Topological quantum computation
-from topological order to fault-tolerant quantum computation-

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Outline

(1) Introduction: what is topological order?

(2) Majorana fermions & 2D Kitaev model

(3) Thermal instability of topological order

(4) Error correction on (Kitaev’s toric code) surface code

(5) Topological quantum computation
defect qubits/ braiding /magic state
distillation/ implementations

condensed matter physics
quantum information processing
What is topological order?

Topological order is.........

-a new kind of order in zero-temperature phase of matter.

cannot be described by Landau’s symmetry breaking argument.

-ground states are degenerated and it exhibits long-range quantum entanglement.

-the degenerated ground states cannot be distinguished by local operations.

-topologically ordered states are robust against local perturbations.

-related to quantum spin liquids, fractional quantum Hall effect, fault-tolerant quantum computation.
Landau’s symmetry breaking argument

Ising model (e.g. two dimension):

Pauli operators:

\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

Computational basis (qubit):

\[
\{ |0\rangle, |1\rangle \} \rightarrow Z|0\rangle = |0\rangle, \quad Z|1\rangle = -|1\rangle
\]

\[
\{ |+\rangle, |-\rangle \} \rightarrow X|+\rangle = |+\rangle, \quad X|-\rangle = -|-\rangle
\]

Multi-qubit system \(\{ |0\rangle, |1\rangle \}^N\):

\[
A_i = I_1 \otimes ... \otimes I_{i-1} \otimes A_i \otimes I_{i+1} \otimes ... I_N
\]

\[
(A = X, Y, Z)
\]
Landau’s symmetry breaking argument

Ising model (e.g. two dimension):

\[ H = -J \sum_{\langle ij \rangle} X_i X_j \]

The Ising Hamiltonian is invariant under spin flipping \( Z \) w.r.t. \( X \)-basis.

The ground states: \( \{|++\cdots+\},|--\cdots-\} \) (\( Z_2 \) symmetry)

\[ \text{longitudinal magnetic field} \rightarrow \text{ground state degeneracy is lifted by longitudinal magnetic field.} \]

\[ \text{ground state degeneracy is not robust against local perturbation.} \]
Topologically ordered states

In topologically ordered system....

degenerated ground states:

\[ |\Psi_{GS1}\rangle \quad \text{well separated} \quad |\Psi_{GS2}\rangle \]

any local perturbation

Only nonlocal operators (high weight operator \( A \otimes O(L) \)) can exchange the ground states.

\[ |\Psi_1\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2} \]
\[ |\Psi_2\rangle = (|0011\rangle + |1100\rangle)/\sqrt{2} \]

e.g.

two orthogonal state cannot be distinguished by measuring a single qubit in any basis \( \rightarrow \) second order perturbation first lifts the degeneracy
Topologically ordered states

How can we describe topologically ordered states efficiently?

→ Theory of quantum error correction is a very useful tool to describe topologically ordered system.

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Majorana fermions

Ising model in one dimension (Ising chain):

\[ H = -J \sum_{\langle ij \rangle} X_i X_j \]

the Ising Hamiltonian is invariant spin flipping \( Z \) w.r.t. \( X \)-basis.

The ground states:
\[ \{ | ++ \cdots + \rangle, | -- \cdots - \rangle \} \quad (Z_2 \text{ symmetry}) \]

longitudinal magnetic field

\[ g_s \quad | ++ \cdots + \rangle \quad | -- \cdots - \rangle \]

\[ h \sum_i X \]

\[ 2Nh \]

a repetition code \( \langle X_i X_{i+1} \rangle \) cannot correct any \( X \) error, since any single bit-flip error \( X_i \) changes the code space non-trivially.
Majorana fermions

Let us consider a mathematically equivalent but physically different system.

**Ising chain:** \( H_{\text{Ising}} = -J \sum_{i=1}^{N-1} X_i X_{i+1} \)

**Jordan-Wigner transformation**

<table>
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<th>spin ↔ fermion</th>
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\[
\begin{align*}
    c_{2i-1} &= Z_1 \ldots Z_{i-1} X_i \\
    c_{2i} &= Z_1 \ldots Z_{i-1} Y_i \\
    \{c_i, c_j\} &= \delta_{ij} I, \quad c_i^\dagger = c_i
\end{align*}
\]

(Majorana fermion operator)

**2N spinless fermions:** \( H_{\text{Maj}} = -J \sum_{2}^{N-1} (-i)c_{2i}c_{2i+1} \)

Superconductor, topological insulator, semiconducting heterostructure (see A. Kitaev and C. Laumann, arXiv:0904.2771 for review)
**Majorana fermions**

$$H_{\text{Maj}} = -J \sum_{2}^{N-1} (-i)c_{2i}c_{2i+1}$$

paired

$\bullet$ $\circ$ $\circ$ $\circ$ $\cdots$ $\circ$

$\circ$ $c_{1}$ $c_{2}$ $c_{3}$ $c_{4}$ $\cdots$ $c_{2N-1}$ $c_{2N}$

ground states: $(-i)c_{2i}c_{2i+1}|\Psi\rangle = |\Psi\rangle$ for all $i$.

unpaired Majorana fermions $\bullet$ at the edges of the chain

$\rightarrow$ “zero-energy Majorana boundary mode” $\{|\bar{0}\rangle, |\bar{1}\rangle\}$

$$(-i)c_{1}c_{2N}|\bar{0}\rangle = |\bar{0}\rangle, \quad (-i)c_{1}c_{2N}|\bar{1}\rangle = -|\bar{1}\rangle, \quad c_{1}|\bar{0}\rangle = |\bar{1}\rangle$$

$Y_{1}Z_{2} \ldots Z_{N-1}Y_{N}$ (Z2 symmetry)

If unpaired Majorana fermions are well separated, this operator would not act.

$$X_{1}$$

(act on the ground subspace nontrivially)

But! $c_{1}$ or $C_{2N}$ (odd weight fermionic operators) require coherent creation/annihilation of a single fermion, which is prohibited by superselection rule.

$\rightarrow X$ errors are naturally prohibited by the fermionic superselection rule.

$\rightarrow$ Unpaired Majorana fermion is robust against any “physical” perturbation.
Majorana fermions are non-Abelian, but do not allow universal quantum computation.

**Topologically protected gates**

**Magic state distillation**

[Bravyi-Kitaev PRA 71, 022316 (2005)]

**Universal quantum computation**

pair creation of Majorana fermions

exchanging Majorana fermions via T-junction
Majorana fermions are non-Abelian, but do not allow universal quantum computation.

What if your system has no superselection rule, such as the fermionic parity preservation?
Kitaev’s honeycomb model


Honeycomb model:

\[ H_{hc} = -J_x \sum_{x\text{-link}} X_i X_j - J_y \sum_{y\text{-link}} Y_i Y_j - J_z \sum_{z\text{-link}} Z_i Z_j \]

Dimerization:

\[ J_x, J_y \ll J_z \]

Efficient Hamiltonian:

\[ H_{eff} = -\frac{J_x^2 J_y^2}{16|J_z|^3} \sum_p Y_{\text{left}(p)} Y_{\text{right}(p)} X_{\text{up}(p)} X_{\text{down}(p)} \]

Toric code Hamiltonian:

\[ H_{TC} = -J \sum_f Z_{l(f)} Z_{r(f)} Z_{d(f)} Z_{u(f)} - J \sum_v X_{l(v)} X_{r(v)} X_{d(v)} X_{u(v)} \]

Kitaev’s toric code model

Kitaev’s toric code model is a representative example of topologically ordered system.

Toric code Hamiltonian:

\[ H = -J \sum_f A_f - J \sum_v B_v \]

Face operator:

\[ A_f = \prod_{i \in \text{face } f} Z_i \]

Vertex operator:

\[ B_v = \prod_{i \in \text{vertex } v} X_i \]

Note that these operators are commutable:

\[ \text{anti-commute \times anti-commute} = \text{commute} \]

The ground states are given by simultaneous eigenstate of all face & vertex operators (gapped and frustration-free):

\[ A_f |\Psi\rangle = |\Psi\rangle, \quad B_v |\Psi\rangle = |\Psi\rangle \]
Kitaev’s toric code model is a representative example of topologically ordered systems.

Kitaev’s toric code Hamiltonian:

\[ H = -J \sum f A_f - J \sum v B_v \]

Toric code face operator:

\[ A_f = \prod_{i \in \text{face}} Z_i \]

Toric code vertex operator:

\[ B_v = \prod_{i \in \text{vertex}} X_i \]

Note that these operators are commutative:

\[ \{ A_f, B_v \} = 0 \]

The ground states are given by simultaneous eigenstates of all face and vertex operators (gapped and frustration-free):

\[ A_f |\Psi\rangle = |\Psi\rangle, \quad B_v |\Psi\rangle = |\Psi\rangle \]

★ Short note on the stabilizer formalism

- n-qubit Pauli group: \( \{ \pm 1, \pm i \} \times \{ I, X, Y, Z \} \otimes_n \)

- Stabilizer group: \( S = \{ S_i \} \), where \( [S_i, S_j] = 0 \) and \( S_i = S_i^\dagger \) (commutative) (hermitian)

- Stabilizer generators: minimum independent set of stabilizer elements

- Stabilizer state: \( S_i |\Psi\rangle = |\Psi\rangle \) for all stabilizer generators \( S_i \)

- Example: \( \langle X_1 X_2, Z_1 Z_2 \rangle \rightarrow (|00\rangle + |11\rangle)/\sqrt{2} \)

- Dimension of the stabilizer subspace: \( 2^{(\text{# qubits} - \text{# generators})} \)
Kitaev toric code model

Kitaev’s toric code model is a representative example of topologically ordered system.

\[ H = -J \sum_f A_f - J \sum_v B_v \]

face operator: \[ A_f = \prod_{i \in \text{face } f} Z_i \]

vertex operator: \[ B_v = \prod_{i \in \text{vertex } v} X_i \]

Toric code Hamiltonian: \[ H = -J \sum_f A_f - J \sum_v B_v \]

Note that these operators are commutale:

anti-commute × anti-commute = commute

The ground states are given by simultaneous eigenstate of all face & vertex operators (gapped and frustration-free):

\[ A_f |\Psi\rangle = |\Psi\rangle, \quad B_v |\Psi\rangle = |\Psi\rangle \]
Degeneracy of the ground subspace:

[Torus]

# qubits: (edges) on $N \times N$ torus = $2N^2$

# stabilizer generators:

(faces + vertexes - 2) = $2N^2 - 2$

# dimension of ground subspace: $2^2 = 4$

# logical qubits $\rightarrow$ 2 (two logical qubits)

[General surface]

$$\text{(face)} + (\text{vertex}) - (\text{edge}) = 2 - 2g$$

$g = \text{genus}$

Euler characteristic

# logical qubits $\rightarrow (\text{edge}) - [(\text{face}) + (\text{vertex}) - 2] = 2g$
Degeneracy of the ground subspace:

[Torus]

- # qubits: (edges) in $N \times N$ torus = $2N^2$
- # stabilizer generators:
  (faces + vertexes - 2) = $2N^2 - 2$
- # dimension of ground subspace: $2^2 = 4$
- # logical qubits $\rightarrow$ 2 (two logical qubits)

[General surface]

How is the ground state degeneracy described?

$\rightarrow$ Find a good quantum number! The operator that acts on the ground subspace nontrivially, “logical operator”.
The operators on non-trivial cycles $Z(c_1^L), X(c_1^L)$ are commutable with all face and vertex operators, but cannot given by a product of them.

$$\{ Z(c_1^L), X(c_1^L) \} = 0$$

$\rightarrow$ logical Pauli operators.

$g=1 \rightarrow$ # of logical qubit = 2:

$$\{ Z(c_1^L), X(c_1^L) \}, \{ Z(c_1^{L'}), X(c_1^{L'}) \}$$

(The action of logical operators depend only on the homology class of the cycle.)

The logical operators have weight $N$.

$\rightarrow$ $N$-th order perturbation shifts the ground energy.
Stability against local perturbations

$$H = H_{TC} + h_x \sum_i X_i + h_z \sum_i Z_i$$

local field terms

quantum/classical mapping by Trotter-Suzuki expansion

Z2 Ising gauge model (dual of 3D Ising model)

topologically ordered (Higgs phase)

Tupitsyn et al., PRB 82, 085114 (2010)
Stability against local perturbations

\[ H = H_{TC} + h_x \sum_i X_i + h_z \sum_i Z_i \]

local field terms

quantum/classical mapping

\( Z_2 \) Ising gauge model (dual of 3D Ising model)

Is stability against perturbations enough for fault-tolerance?

No. Stability against thermal fluctuation is also important!

Tupitsyn et al., PRB 82, 085114 (2010)
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Thermal instability of topological order

Majorana fermion:

\[ c_1 c_2 c_3 c_4 \cdots c_{2N-1} c_{2N} \]

Excitation (domain-wall) is a point-like object.

\[ Z_i = c_{2i-1} c_{2i} \]

There is no large energy barrier between the degenerated ground states.
Thermal instability of topological order

Kitaev’s toric code model:

- Anyonic excitation (Abelian) → excitation is a point-like object.
- Anyon can move freely without any energetic penalty.
- Pair creation
- Pair annihilation
- 1st excitation
- Domain growth
- Ground state (gs)
Thermal instability of topological order

More generally...

Topological order in any local and translation invariant stabilizer Hamiltonian systems in 2D and 3D do not have thermal stability.


(Excitation has to be two-dimensional object for each non-commuting errors, X and Z. →4D)

Existence/non-existence of thermally stable topological order (= self-correcting quantum memory) in 3 or lower dimensions is one of the open problems in physics! (see list of unsolved problem in physics in wiki)

Non-equilibrium condition (feedback operations) is necessary to observe long-live topological order (many-body quantum coherence) at finite temperature.
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condensed matter physics

quantum information processing
Topological error correction

Toric code Hamiltonian:

- Face stabilizer: \( A_f = \prod_{i \in \text{face } f} Z_i \)
- Vertex stabilizer: \( B_v = \prod_{i \in \text{vertex } v} X_i \)

Toric code Hamiltonian: \( H = -J \sum_f A_f - J \sum_v B_v \)

The code state is defined by

\[ A_f |\Psi\rangle = |\Psi\rangle, \quad B_v |\Psi\rangle = |\Psi\rangle \]

for all face and vertex stabilizers.
Errors on the surface code

If a chain of $X$ (bit-flip) errors occurs, the eigenvalues of the face stabilizers become -1 at the boundary of the error chain.

(In the toric code Hamiltonian, they correspond to the anyonic excitations)
Errors on the surface code

Similarly if a $Z$ (phase-flip) error chain occurs, the eigenvalues of the vertex stabilizers become -1 at boundary of the error chain.

(that is, toric code model have two types of anyonic excitations)

For simplicity, we only consider $X$ errors correction below.
Errors on the surface code

If a chain of $X$ (bit-flip) errors occurs, the eigenvalues of the face stabilizers become -1 at the boundary of the error chain.

(In the toric code Hamiltonian, they correspond to the anyonic excitations)
Syndrome measurements

Measure the eigenvalues of the stabilizer operators.

Projective measurement for an operator $A$ (hermitian & eigenvalues ±1)

$$|+\rangle \rightarrow X$$

$$|\psi\rangle \rightarrow A$$

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes A$$

(In the toric code Hamiltonian, the syndrome measurements correspond to measurements of the local energy.)
The syndrome measurements do not tell us the actual location of errors, but boundaries of them.

(It tells location of excitations, but does not tell the trajectory of the excitations)

Then we have to infer a recovery chain, to recover from errors.

In the toric code Hamiltonian, this can be viewed as finding an appropriate way to annihilate pairs of anyones.
Topological error correction

If error and recovery chains result in a trivial cycle, the error correction succeeds.

Trivial cycle = stabilizer element

Actual and estimated error locations are the same.
Topological error correction

If the estimation of the recovery chain is bad ....
Topological error correction

If the estimation of the recovery chain is bad ....

The error and recovery chains result in a non-trivial cycle, which change the code state.
The error chain which has the highest probability conditioned on the error syndrome.

→ minimum-weight-perfect-match (MWPM) algorithm (polynomial algorithm)

Blossom 5 by V. Kolmogorov, Math. Prog. Comp. 1, 43 (2009).

[Improved algorithms]

Algorithm for error correction

The inference problem can be mapped to a ferro-para phase transition of random-bond Ising model.

Noise model and threshold values

**Code performance:**
Independent X and Z errors with perfect syndrome measurements.

[10.3-10.9%]

**Phenomenological noise model:**
Independent X and Z errors with noisy syndrome measurements.

[2.9-3.3%]

**Circuit noise model:**
Errors are introduced by each elementary gate.

[0.75%]

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distillation/ implementations
p- and d-type defects

introduce “defects” on the planer surface code

(defect = removal of the stabilizer operator from the stabilizer group, which introduce a degree of freedom)
dynamics of defects

• preparation of logical qubit → creation of defect pair

• moving the defect

• measurement of logical qubit pair annihilation of defects

• braiding p-defect around d-defect → Controlled-Not gate between p-type (control) and d-type (target) qubits.

p-type qubit

![p-type qubit image]

d-type qubit

![d-type qubit image]
Prepare & move the defect

Preparation of eigenstate $|+\rangle^p_L$ of $L^p_X$:

X basis measurement

Moving the defect:

expand

logical operator

surface code

shrink

primal defect pair creation

primal defect pair

time
Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.
Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.
CNOT gate by braiding

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Trivial cycle is a stabilizer operator, and hence acts trivially on the code space.
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CNOT gate by braiding

Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.

\[ L_X^p \otimes I^d \] is transformed into \[ L_X^p \otimes L_X^d \]!
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Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.
Observe the time evolution of the logical operator under the braiding operation of the primal defect around the dual defect.

\[ I^p \otimes L^d_Z \text{ is transformed into } L^p_Z \otimes L^d_Z \]
CNOT gate by braiding

That is, the braiding operation is equivalent to the CNOT gate from the primal to the dual qubits.
CNOT gate by braiding Abelian anyon

The p-type and d-type defect qubits are always control and target, respectively.

\[
\begin{align*}
\text{p-type qubit} & \quad \bullet \quad \bullet \\
\text{d-type qubit} & \quad \circ \quad \circ \\
\end{align*}
\]

\[
\begin{align*}
\text{control in} & \quad \text{control out} \\
|0\rangle^d_L & \quad \text{d} \quad Z \\
|+\rangle^p_L & \quad \text{target out} \\
\text{target in} & \quad \text{p} \quad X \\
\end{align*}
\]

→ The anyonic excitation in the Kitaev toric code is Abelian.
Universal quantum computation by magic state distillation

CNOT gate (Clifford gate) is not enough for universal quantum computation. (This is also the case for the Ising anyon.)

Topologically protected CNOT gate + Noisy ancilla state

Magic state distillation → universal quantum computation with an arbitrary accuracy

Bravyi-Kitaev PRA 71, 022316 (2005)

Over 90% of computational overhead is consumed for magic state distillation!

Improved magic state distillation protocols

Bravyi-Haah, PRA 86, 052329 (2012)
Eastin, PRA 87, 032321 (2013)

Raussendorf-Harrington-Goyal, NJP 9, 199 (2007).
Non Clifford gates

State injection:

\[ e^{i\theta X} |\Psi_{\text{vac}}\rangle = (\cos \theta I + i \sin \theta L_X^P) |0\rangle_L^P = \cos \theta |0\rangle_L^P + i \sin \theta |1\rangle_L^P \]

\[ e^{i\theta Z} |\Psi_{\text{vac}}\rangle = \cos \theta |+\rangle_L^d + i \sin \theta |-\rangle_L^d = e^{i\theta} (|0\rangle_L^d + e^{-i2\theta} |1\rangle_L^d) \]

One-bit teleportation for non-Clifford gate

\[ |\psi\rangle_L^p \xrightarrow{X} \cos \theta |0\rangle_L^p + i \sin \theta |1\rangle_L^p \]

\[ e^{i\theta L_Z^p} |\psi\rangle_L^p \]

\[ e^{i\theta L_Z^p} |\psi\rangle_L^p \]
Implementations (circuit)

- data qubit which constitutes the surface code
- ancilla qubit for the face syndrome measurement
- ancilla qubit for the vertex syndrome measurement

Qubits on the square lattice/ nearest-neighbor two-qubit gates/ initialization and projective measurement of individual qubits $\rightarrow$ fault-tolerant universal QC

[On-chip monolithic architectures]


  factorization of 1024-bit composite number: $\sim 10^8$ qubits, gates $\sim 10$[ns], error rate 0.1% $\rightarrow$ 1.8 day

  (768-bit takes 1500 CPU years with classical computer)


[distributed architectures]

- DQC-1: Y. Li et al., PRL 105, 250502 (2010); KF & Y. Tokunaga, PRL 105, 250503 (2010).

  fidelity of quantum channel $\sim 0.9$, error rate of local operations $\sim 0.1$
Implementation

Measurement-based quantum computation (MBQC):
After generating a many-body entangled state, we only need to readout the state of the particles.

Topologically protected MBQC on thermal state:

[Thermal state of two-body Hamiltonian (no phase transition)]
spin-2 & spin-3/2 particles: Li et al., PRL 107, 060501 (2011)

[Symmetry breaking thermal state (ferromagnetic phase transition)]
Summary

Topological order & topological quantum computation (extremely long-live quantum coherence)

Equilibrium/ local/ translationally invariant (without control)

Non-equilibrium (error correction by feedback operation)

Selective addressing (measurement and control) of individual particle

Topological QEC by global control and dissipative dynamics

No selective addressing! No measurement!

See poster session

Zero temperature 4- or higher dimension

Non-Abelian anyones Ising anyon + Magic state distillation Fibonacci anyon

Topologically protected quantum computation in 2D

Topologically protected MBQC in 3D

Abelian anyon (Kitaev’s toric code model) is enough for universal quantum computation.

Thank you for your attention!
**List of my works**

**1.** "Measurement-Based Quantum Computation on Symmetry Breaking Thermal States" (Editors’ suggestion)  

**2.** "Duality analysis on random planar lattice"  

**3.** "Blind topological measurement-based quantum computation"  

**4.** "Error- and Loss-Tolerances of Surface Codes with General Lattice Structures"  

**5.** "Not all physical errors can be linear CPTP maps in a correlation space"  

**6.** "Computational Power and Correlation in Quantum Computational Tensor Network"  

**7.** "Topologically protected measurement-based quantum computation on the thermal state of a nearest-neighbor two-body Hamiltonian with spin-3/2 particles"  

**8.** "Robust and Scalable Scheme to Generate Large-Scale Entanglement Webs"  

**9.** "Fault-Tolerant Topological One-Way Quantum Computation with Probabilistic Two-Qubit Gates"  

**10.** "Topological One-Way Quantum Computation on Verified Logical Cluster States"  

**11.** "Anti-Zeno effect for quantum transport in disordered systems"  

**12.** "Cluster-based architecture for fault-tolerant quantum computation"  

**13.** "Entanglement purification with double-selection"  