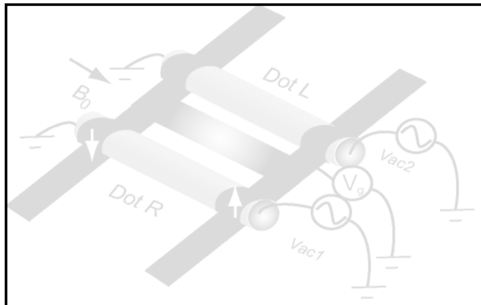


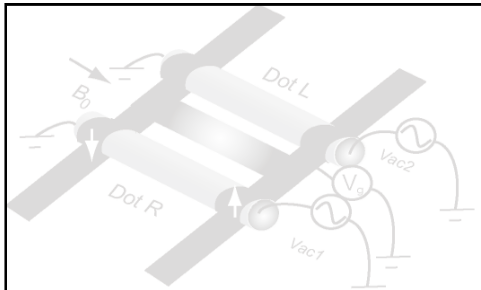
半導体を用いた量子情報処理
Quantum information processing in semiconductors

Yasuhiro Tokura (University of Tsukuba, NTT BRL)

Part I – August 14, afternoon I

Part II – August 15, morning I

Part III – August 15, morning II



自己紹介

1985年　東京大学教養学部相関理化学卒
　　　　Peierls transition、量子Monte Carlo
1985年　NTT基礎研究所 電子波干渉現象、結晶成長理論
1996年　共鳴トンネル、量子細線弾道伝導、少数電子厳密対角化
1998年　オランダ・デルフト工科大　客員研究員 CNT*伝導
2002年　量子ドット伝導、Pauliスピン閉塞、スピン緩和時間
2005年　量子鍵配送、量子もつれ光子
2006年　スピン量子ビット操作(ICORP)
2012年　筑波大学　研究室立ち上げ中

趣味 バドミントン、ギター

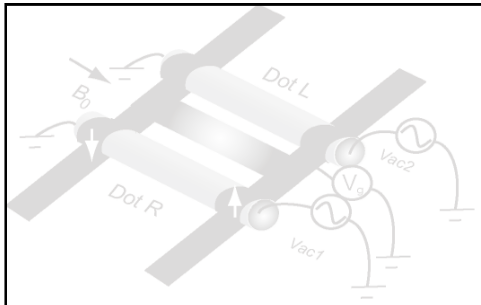
NTT → 筑波大学 University of Tsukuba


デルフト工科大の図書館

*CNT: Carbon nanotube

Plan of this lecture

- Part I (Aug. 14, afternoon I)
    - Basics of semiconductor system +CNT, Graphene
        - Quantum dots, Double quantum dots: Hubbard model
        - Quantum point contacts: charge detection
    - Charge qubits
    - Charge detection
- Part II (Aug. 15, morning I)
    - Which path detector, continuous weak measurement
    - Spin detection - Spin to charge conversion
    - Exchange based (only) qubits
        - Single qubit manipulations, Hybrid qubit
        - Two-qubit interaction
- Part III (Aug. 15, morning II)
    - Single spin qubits
        - Single spin manipulations: magnetic, electric (μ-magnet, SOI, etc)
        - Two or more qubit manipulation
    - Hyperfine interaction and material issues
    - Coupling remote qubits (Resonator coupling, Flying qubits)
    - Prospective

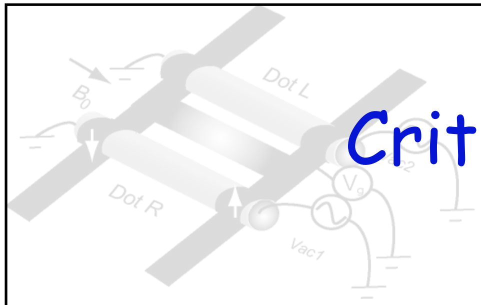


Part I

Semiconductor

Quantum Dots

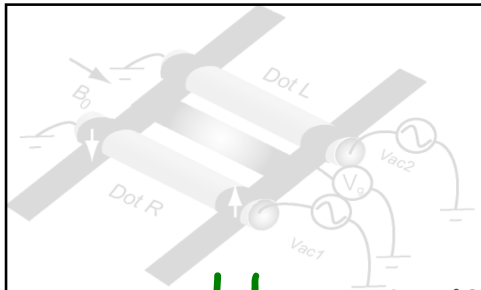
Quantum Point Contacts



Criteria of realizing quantum computers

D. P. DiVincenzo Fortschr. Phys. (2000).

	Electrically controlled spin qubits
1. *A scalable physical system with well characterized qubits* (スケーラビリティ)	?
2. *The ability to initialize the state of the qubits to a simple fiducial state* （初期化）	✓
3. *Long relevant decoherence times, much longer than the gate operation time* （良いコヒーレンス）	△
4. *A "universal" set of quantum gates* （量子演算）	✓
5. *A qubit-specific measurement capability* （読み出し）	△



Main subject of this talk

- How can we realize coherent system in semiconductor ?
- What is the current status of the research ? (How good, how many …)
- To which direction can we go ?
  - Can we see straightforward milestones?
  - Or do we need another big breakthrough?

One sheet summary of semiconductor

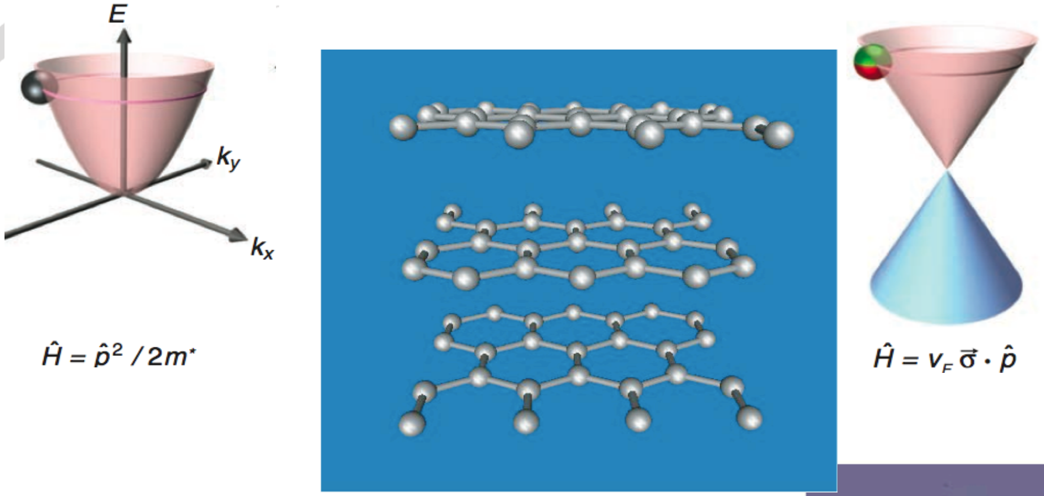
We can enjoy the variety of material features and their combinations.

Band gap E_{gap}, - Important for optical interface
Effective mass m^* - scales `Quantum confinement', zero - metallic CNT/Graphene
Multi-valley (Silicon, CNT, Graphene) – additional quantum index ?

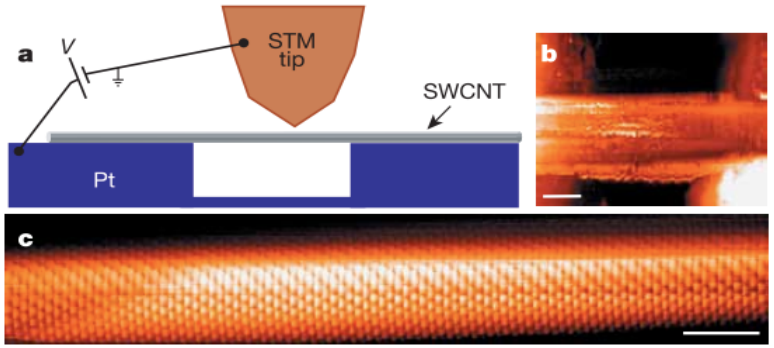
Lande g-factor g^* - magnetic coupling of spin, electrically tunable
Spin-orbit interaction (SOI) α,β
− enabling electric control of spin / topological states, Majorana

Hyperfine coupling A – enemy of spin coherence, isotope engineering
Deformation/Piezoelectric Phonon Ξ, h_{14} – another source of decoherence

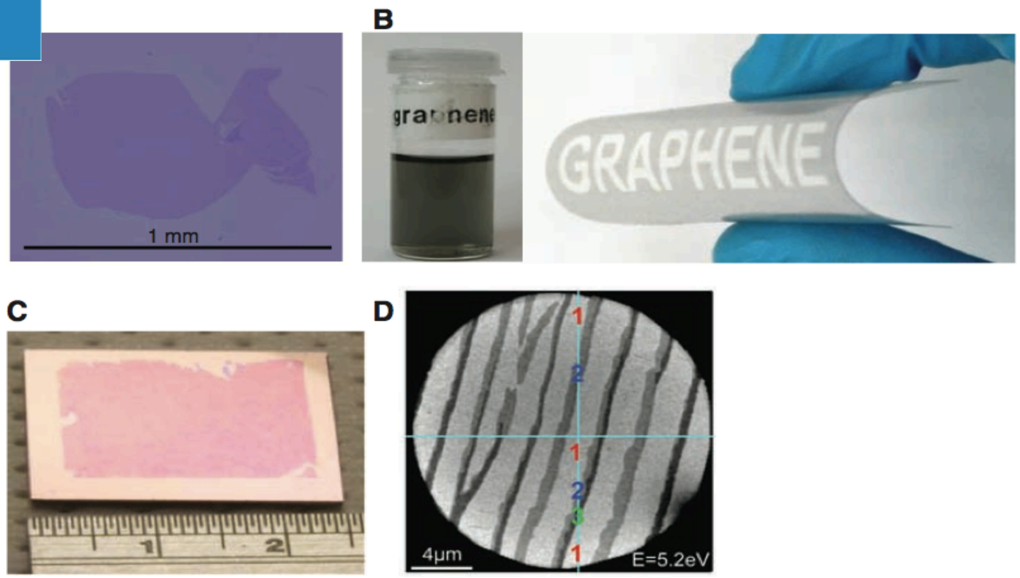
Carbon nanotube and Graphene



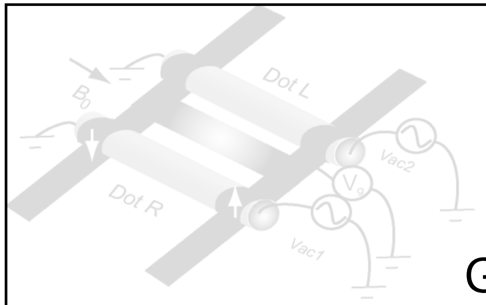
Massless Dirac Fermion
K and K′ valleys



B. J. LeRoy, et al., Nature 432, 371 (2004).



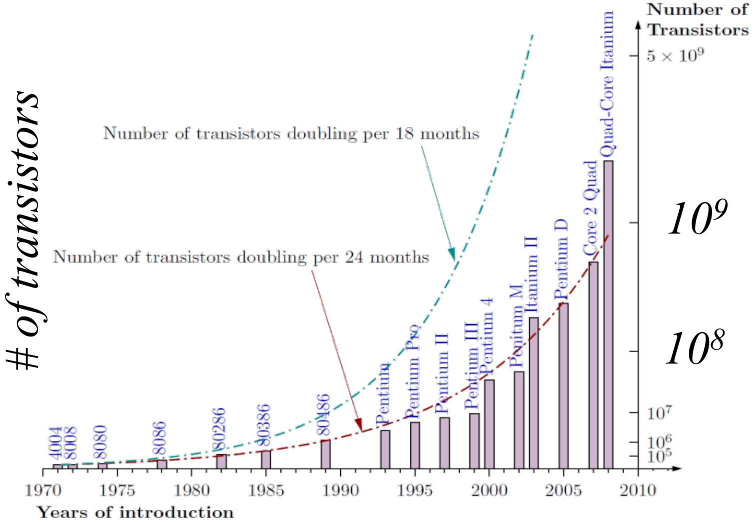
A. K. Geim, et al., Science 324, 1530 (2009).



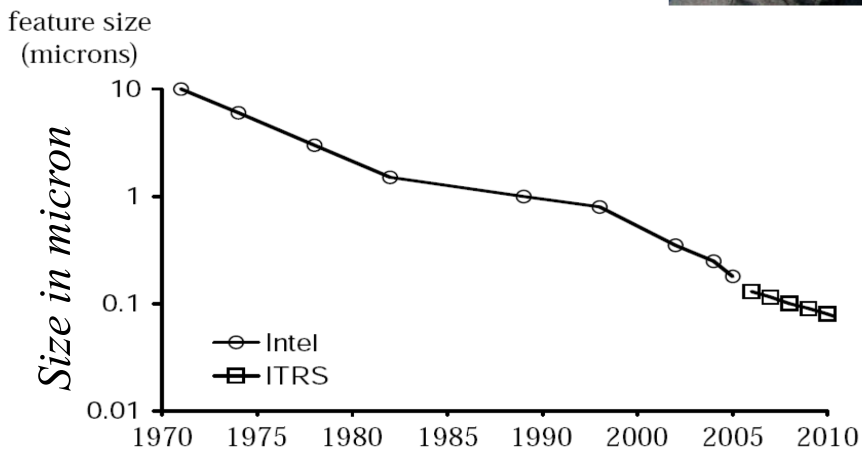
Nano-technology

Gordon E. Moore（Chairman Emeritus of Intel）

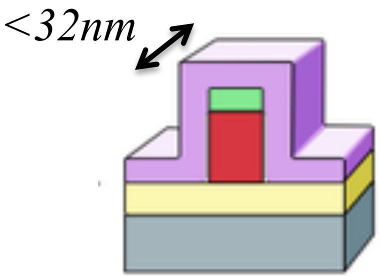
Moore's law



Minimum Feature Size



The decreasing minimum feature size of transistor components is shown for both Intel products and data reported by the International Technology Roadmap for Semiconductors (ITRS).



Fin-FET

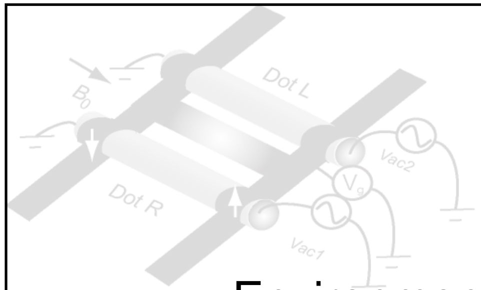
Potentially, the developed nano-technology for the semiconductor devices may help also to realize scalable quantum system.

Isolation of single charge and spin

In contrast to naturally well-isolated systems like cold-atoms, ions, and photons, forming quantum two-level systems (qubits) in condensed matter is not a easy task.

Controlling single charge one-by-one had been achieved in metallic small grains, but these systems cannot be a candidate of qubits, except for the superconducting states, where finite gap is formed and macroscopic quantum coherence is maintained.

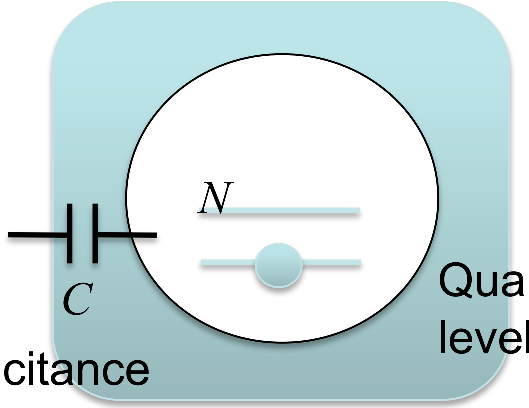
Therefore, isolation of single electron (artificial atom) is an important milestone to realize well-defined qubits in condensed matter.



Quantum dots (QDs)

Environment in thermal equilibrium

$k_B T \qquad \mu$



Total energy of *N* electrons

$$E(N) \sim \sum_{i=1}^{N} \varepsilon_i + {}_N C_2 U$$

Constant interaction model:

$$U \equiv \frac{e^2}{2C}$$

Stability condition of N electrons in the QD:

No addition $\quad \mu + \frac{1}{2}k_B T \ll E(N+1) - E(N) \quad \sim UN+\varepsilon_{N+1}$

No escape $\quad \mu - \frac{1}{2}k_B T \gg E(N) - E(N-1) \quad \sim U(N-1)+\varepsilon_N$

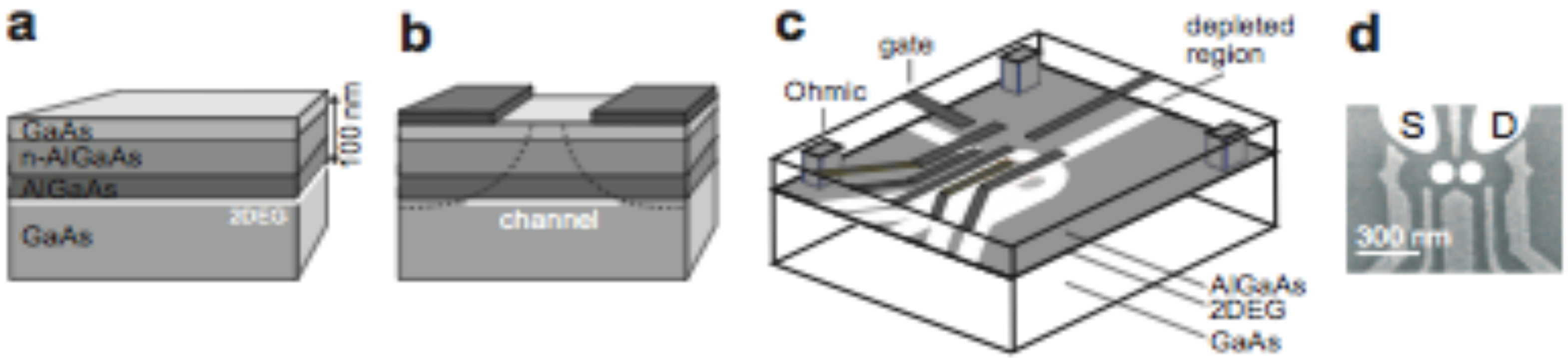
$$\Rightarrow \quad \boxed{U + \varepsilon_{N+1} - \varepsilon_N} \gg k_B T$$

Addition energy

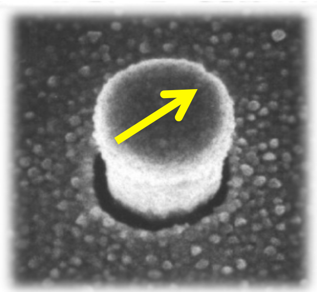
Coulomb blockade for
very low temperatures,
small capacitance,
large quantization energy

Fabrication of QDs

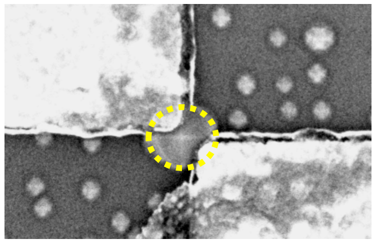
Typical top-down approach, starting from two-dimensional (2D) electron gas formed at the hetero-interface, and depleting selective areas by the surface metallic gates negatively biased.



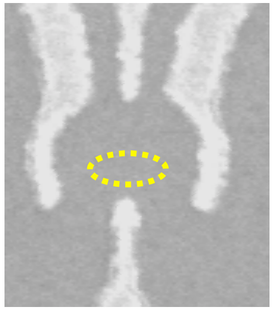
Advent of one-electron single QDs



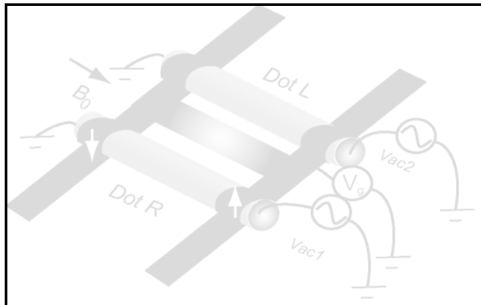
Tarucha et al. PRL 96



Jung et al. APL05

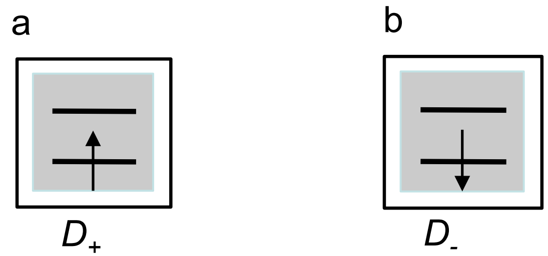


Ciorga et al. PRB 02

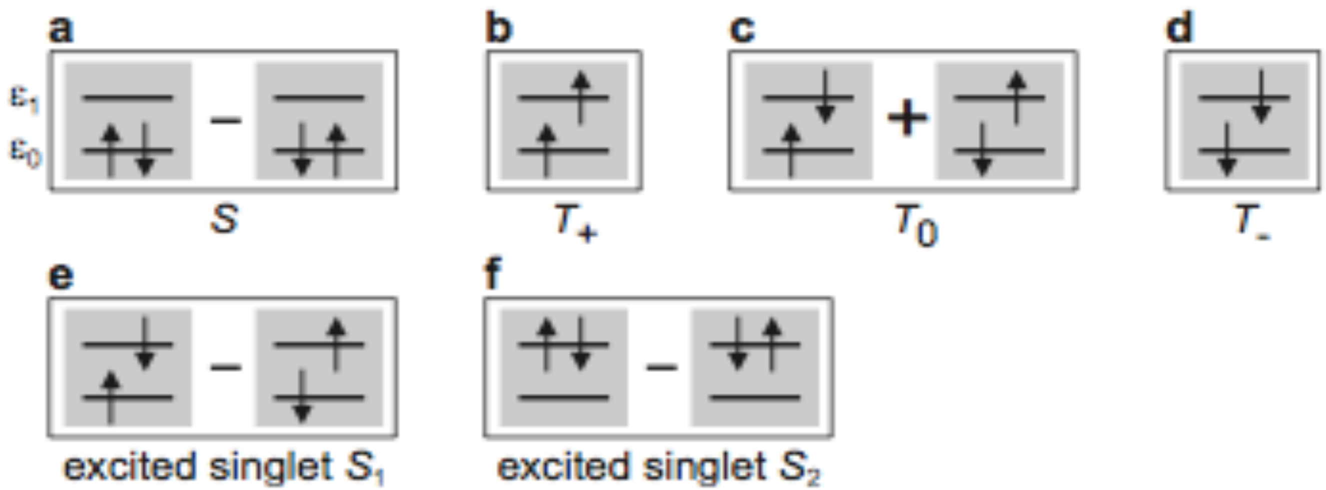


Spins in a QD

N=1

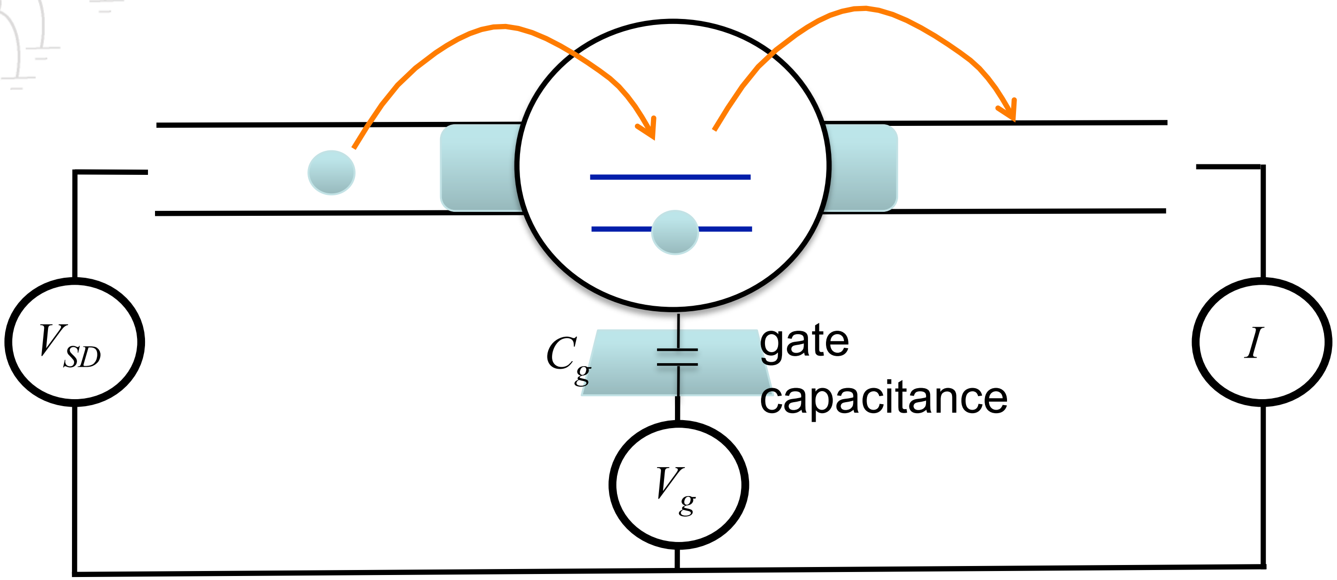


N=2



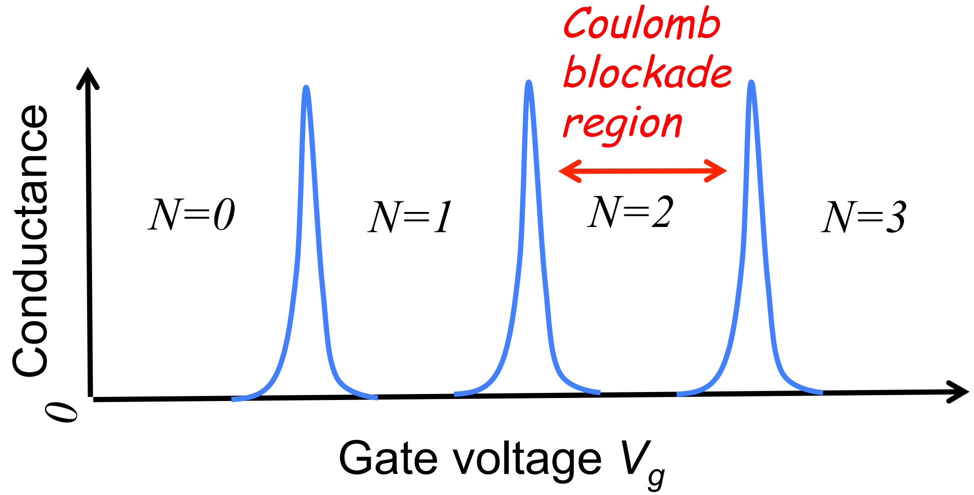
Simple… But, how can we probe these ?

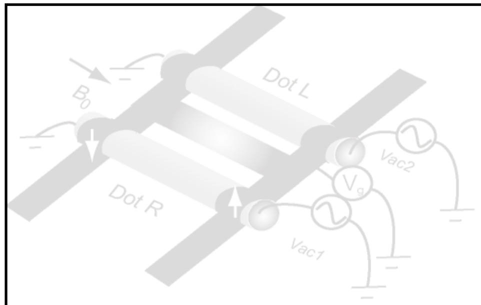
QDs coupled to the leads



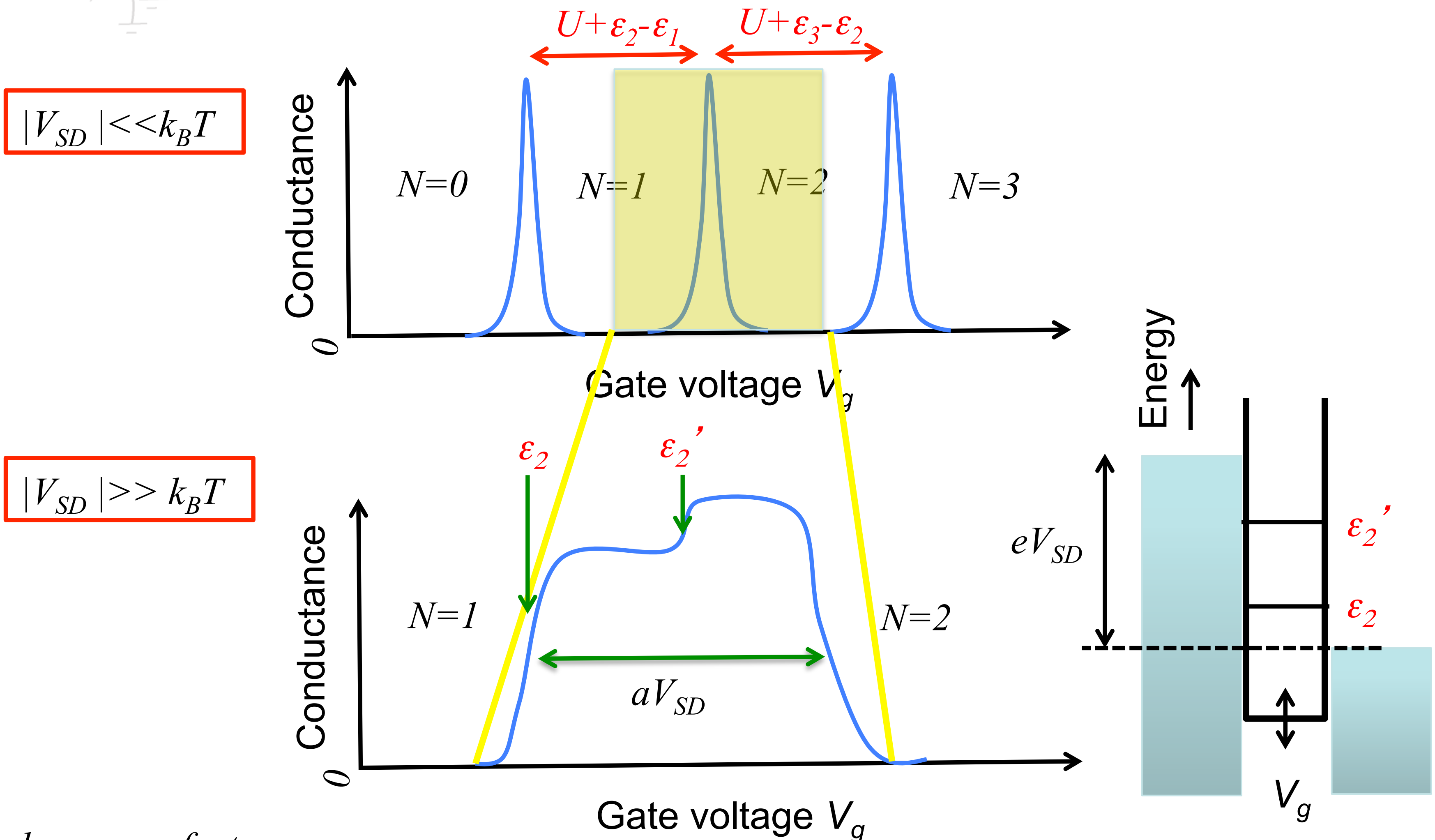
Conductance $G=I/V_{SD}$ is peaked when ($|V_{SD}| << k_B T$)

$$\begin{aligned} \mu &= E(N) - E(N-1) \\ &\sim U(N-1) + \varepsilon_N - \frac{C_g}{C} e V_g \end{aligned}$$



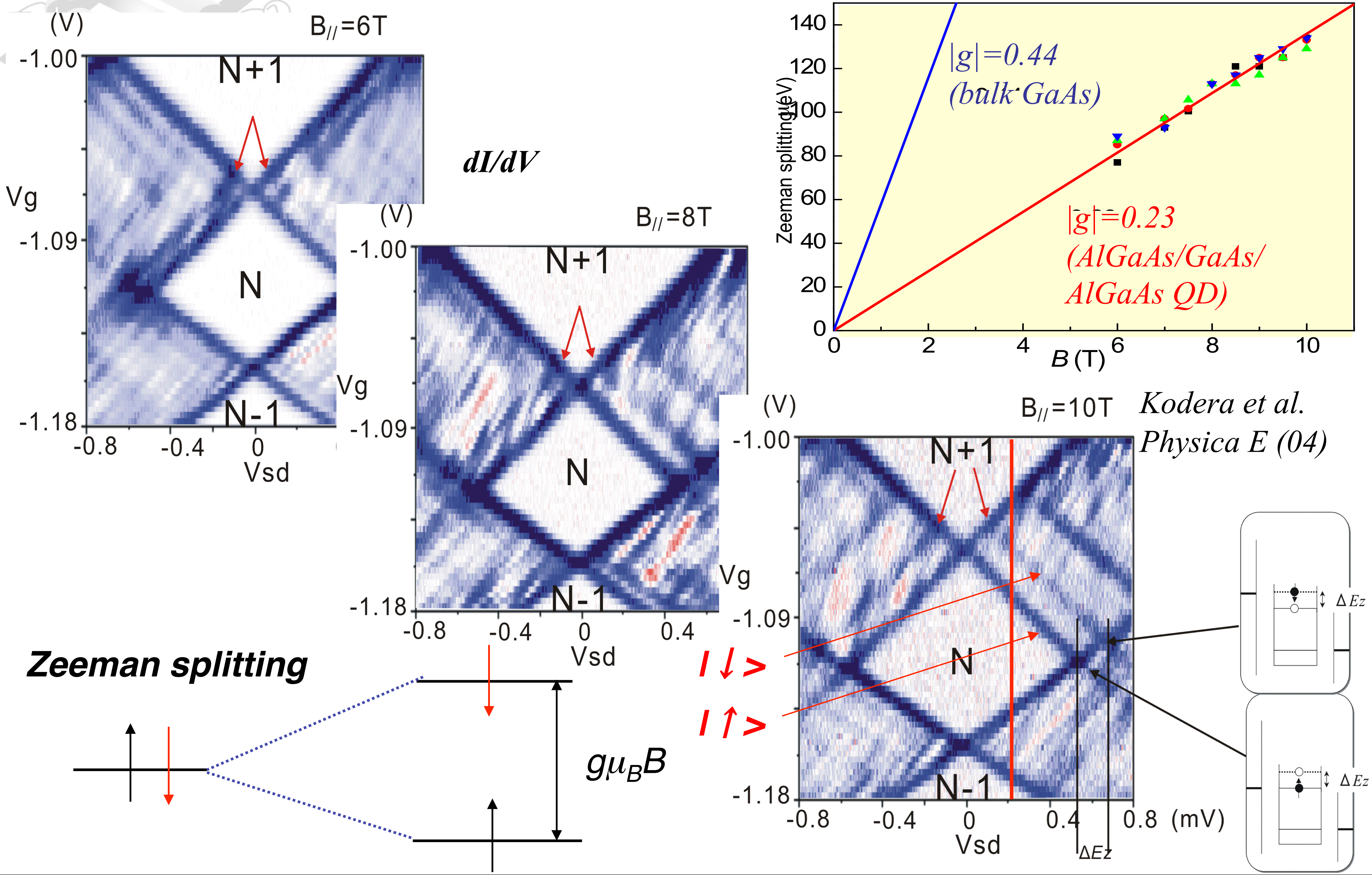


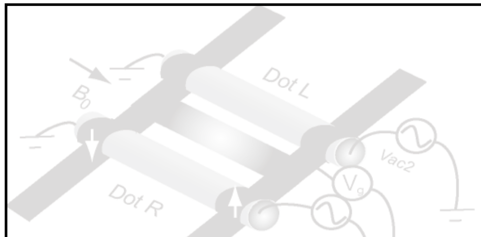
Tunneling spectroscopy



*a: lever-arm factor

g-factor in Quantum Dot: Excitation spectroscopy

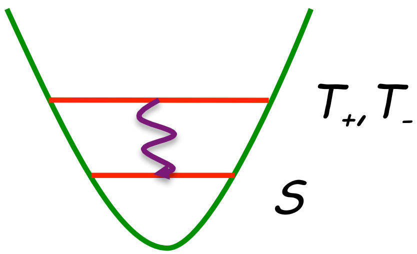


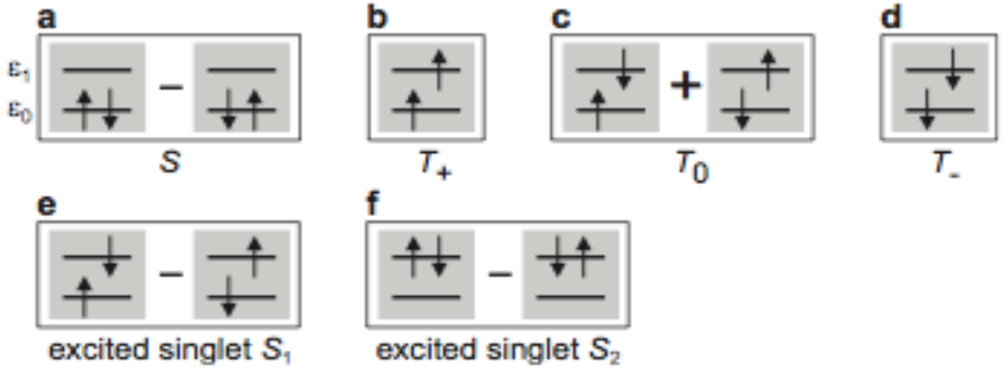


Relaxation time (T_1)

Energy relaxation time without changing spin state is very fast ~ 1ps (typical), by electron-phonon scatterings.

Spin flip relaxation is forbidden in the lowest order, but is possible by spin-orbit interaction, hyperfine coupling etc.





$$
\begin{aligned}
|T'_\pm\rangle &= |T_\pm\rangle + \frac{\Delta_{SOI}}{\delta E_{ST}}|S\rangle, \\
|S'\rangle &= |S\rangle - \frac{\Delta_{SOI}}{\delta E_{ST}}|T_\pm\rangle
\end{aligned}
$$

Fermi's golden rule $\Lambda_q^x \sim \langle S'|\mathcal{H}_{e-p}|T'_\pm\rangle$

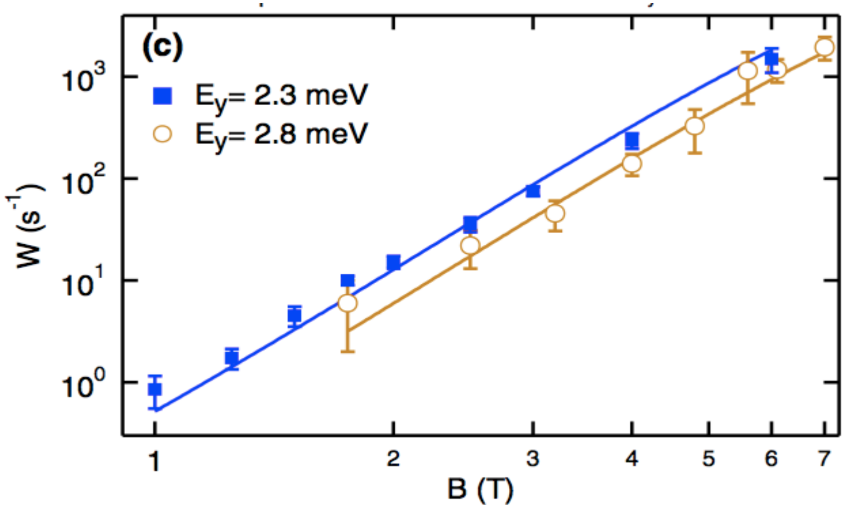
$$
\frac{1}{T_1} = \frac{2\pi}{\hbar}\sum_q |\Lambda_q^x|^2 \delta(\varepsilon_z - \hbar\omega_q)\coth\frac{\beta\varepsilon_z}{2}
$$

Determining spin T_1

Spin relaxation time is evaluated by "pump-and-probe" method

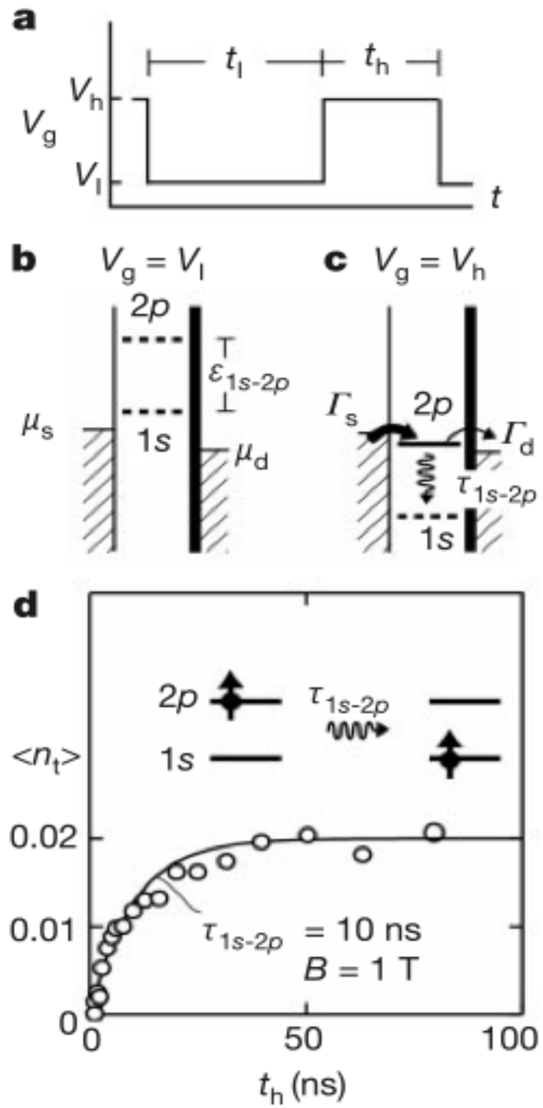
$T_1 > 200\mu s$

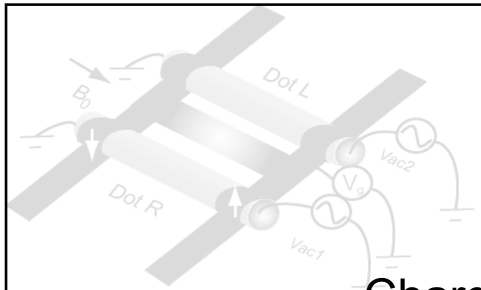
T. Fujisawa et al., Nature 419, 278 (2002).



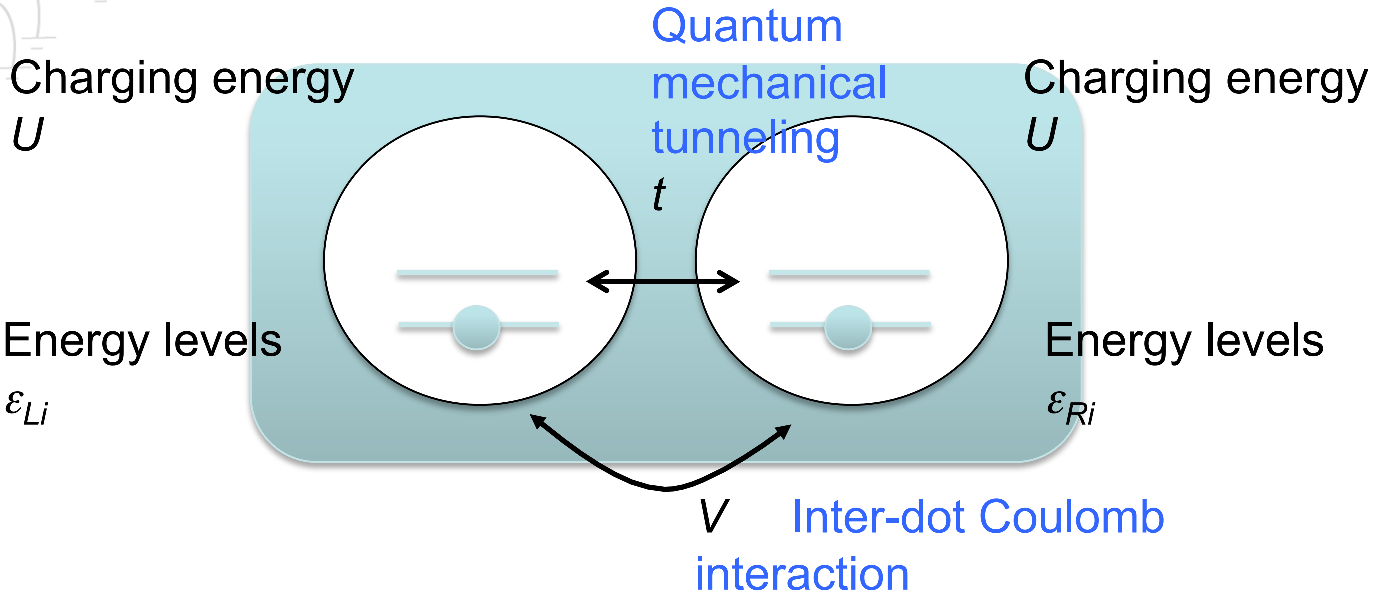
$T_1 \sim 1s$

S. Amasha, et al., Phys. Rev. Lett. 100, 046803 (2008).





Coupled quantum dots



Minimum realization of Hubbard model:

$$\mathcal{H}_{DQD} = \sum_{\mu=L,R}\sum_{\sigma}\varepsilon_{\mu}\hat{a}^{\dagger}_{\mu,\sigma}\hat{a}_{\mu,\sigma} - t(\hat{a}^{\dagger}_{L,\sigma}\hat{a}_{R,\sigma} + \mathrm{H.c.})$$

$$+U\sum_{\mu=L,R}\hat{n}_{\mu,\uparrow}\hat{n}_{\mu,\downarrow} + V\hat{n}_{L}\hat{n}_{R}$$

$$\hat{n}_{\mu,\sigma} \equiv \hat{a}^{\dagger}_{\mu,\sigma}\hat{n}_{\mu,\sigma}$$

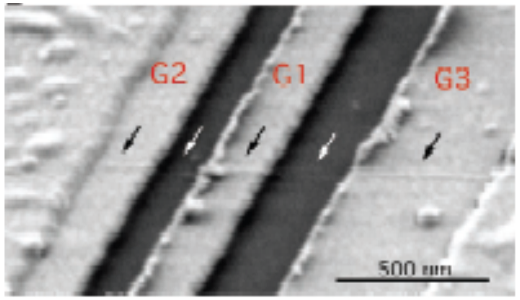
$$\hat{n}_{\mu} \equiv \sum_{\sigma}\hat{n}_{\mu,\sigma}$$

Double QDs holiding few electrons

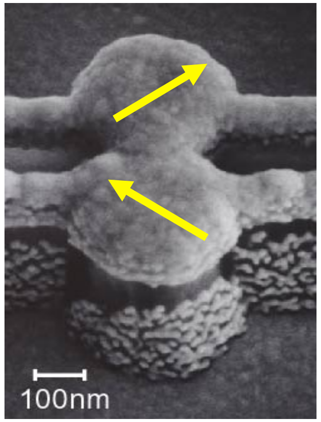
Fabrication of two QDs is straightforward extension in top-down approach, but realizing tunable coupling between the two QDs and going into few electron regime is not a simple task.

Advent of two-electron double QDs

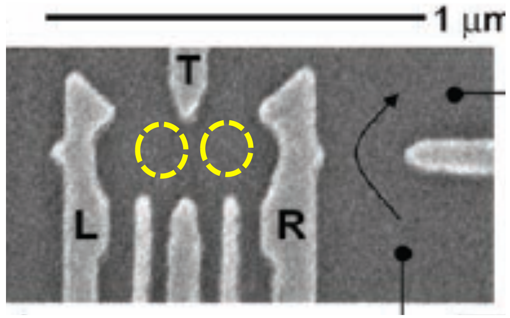
nanotube



Mason et al. Science 04



Hatano et al. Science 05

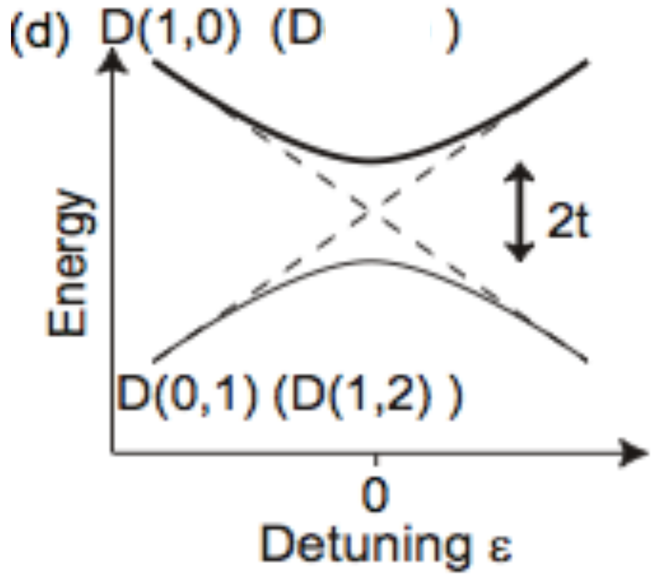


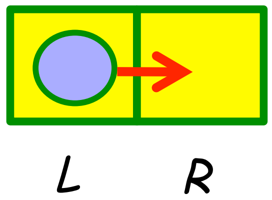
Petta et al. Science 04

Charge qubits

Effectively one electron in coupled QDs is simple two level system: charge qubit.

$$\mathcal{H}_{DQD} = \sum_{\mu=L,R} \varepsilon_\mu \hat{a}_\mu^\dagger \hat{a}_\mu - t(\hat{a}_L^\dagger \hat{a}_R + \mathrm{H.c.})$$



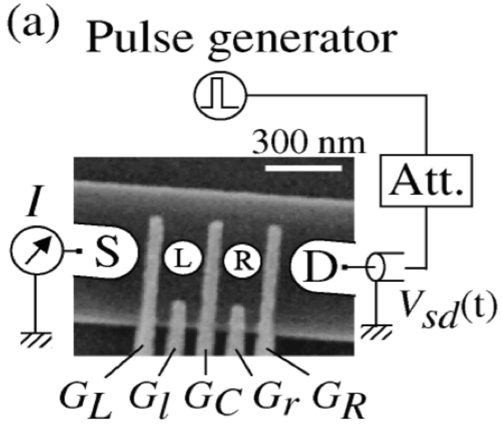


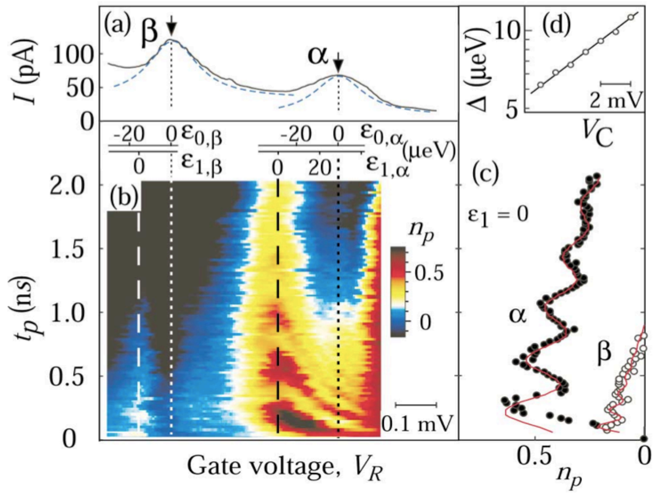
Detuning energy

$$\varepsilon \equiv \varepsilon_L - \varepsilon_R$$

(n_L,n_R) represents n_L and n_R electrons in the left and right QDs, resp.

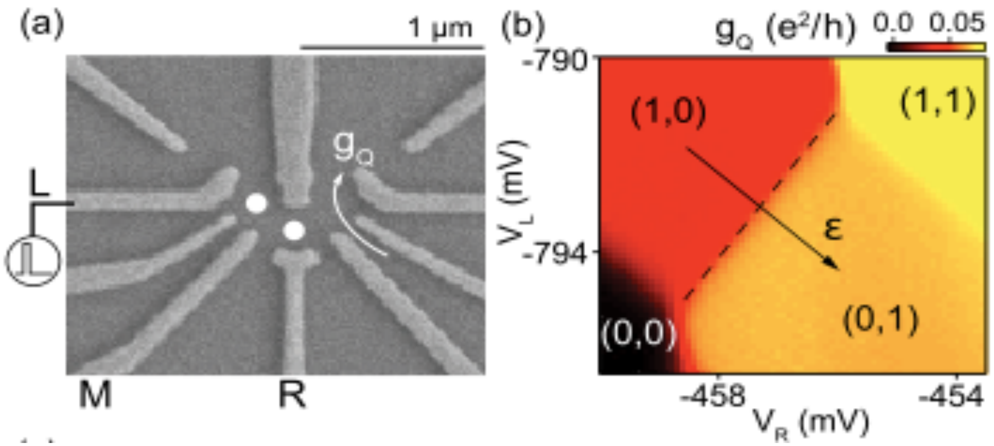
Charge qubit experiments

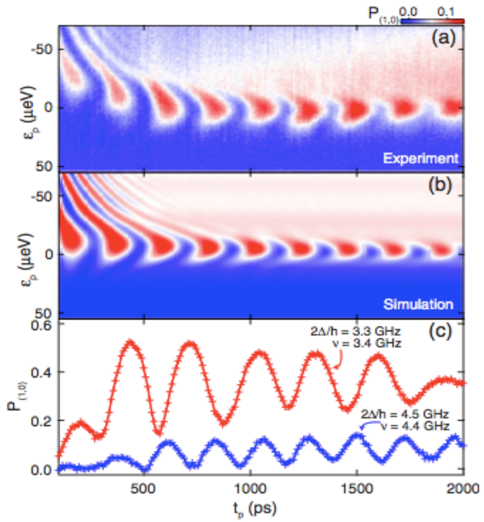




T. Hyashi, et al., Phys. Rev. Lett. 91, 226804 (2002).

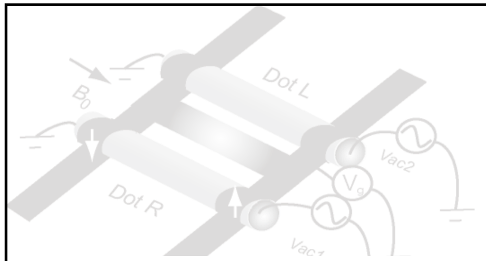
T^*_{Rabi}~1 ns
Origin:
Cotunneling
Phonon



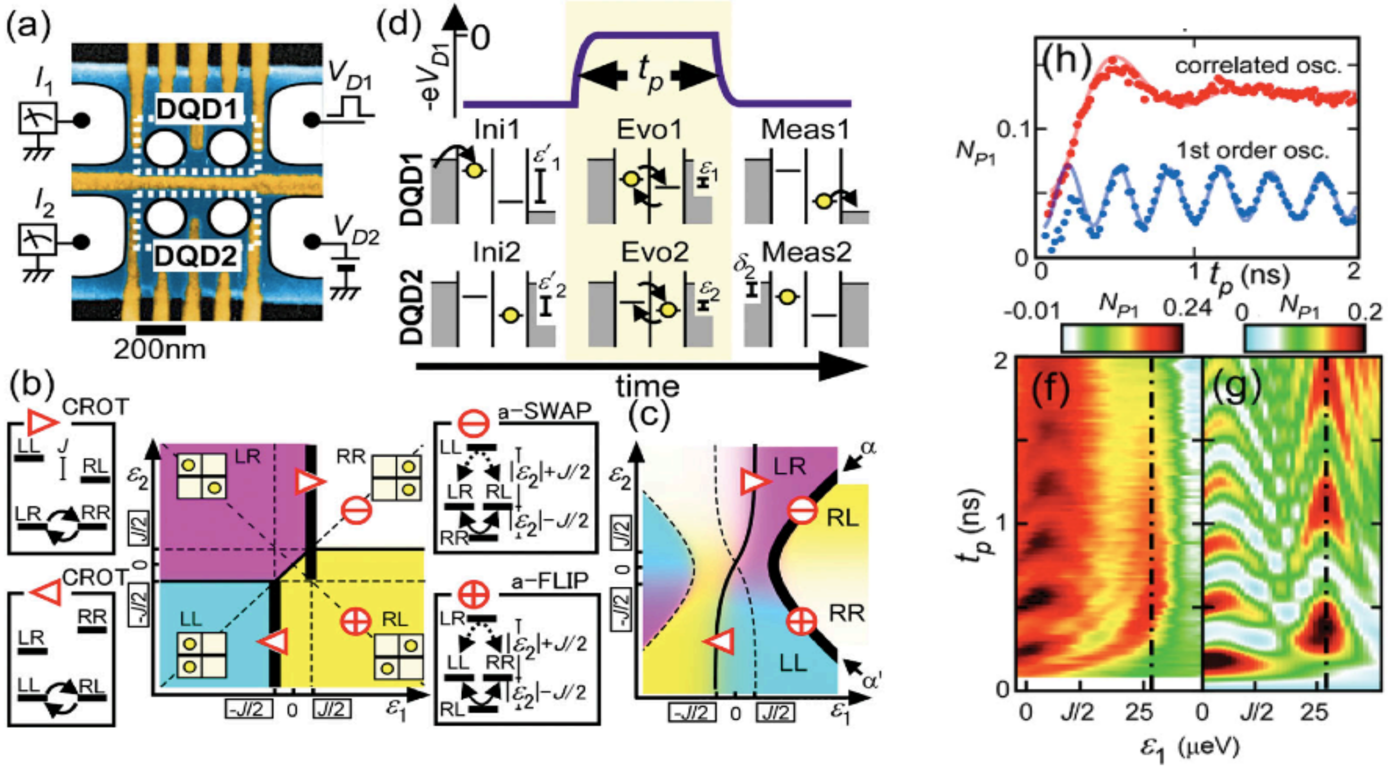


K. D. Petersson, et al., Phys. Rev. Lett. 105, 246804 (2010).
Y. Dovzhenko, et al., Phys. Rev. B 84, 161302 (2011).

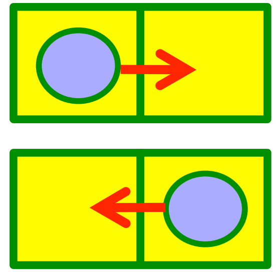
Ramsey T^*_2~60ps



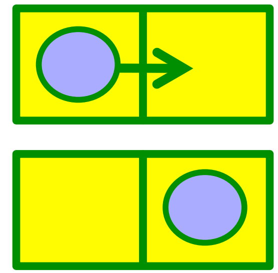
Coupled charge qubits



Mutual coherent osc.

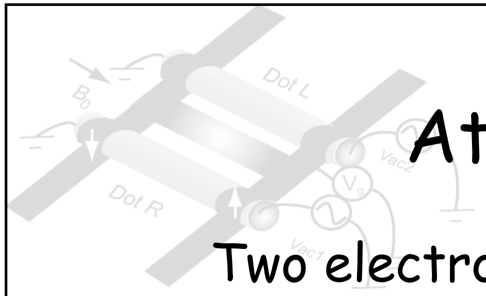


Conditional coherent osc.



$$\mathcal{H}_{2DQD} = \frac{1}{2}\sum_i(\varepsilon_i\sigma_z^{(i)} - t_i\sigma_x^{(i)}) + \frac{J}{4}\sigma_z^{(1)}\otimes\sigma_z^{(2)}$$

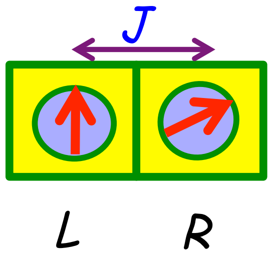
G. Shinkai, et al., Phys. Rev. Lett. 103, 056802 (2009).



Atomic limit of two electron in DQD

Two electron problem in double dot system:
Heisenberg Hamiltonian that enable swap operations

$$\mathcal{H}_S = J\mathbf{S_R}\cdot\mathbf{S_L} + \mathbf{g}\mu_\mathbf{B}\mathbf{B_0}(\mathbf{S_R}+\mathbf{S_L})$$



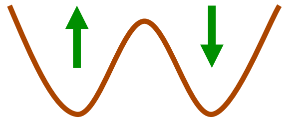
This is from an ideal situation of two identical dots in the atomic limit (t<<U). Here, we generalize the argument to finite detuning energy ε.

$$\mathcal{H}_{DQD} = \frac{\varepsilon}{2}(\hat{n}_L - \hat{n}_R) - t(\hat{a}^\dagger_{L,\sigma}\hat{a}_{R,\sigma} + \mathrm{H.c.}) + U\sum_{\mu=L,R}\hat{n}_{\mu,\uparrow}\hat{n}_{\mu,\downarrow} + V\hat{n}_L\hat{n}_R$$

Two electron basis functions

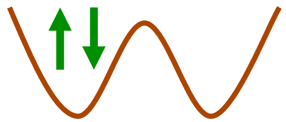
There are six two electron basis functions.

$$|S(1,1)\rangle \;=\; \frac{1}{\sqrt{2}}(a^{\dagger}_{L\uparrow}a^{\dagger}_{R\downarrow} - a^{\dagger}_{L\downarrow}a^{\dagger}_{R\uparrow}|0\rangle,$$

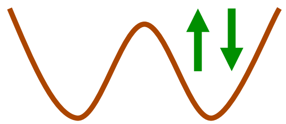


$$|S(2,0)\rangle \;=\; a^{\dagger}_{L\uparrow}a^{\dagger}_{L\downarrow}|0\rangle,$$

Spin singlets



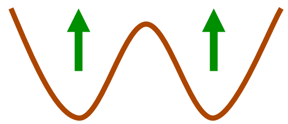
$$|S(0,2)\rangle \;=\; a^{\dagger}_{R\uparrow}a^{\dagger}_{R\downarrow}|0\rangle$$



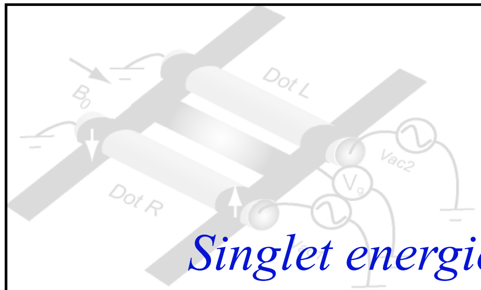
$$|T^{1}\rangle \;=\; a^{\dagger}_{L\uparrow}a^{\dagger}_{R\uparrow}|0\rangle,$$

Spin triplets

$$|T^{0}\rangle \;=\; \frac{1}{\sqrt{2}}(a^{\dagger}_{L\uparrow}a^{\dagger}_{R\downarrow} + a^{\dagger}_{L\downarrow}a^{\dagger}_{R\uparrow}|0\rangle,$$



$$T^{-1}\rangle \;=\; a^{\dagger}_{L\downarrow}a^{\dagger}_{R\downarrow}|0\rangle$$



Eigen energies

Singlet energies are the eigenvalues of the 3x3 matrix in the basis of (|S(1,1)>, |S(2,0)>, |S(0,2)>).

$$\mathcal{H}_S = \begin{pmatrix} 0 & \sqrt{2}t & \sqrt{2}t \\ \sqrt{2}t & U-V+\varepsilon & 0 \\ \sqrt{2}t & 0 & U-V-\varepsilon \end{pmatrix}$$

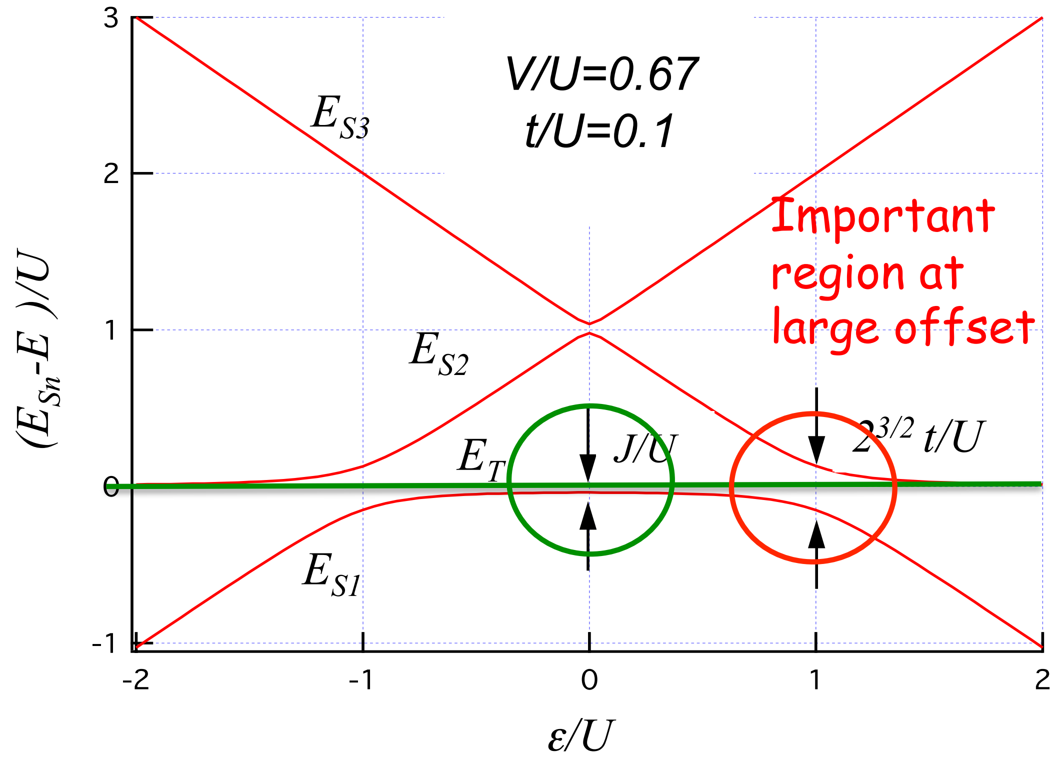
Triplet energies are degenerate up to the Zeeman energy

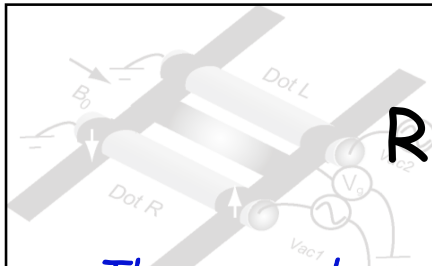
$$E_T = V$$

Exchange energy is defined by the difference of energy of spin triplet to spin singlet ground states

$$J \sim \frac{4t^2}{U}$$

for $|\varepsilon| << U-V$

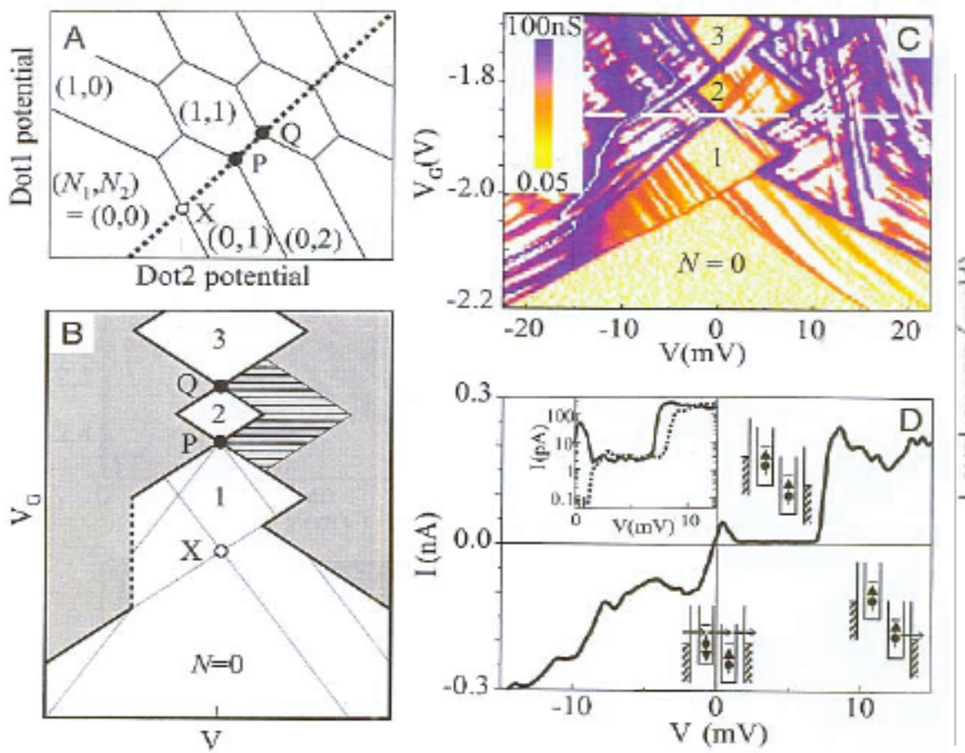


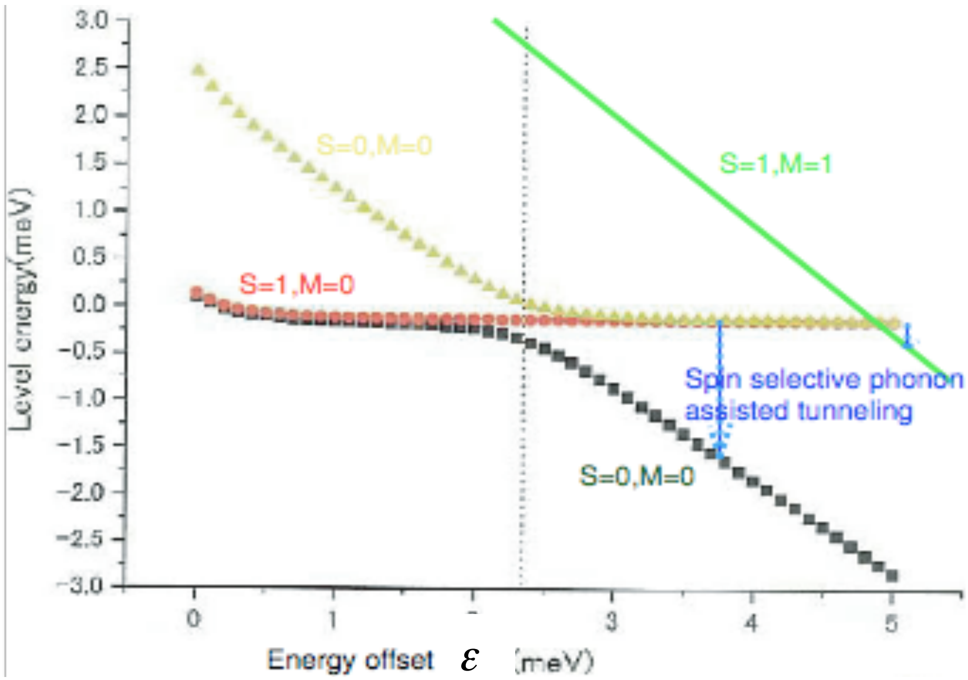


Region of large offset: Pauli blockade

The very slow relaxation process from spin triplet to spin singlet is the origin of Pauli blockade.

Result of exact diagonalization



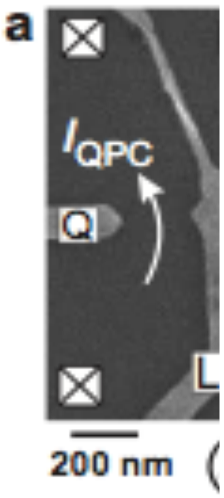


K. Ono et al., Science ***297****, 1313 (2001).*

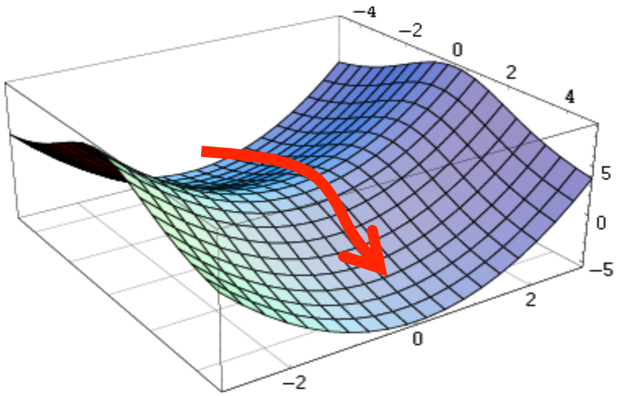
Y. Tokura unpublished (2001).

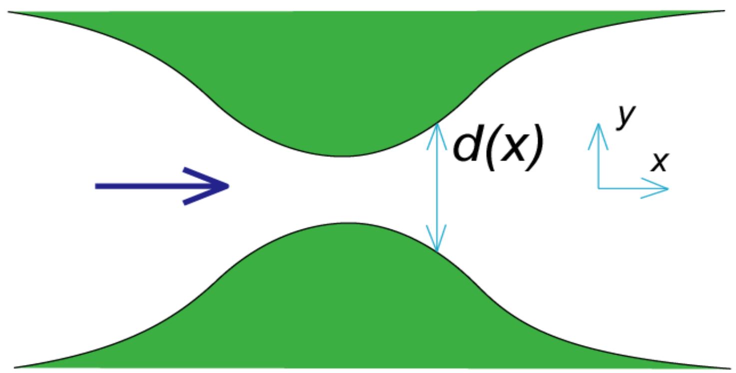
Quantum point contact (QPC)

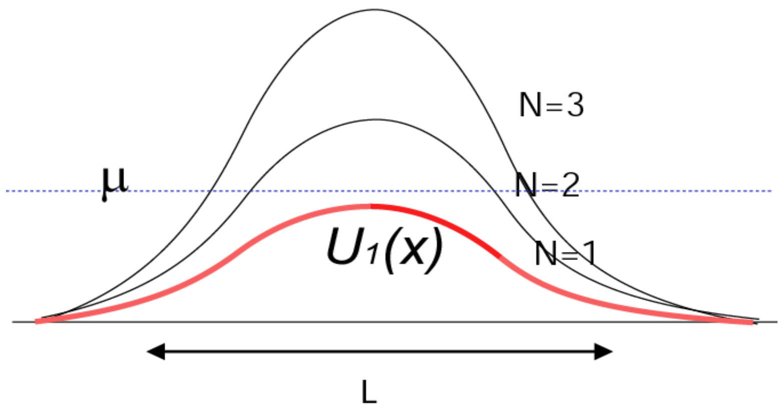
Quantum point contact (QPC) is a vary short and narrow constriction.



Landscape near QPC is the saddle point potential.

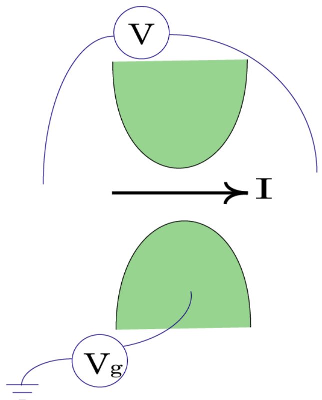


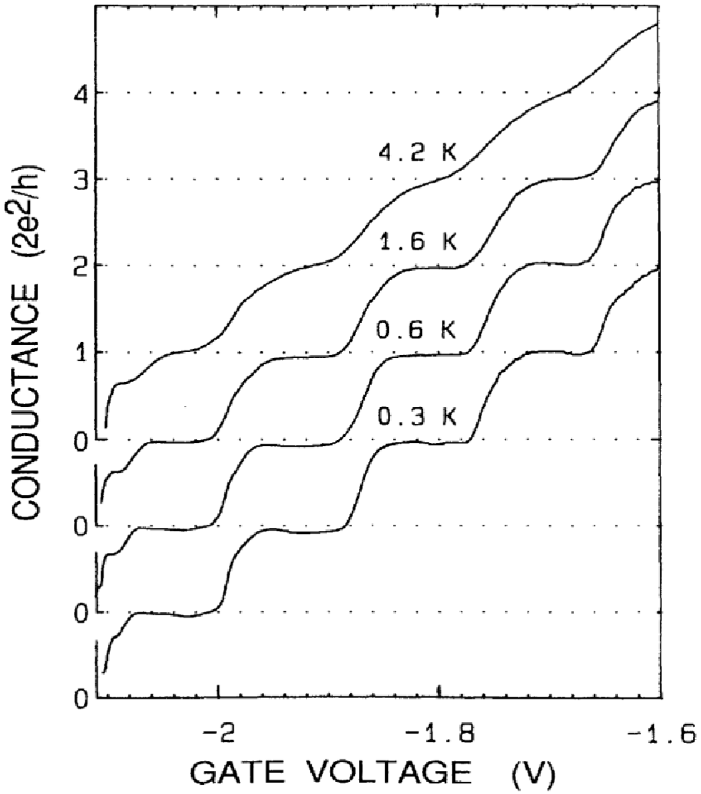


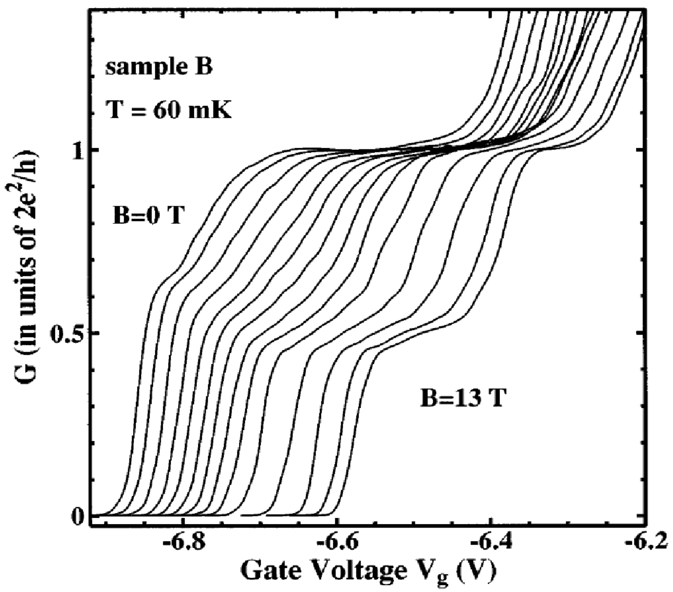


都倉康弘、固体物理 37 (2002) 363.

Condactance quantization



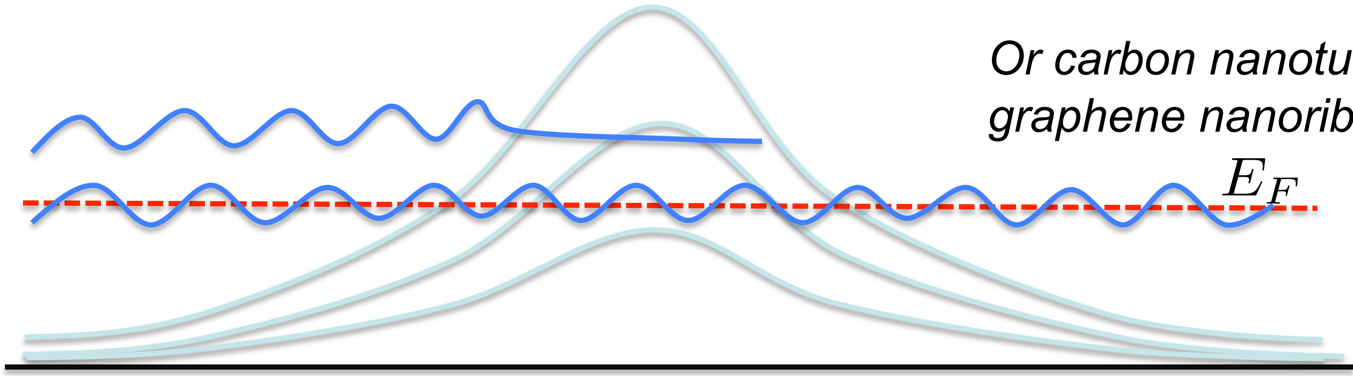




B.J.van Wees, et al, Phys. Rev. ***B 43****, 12431 (1991).*

K.J.Thomas, et al, Phys. Rev. Lett. 77, 135 (1996).

Adiabatic transport - no channel mixing



Or carbon nanotube, graphene nanoribbon,…

Conductance Landauer's formula

$$G = \frac{2e^2}{h}\sum_n T_n$$

Transmission probability of mode n T_n

$T_n = 1$ *Noiseless mode*

Statistics of transmitted current

Input flux from degenerate Fermi sea with bias voltage V_{SD}:

$$J = e v_F (e V_{SD}) \rho_F = \frac{e^2 V_{SD}}{\pi \hbar}$$

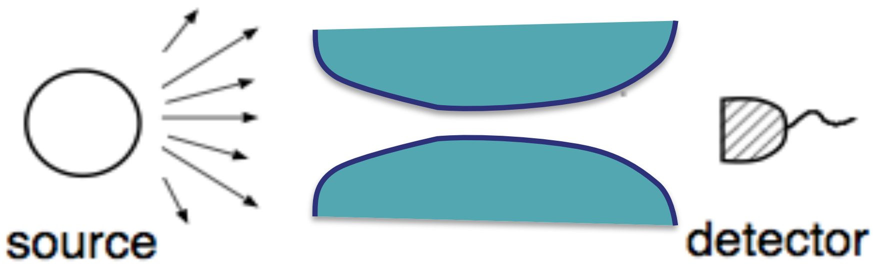
Here we used the density of states at the Fermi energy

$$\rho_F = \frac{2}{2\pi} \frac{\partial k}{\partial E} = \frac{1}{\pi \hbar v_F}$$

the Fermi velocity v_F

spin

Counting statistics



For each input electron, there are only two possible events, Transmitted (+) and Reflected (-).

For N inputs (in other words, for a finite time τJ), one possible series of events is (+,+,+,-,-,+,-,+,-,-,-,-).

N *# of "+" is Q*

The probability of this particular series, where Q out of N electrons are transmitted, is given by Binomial distribution function

$$P_N(Q) = \frac{N!}{Q!(N-Q)!}\mathcal{T}^Q(1-\mathcal{T})^{N-Q}$$

Characteristic function

It is very useful to introduce the characteristic function, $C_N(\lambda)$, with a counting field λ:

$$
\begin{aligned}
C_N(\lambda) &= \sum_{Q=0}^{N} P_N(Q) e^{-i\lambda Q} \\
&= (1-\mathcal{T})^N + N(\mathcal{T}e^{-i\lambda})(1-\mathcal{T})^{N-1} \\
&\quad + \frac{N(N-1)}{2!}(\mathcal{T}e^{-i\lambda})^2(1-\mathcal{T})^{N-2} + \cdots + (\mathcal{T}e^{-i\lambda})^N \\
&= (1-\mathcal{T}+\mathcal{T}e^{-i\lambda})^N.
\end{aligned}
$$

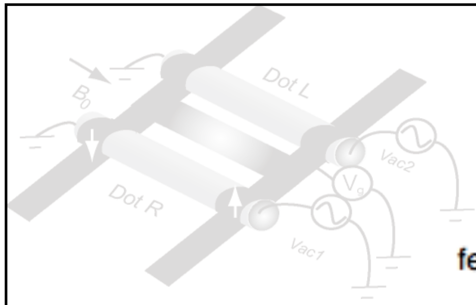
With this function, we can obtain l-th cumulant

$$
\langle\langle Q^\ell \rangle\rangle = i^\ell \frac{d^\ell}{d\lambda^\ell} \ln C_N(\lambda)|_{\lambda=0}
$$

For example, the average and its variance are

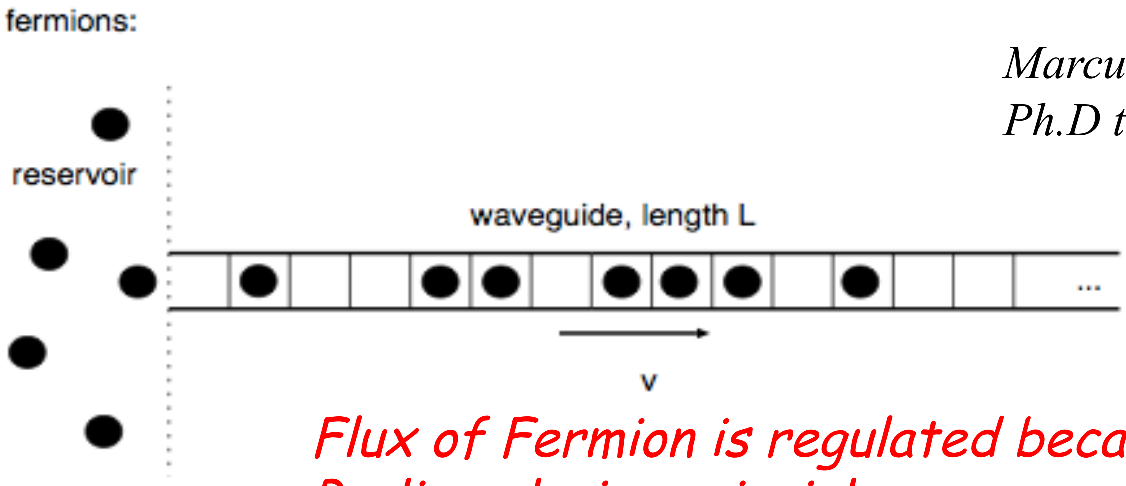
$$
\begin{aligned}
\langle\langle Q \rangle\rangle &= N\mathcal{T} \\
\langle\langle Q^2 \rangle\rangle &\equiv V = N\mathcal{T}(1-\mathcal{T}) \\
&= \langle Q^2 \rangle - \langle Q \rangle^2
\end{aligned}
$$

Zero at T=1!

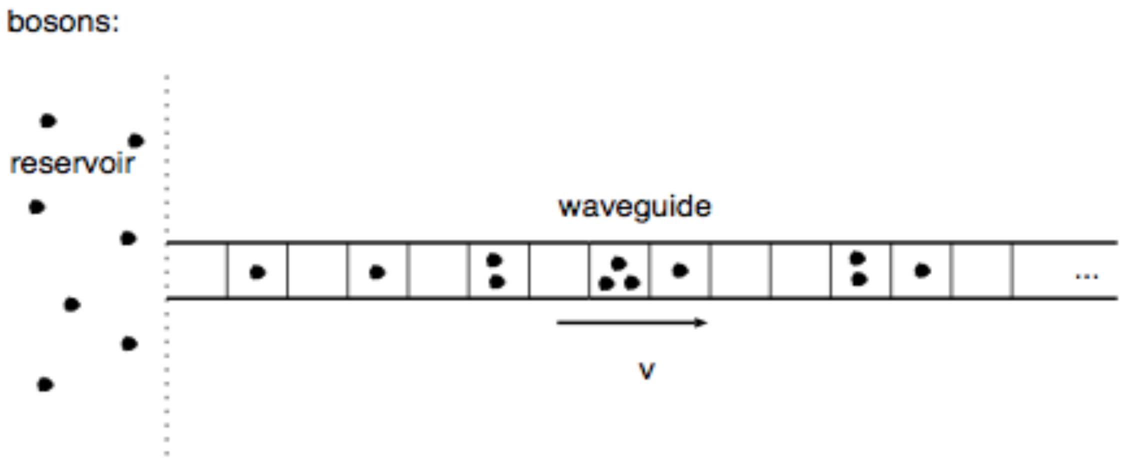


Dependence on statistics

Marcus Kindermann, Ph.D thesis



Flux of Fermion is regulated because of Pauli exclusion principle.



Shot noise

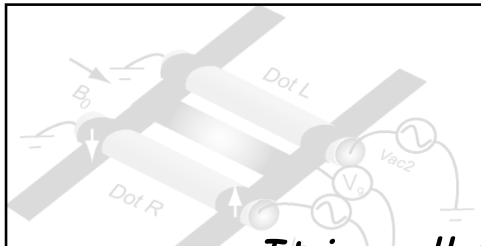
Zero frequency current noise for $eV_{SD} >> k_B T$ *is given by the variance for large* τ

$$N = \tau J \qquad V = N\mathcal{T}(1-\mathcal{T})$$

$$S \sim \frac{V}{\tau} = J\mathcal{T}(1-\mathcal{T}) = \langle I \rangle (1 - T)$$

Average current is given by Landauer's formla: $\langle I \rangle = J\mathcal{T}$

L. S. Levitov and G. B. Lesovik, JETP Lett. 58, 230 (1993).



Poisson distribution

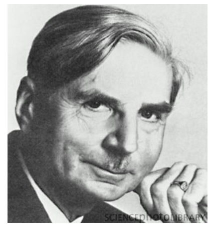
It is well-known that the Binomial distribution becomes Poisson distribution by letting N large while keeping average <Q>=NT finite:

$$
\begin{aligned}
C_N(\lambda) &= (1-\mathcal{T}+\mathcal{T}e^{-i\lambda})^N \\
&= (1+\frac{(N\mathcal{T}(e^{-i\lambda}-1)}{N})^N \\
\to_{N\to\infty} & \quad e^{N\mathcal{T}(e^{-i\lambda}-1)} \equiv C_P(\lambda)
\end{aligned}
$$

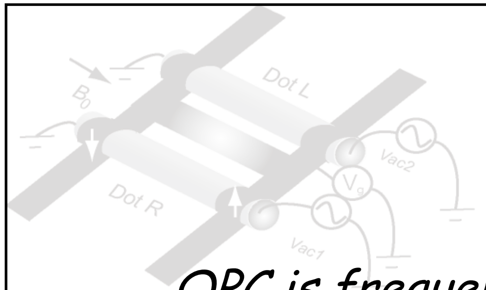
Then all the cumulants are identical.

$$
\begin{aligned}
\langle\langle Q^\ell \rangle\rangle &= i^\ell \frac{d^\ell}{d\lambda^\ell}[N\mathcal{T}(e^{-i\lambda}-1)]|_{\lambda=0} \\
&= N\mathcal{T} i^\ell \frac{d^\ell e^{-i\lambda}}{d\lambda^\ell} \\
&= N\mathcal{T}
\end{aligned}
$$

$$S \sim \langle I \rangle$$



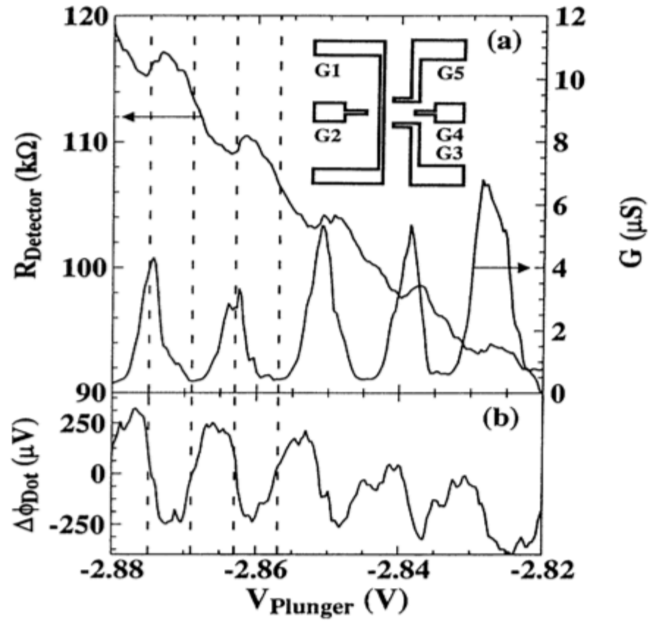
Walter Schottky (1918)



QPC Charge detection

QPC is frequently used as a sensitive charge detector since the current changes with the potential barrier.

M. Field, et al., Phys. Rev. Lett. 70, 1311 (1993).



Necessary condition to the time required to distinguish the change of the QPC current by the change of transmission.

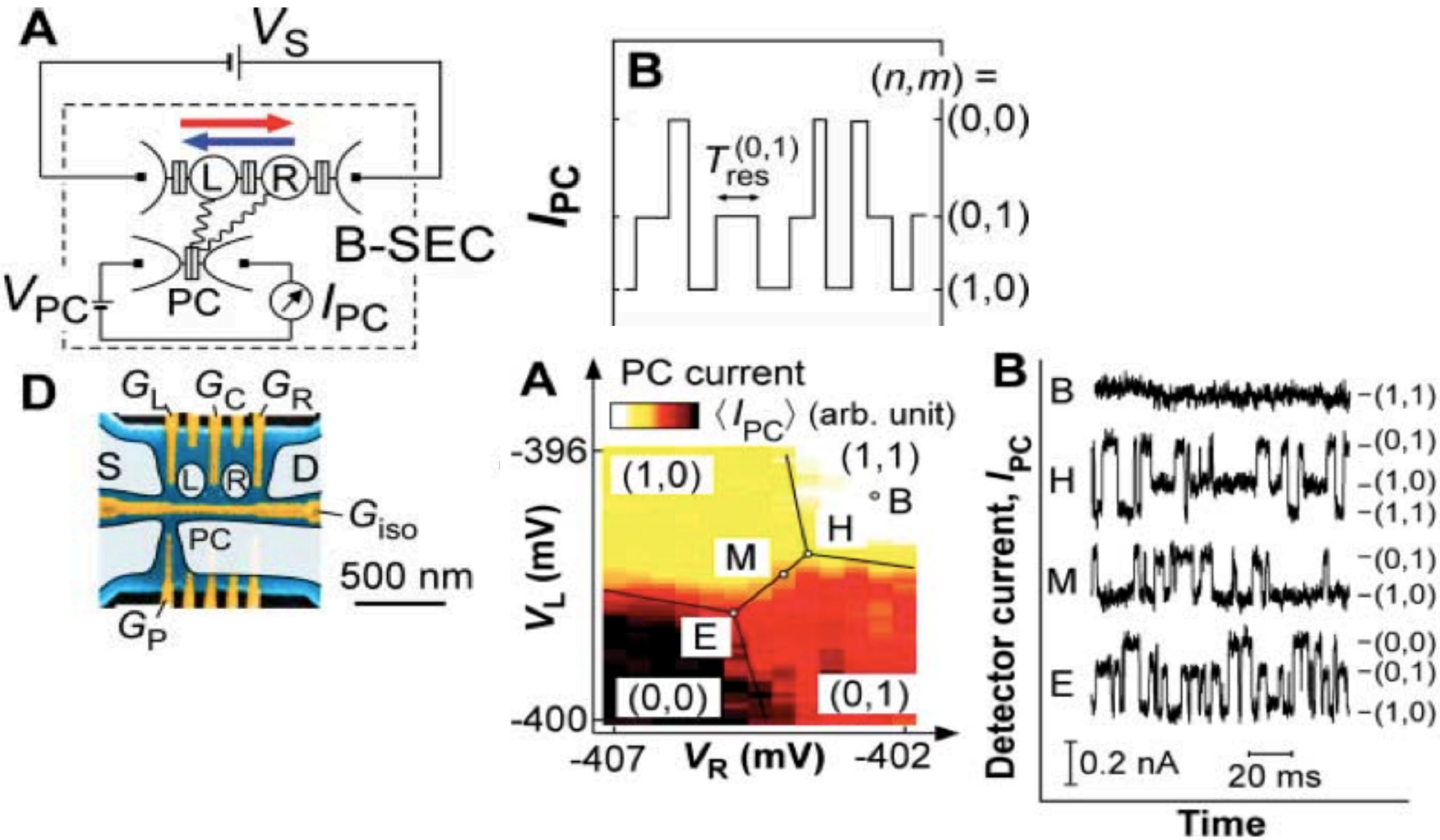
$$t_d \frac{eV_{SD}}{\pi\hbar}\Delta\mathcal{T} \geq \sqrt{t_d \frac{eV_{SD}}{\pi\hbar}\mathcal{T}(1-\mathcal{T})} \Rightarrow \frac{1}{t_d} \sim \frac{eV_{SD}}{h}\frac{(\Delta\mathcal{T})^2}{\mathcal{T}(1-\mathcal{T})}$$

Change of transferred charge

Fluctuation

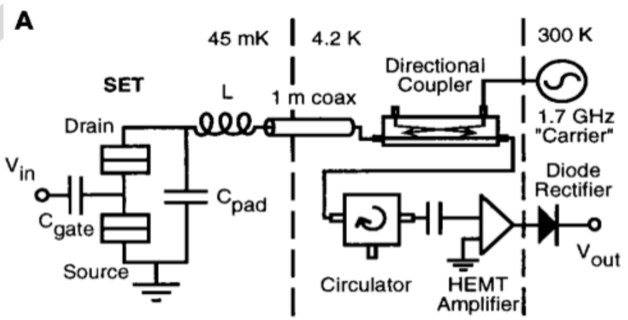
I. L. Aleiner, et al., Phys. Rev. Lett. 79, 3740 (1997).

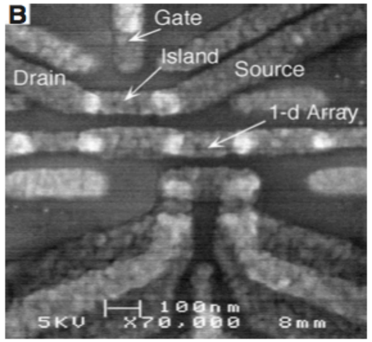
Counting electrons



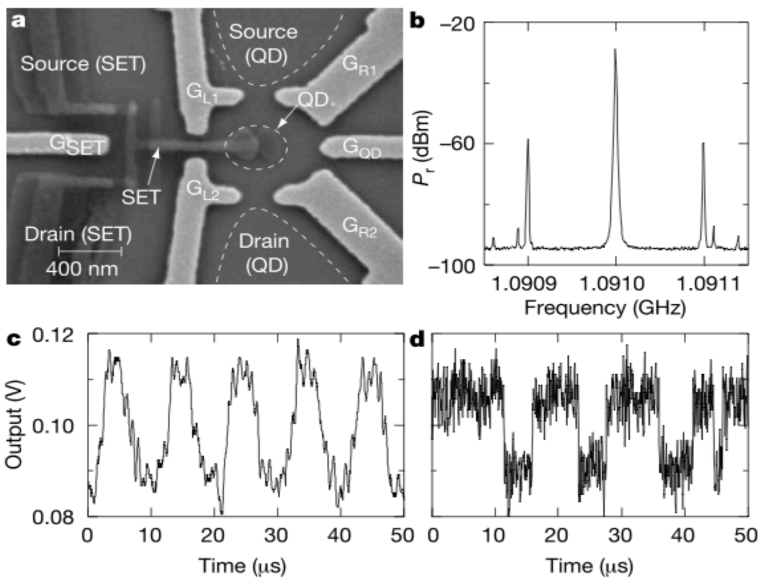
T. Fujisawa, et al., Science 312, 1634 (2006).

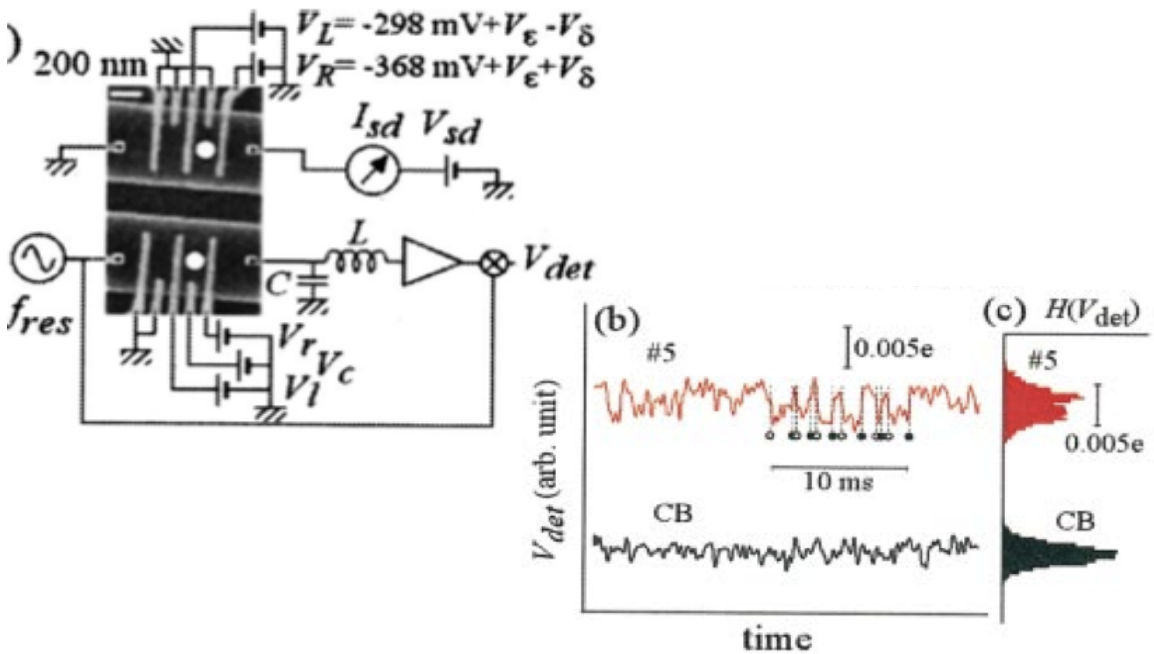
Radio-frequency(rf)-SET





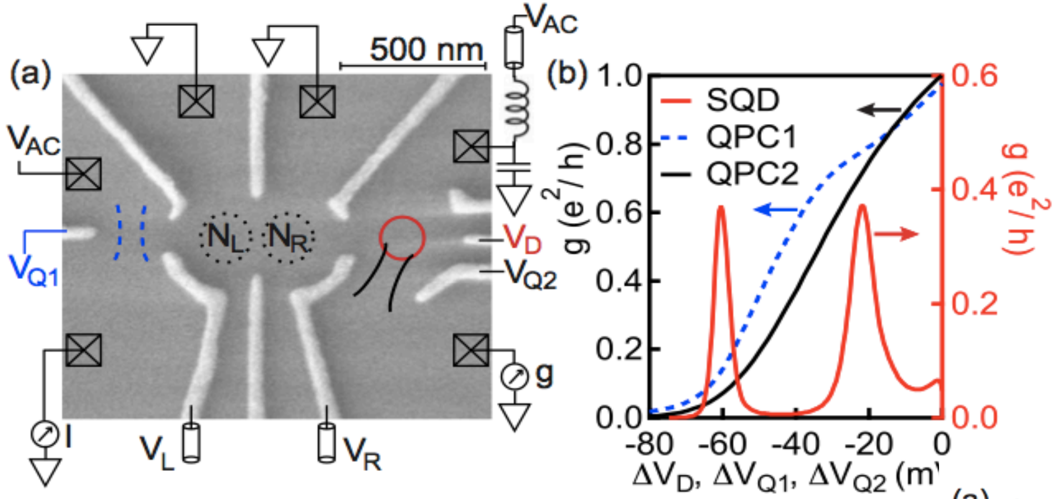
R. J. Schoelkopf, et al., Science 280, 1238 (1998).



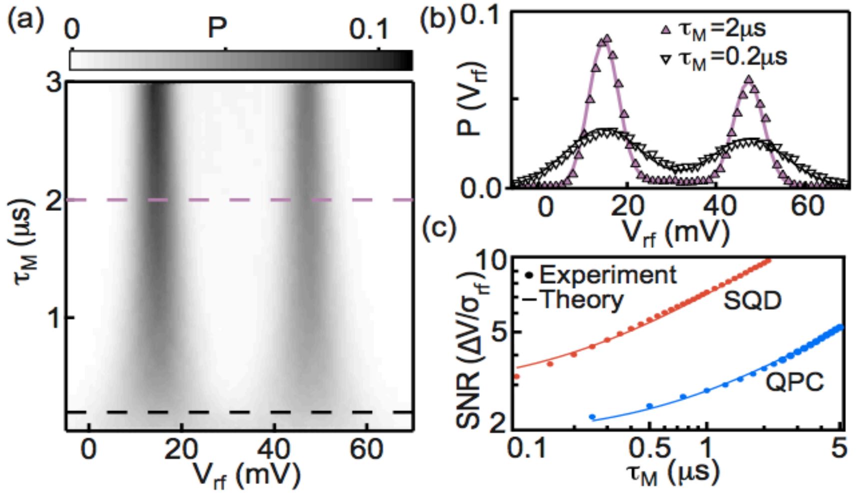


Wei Lu, et al., Nature 425, 422 (2003). *T. Fujisawa, et al., Appl. Phys. Lett. 84, 2343 (2003).*

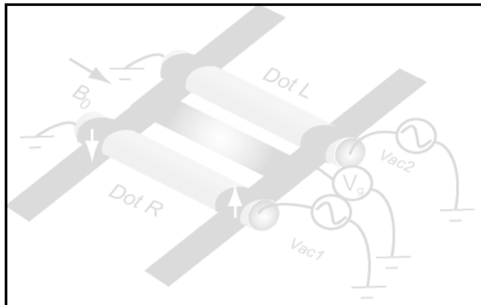
Comparison of QPC/QD detectors



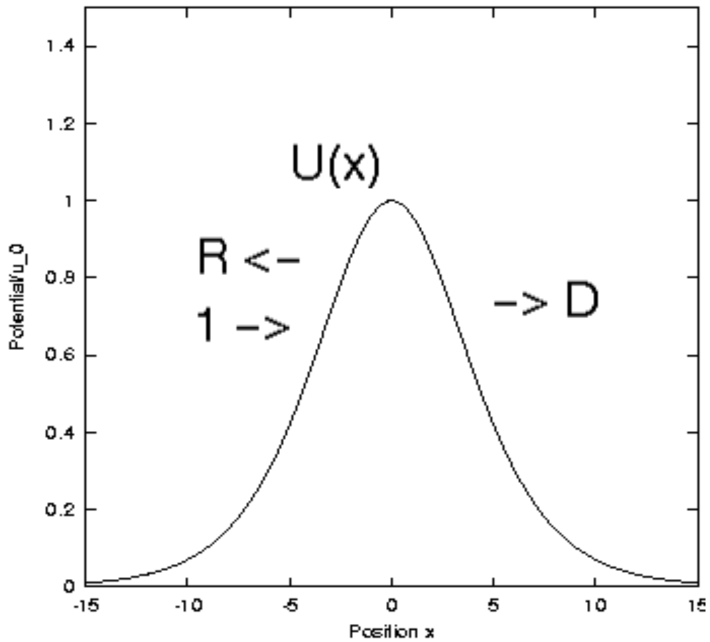
The sensitivity of QD charge detector is superior to that of QPC.



C. Barthel, et al., Phys. Rev. B 81, 161308 (2010).


Dot L
Dot R
Vac2
Vac1

End of Part I


U(x)
R <-
1 ->
-> D
Potential/u_0
Position x
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-15
-10
-5
0
5
10
15