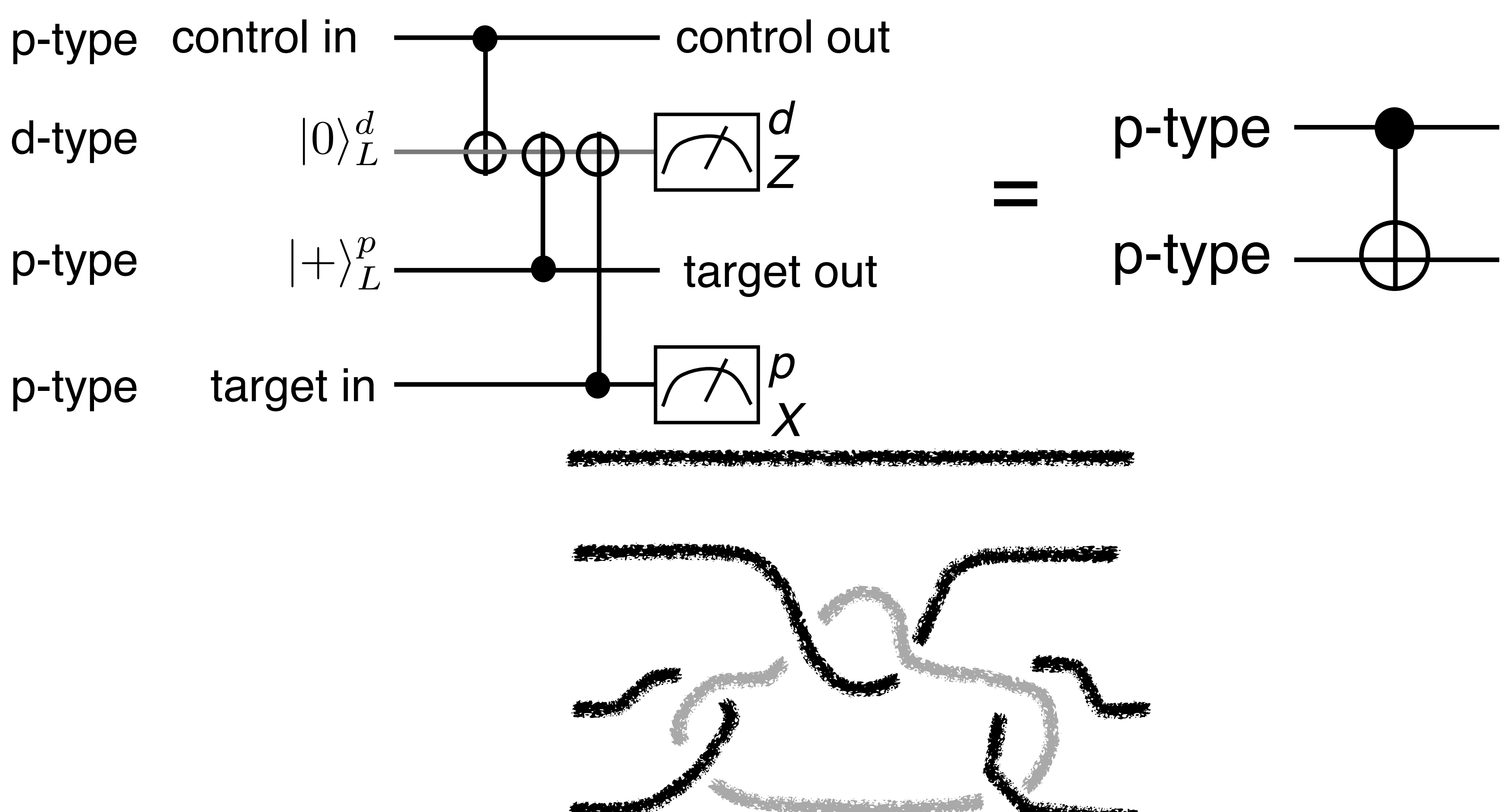
That is, the braiding operation is equivalent to
the CNOT gate from the primal to the dual qubits.

CNOT gate by braiding Abelian anyon

The p-type and d-type defect qubits are always control and target, respectively.



→The anyonic excitation in the Kitaev toric code is Abelian.



Universal quantum computation by magic state distillation

CNOT gate (Clifford gate) is not enough for universal quantum computation.

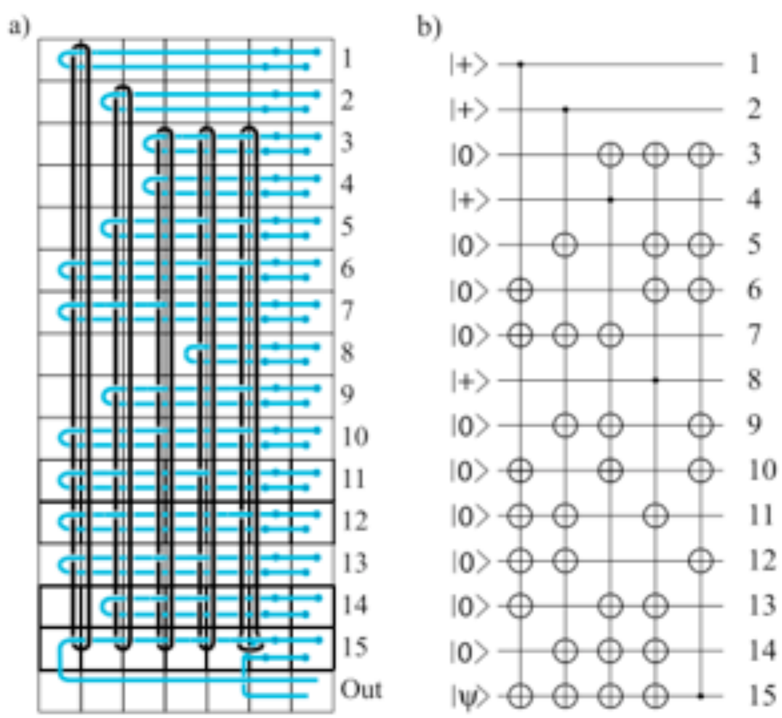
(This is also the case for the Ising anyon.)

Topologically protected CNOT gate + Noisy ancilla state

→ universal quantum computation with an arbitrary accuracy

Magic state distillation

Bravyi-Kitaev PRA **71**, 022316 (2005)



Over 90% of computational overhead is consumed for magic state distillation!

[Improved magic state distillation protocols]

Bravyi-Haah, PRA **86**, 052329 (2012)

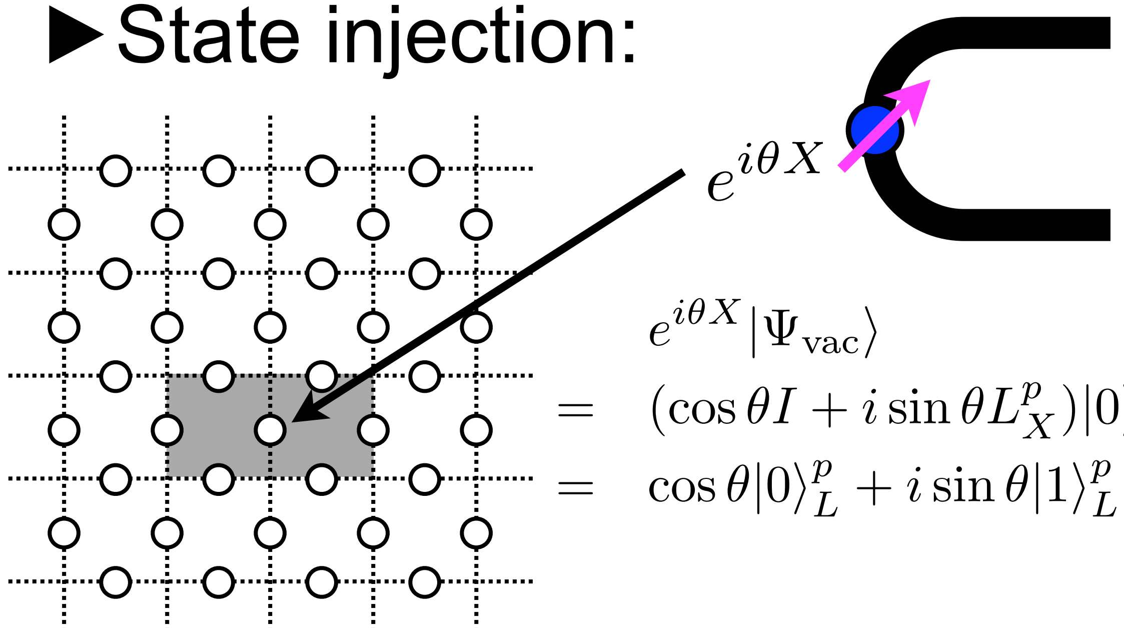
Eastin, PRA **87**, 032321 (2013)

Jones, Phys. Rev. A **87**, 022328 (2013)

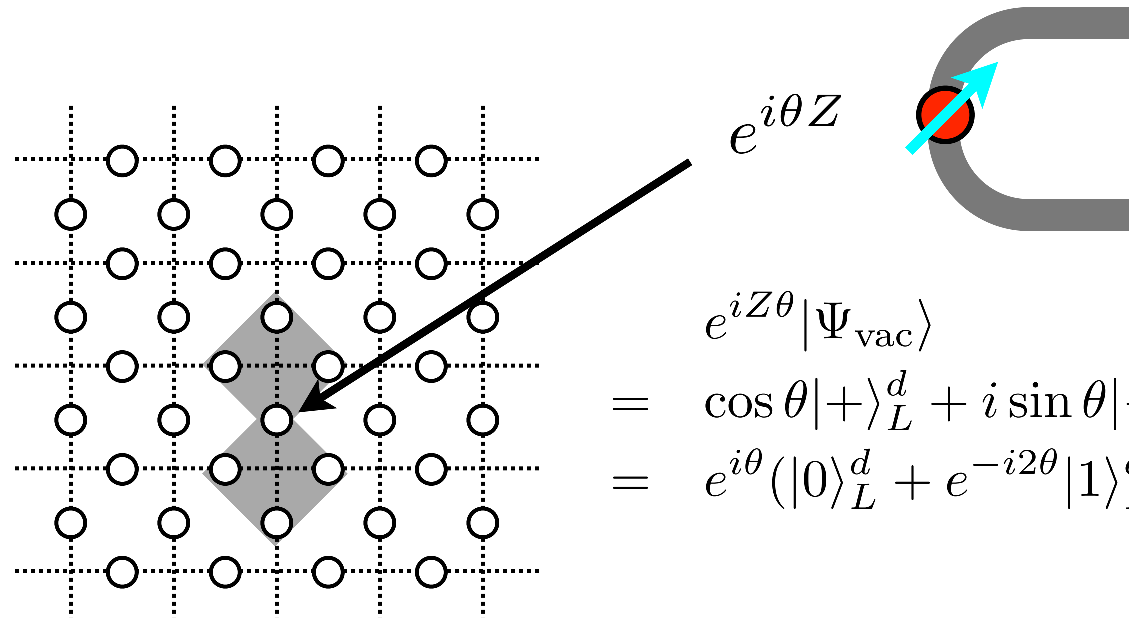
Raussendorf-Harrington-Goyal, NJP **9**, 199 (2007).

Non Clifford gates

▶State injection:



$$e^{i\theta X}|\Psi_{\rm vac}\rangle$$
$$= (\cos\theta I + i\sin\theta L_X^p)|0\rangle_L^p$$
$$= \cos\theta|0\rangle_L^p + i\sin\theta|1\rangle_L^p$$

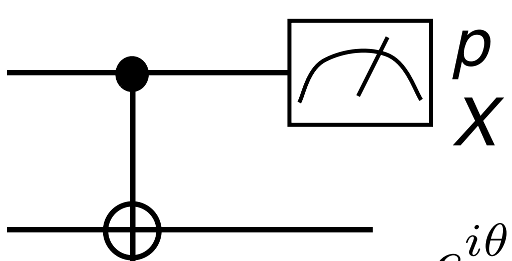


$$e^{iZ\theta}|\Psi_{\rm vac}\rangle$$
$$= \cos\theta|+\rangle_L^d + i\sin\theta|-\rangle_L^d$$
$$= e^{i\theta}(|0\rangle_L^d + e^{-i2\theta}|1\rangle_L^d)$$

▶One-bit teleportation for non-Clifford gate

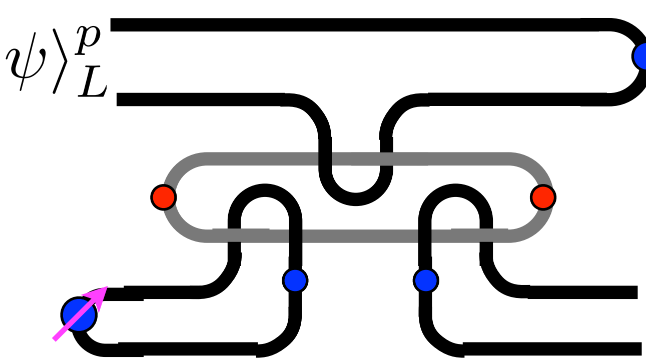
$|\psi\rangle_L^p$

$\cos\theta|0\rangle_L^p + i\sin\theta|1\rangle_L^p$



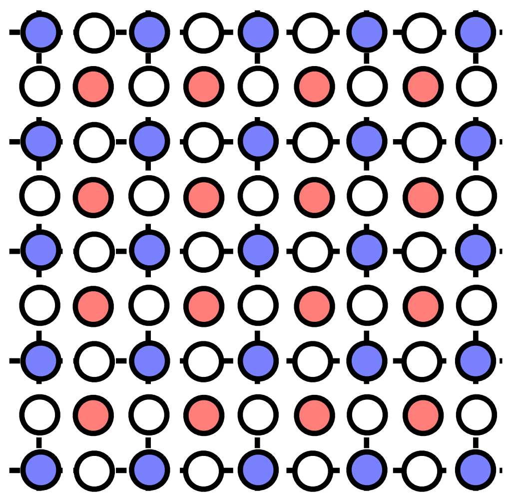
$e^{i\theta L_Z^p}|\psi\rangle_L^p$

$|\psi\rangle_L^p$



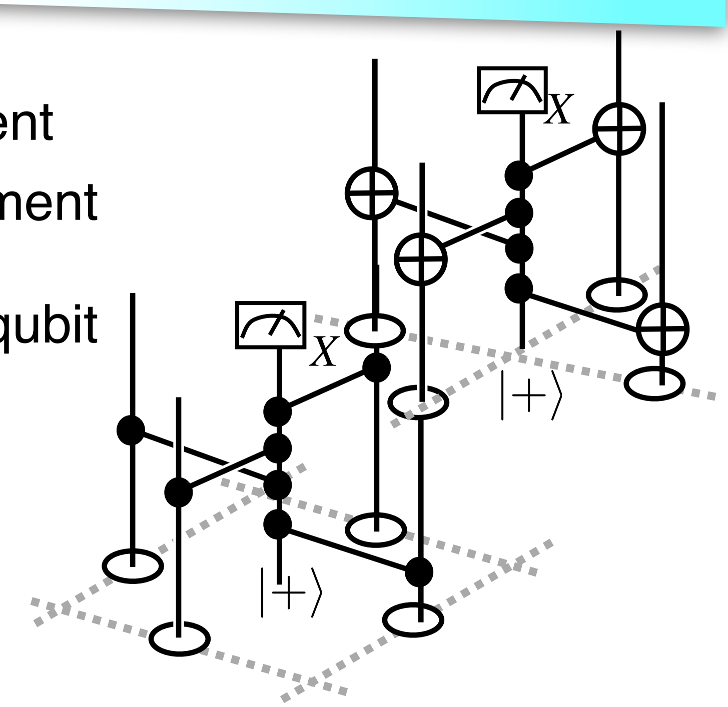
$e^{i\theta L_Z^p}|\psi\rangle_L^p$

Implementations (circuit)



- ○ data qubit which constitutes the surface code
- ● (red) ancilla qubit for the face syndrome measurement
- ● (blue) ancilla qubit for the vertex syndrome measurement

qubits on the square lattice/ nearest-neighbor two-qubit gates/initialization and projective measurement of individual qubits → fault-tolerant universal QC



[On-chip monolithic architectures]

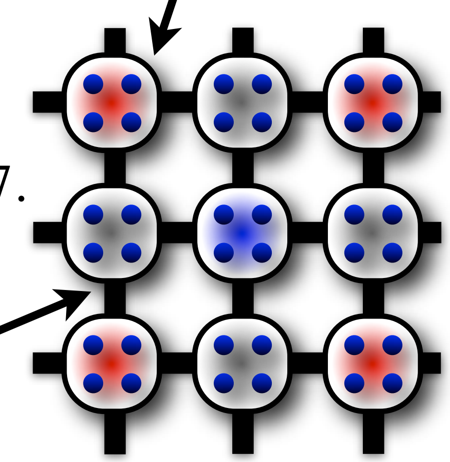
- quantum dot: N. C. Jones *et al.*, PRX **2**, 031007 (2012).

  factorization of 1024-bit composite number: ~10^8 qubits, gates ~10[ns], error rate 0.1% → **1.8 day**
  (768-bit takes 1500 CPU years with classical computer)

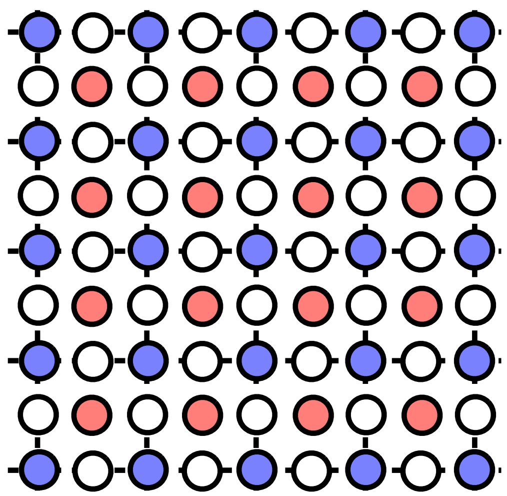
- superconducting qubit: J. Ghosh, A. G. Fowler, M. R. Geller, PRA **86**, 062318 (2012).

[distributed architectures]

- DQC-1:Y. Li *et al.*, PRL **105**, 250502 (2010); KF & Y. Tokunaga, PRL **105**, 250503 (2010).
- DQC-3:Y. Li and S. Benjamin, NJP **14**, 093008 (2012).
- DQC-4:KF *et al.*, arXiv:1202.6588 N. H. Nickerson, Y. Li and S. C. Benjamin, arXiv:1211.2217.
  fidelity of quantum channel ~0.9, error rate of local operations ~0.1%
- Trapped Ions: C. Monroe *et al.*, arXiv:1208.0391.

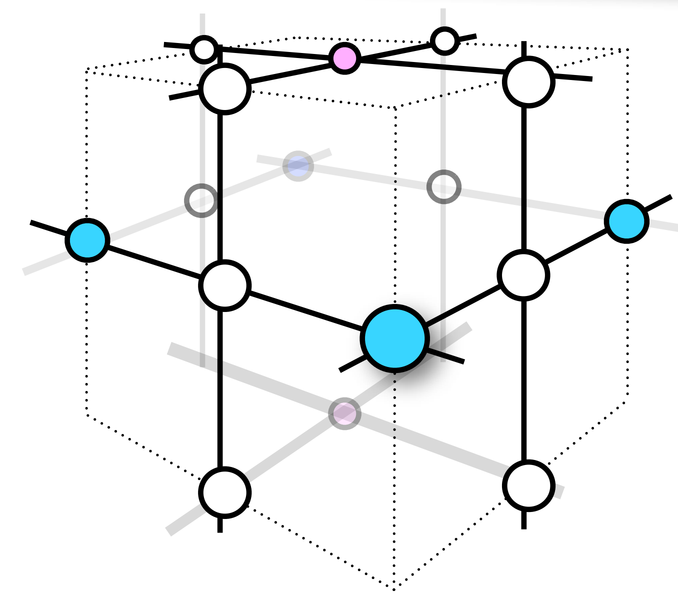


Implementations



2D system

+ (time evolution) → quantum teleportation



3D entangled state & measurement-based QC

Raussendorf-Harrington-Goyal,Annals Phys. **321**, 2242 (2006);NJP **9**, 199 (2007).

Measurement-based quantum computation (MBQC): After generating a many-body entangled state, we only need to readout the state of the particles.

Topologically protected MBQC on thermal state:

[Thermal state of two-body Hamiltonian (no phase transition)]

spin-2 & spin-3/2 particles: Li *et al*., PRL **107**, 060501 (2011)
spin-3/2 particles: KF & T. Morimae, PRA **85**, 010304(R) (2012)

[Symmetry breaking thermal state (ferromagnetic phase transition)]

KF, Y. Nakata, M. Ohzeki, M. Murao , PRL **110**, 120502 (2013).

Summary

topological order & topological quantum computation
(extremely long-live quantum coherence)

Abelian anyon (Kitaev's toric code model) is enough for universal quantum computation.

equilibrium/ local/ translationally invariant (without control)

non-equilibrium (error correction by feedback operation)

Zero temperature

4- or higher dimension

Topological QEC by global control and dissipative dynamics

no selective addressing!
no measurement!

see poster session

selective addressing (measurement and control) of individual particle

Non-Abelian anyones
Ising anyon + Magic state distillation
Fibonacci anyon

topologically protected quantum computation in 2D

topologically protected MBQC in 3D

Thank you for your attention!

List of my works

(category)	
(MBQC)	1."Measurement-Based Quantum Computation on Symmetry Breaking Thermal States" (Editors' suggestion) K. Fujii, Y. Nakata, M. Ohzeki, M. Murao, Phys. Rev. Lett. 110, 120502 (2013) arXiv:1209.1265
(spin glass)	2."Duality analysis on random planar lattice" M. Ohzeki and K. Fujii, Phys. Rev. E 86, 051121 (2012) arXiv:1209.3500
(Blind QC)	3."Blind topological measurement-based quantum computation" T. Morimae and K. Fujii, Nature Communications 3, 1036 (2012). arXiv:1110.5460
(QEC code)	4."Error- and Loss-Tolerances of Surface Codes with General Lattice Structures" K. Fujii and Y. Tokunaga, Phys. Rev. A 86, 020303(R) (2012). arXiv:1202.2743
(MBQC)	5."Not all physical errors can be linear CPTP maps in a correlation space" T. Morimae and K. Fujii Scientific Reports 2, 508 (2012). arXiv:1106.3720 arXiv:1110.4182(supplemental material)
(MBQC)	6."Computational Power and Correlation in Quantum Computational Tensor Network" K. Fujii and T. Morimae, Phys. Rev. A 85, 032338 (2012). arXiv:1106.3377
(MBQC)	7."Topologically protected measurement-based quantum computation on the thermal state of a nearest-neighbor two-body Hamiltonian with spin-3/2 particles" K. Fujii and T. Morimae, Phys. Rev. A 85, 010304(R) (2012) arXiv:1111.0919
(Entanglement web)	8."Robust and Scalable Scheme to Generate Large-Scale Entanglement Webs" K. Fujii, H. Maeda and K. Yamamoto, Phys. Rev. A 83, 050303(R) (2011) arXiv:1102.4682
(Fault-tolerance)	9." Fault-Tolerant Topological One-Way Quantum Computation with Probabilistic Two-Qubit Gates" K. Fujii and Y. Tokunaga, Phys. Rev. Lett. 105, 250503 (2010). arXiv:1008.3752
(Fault-tolerance)	10."Topological One-Way Quantum Computation on Verified Logical Cluster States" K. Fujii and K. Yamamoto, Phys. Rev. A 82, 060301(R) (2010). arXiv:1008.2048
(Zeno effect)	11."Anti-Zeno effect for quantum transport in disordered systems" K. Fujii and K. Yamamoto, Phys. Rev. A 82, 042109 (2010). arXiv:1003.1804
(Fault-tolerance)	12."Cluster-based architecture for fault-tolerant quantum computation" K. Fujii and K. Yamamoto, Phys. Rev. A 81, 042324 (2010). arXiv:0912.5150
(Entanglement purification)	13."Entanglement purification with double-selection" K. Fujii and K. Yamamoto, Phys. Rev. A 80, 042308 (2009). arXiv:0811.2639