FIRST Quantum Information Processing Project
Summer School 2012

18 August 2012 Miyakojima

Quantum Simulation Using Ultracold Atoms

Kyoto University

Y. Takahashi

Introduction

Education：

Ohta High-School

Kyoto University,　Faculty of Science

Kyoto University,　Graduate School of Science

Degree:

Anomalous Behavior of Raman Heterodyne Signal in $Pr^{3+}:LaF_3$

Employment：

Kyoto University,

Research Associate: Atoms in Superfluid Helium

Lecturer: Photo-excited triplet DNP

Associate Professor: Laser Cooling

Professor: Optical Lattice

趣味　散歩

Introduction

Current Research Interest:

Quantum Information Science Using Cold Atoms

Quantum Simulation (of Hubbard Model)

Spin Squeezing by QND Measurement

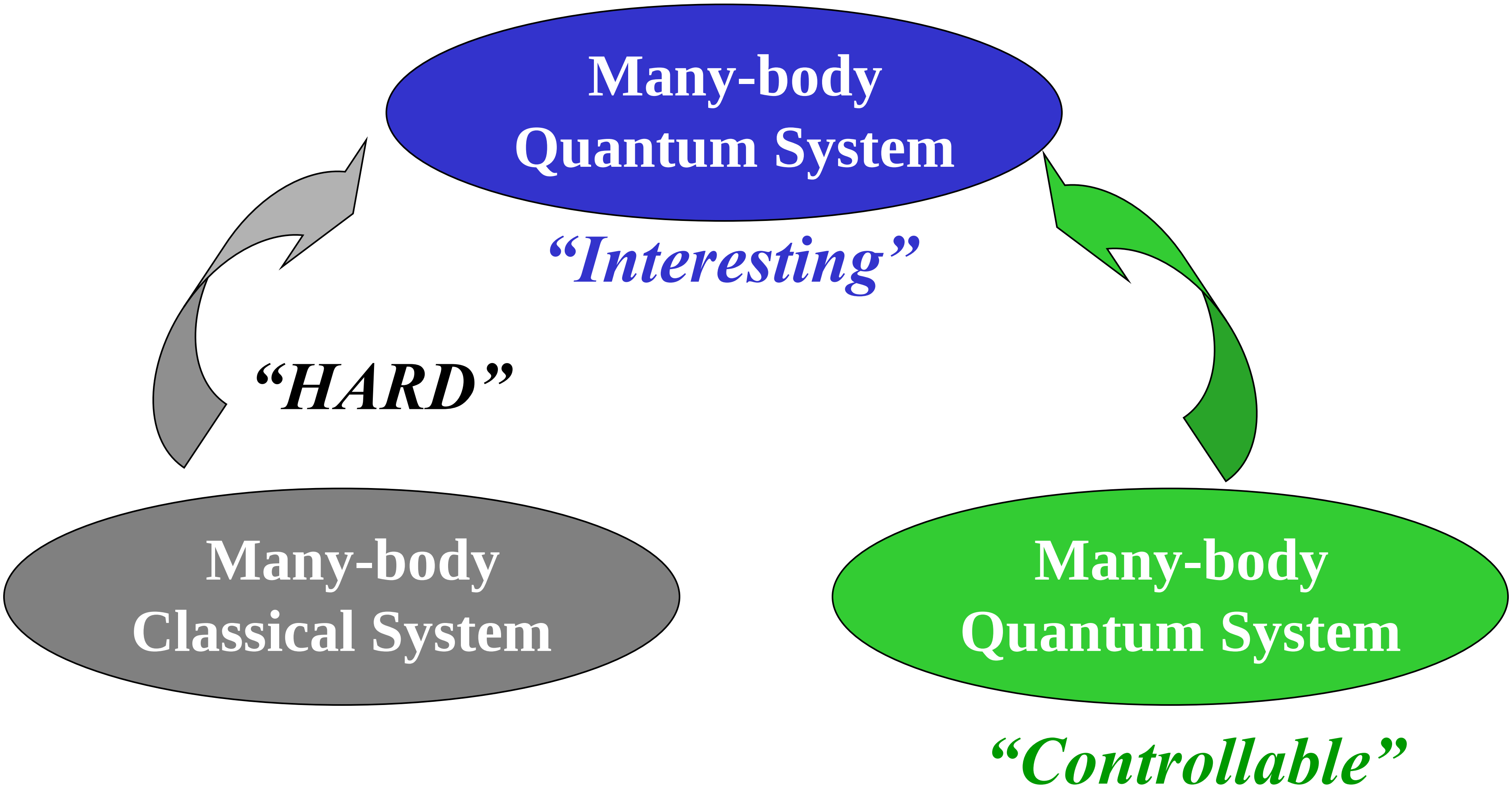
Fundamental Physics Using Cold Atoms or Molecules:

Searching for Permanent Electric Dipole Moment

Test of Newton Gravity at Short Distance:

$$V = -G\frac{M_1 M_2}{r}(1+\alpha\exp(-\frac{r}{\lambda}))$$

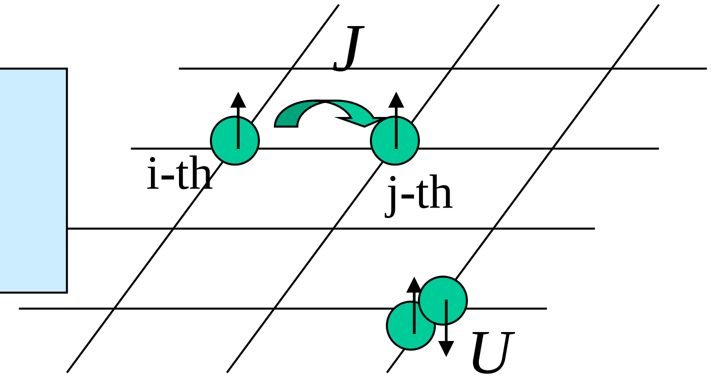
Quantum Simulation



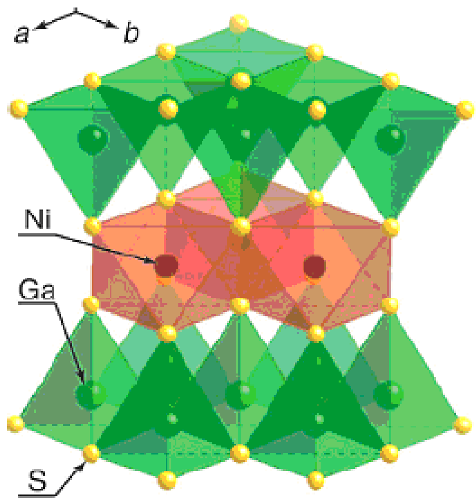
Quantum Simulation

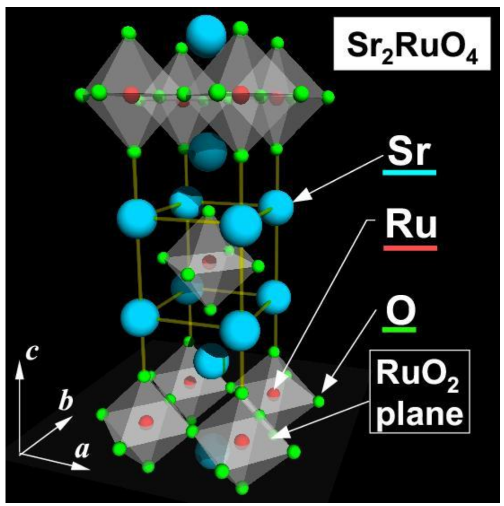
Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Magnetism, Superconductivity

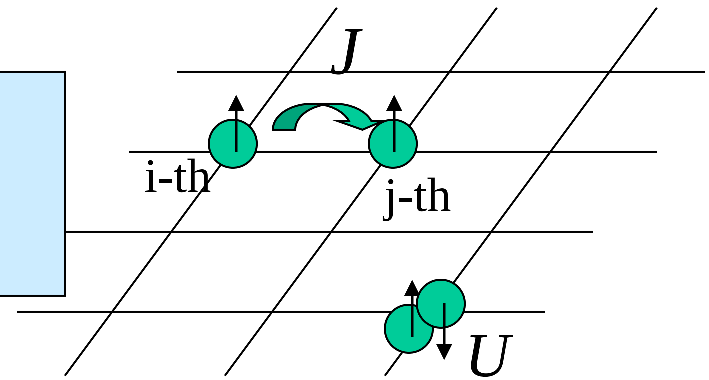




Quantum Simulation

Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



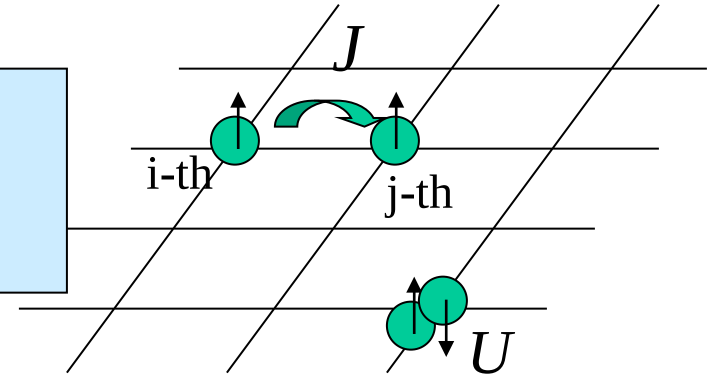
→ Numerical Calculation

DMFT(動的平均場)
Gutzwiller
QMC(量子モンテカルロ)
DMRG(密度行列繰り込み群)
Exact Diagonalization (厳密対角化)

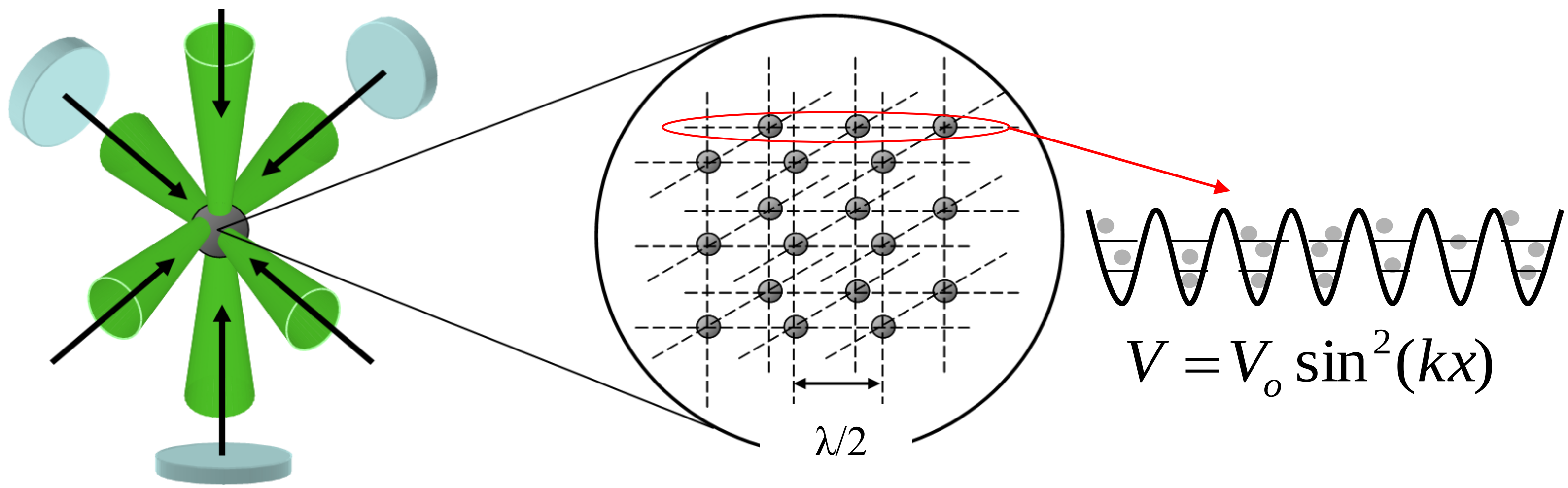
Quantum Simulation

Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Cold Atoms in Optical Lattice



$$V = V_o \sin^2(kx)$$

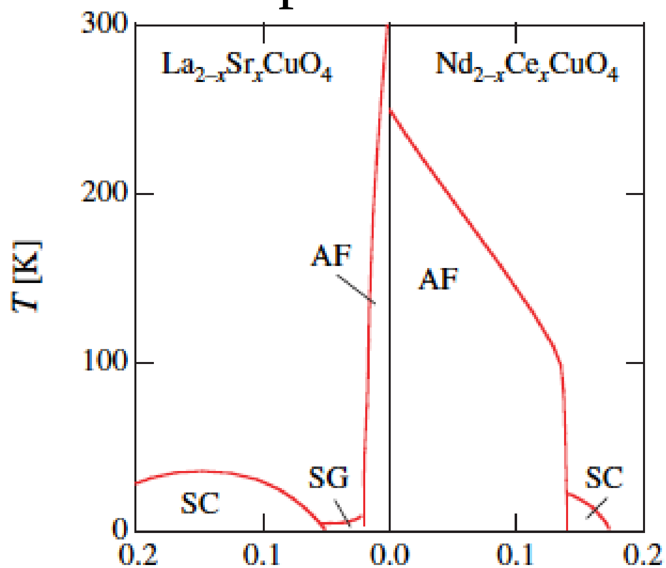
→ also “exciton-polariton”

Resolving controversy

"Phase Diagram of High-T_c Cuprate Superconductor"

[from T. Moriya and K. Ueda, Rep. Prog.Phys.66(2003)1299]

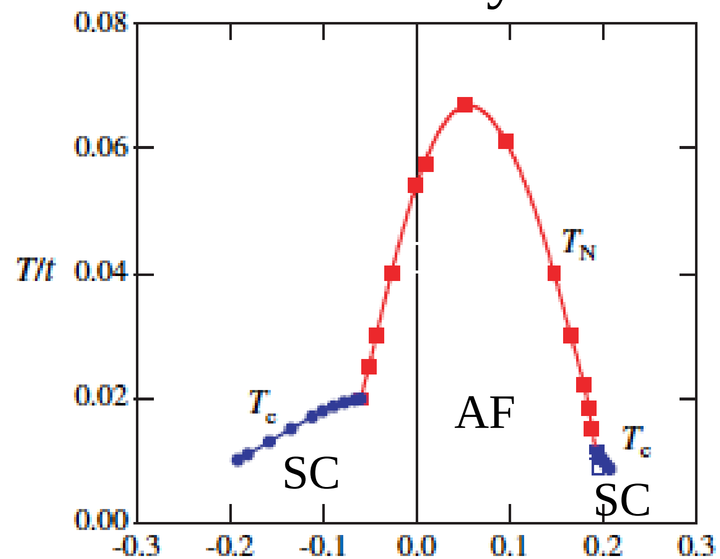
experiment



hole ← x → electron

(carrier doping)

theory

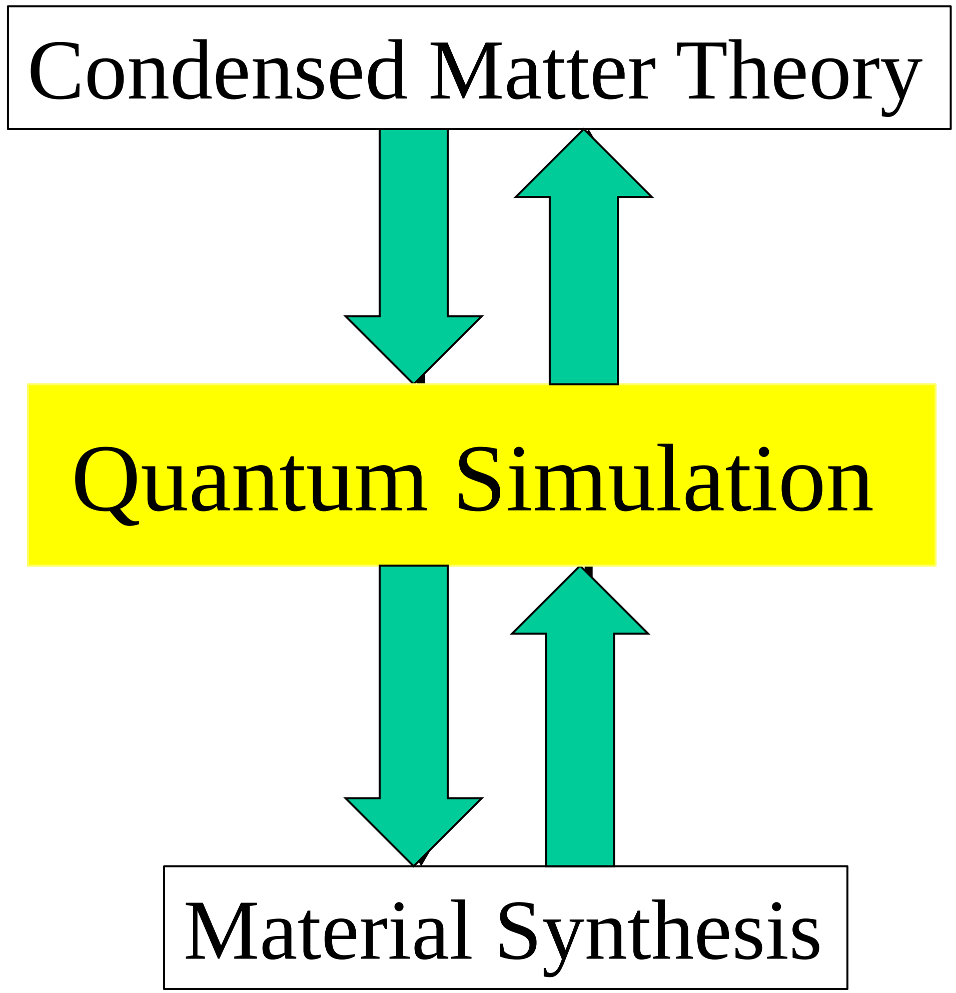


hole ← x → electron

(carrier doping)

Providing a Guideline for Material Synthesis

need not heavily rely on “emergence”



Outline

Atom Manipulation Technique

Optical Trapping, Optical Lattice, (anisotopy-induced)Feshbach Resonance

Bose-Hubbard Model

Superfluid-Mott Insulator Transition, Spectroscopy,
Quantum Gas Microscope

Fermi-Hubbard Model

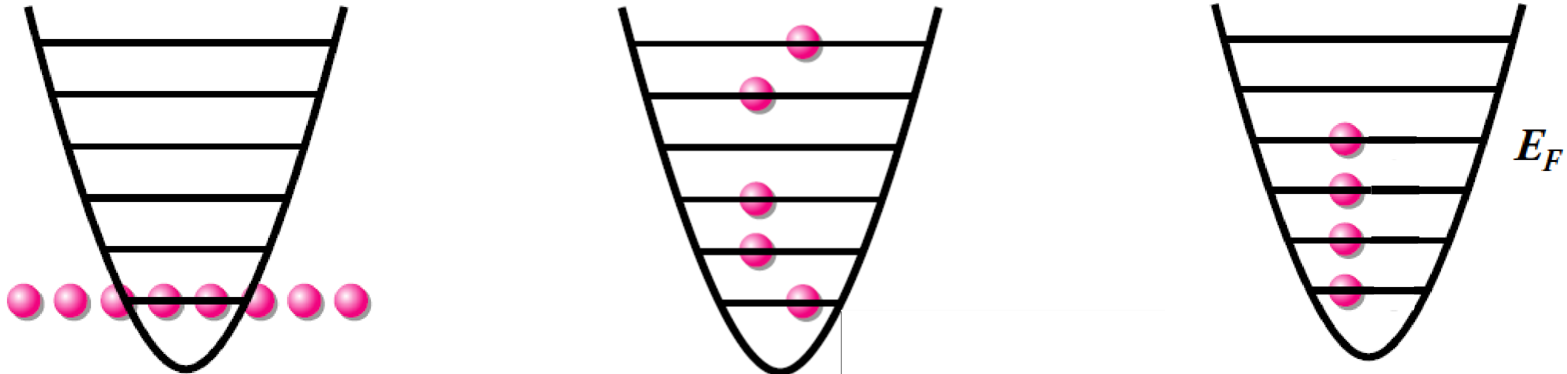
SU(2) & SU(6) Mott insulator, Pomeranchuk cooling

Bose-Bose/Bose-Fermi Hubbard Model

Anderson-Hubbard model, Dual Mott insulators

Atomic Gases Reach the Quantum Degenerate Regime

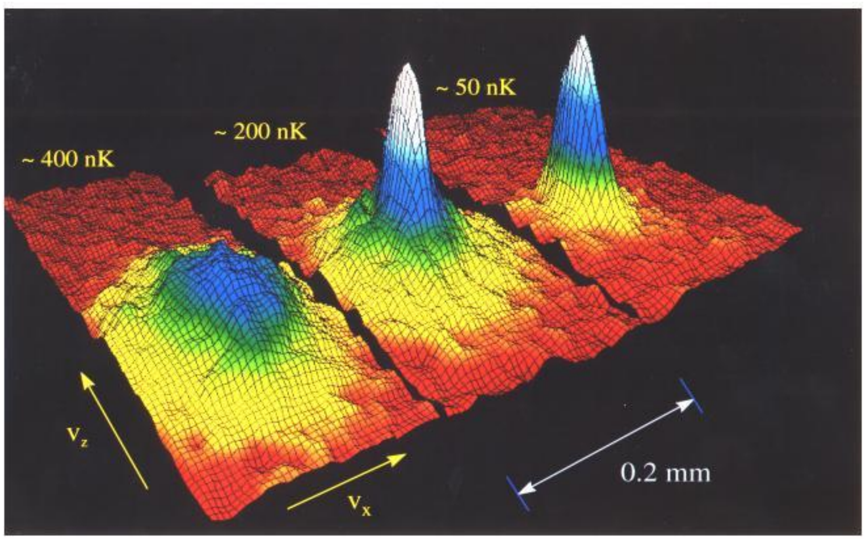
"Boson versus Fermion"



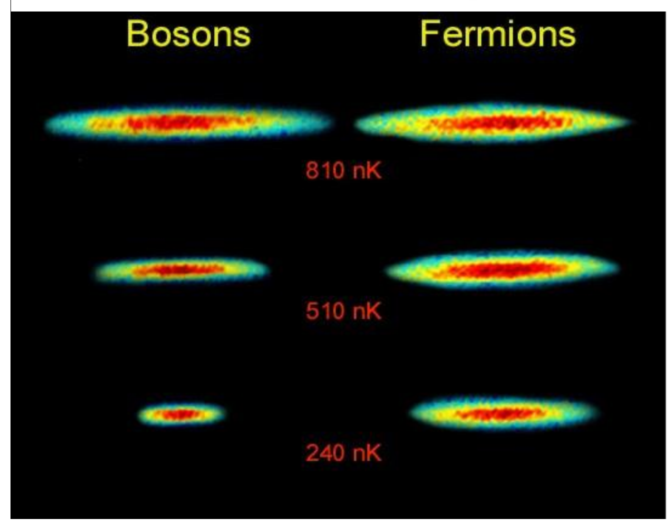
"Bose-Einstein Condensation"

"Fermi Degeneracy"

^{87}Rb



^{6}Li and ^{7}Li



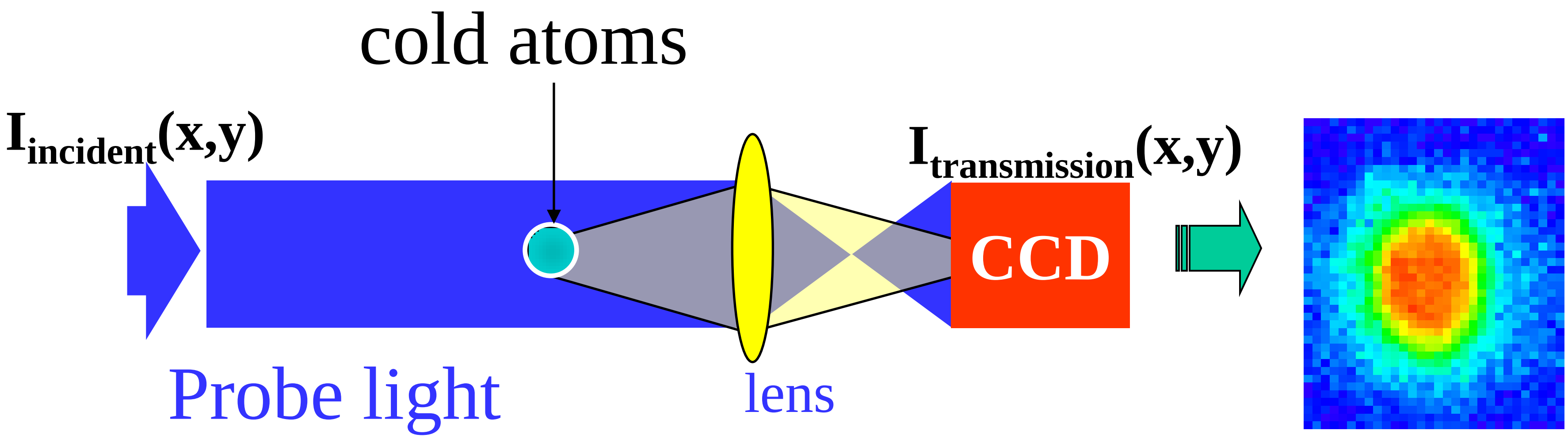
Momentum Distribution

[E. Cornell et al, (1995)]

Spatial Distribution

[R. Hulet et al, (2000)]

Optical Absorption Imaging of Atoms



- *In-Situ* Image: ⟶ Reflect "**density**" distribution in a trap

- Time-of-Flight Image: ⟶ Reflect "**momentum**" distribution in a trap

  t=0 release atoms from a trap

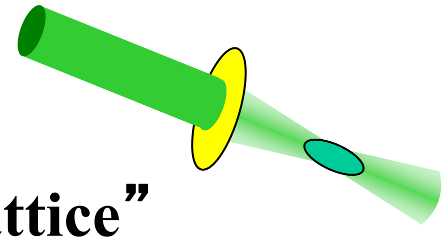
  $t=t_{TOF}$ observe atom density distribution

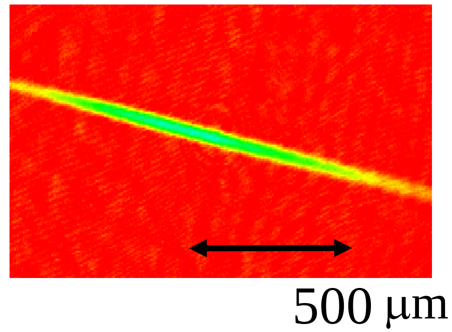
$$x = p / M \cdot t_{TOF}$$

Optical Trap & Optical Lattice

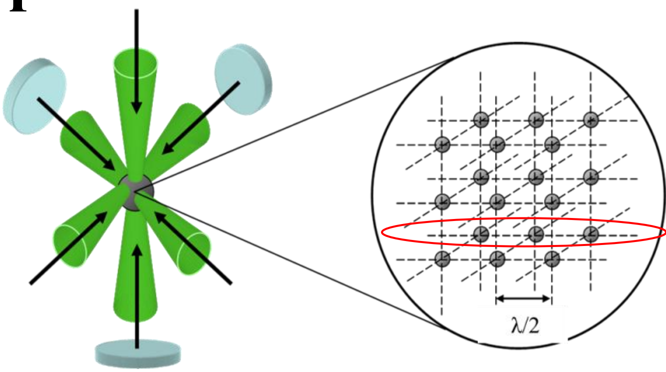
"optical trap"

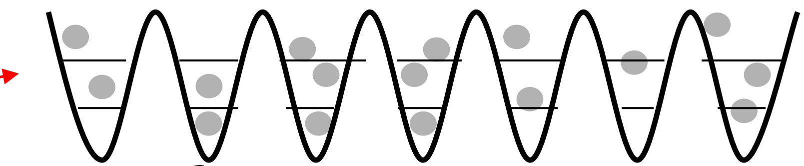
$$V_{\text{int}} = -p \cdot E \qquad U_{pot}(r) = -\frac{\chi E(r)^2}{2} \propto I(r)$$





"optical lattice"

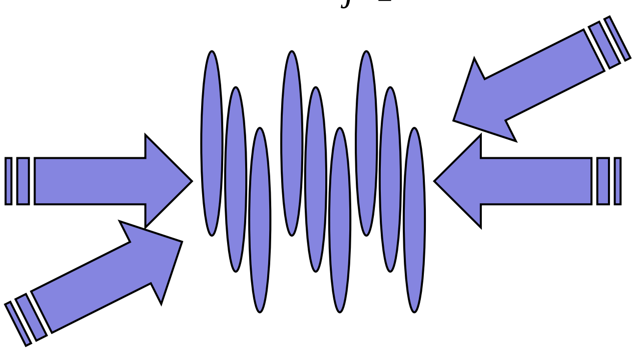




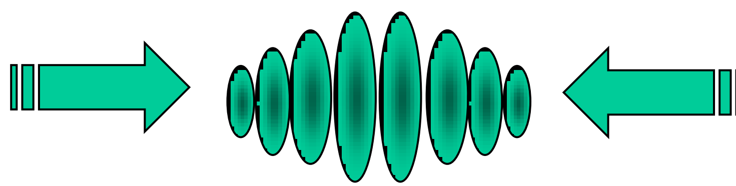
$$V_o(x) = V_o \sin^2(k_L x)$$

$$E_R = \frac{(\hbar k_L)^2}{2m}, s = \frac{V_0}{E_R}$$

$$V_o(\mathbf{x}) = \sum_{j=1}^{3} V_{oj} \sin^2(k_L x_j) = V_o \sum_{j=1}^{3} \sin^2(k_L x_j)$$



1D gas
(tube)



2D gas
(pancake)

band structure of square lattice

“tight-binding model”

$$H_0 = -J \sum_{i,j,\sigma=\uparrow,\downarrow} c^{+}_{i,\sigma} c_{j,\sigma} \longrightarrow H_0 = \sum_{k,\sigma=\uparrow,\downarrow} c^{+}_{k,\sigma} c_{k,\sigma} \varepsilon(k)$$

,where

$$\varepsilon(k) = -J \sum_{<i,j>} \exp(-ik\cdot(x_i - x_j))$$

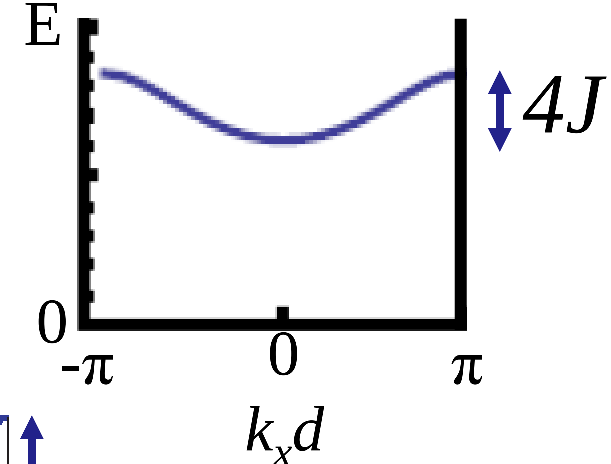
$$c_{j,\sigma} = \frac{1}{\sqrt{N}} \sum_k c_{k,\sigma} \exp(ik\cdot x_j)$$

$c_{k,\sigma}$: annihilation operator of atom with spin σ for the wavevector *k*

1D case:

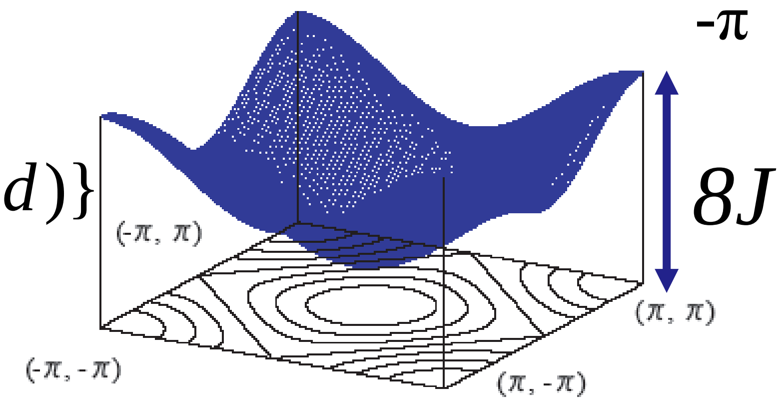
$$\varepsilon(k) = -J\{\exp(-ik_x d) + \exp(+ik_x d)\} = -2J\cos(k_x d)$$

(d :lattice constant)



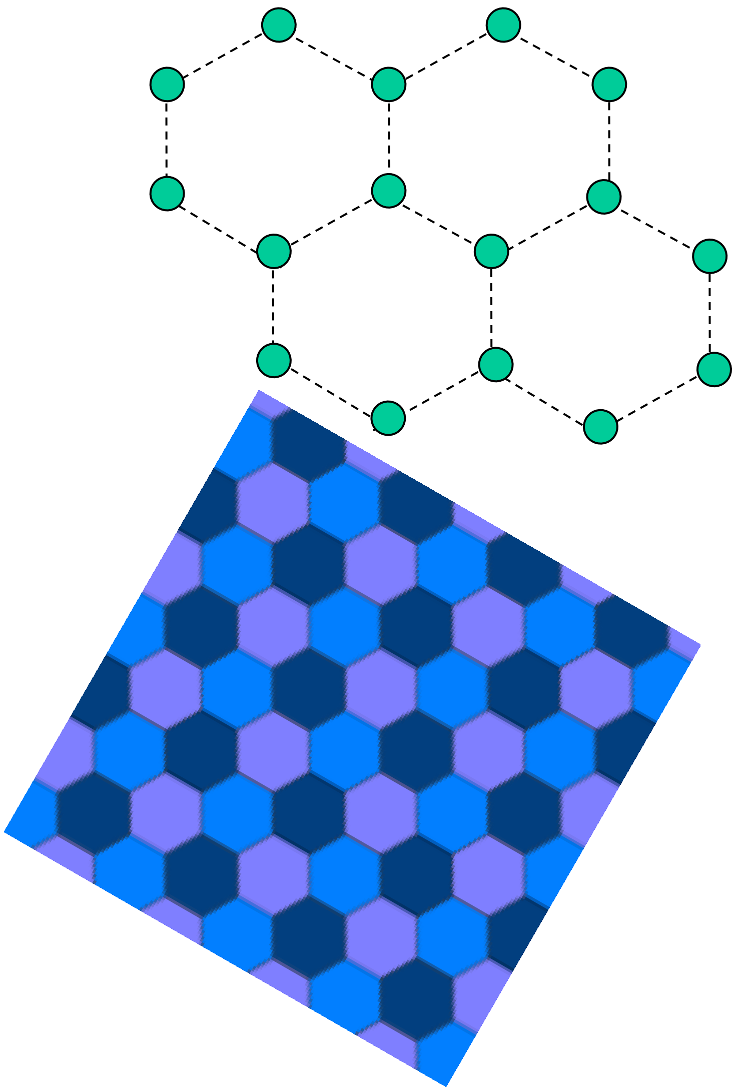
2D case:

$$\varepsilon(k) = -2J\{\cos(k_x d) + \cos(k_y d)\}$$

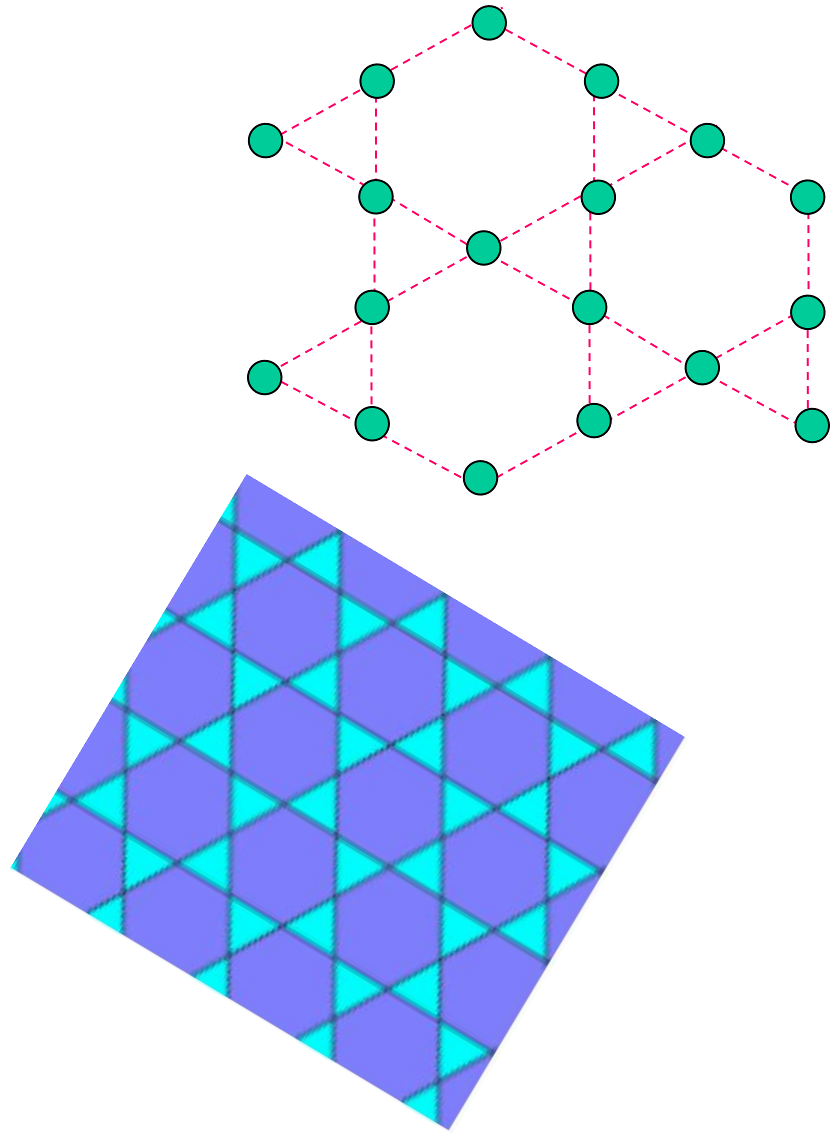


“Non-Standard Lattice”

Honeycomb (hexagonal)

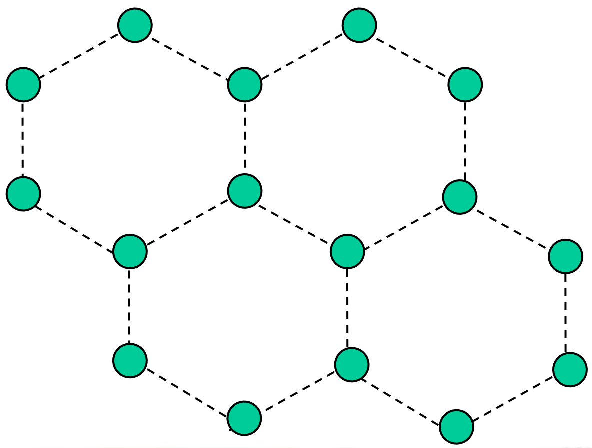


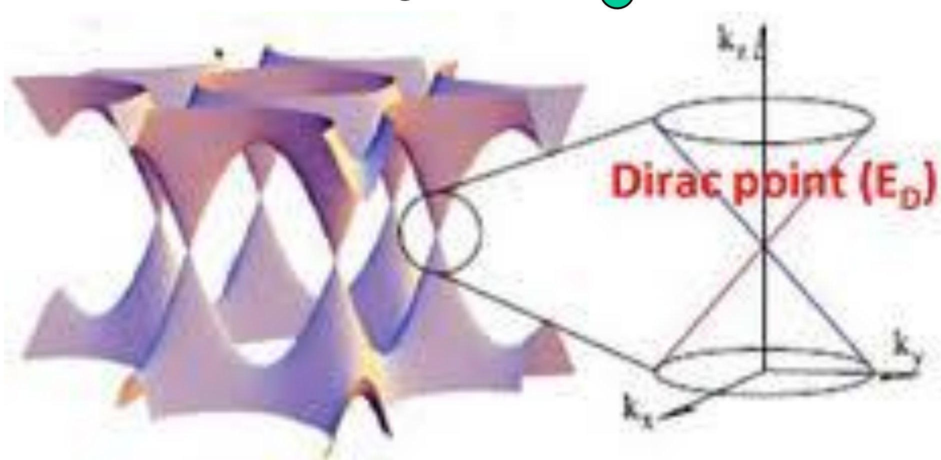
Kagome



“Non-Standard Lattice”

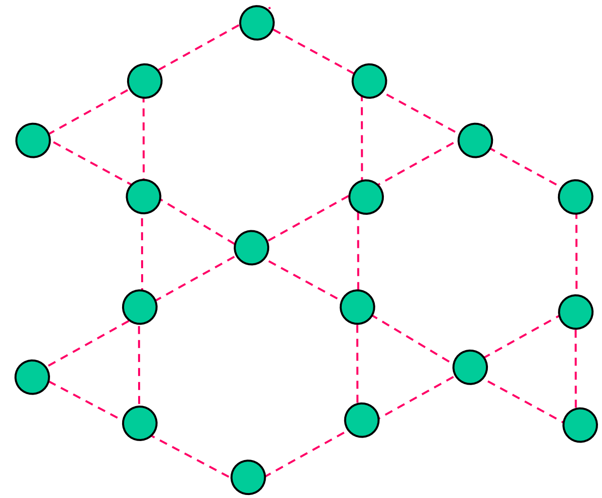
Honeycomb (hexagonal)

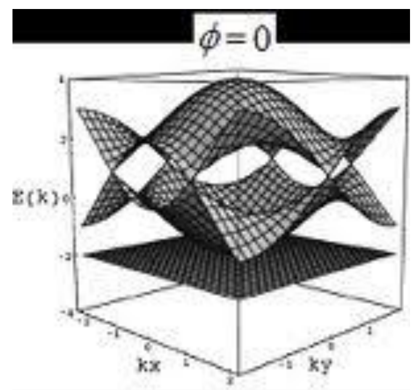




Dirac point
:Linear dispersion (realized in graphene)

Kagome



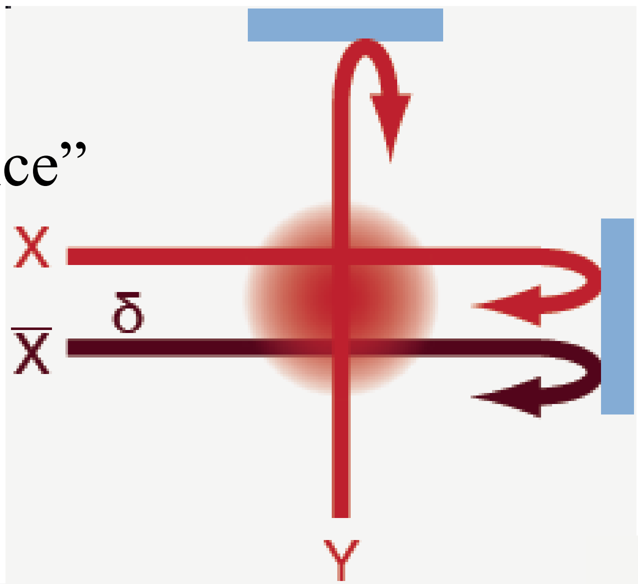


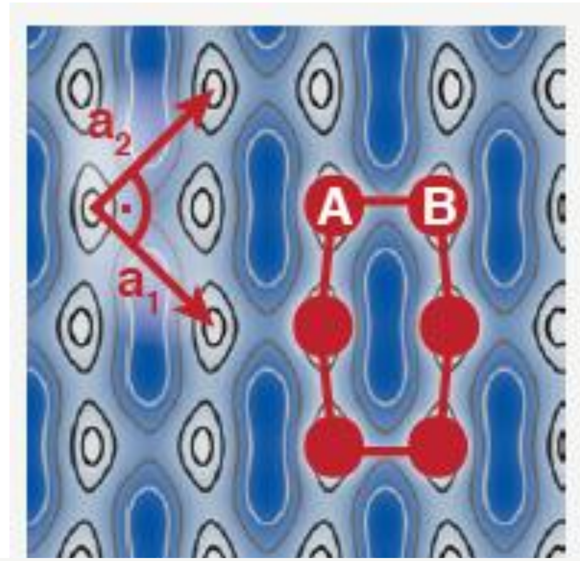
Flat-band:
    Itinerant ferromagnetism
Frustration

“Non-Standard Lattice-Honeycomb Lattice-”

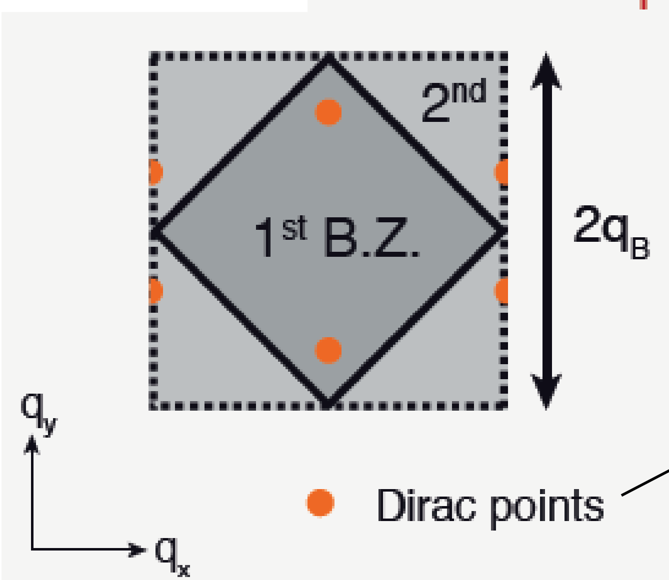
“Creating, moving, and merging Dirac points with a Fermi gas in a tunable honeycomb lattice”
arXiv. 1111.5020v1 L. Tarruell, *et al*

“Lasers for Optical Lattice”

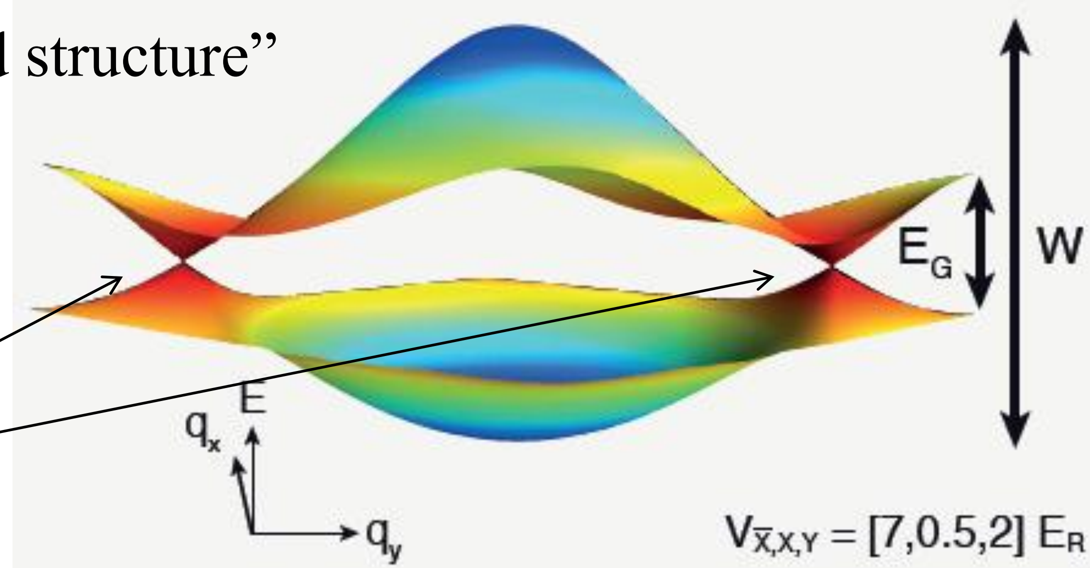




“Optical Lattice Potential”



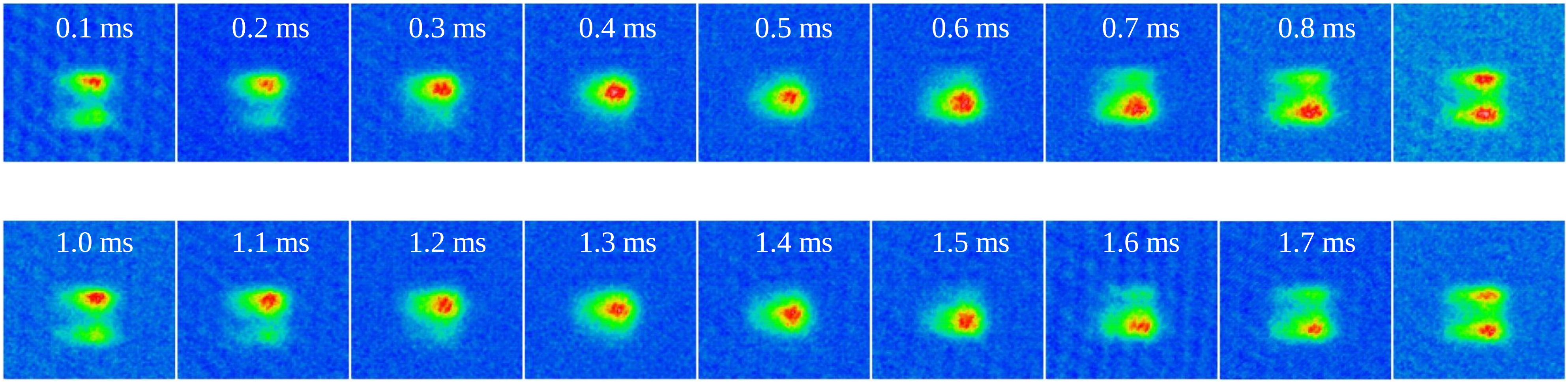
“band structure”



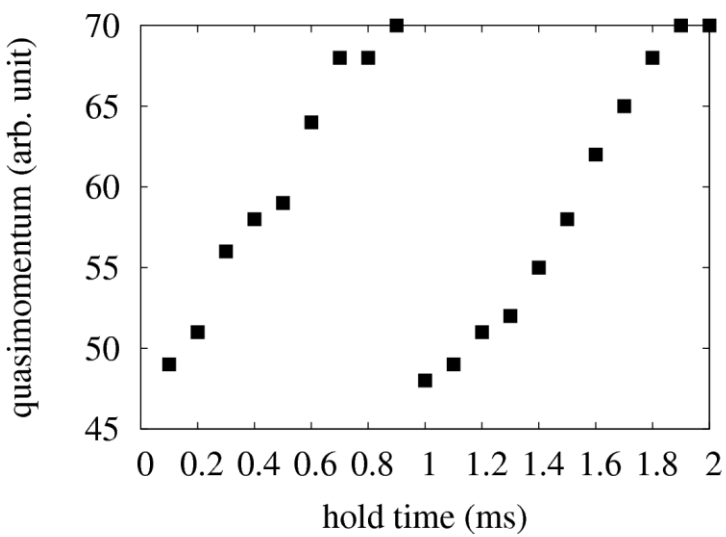
“Non-Standard Lattice-Honeycomb Lattice-”

“Creating, moving, and merging Dirac points with a Fermi gas in a tunable honeycomb lattice”
arXiv. 1111.5020v1 L. Tarruell, *et al*

“Performing Bloch Oscillation” $\frac{dq}{dt} = F$



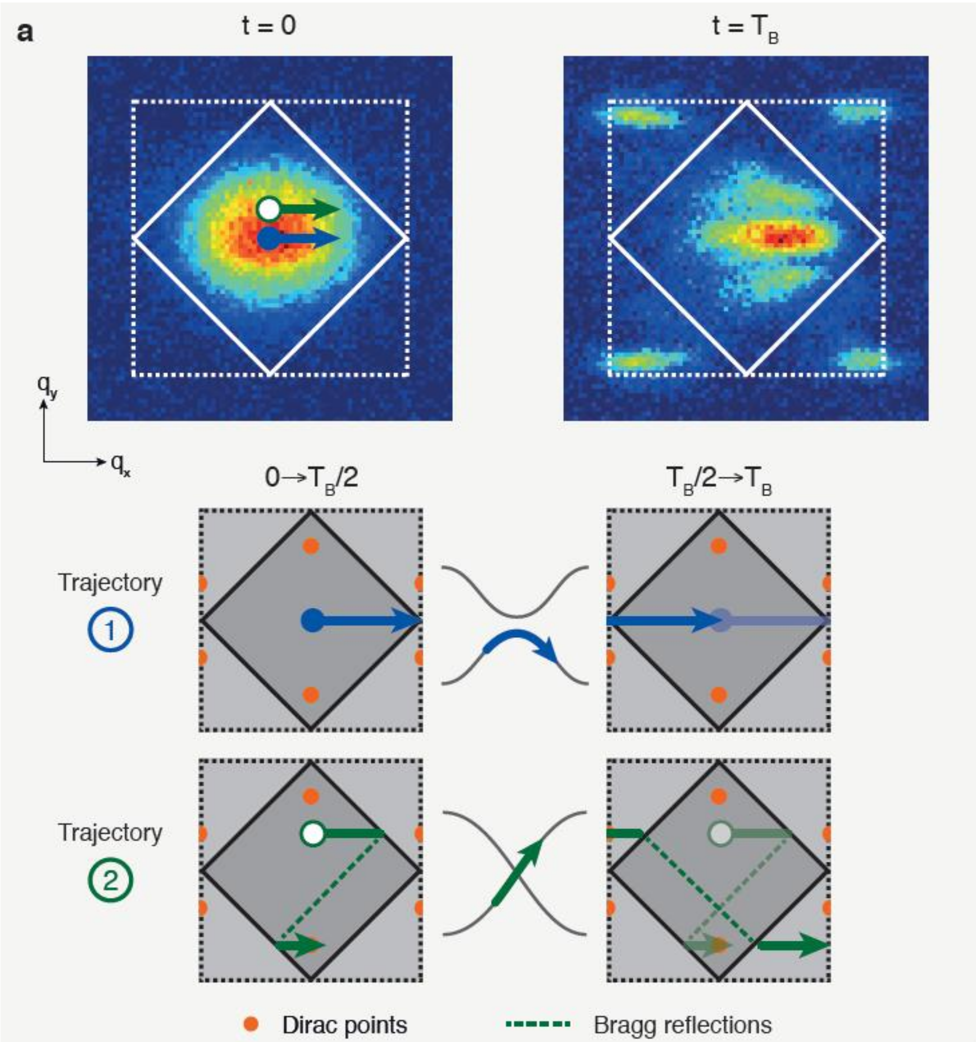
^{171}Yb :3E$_R$

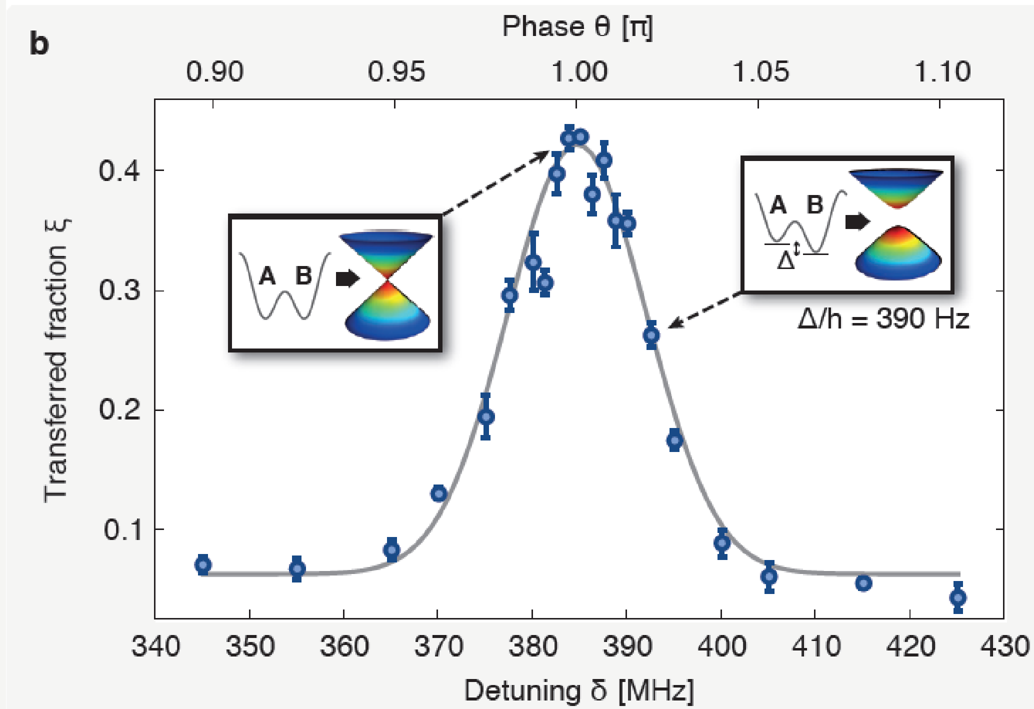


“Non-Standard Lattice-Honeycomb Lattice-”

“Creating, moving, and merging Dirac points with a Fermi gas in a tunable honeycomb lattice”
arXiv. 1111.5020v1 L. Tarruell, *et al*

“Performing Bloch Oscillation” $\frac{dq}{dt} = F$ Higher band fraction





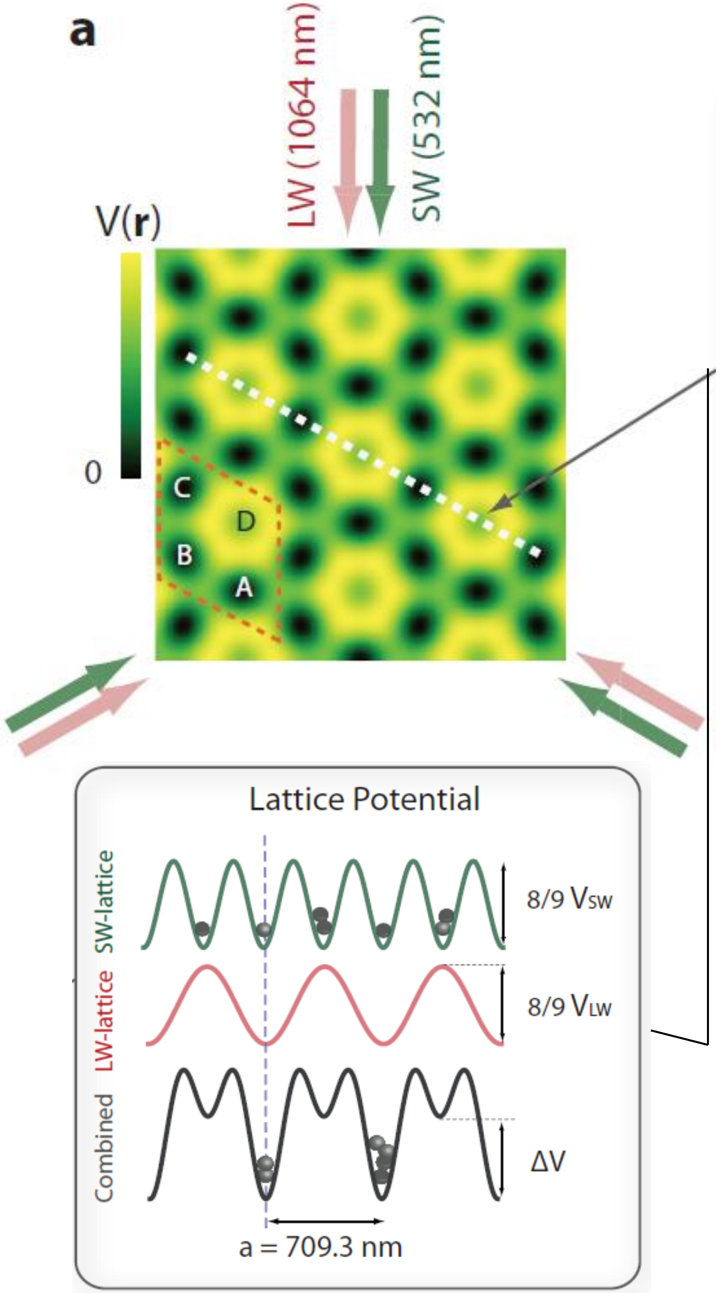
→ Relative phase θ between X and $\overline{X}$

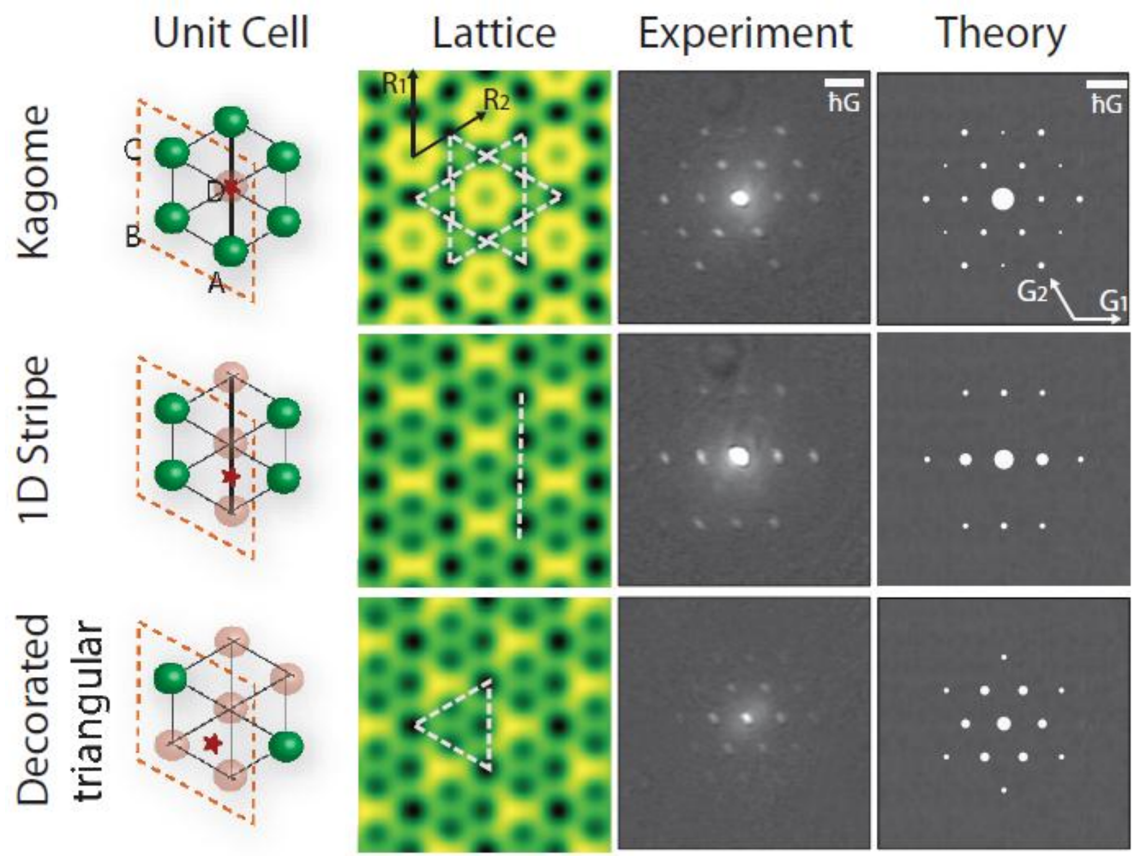
→ Potential diff. Δ between A and B

→ Band gap

“Non-Standard Lattice-Kagome Lattice-”

“Ultracold atoms in a Tunable Optical Kagome Lattice”
arXiv. 1109.1591v1 Gyu-Boong Jo, *et al*

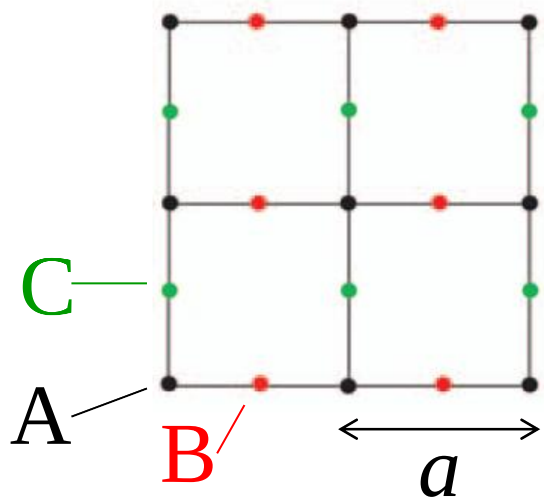




“Non-Standard Lattice-Lieb Lattice-”

Single Dirac cone with a flat band touching on line-centered-square optical lattices
R. Shen et al., PRB**81**, 041410R,2010

$$V(x,y) = V_1(\sin^2 k^L x + \sin^2 k^L y + \sin^2 2k^L x + \sin^2 2k^L y) + V_2\left(\sin^2\left[k^L(x+y)+\frac{\pi}{2}\right] + \sin^2\left[k^L(x-y)+\frac{\pi}{2}\right]\right)$$



“tight-binding model”

$$H_0 = \begin{pmatrix} \Delta & -2t\cos(k_x a/2) & 0 \\ -2t\cos(k_x a/2) & -\Delta & -2t\cos(k_y a/2) \\ 0 & -2t\cos(k_y a/2) & \Delta \end{pmatrix} \begin{matrix} |B,k\rangle \\ |A,k\rangle \\ |C,k\rangle \end{matrix}$$

$$\Delta = (\varepsilon_B - \varepsilon_A)/2 = (\varepsilon_C - \varepsilon_A)/2$$

$$\longrightarrow \quad E_0 = \Delta \quad ,\langle A,k | E_0 \rangle = 0$$

“flat band”

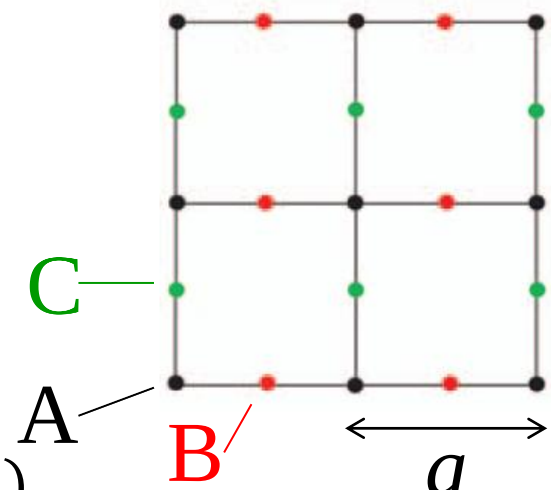
$$E_\pm = \pm\sqrt{\Delta^2 + 4t^2\{\cos^2(k_x a/2) + \cos^2(k_y a/2)\}}$$

“Dirac fermion”

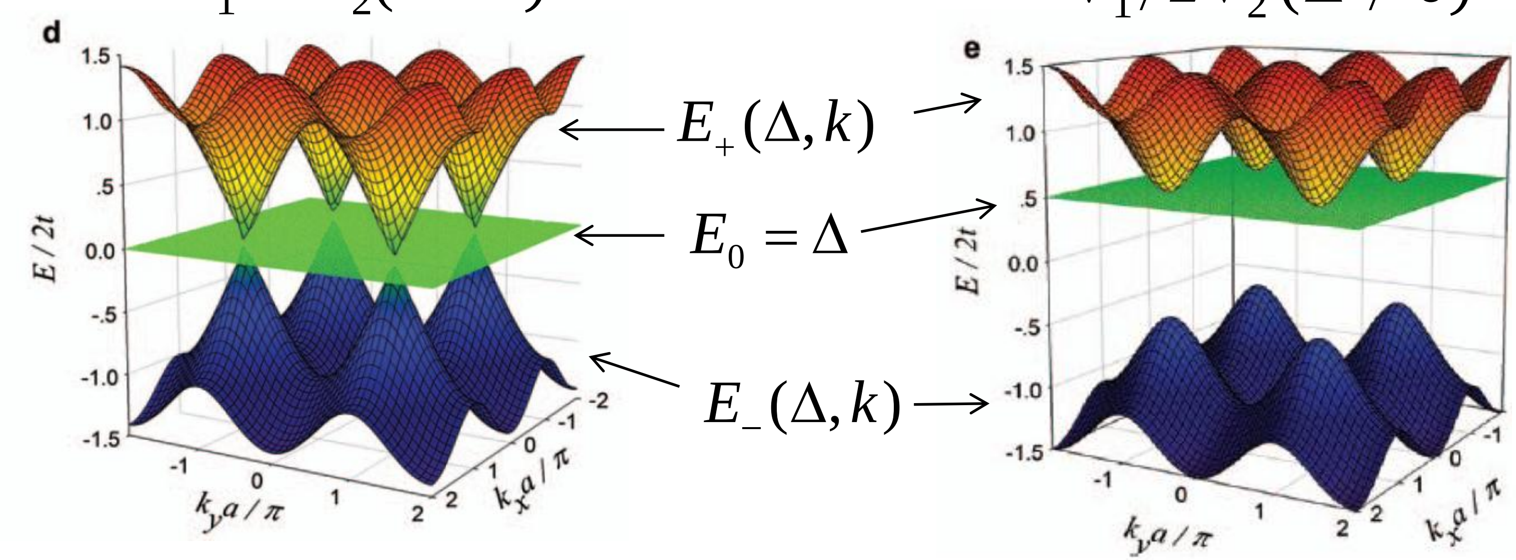
“Non-Standard Lattice-Lieb Lattice-”

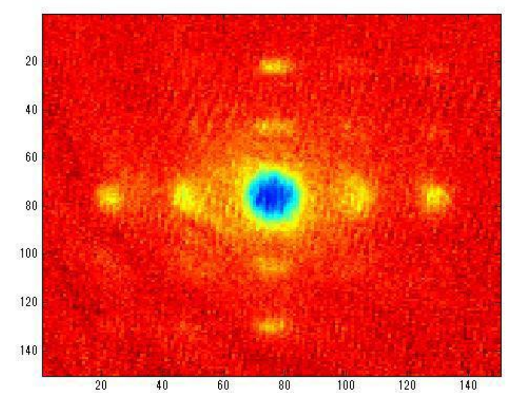
Single Dirac cone with a flat band touching on line-centered-square optical lattices
R. Shen et al., PRB**81**, 041410R,2010

$$V(x,y) = V_1(\sin^2 k^L x + \sin^2 k^L y + \sin^2 2k^L x + \sin^2 2k^L y) + V_2\left(\sin^2\left[k^L(x+y)+\frac{\pi}{2}\right] + \sin^2\left[k^L(x-y)+\frac{\pi}{2}\right]\right)$$



$V_1=2V_2(\Delta=0)$ $V_1\neq 2V_2\,(\Delta \neq 0)$





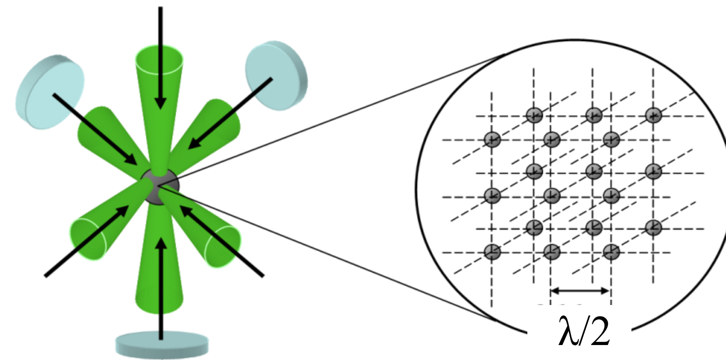
“Super-Lattice for Yb atoms”

$$E_\pm = \pm\sqrt{\Delta^2 + 4t^2\{\cos^2(k_x a/2) + \cos^2(k_y a/2)\}}$$

Quantum Simulation of Hubbard Model using "Cold Atoms in Optical Lattice"

[D. Jaksch *et al.*, PRL, **81** , 3108(1998)]

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



$$J = E_R (2/\sqrt{\pi}) s^{3/4} \exp(-2\sqrt{s})$$

$$U = E_R a_s k_L \sqrt{8/\pi} s^{3/4}$$

$s \equiv V_o / E_R$, $E_R \equiv (\hbar k_L)^2 / 2m$, a_s : scattering length

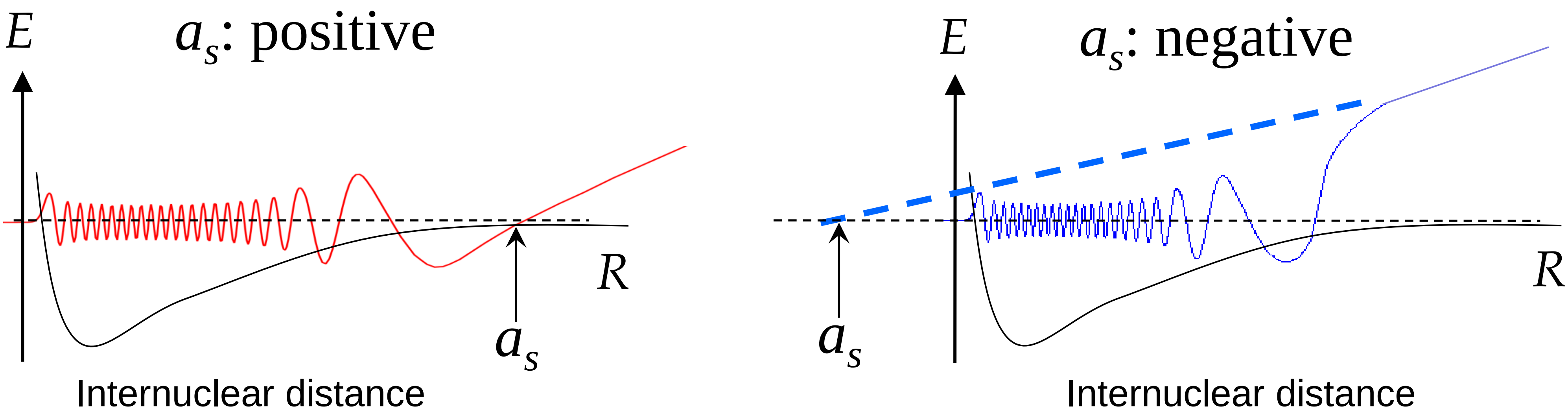
Controllable Parameters

hopping between lattice sites	: J	→	lattice potential	: V_0
On-site interaction	: U		Feshbach Resonance	: a_s
filling factor (e- or h-doping)	: n		atom density	: n

Various geometry

What is *Scattering Length* ?

$$\psi_{SC}(R) \underset{R\to\infty}{\propto} \frac{\sin(kR+\delta_0)}{kR} = \frac{\sin(k(R-a_s))}{kR}$$



$$V_{\text{int}} = \frac{4\pi\hbar^2 a_s}{m}\delta(\vec{r}_1 - \vec{r}_2) \qquad U = \frac{4\pi a_s \hbar^2}{m}\int d^3x \left|w(x - x^{(i)})\right|^4$$

Feshbach Resonance:

ability to tune an inter-atomic interaction

Collision is in Quantum Regime

It is described by s-wave scattering length a_s

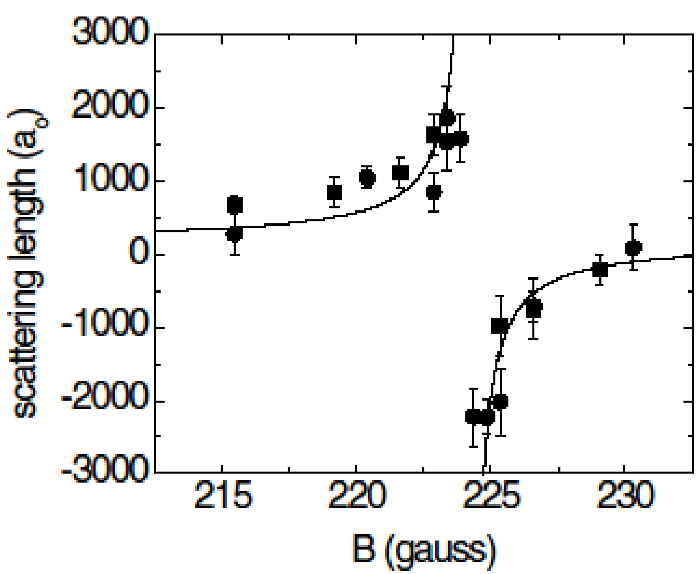
$$a_s = -\delta_l / k$$

$$\sigma_0 = 4\pi |f_0|^2 = 4\pi |a_s|^2$$

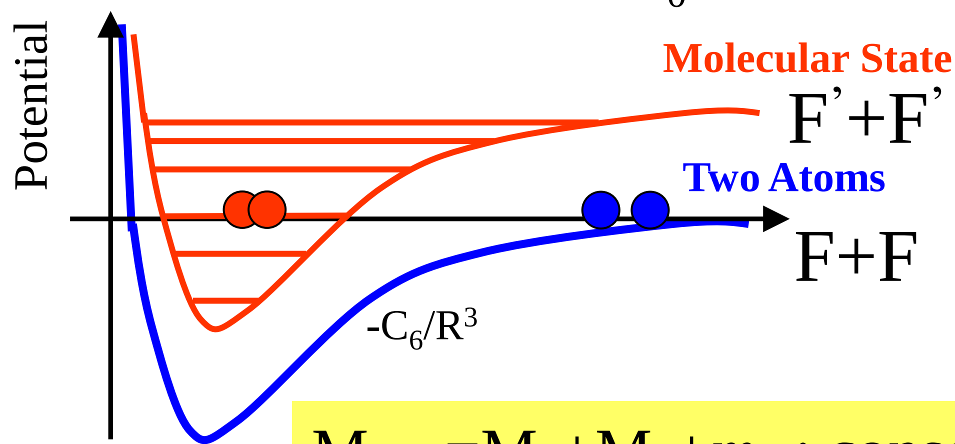
Coupling between "**Open Channel**" and "**Closed Channel**"

→ Control of Interaction(a_s)

$$a_s(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



[C. Regal and D. Jin, PRL**90**, 230404(2003)]



$M_{total} = M_1 + M_2 + m_l$: conserved

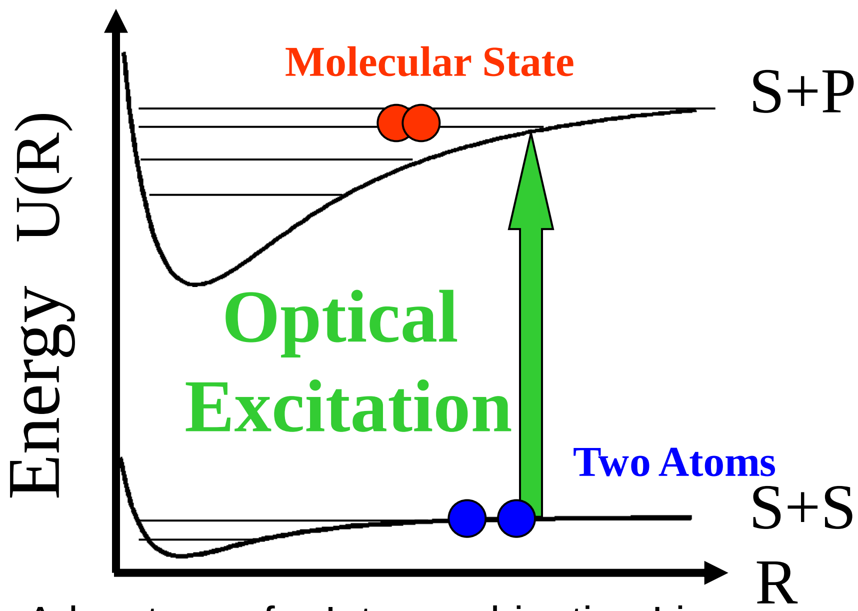
$l_{open} = l_{closed}$, $l_{open} \neq l_{closed}$ if $V_{ss} \neq 0$

[C. Chin et al., RMP**82**, 1225(2010)]

TABLE IV. Properties of selected Feshbach resonances. The first column describes the atomic species and isotope. The next three columns characterize the scattering and resonance states, which include the incoming scattering channel (ch.), partial wave ℓ, and the angular momentum of the resonance state ℓ_c. This is followed by the resonance location B_0, the width Δ, the background scattering length a_{bg}, the differential magnetic moment $\delta\mu$, the dimensionless resonance strength s_{res}, the background scattering length in van der Waals units $r_{bg}=a_{bg}/\bar{a}$, and the bound state parameter ζ from Eq. (52). Here a_0 is the Bohr radius and μ_B is the Bohr magneton. Definitions are given in Sec. II. The last column gives the source. A string "na" indicates that the corresponding property is not defined. For example a_{bg} is not defined for p-wave scattering.

Atom	ch.	ℓ	ℓ_c	B_0 (G)	Δ (G)	a_{bg}/a_0	$\delta\mu/\mu_B$	s_{res}	r_{bg}	ζ	Reference
^{23}Na	*cc*	*s*	*s*	1195	−1.4	62	−0.15	0.0050	1.4	0.004	Inouye *et al.*, 1998; Stenger *et al.*, 1999[a]
	aa	*s*	*s*	907	1	63	3.8	0.09	1.5	0.07	Inouye *et al.*, 1998; Stenger *et al.*, 1999[a]
	aa	*s*	*s*	853	0.0025	63	3.8	0.0002	1.5	0.0002	Inouye *et al.*, 1998; Stenger *et al.*, 1999[a]
^{39}K	*aa*	*s*	*s*	402.4	−52	−29	1.5	2.1	−0.47	0.49	D'Errico *et al.*, 2007
^{40}K	*bb*	*p*	*p*	198.4	na	na	0.134	na	na	na	Regal *et al.*, 2003b; Ticknor *et al.*, 2004[a]
	bb	*p*	*p*	198.8	na	na	0.134	na	na	na	Regal *et al.*, 2003b; Ticknor *et al.*, 2004[a]
	ab	*s*	*s*	202.1	8.0	174	1.68	2.2	2.8	3.1	Regal *et al.*, 2004[a]
	ac	*s*	*s*	224.2	9.7	174	1.68	2.7	2.8	3.8	Regal and Jin, 2003[a]
^{85}Rb	*ee*	*s*	*s*	155.04	10.7	−443	−2.33	28	−5.6	80	Claussen *et al.*, 2003
^{87}Rb	*aa*	*s*	*s*	1007.4	0.21	100	2.79	0.13	1.27	0.08	Volz *et al.*, 2003; Dürr, Volz, and Rempe, 2004[a]
	aa	*s*	*s*	911.7	0.0013	100	2.71	0.001	1.27	0.0006	Marte *et al.*, 2002[a]
	aa	*s*	*s*	685.4	0.006	100	1.34	0.006	1.27	0.004	Marte *et al.*, 2002; Dürr, Volz, and Rempe, 2004[a]
	aa	*s*	*s*	406.2	0.0004	100	2.01	0.0002	1.27	0.0001	Marte *et al.*, 2002[a]
	ae	*s*	*s*	9.13	0.015	99.8	2.00	0.008	1.27	0.005	Widera *et al.*, 2004
^{133}Cs	*aa*	*s*	*s*	−11.7	28.7	1720	2.30	560	17.8	5030	Chin, Vuletić, *et al.*, 2004; Lange *et al.*, 2009[a]
	aa	*s*	*d*	47.97	0.12	926	1.21	0.67	9.60	3.2	Chin, Vuletić, *et al.*, 2004; Lange *et al.*, 2009[a]
	aa	*s*	*g*	19.84	0.005	160	0.57	0.002	1.66	0.002	Chin, Vuletić, *et al.*, 2004[a]
	aa	*s*	*g*	53.5	0.0025	995	1.52	0.019	10.3	0.1	Chin, Vuletić, *et al.*, 2004; Lange *et al.*, 2009[a]
	aa	*s*	*s*	547	7.5	2500	1.79	170	26	2200	[a]
	aa	*s*	*s*	800	87.5	1940	1.75	1470	20	15000	[a]

Optical Feshbach Resonance



Advantages for Intercombination Lines

R. Ciurylo, *et al.* *Phys. Rev. A* **70**. 062710 (2004)

$$S_{00} = \frac{\Delta - i\Gamma_S/2 + i\gamma/2}{\Delta + i\Gamma_S/2 + i\gamma/2}$$

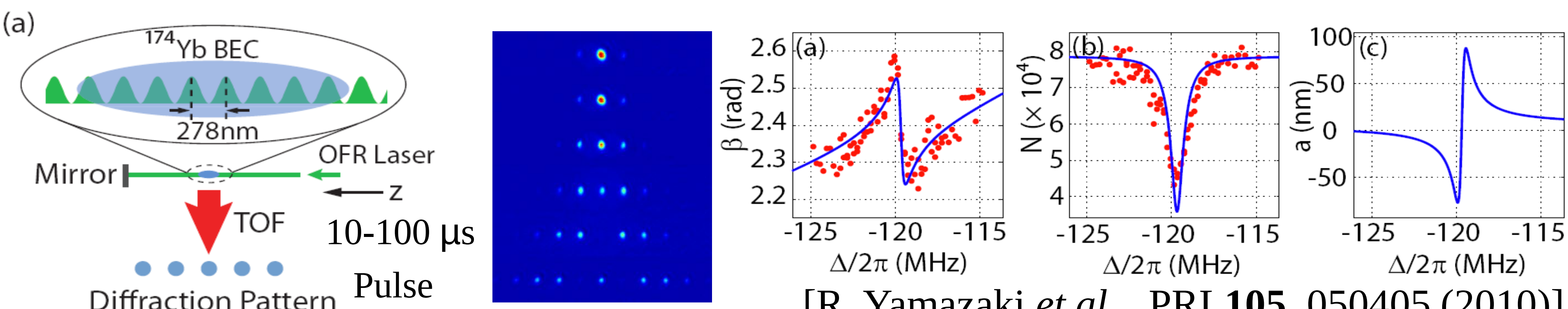
$$\Gamma_S \propto \left|\langle b|V_{las}|f\rangle\right|^2$$

γ :spontaneous decay rate

Δ :detuning from the PA resonance

[J. Bohn and P. Julienne PRA(1999)]

Nanometer-scale Spatial Modulation



[R. Yamazaki *et al* ., PRL**105**, 050405 (2010)]

Quantum Simulation of Hubbard Model using "Cold Atoms in Optical Lattice"

[D. Jaksch *et al.*, PRL, **81** , 3108(1998)]

provided by Polar Molecules

~~RbK,~~
RbCs,
NaK,...

$$H = -J\sum_{<i,j>} c_i^{+} c_j + U\sum_{i} n_{i\uparrow} n_{i\downarrow} + V\sum_{<i,j>} n_i n_j$$

$$J = E_R (2/\sqrt{\pi}) s^{3/4} \exp(-2\sqrt{s})$$

$$U = E_R a_s k_L \sqrt{8/\pi}\, s^{3/4}$$

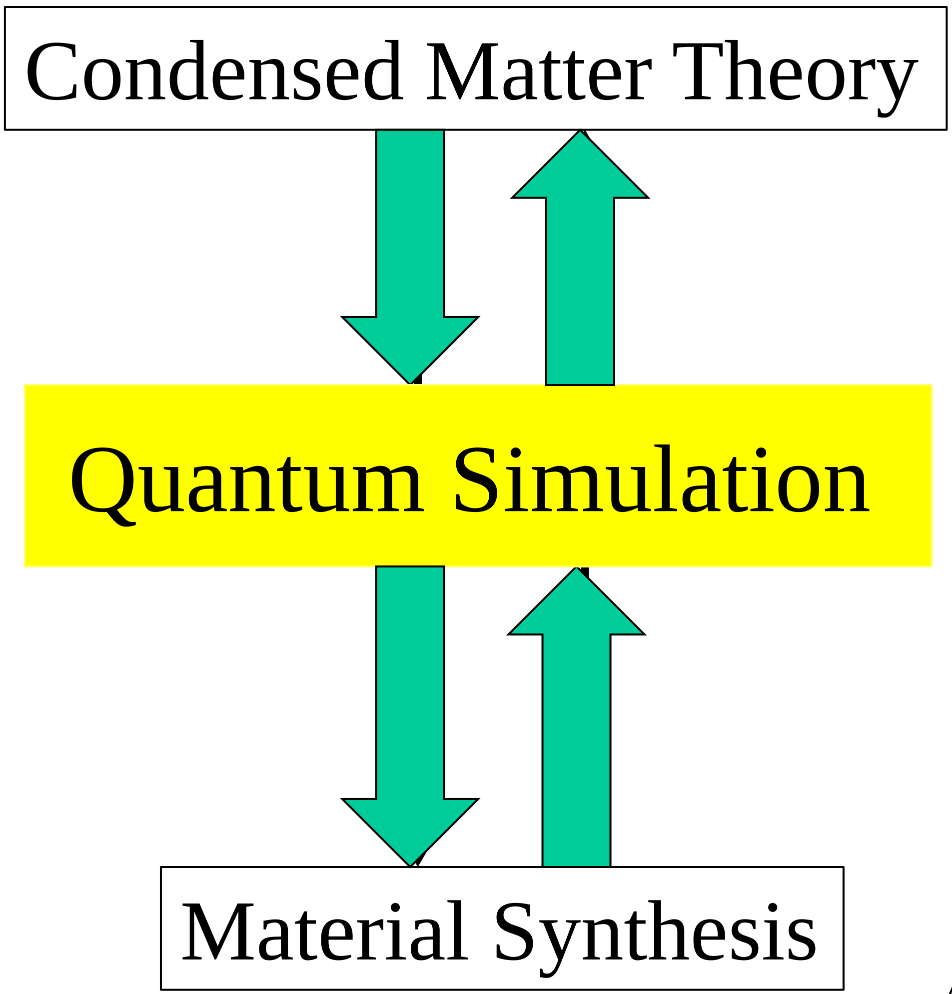
$s \equiv V_o / E_R$, $E_R \equiv (\hbar k_L)^2 / 2m$, a_s : scattering length

Controllable Parameters

hopping between lattice sites	: J	lattice potential	: V_0
On-site interaction	: U	Feshbach Resonance	: a_s
filling factor (e- or h-doping)	: n	atom density	: n

Various geometry

“Quantum Simulation *Business*”



1.Lattice Geometry
standard: $10k
X non-standard: $100k +
1 $100k per Dirac Point
1 $100k per Flat Band

2. Quantum Statistics
boson: : $30k
X fermion: $50k

3. Interaction
repulsive/attractive :$10k
X Feshbach Resonance:$100k
long-range: $500k
spin-orbit: $500k

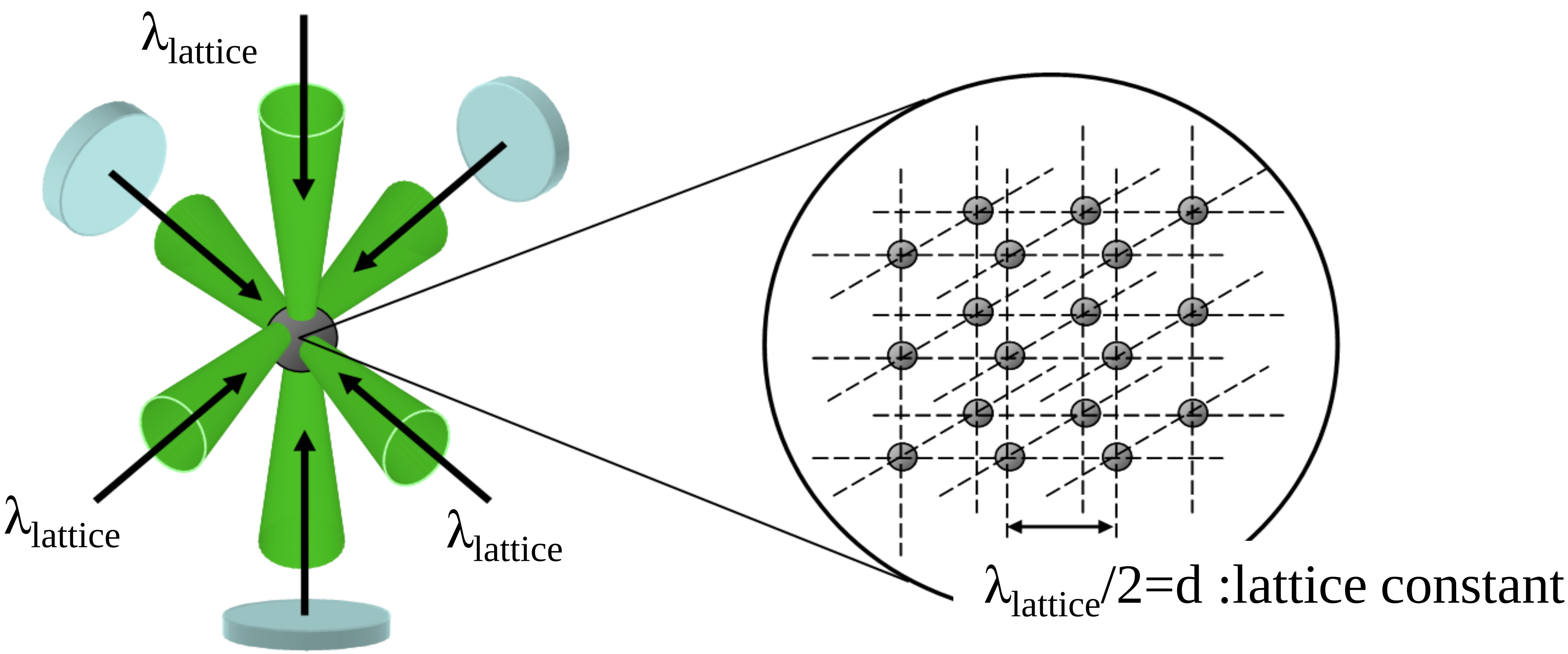
4. Quantum Gas Microscope
X:$1M

Total : $ 1.45M

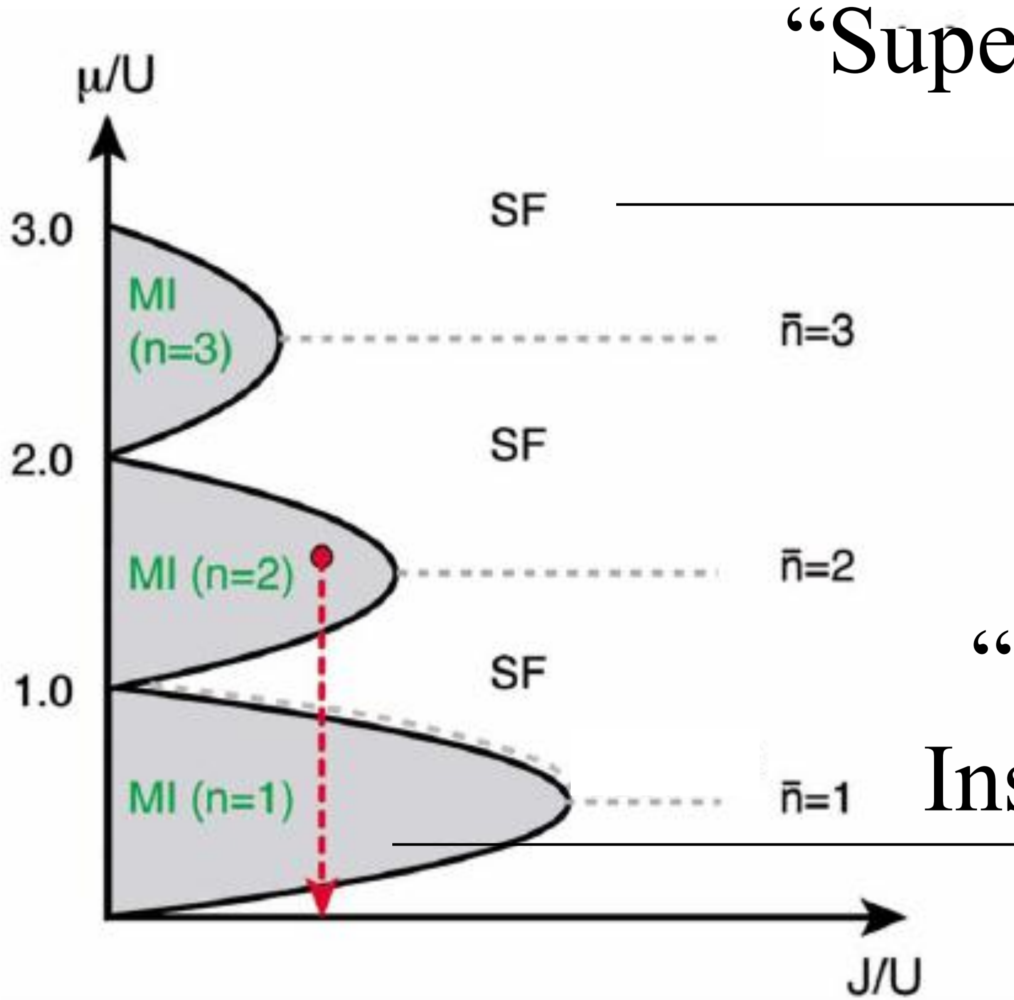
Bosons in a 3D optical lattice

$$H = -J \sum_{<i,j>} a_i^{+} a_j + \frac{U}{2} \sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$

"Bose-Hubbard Model"

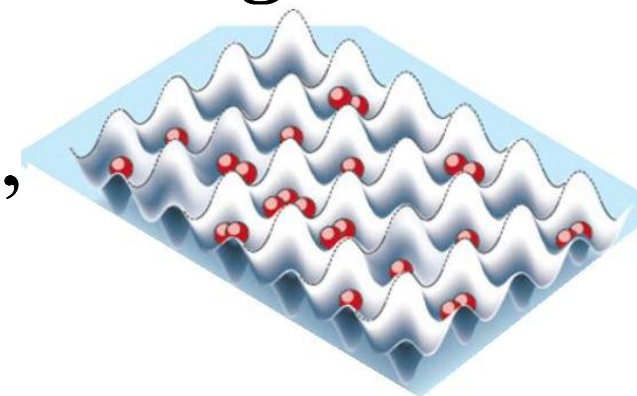


Phase Diagram of Repulsively Interacting Bosons



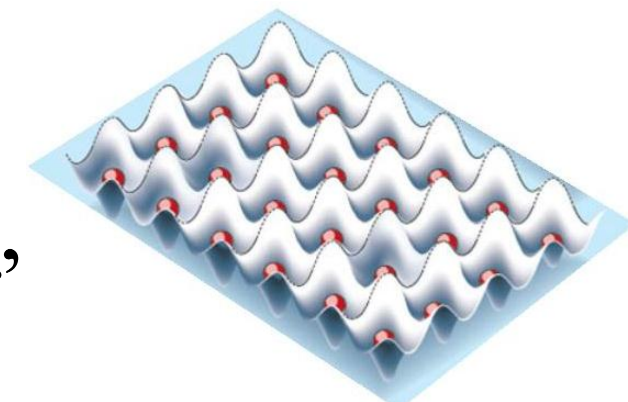
[RMP**80**,885(2008)]

"Superfluid"



$$\left|\Psi_{SF}\right\rangle \propto \left(\frac{1}{\sqrt{M}}\sum_{i=1}^{M} a_i^{+}\right)^{N}\left|0\right\rangle$$

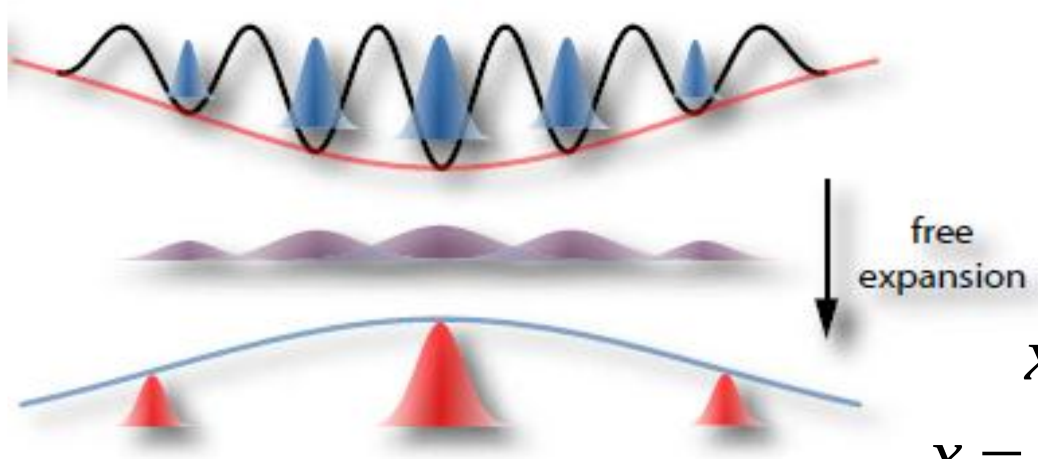
"Mott Insulator"



$$\left|\Psi_{MI}\right\rangle \propto \prod_{i=1}^{M}\left(a_i^{+}\right)^{n}\left|0\right\rangle$$

Interference Fringe : the direct signature of the phase coherence

"Sudden Release"



t_{TOF}

$x \leftrightarrow \hbar k$

$x = (\hbar k / M) t_{TOF}$

$$n(k) \propto \left|\tilde{w}(k)\right|^2 G(k)$$

Fourier Transform of the Wannier function

$$G(k) = \sum_{R,R'} \exp(ik \cdot (R - R')) \left\langle \hat{a}_R^+ \hat{a}_{R'} \right\rangle$$

no long-range order: $\left\langle \hat{a}_R^+ \hat{a}_{R'} \right\rangle = \delta_{R,R'} \rightarrow G(k) = N$

uniform long-range order: $\left\langle \hat{a}_R^+ \hat{a}_{R'} \right\rangle = 1 \rightarrow G(k) = \dfrac{\sin^2(kdN/2)}{\sin^2(kd/2)} = N^2$

at $k = \pm 2nk_L$ (n=0,1,2...)

$d = \lambda / 2 = \pi / k_L$

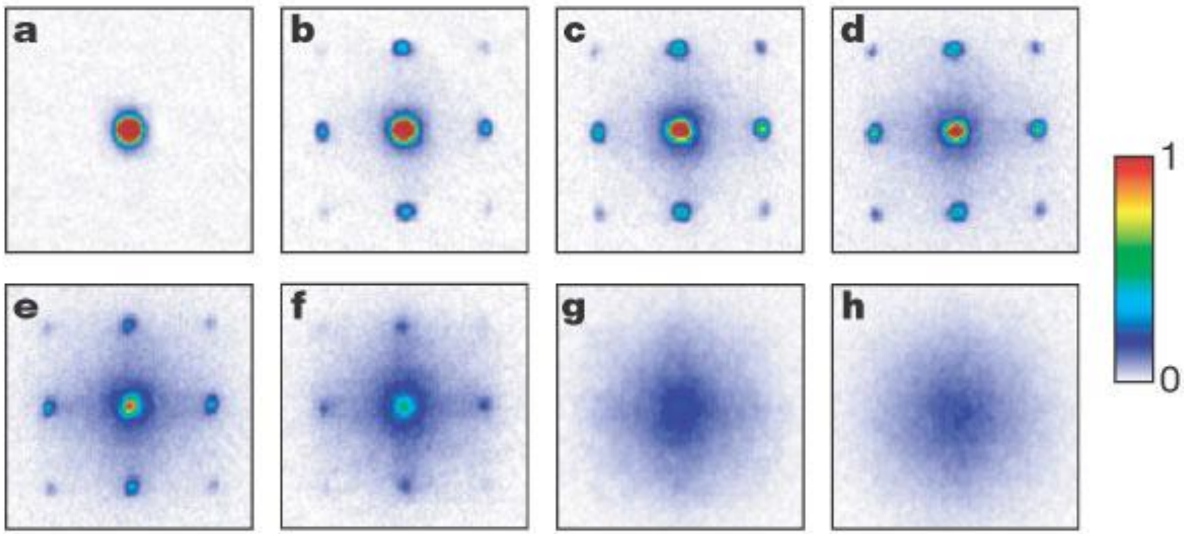
[I. Bloch et al, RMP80, 885(2008)]

Bose-Hubbard Model:
“Superfluid - Mott-insulator Transition”

[M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415,39 (2002)]

No lattice $V_0 / E_R =$ 3 7 10

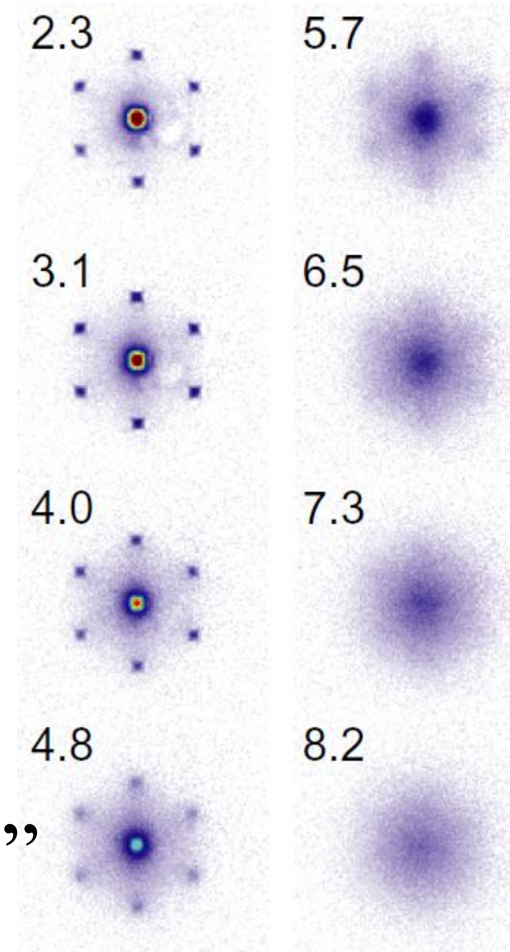
^{87}Rb



13 14 16 20

“cubic lattice”

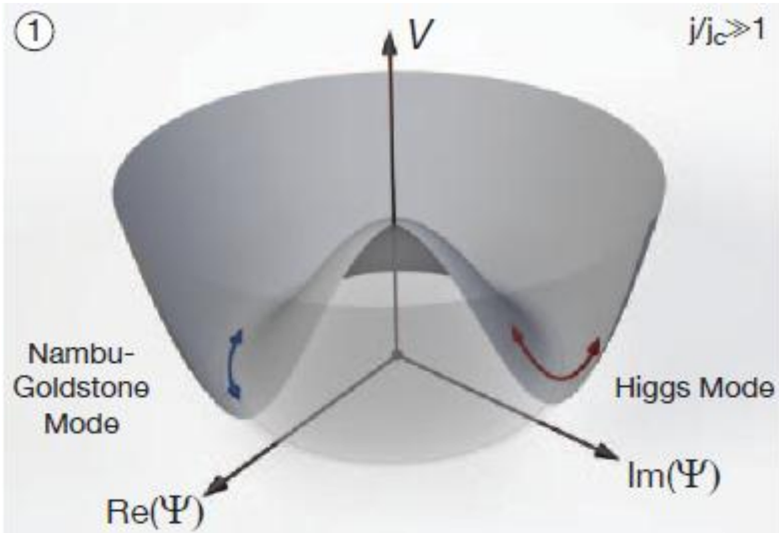
$s=$



“triangular lattice”

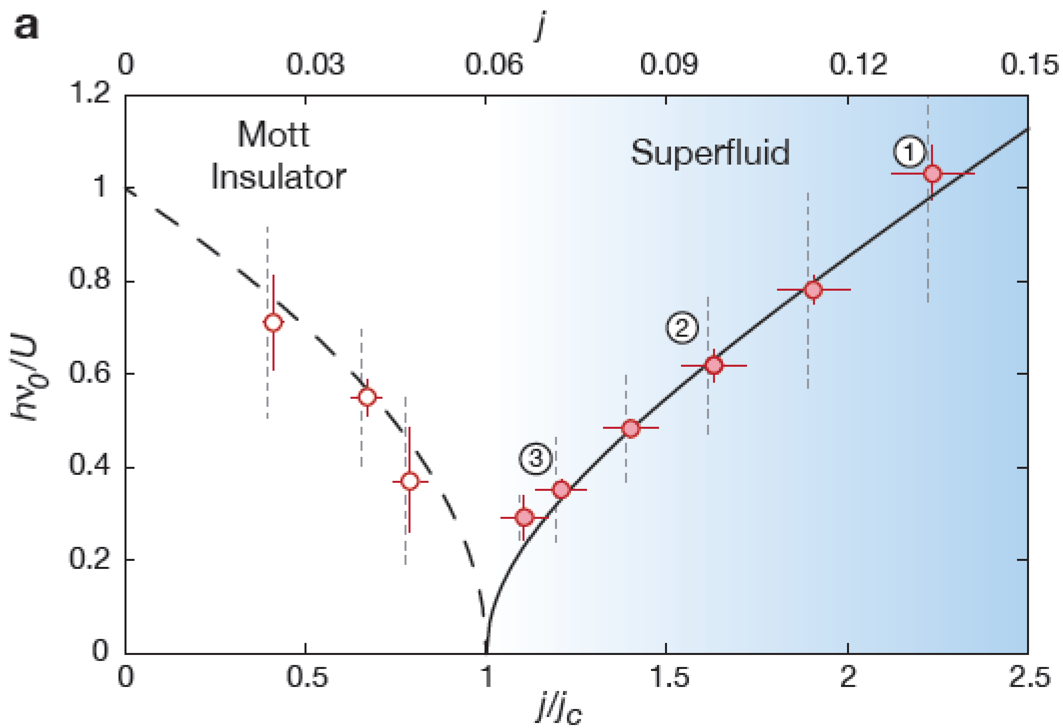
[C. Becker *et al.*, New J. Phys. **12** 065025(2010)]

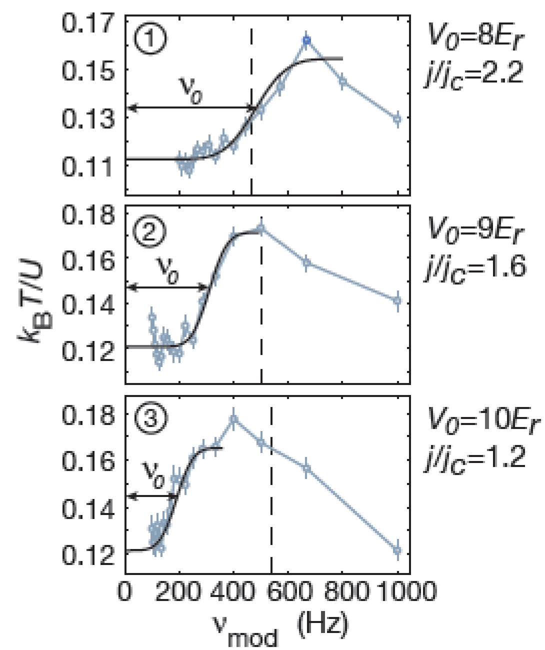
“amplitude-(Higgs-)mode”



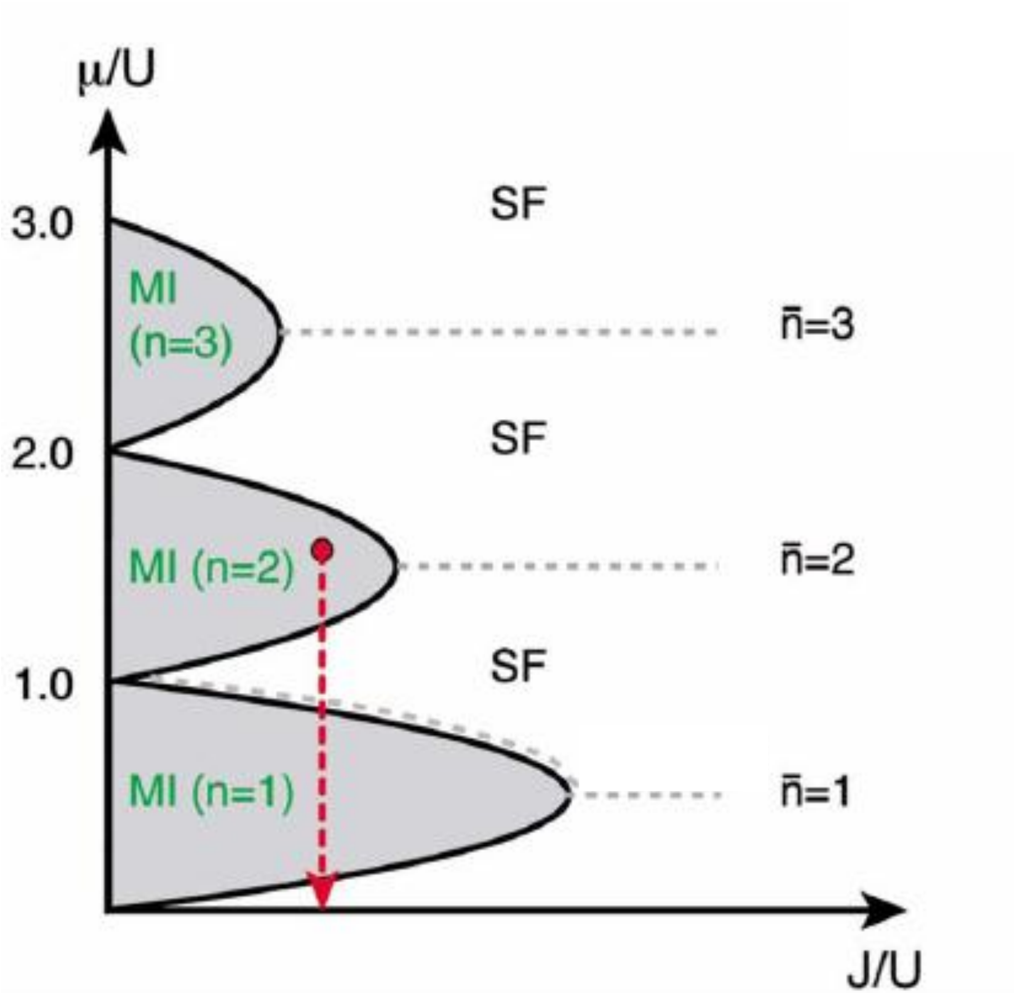
The `Higgs' Amplitude Mode at the Two-Dimensional Supefruid-Mott Insulator Transition

M. Endres *et al.*, arXiv:1204.5183v2

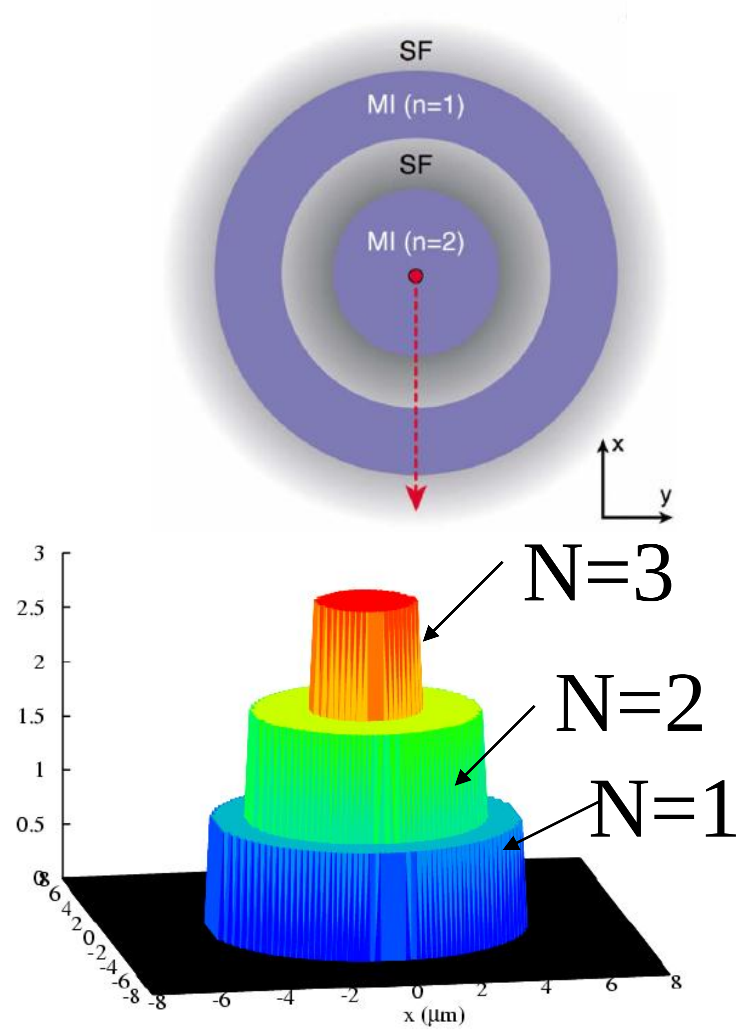




Phase Diagram of Repulsively Interacting Bosons



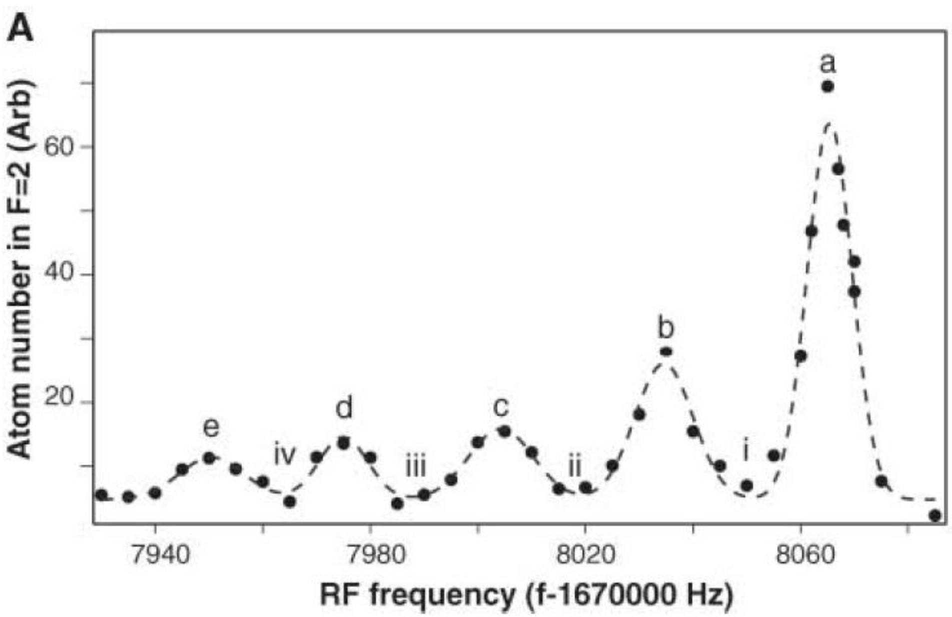
[RMP**80**,885(2008)]



Shell Structure of Mott States

High-Resolution RF Spectroscopy: Observation of Mott Shell Structure

[G. K. Campbell et al., Science 313, 649 (2006)]



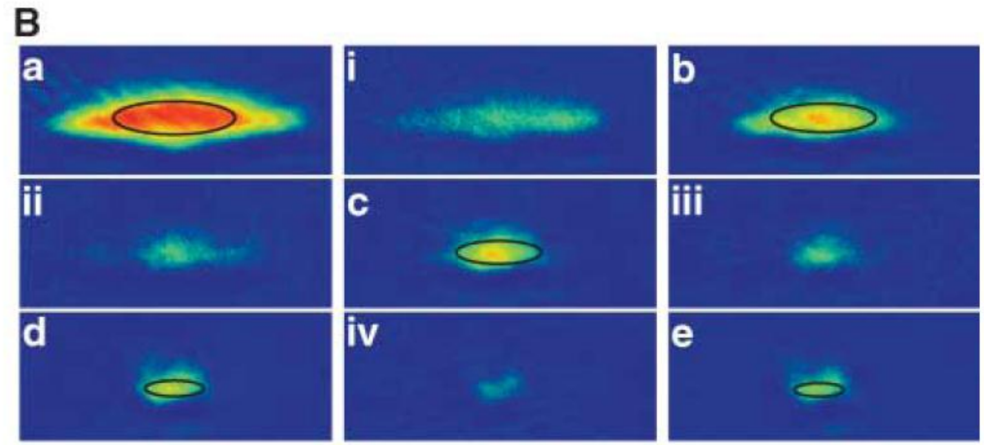
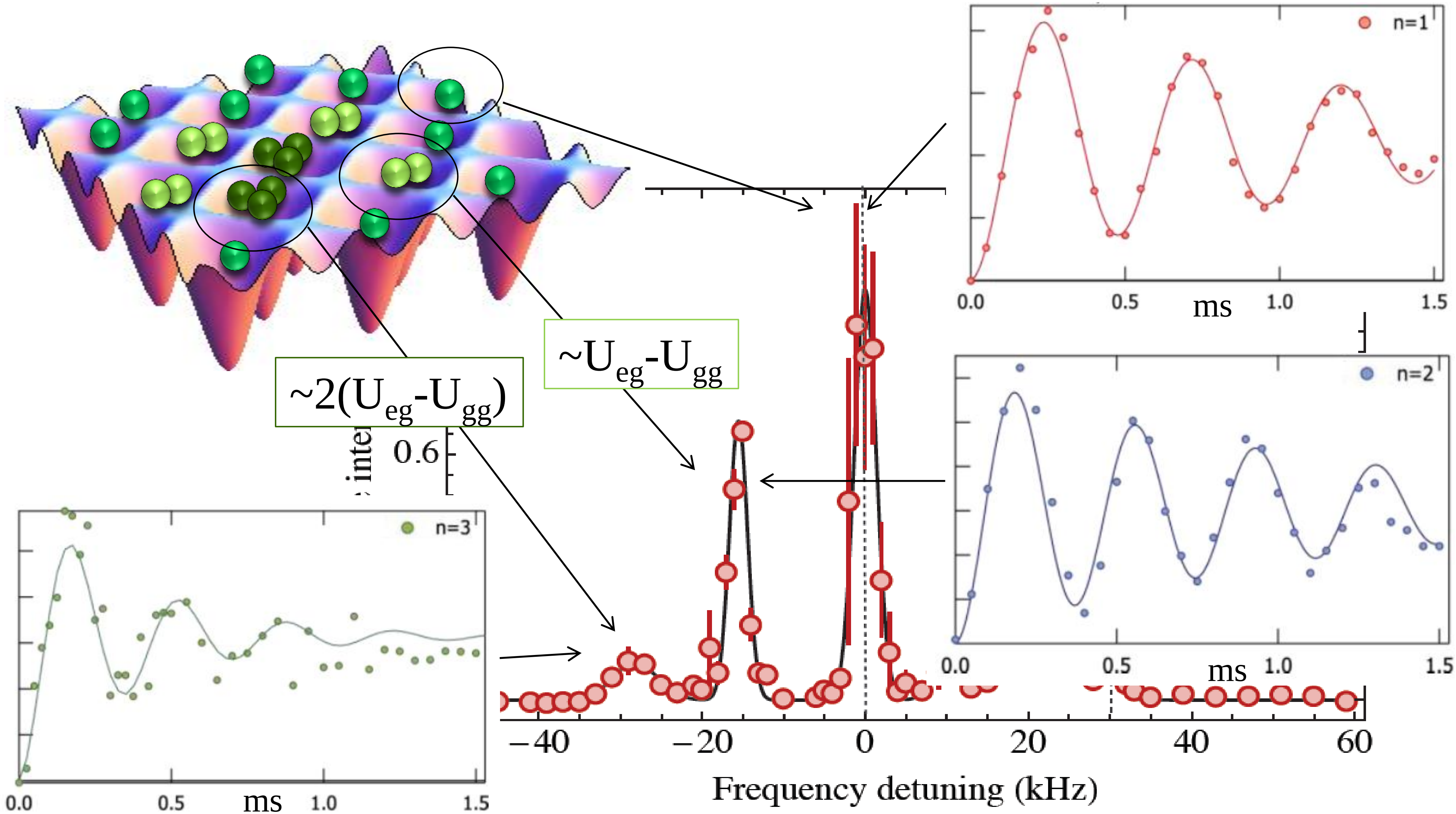


Fig. 3. Imaging the shell structure of the MI. (**A**) Spectrum of the MI at V = $35E_{\mathrm{rec}}$. (**B**) Absorption images for for decreasing rf frequencies. Images a to e were taken on resonance with the peaks shown in (A) and display the spatial distribution of the n =1 to n = 5 shells. The solid lines shows the predicted contours of the shells. Absorption images taken for rf frequencies between the peaks (images i to iv) show a much smaller signal. The field of view was 185 μm by 80 μm.

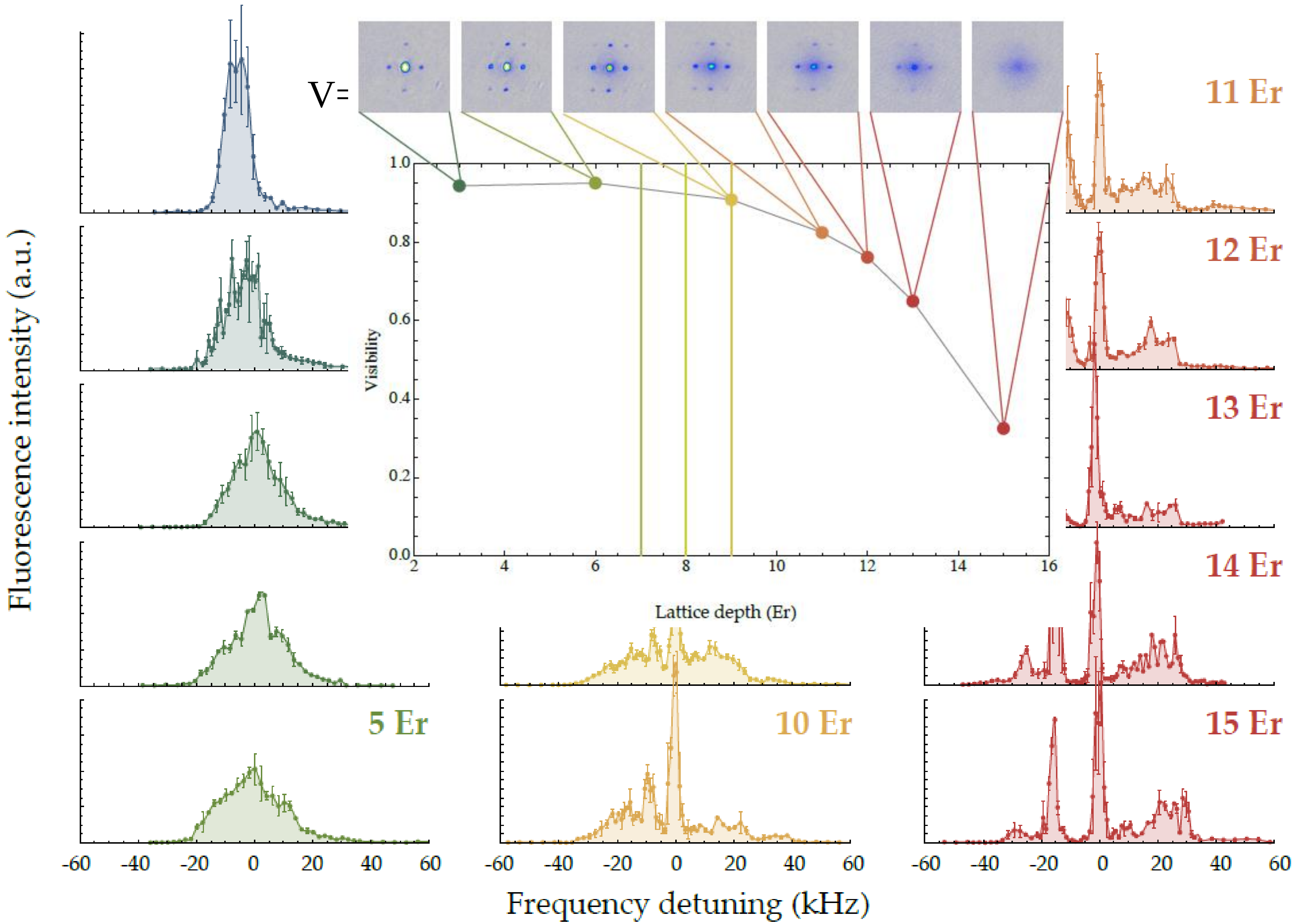
$$h\nu_n = \frac{U}{a_{11}}(a_{12} - a_{11})(n-1)$$

Laser Spectroscopy of Yb Atoms in a Mott Insulating State

"independent control of the single, double, and triple occupancy"



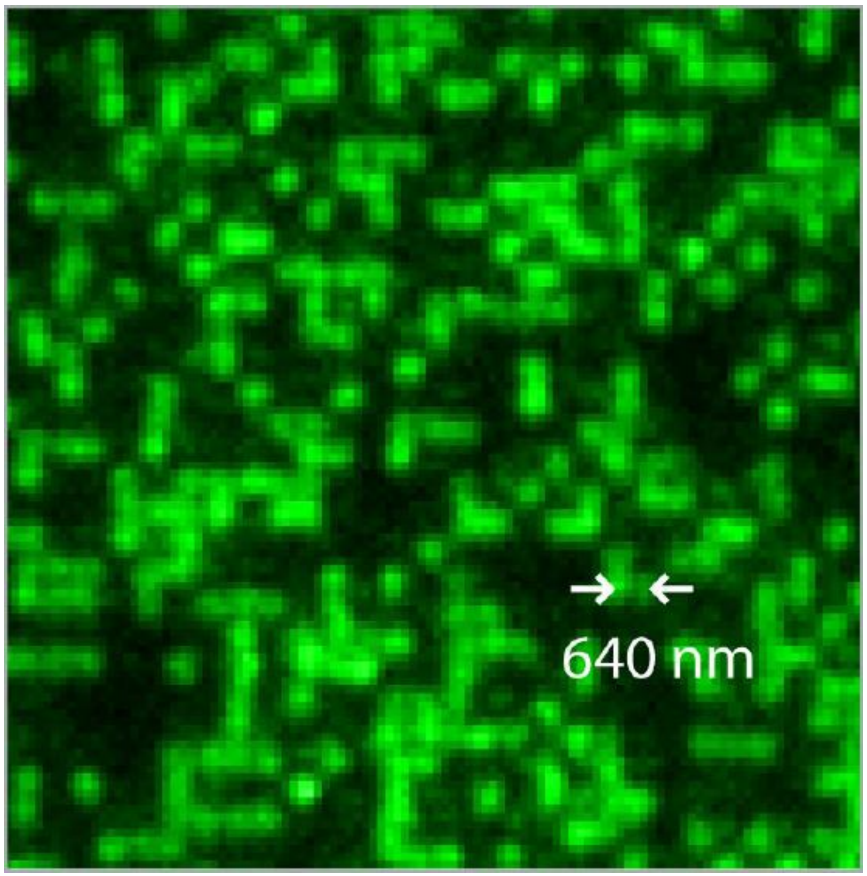
Spectroscopy of Superfluid-Mott Insulator Transition

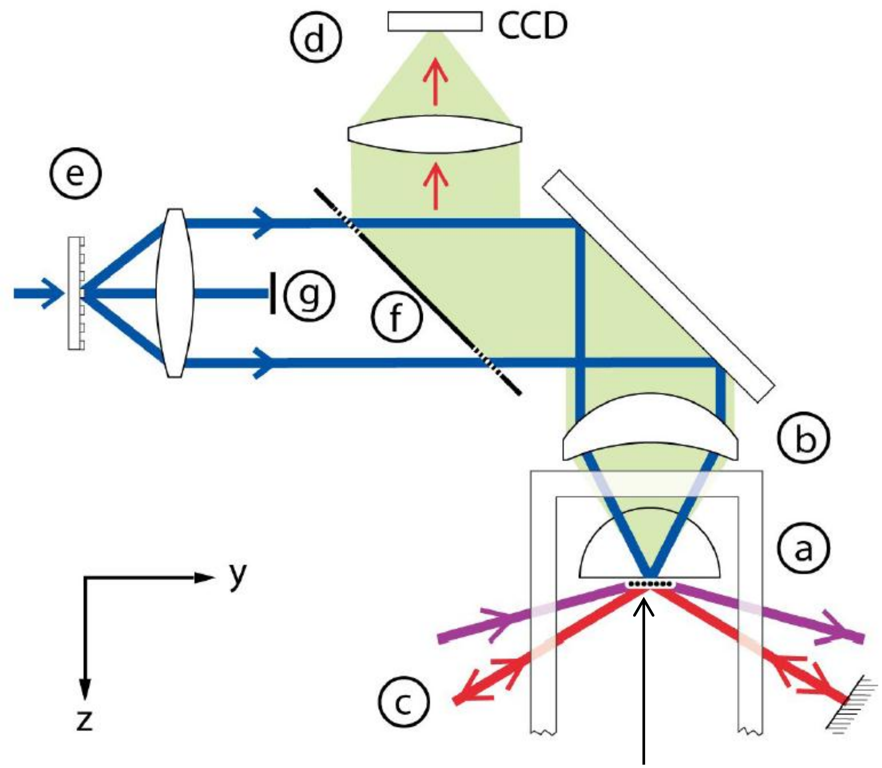


Quantum Gas Microscope
: Single Site Observation

[WS. Bakr, I. Gillen, A. Peng, S. Folling, and M. Greiner, Nature 462(426), 74-77(2009)]

Fluorescence Imaging

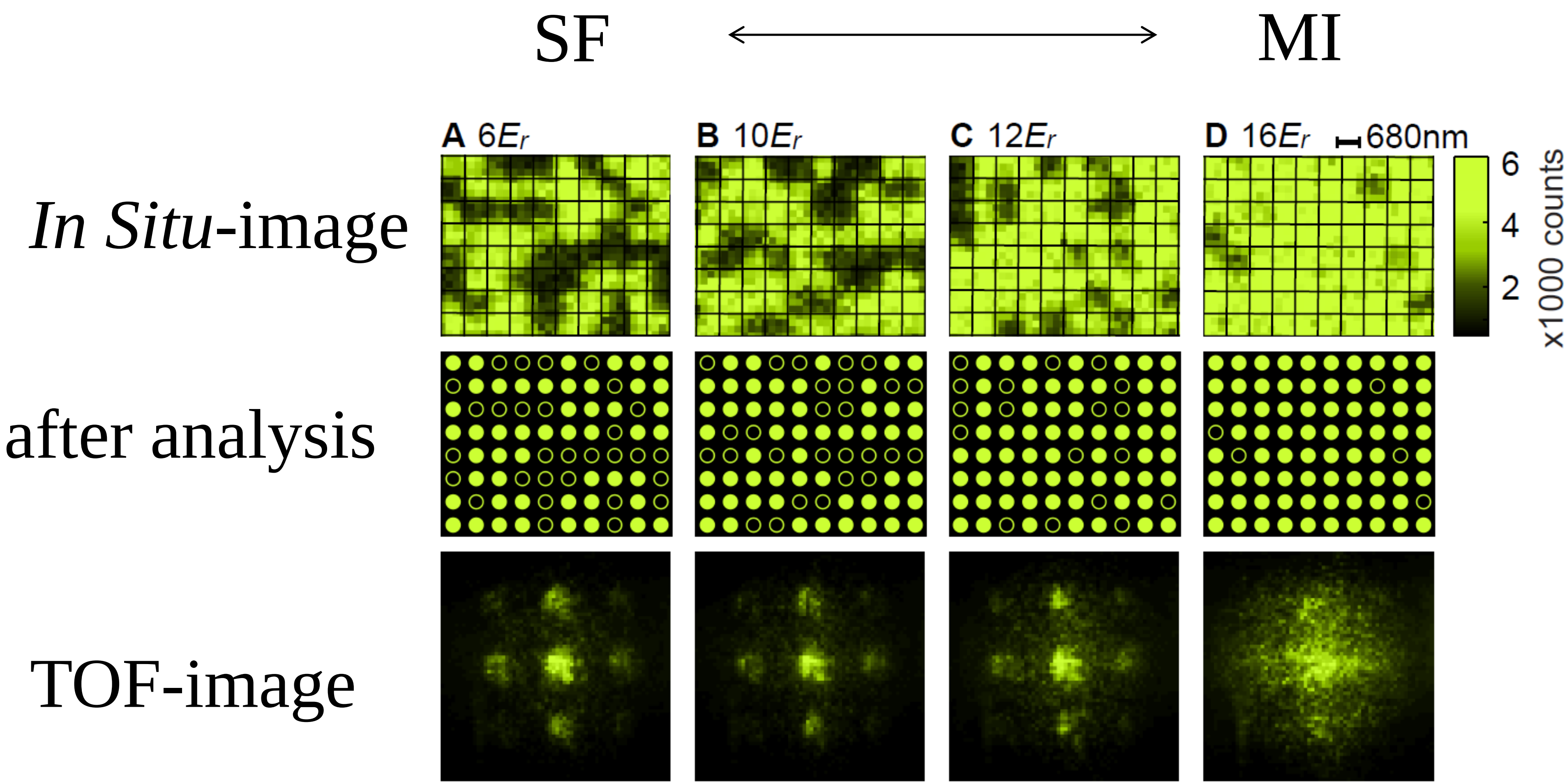




^{87}Rb

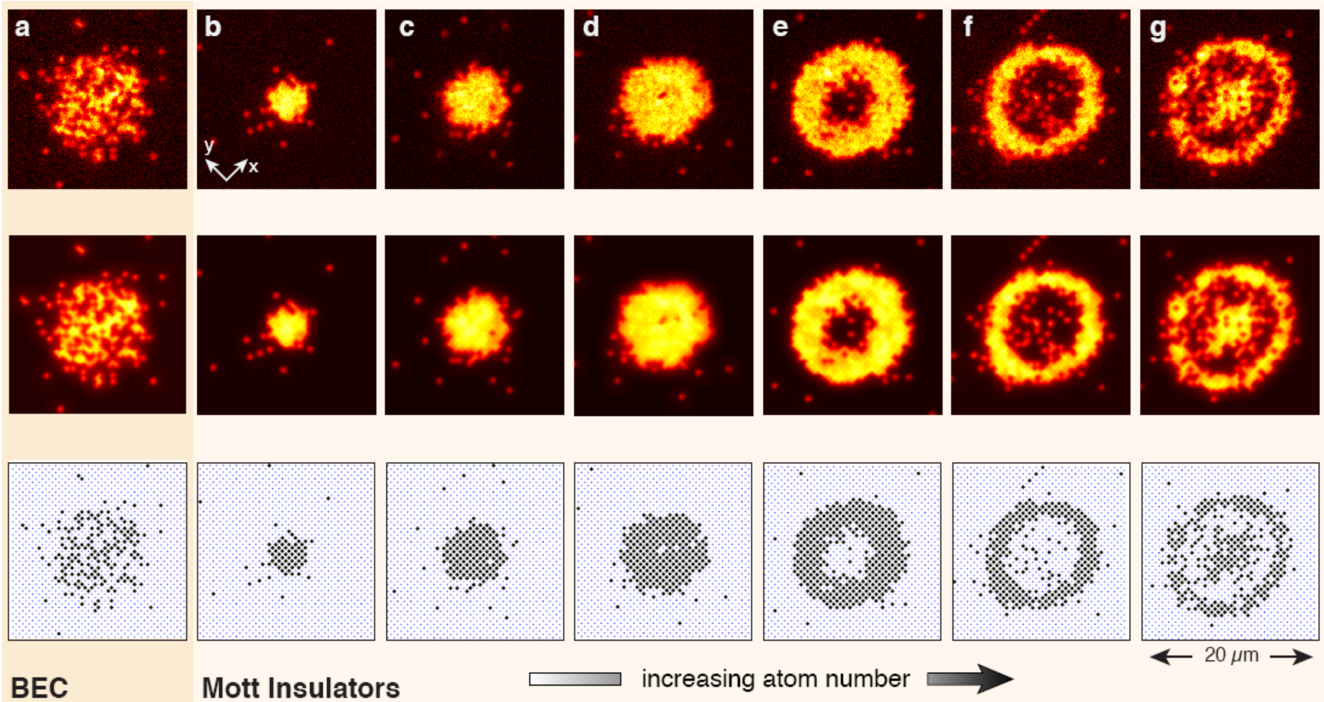
Single Site Resolved Detection of MI

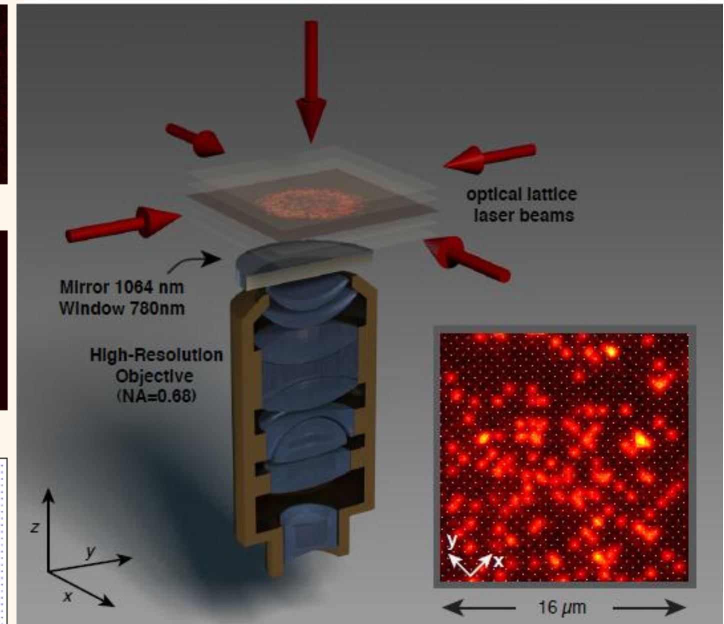
[WS Bakr, et al., Science 329, 547(2010)]



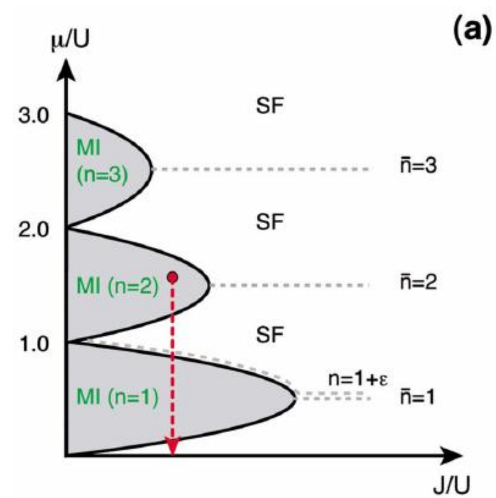
Single Site Resolved Detection of MI

[J. F. Sherson, et al., Nature 467, 68(2010)]



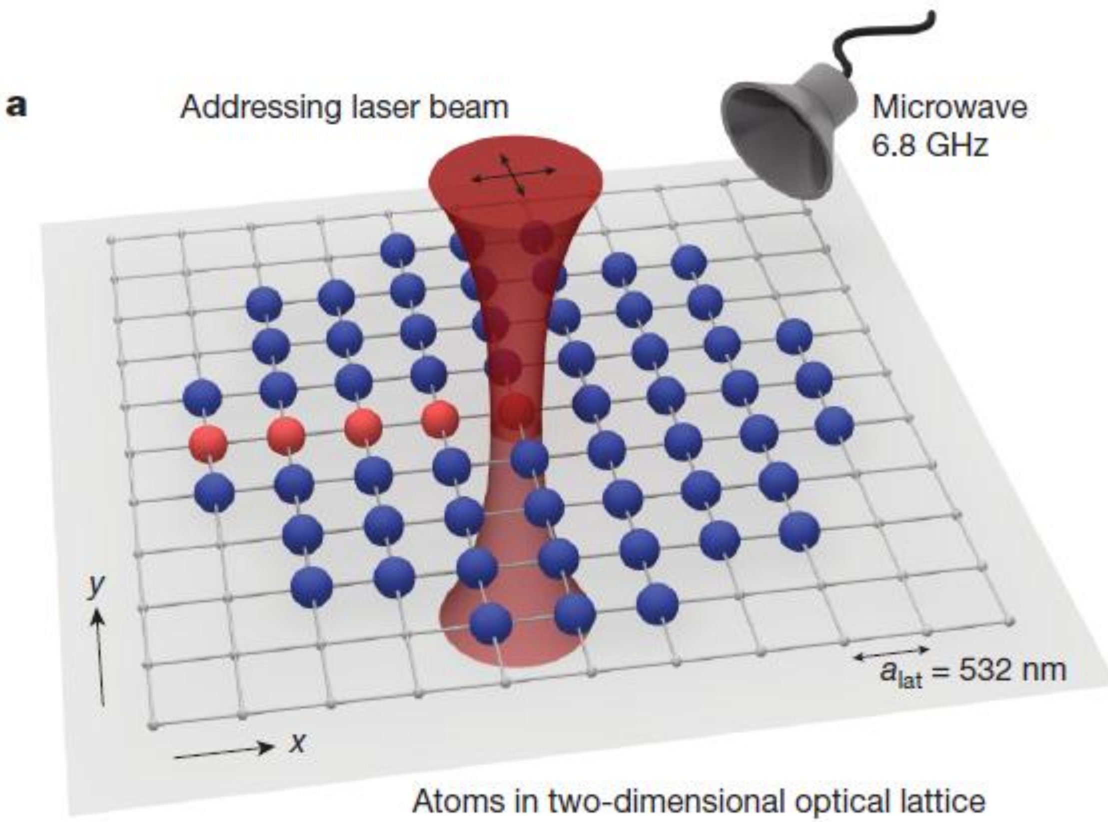


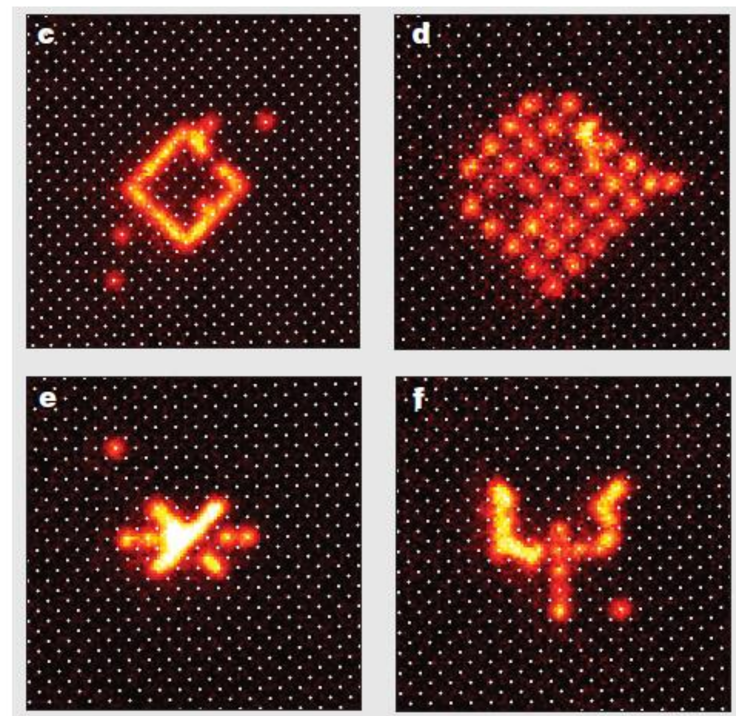
IG. 2: **High resolution fluorescence images of a BEC and Mott insulators.** Top row: Experimentally obtained images f a BEC (a) and Mott insulators for increasing particle numbers (b-g) in the zero-tunneling limit. Middle row: Numerically econstructed atom distribution on the lattice. The images were convolved with the point-spread function of our imaging ystem for comparison with the original images. Bottom row: Reconstructed atom number distribution. Each circle indicates single atom, the points mark the lattice sites.



Single Spin Manipulation in Mott Insulator

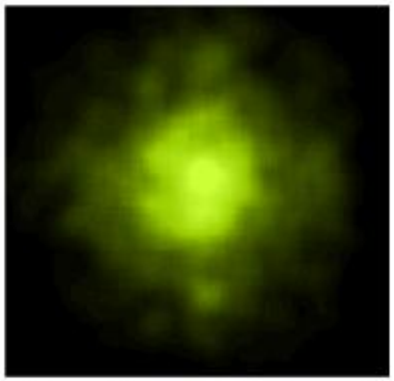
[C. Ewitenberg *et al*, Nature 471, 319(2011)]



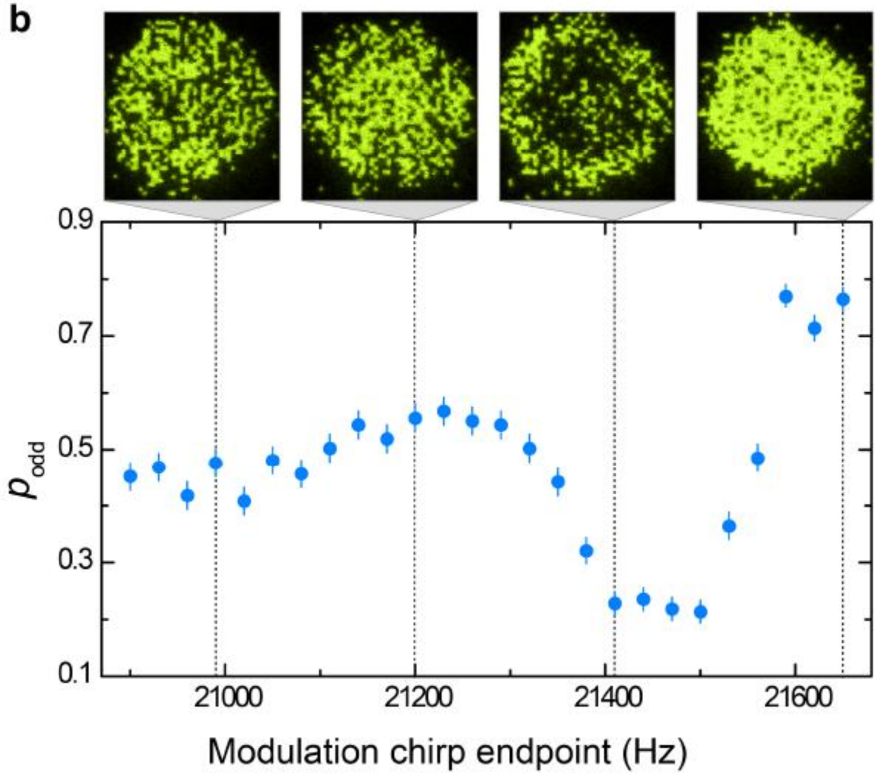


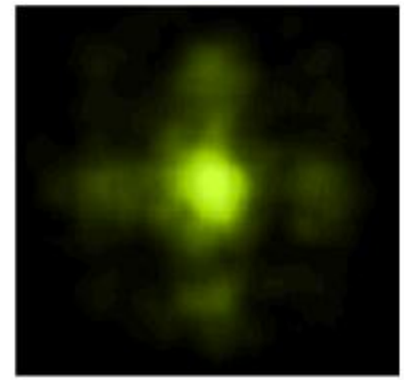
Manipulation of Mott Shell / Filter(Algorithmic) Cooling

[arXiv:1105.5834v1, W. S. Bakr, *et al*.,]



Dephased cloud



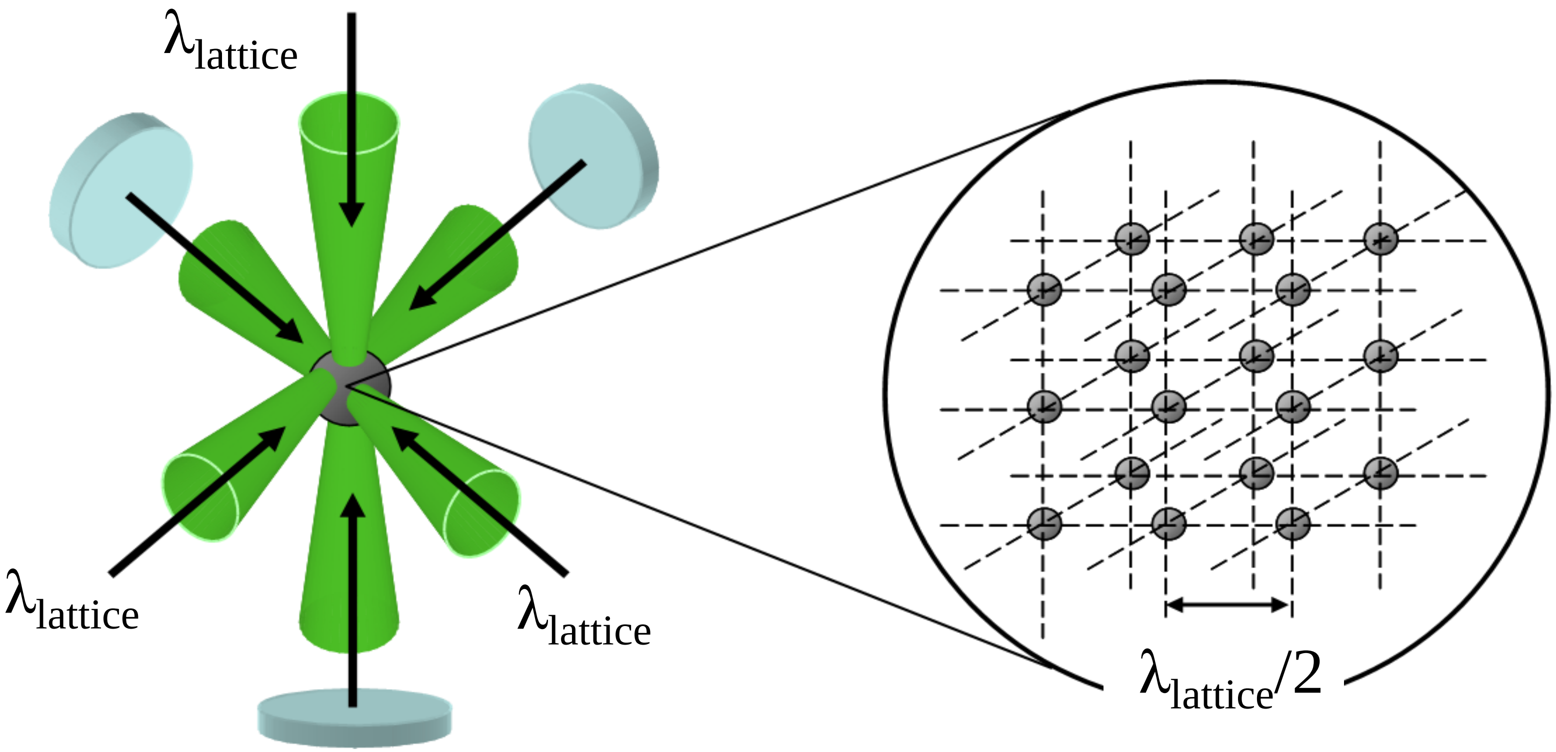


Recooled superfluid

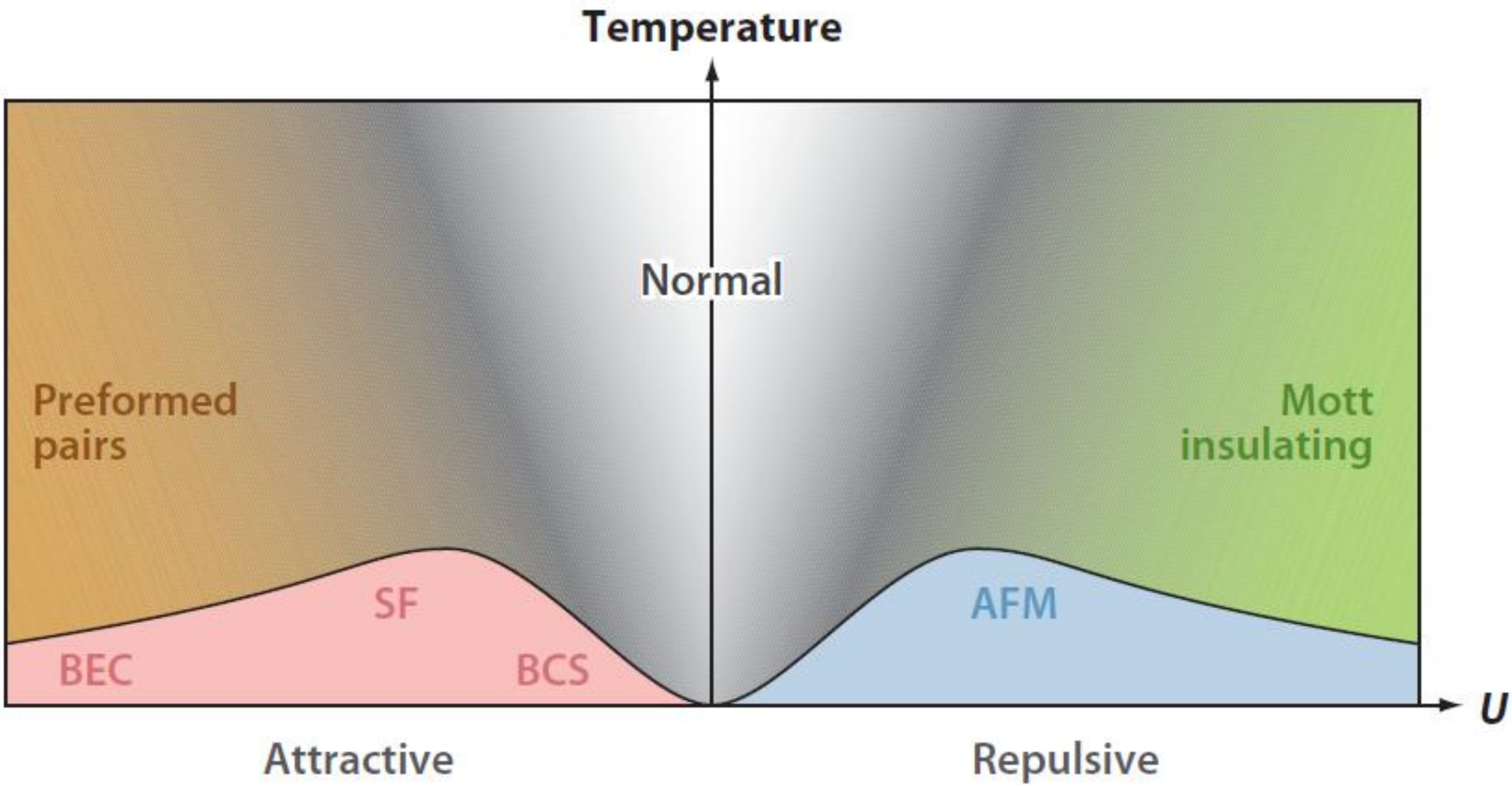
Fermions in a 3D optical lattice

$$H = -J\sum_{<i,j>} c_i^{+} c_j + U\sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_i \varepsilon_i n_i$$

"Fermi-Hubbard Model"

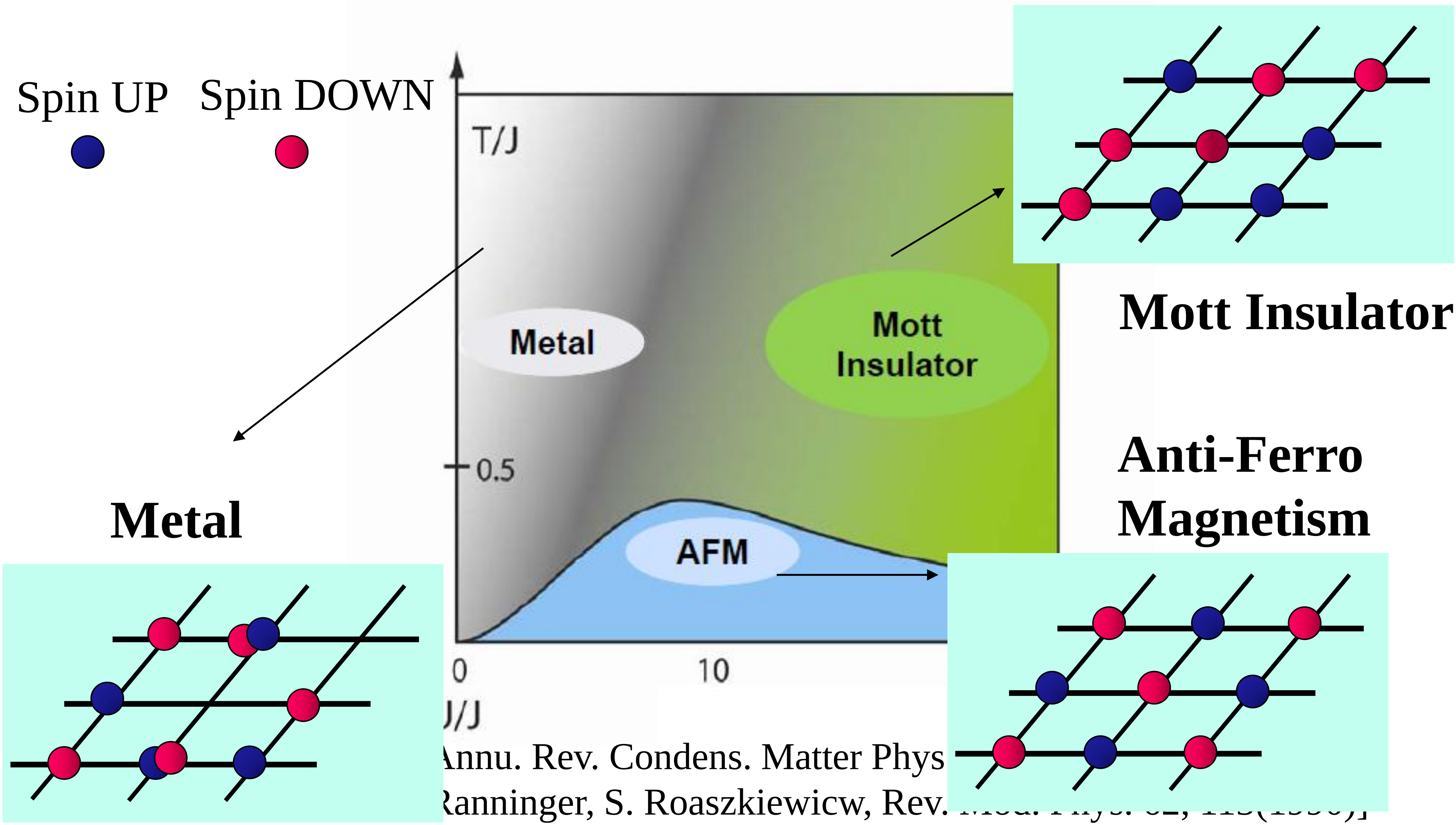


Phase Diagram of Repulsively and Attractively Interacting Fermions



[T. Esslinger, Annu. Rev. Condens. Matter Phys. 2010. 1:129-152,
R. Micnas, J. Ranninger, S. Roaszkiewicw, Rev. Mod. Phys. 62, 113(1990)]

Phase Diagram of Repulsive Fermi-Hubbard Model

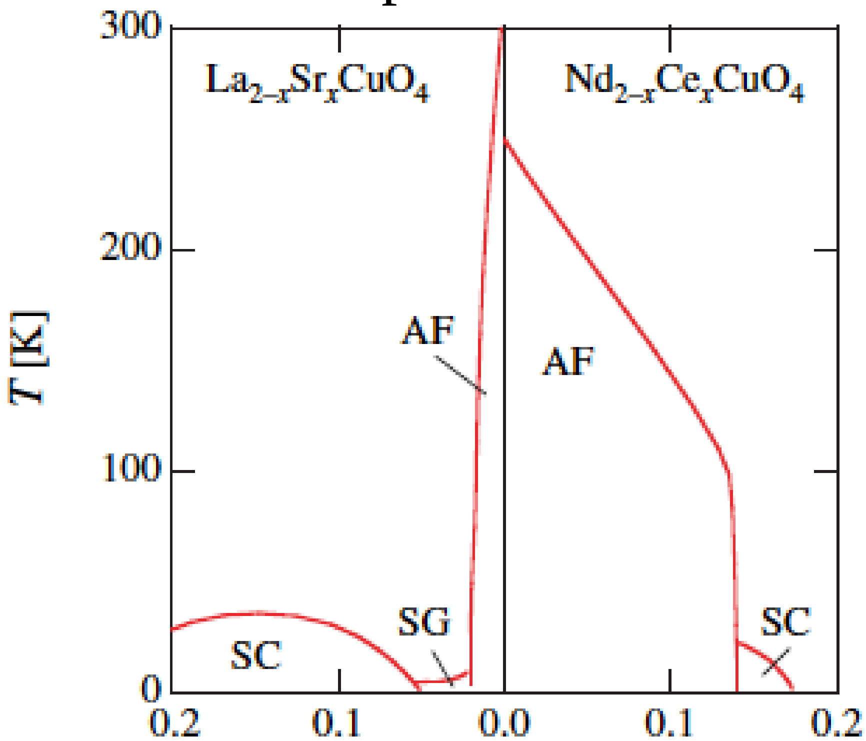


Annu. Rev. Condens. Matter Phys

Ranninger, S. Roaszkiewicw, Rev.

Phase Diagram of High-T_c Cuprate Superconductor

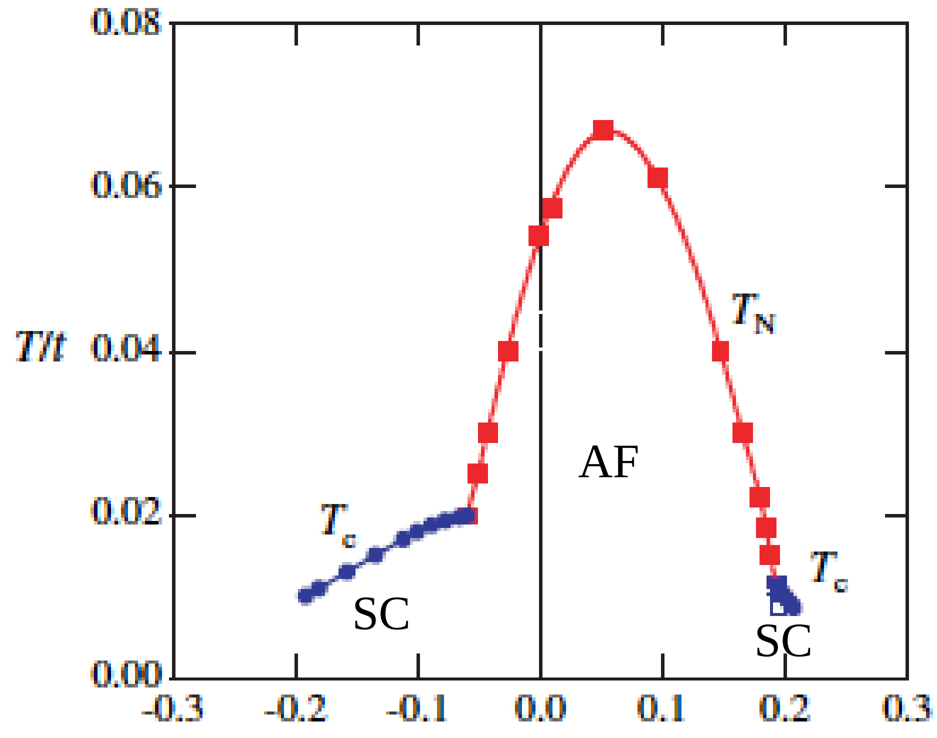
experiment



hole ⟵ x ⟶ electron

(carrier doping)

theory



hole ⟵ x ⟶ electron

(carrier doping)

[in T. Moriya and K. Ueda, Rep. Prog.Phys.66(2003)1299]

There is controversy in the under-dope region

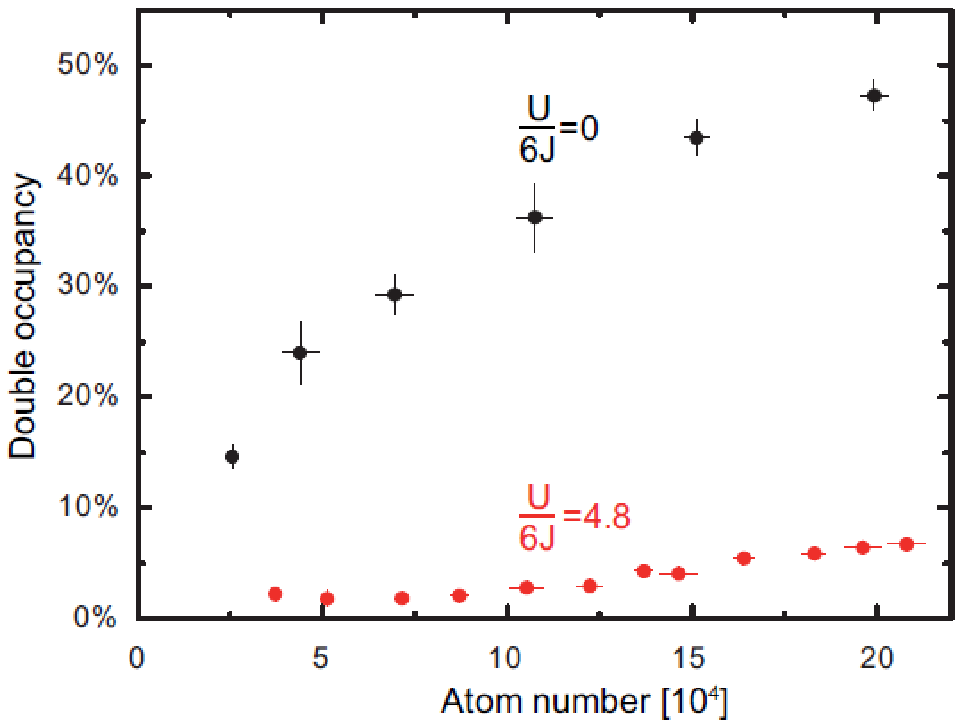
Current Status of

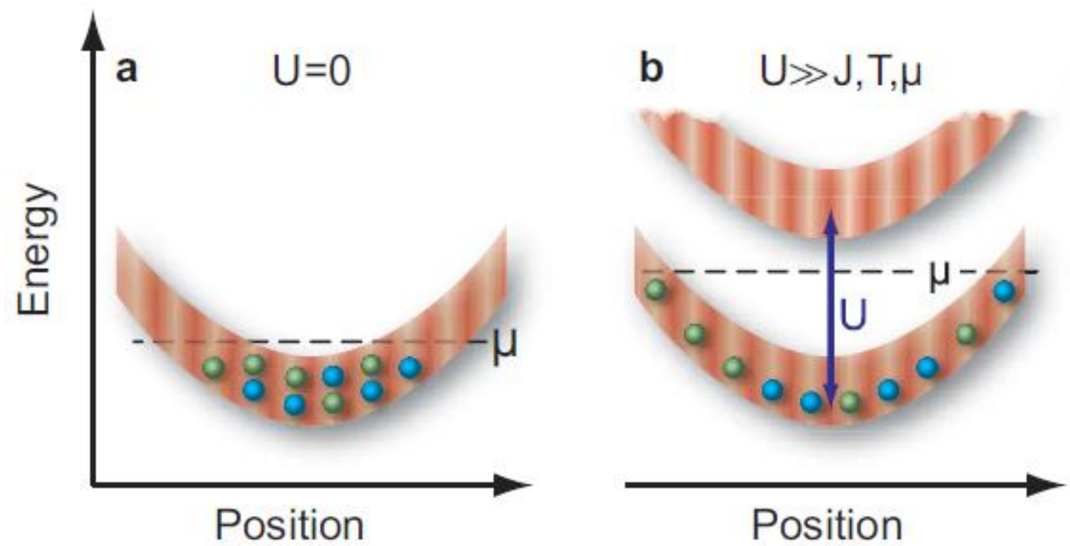
Quantum Simulation of Fermi Hubbard Model:

"Formation of (paramagnetic) Mott insulator"

"A Mott insulator of ^{40}K atoms (2-component) "

[R. Jördens *et al.*, Nature **455**, 204 (2008)] [U. Schneider, *et al.*, Science **322**,1520(2008)]

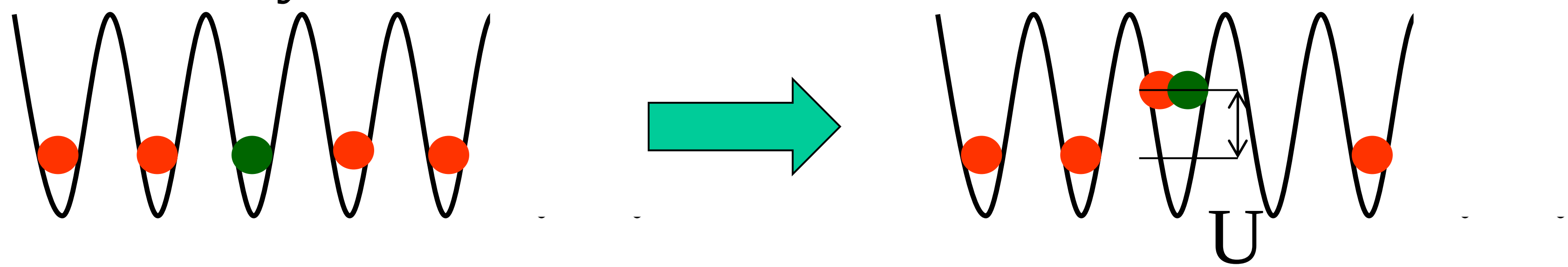


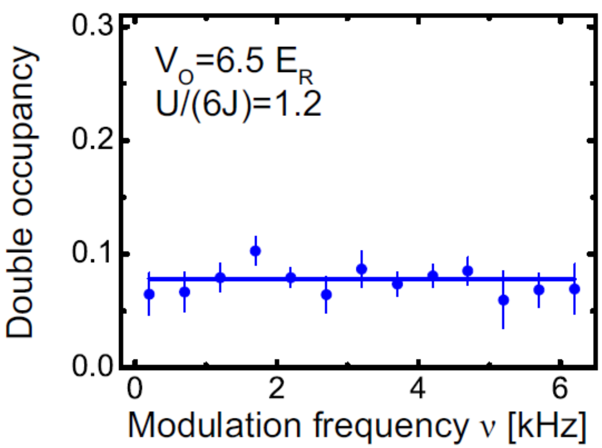


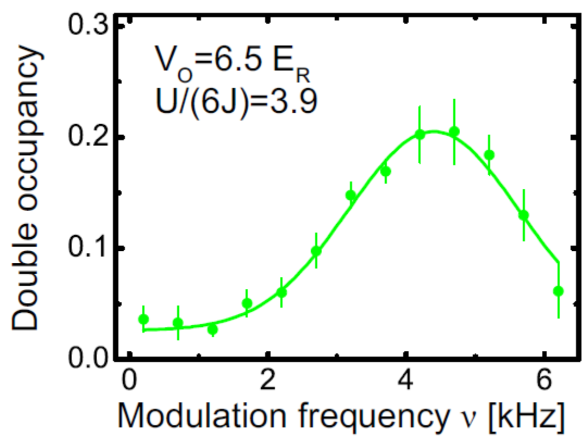
Current Status of
Quantum Simulation of Fermi Hubbard Model:
"Formation of (paramagnetic) Mott insulator"

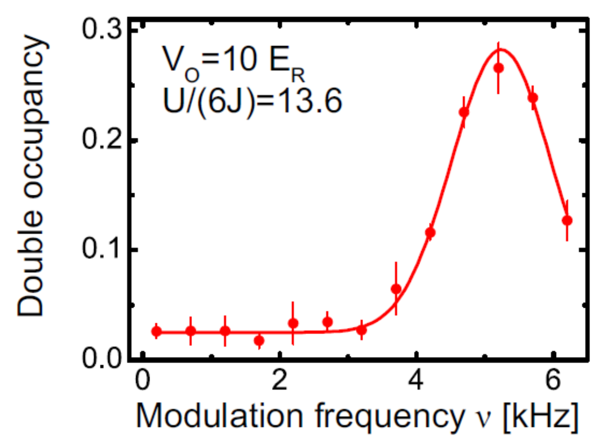
<u>*Modulation Spectroscopy* of Mott Gap:</u>

lattice intensity modulation results in creation of doublon





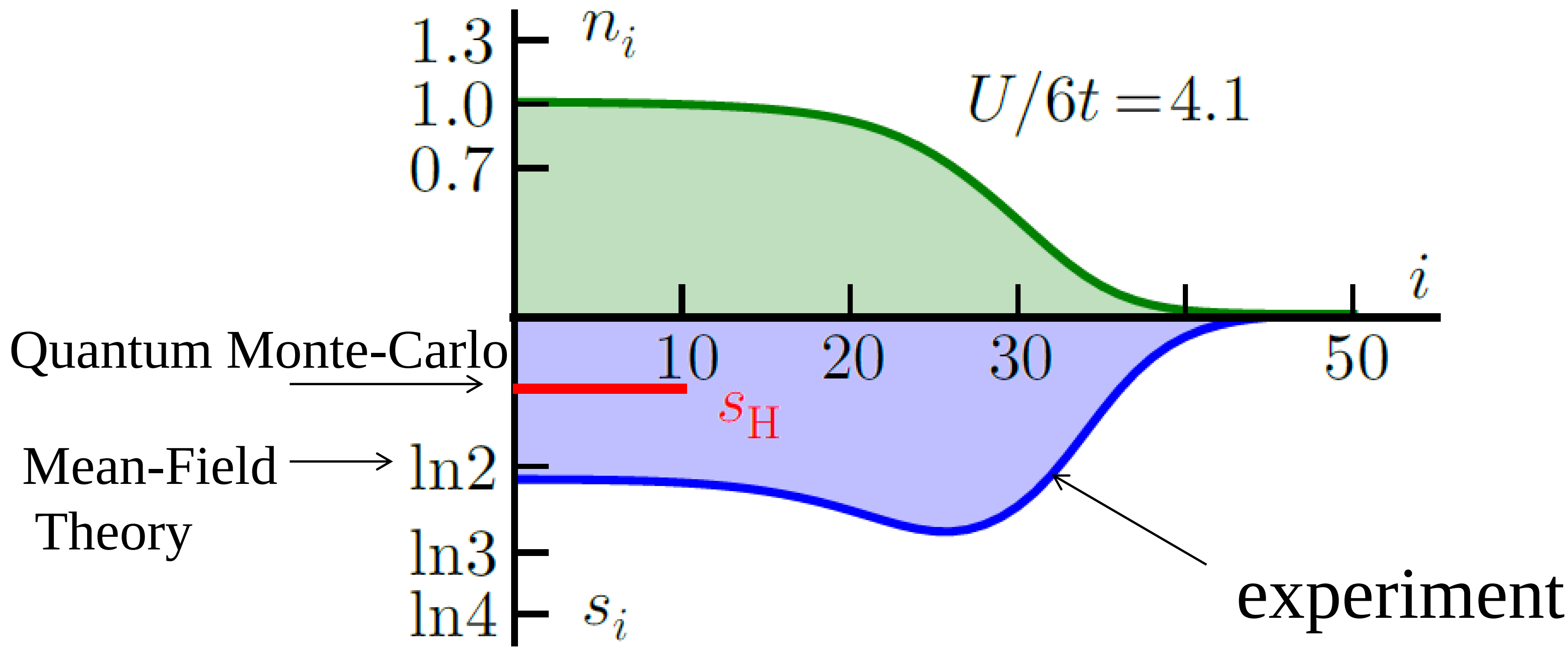




Current Status of
Quantum Simulation of Fermi Hubbard Model:
"Formation of (paramagnetic) Mott insulator"

[R. Jördens *et al.*, PRL **104**, 180401 (2010)]

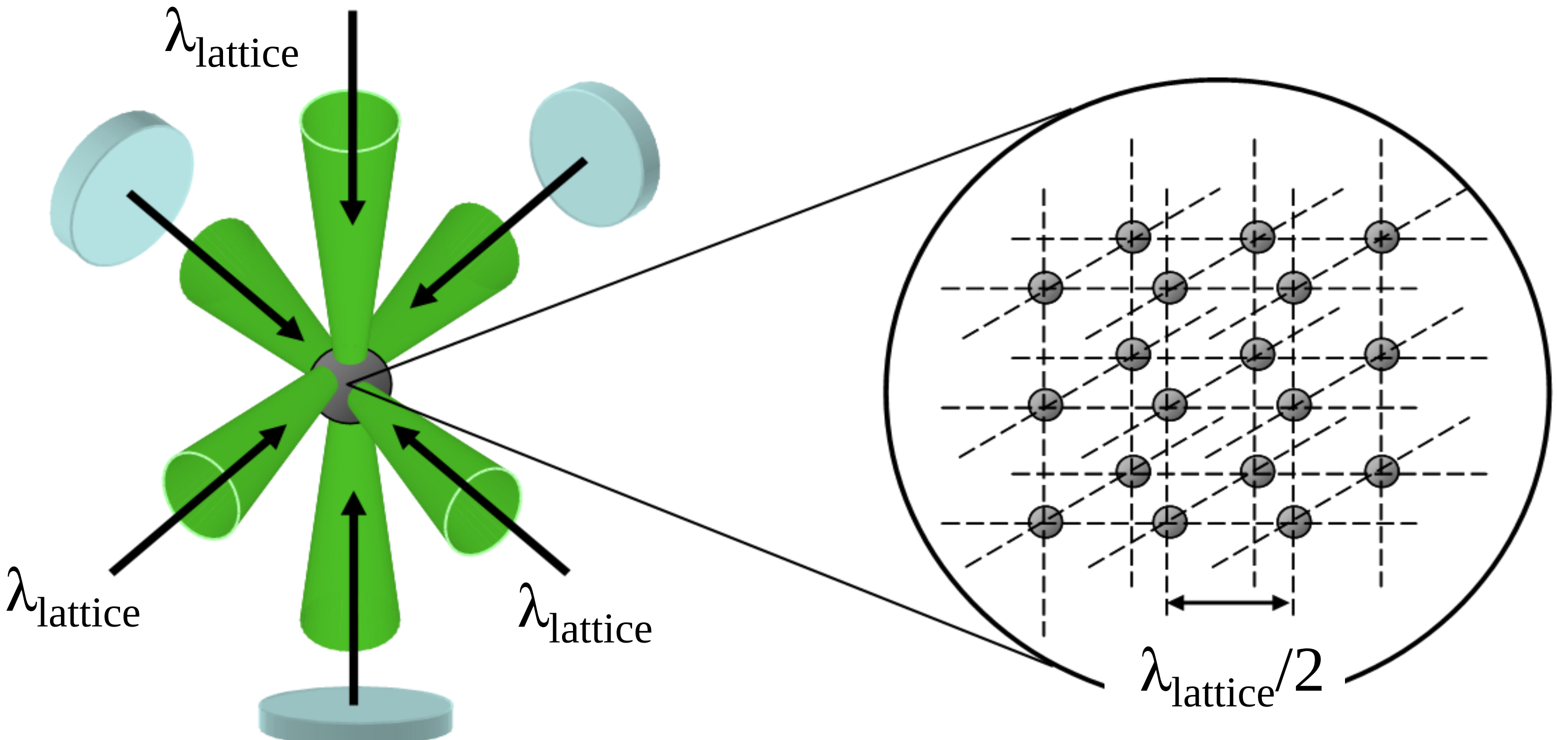
^{40}K atoms (2-component)



Bose-Fermi Mixture in a 3D optical lattice

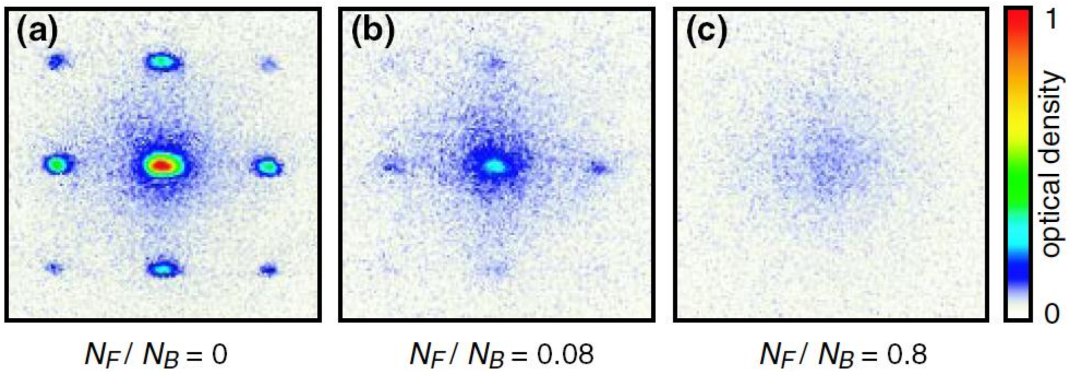
$$H = -t_B \sum_{<i,j>} a_i^+ a_j + \frac{U_{BB}}{2} \sum_i n_{Bi}(n_{Bi} - 1) - t_F \sum_{<i,j>} c_i^+ c_j + U_{BF} \sum_i n_{Bi} n_{Fi}$$

"Bose-Fermi Hubbard Model"



Bose-Fermi Mixture in a 3D optical lattice

Superfluidity of Boson affected by Fermion:



“ ^{40}K(Fermion)-^{87}Rb(Boson)”

[K. Günter, et al, PRL**96**, 180402 (2006)]

[S. Ospelkaus, et al, PRL**96**, 180403 (2006)]

[Th. Best, *et al*, PRL**102**, 030408 (2008)]

Dual Mott Insulating Regime of Boson and Fermion:

$$J \ll k_B T < U_{BB} < |U_{BF}| < U_{FF}$$

“ ^{173}Yb(Fermion)-^{174}Yb(Boson)”

“ ^{173}Yb(Fermion)-^{170}Yb(Boson)”

[Sugawa, S. *et al. Nature Phys*. **7,** 642–648 (2011)]

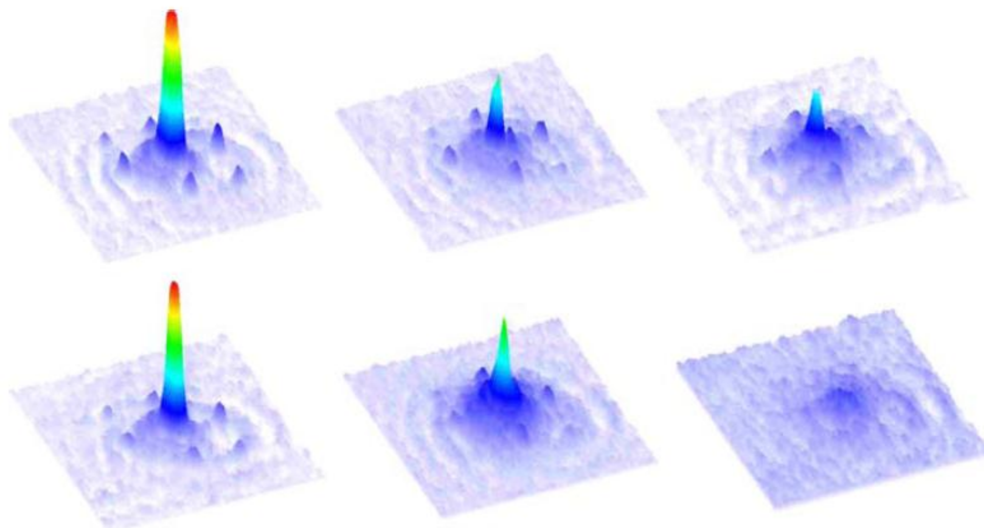
Bose-Bose Hubbard Model

[J. Catani, et al, PRA**77**, 011603(R) (2008)]

“ ^{41}K(Boson)-^{87}Rb(Boson)” **a_{BB} = +8.6 nm**

^{87}Rb only

^{87}Rb mixed with ^{41}K



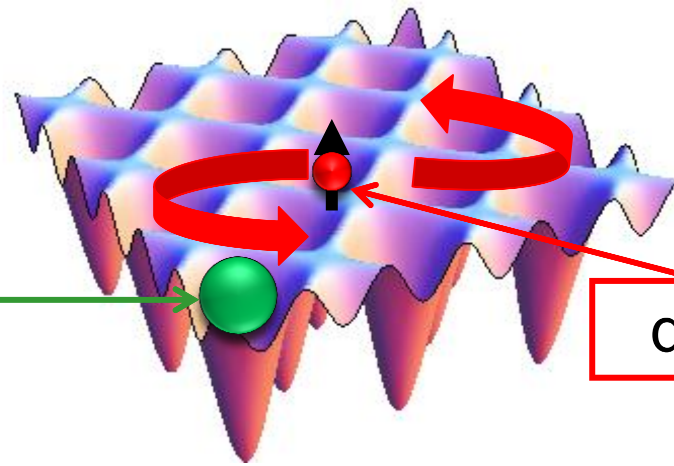
[B. Gadway, et al, PRL**105**, 045303 (2010)]

“^{87}Rb:F=1(Boson)-^{87}Rb:F=2(Boson)”

↑ “delocalized” ↑ “localized”

Simulation of Impurity System

the hopping rate $t_1 \ll t_2$



localized impurity

delocalized carrier

Anderson Hubbard Model **(Binary Alloy Model)**

$$H = -J \sum_{<i,j>, m=\uparrow,\downarrow} c^{+}_{i,m} c_{j,m} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow} + \sum_{i} W_i n_i$$

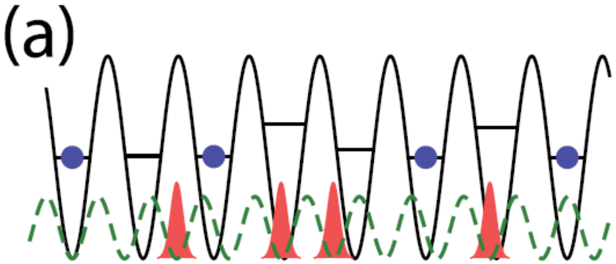
"Random Potential" $W_i = \begin{cases} W & \text{(with atom\#2)} \\ 0 & \text{(without atom\#2)} \end{cases}$

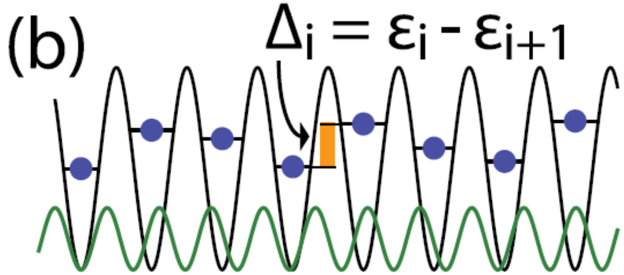
"Randomness and Superfluidity" Anderson (localization) vs Anderson (theorem)

“Glassy Behavior in a Binary Atomic Mixture”

[B. Gadway, et al, PRL**107**, 145306 (2011)]

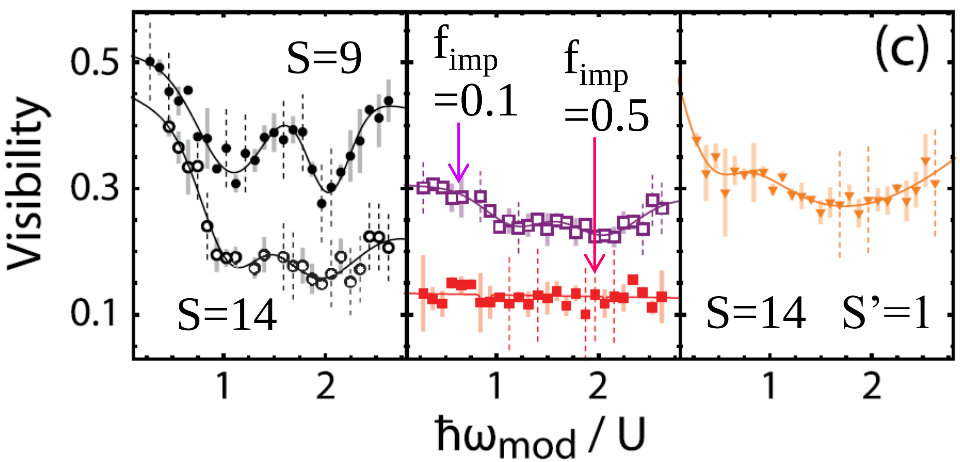
Atomic impurity VS **Bi-chromatic lattice**





“Lattice Modulation”

“No Disorder” “Atomic” “Bi-chromatic”



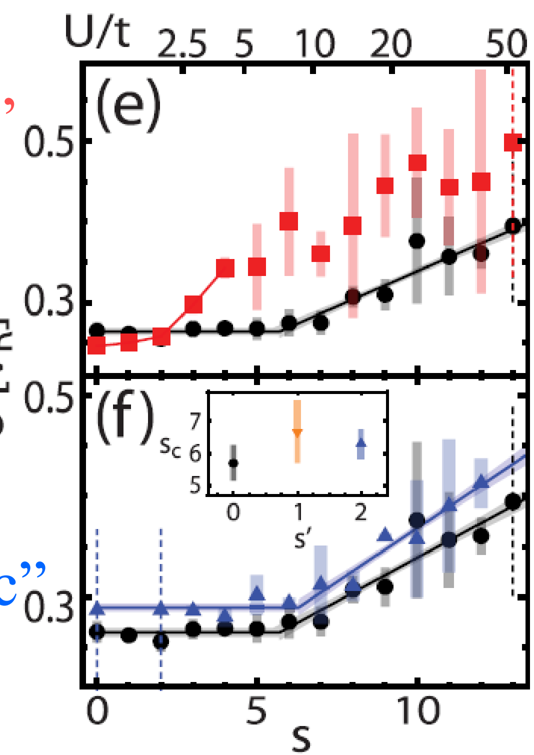
→ Glassy behavior!?

“superfluidity”

“Atomic” ■ f_{imp}=0.5

“No Disorder” ●

“Bi-chromatic” ▲ S’=2



FIRST Quantum Information Processing Project
Summer School 2012

18 August 2012 Miyakojima

Quantum Simulation Using Ultracold Two-Electron Atoms

Kyoto University Y. Takahashi

Quantum Optics Group Members

NTT:
K. Inaba
M. Yamashita

Harvard:
J. M. Doyle

ITAMP:
P. Zhang
H. R. Sadeghpour
A. Dalgarno

Durham:
J. M. Hutson

NIST:
P. Julienne

Niigata:
Y. Yanase

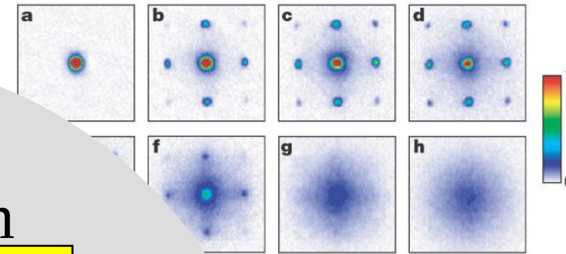
Quantum Simulators using Alkali Atoms

Bose-Hubbard Model:

"Superfluid - Mott-insulat

[M. Greiner, *et al.*, Nat

…



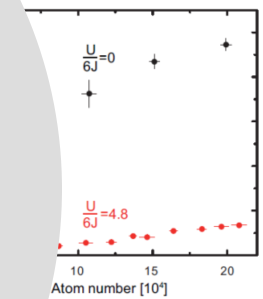
^{87}Rb

Fermi-Hubbard Mo

"Formation of M

[R. Jördens *et al.*,

[U. Schneider, *et*



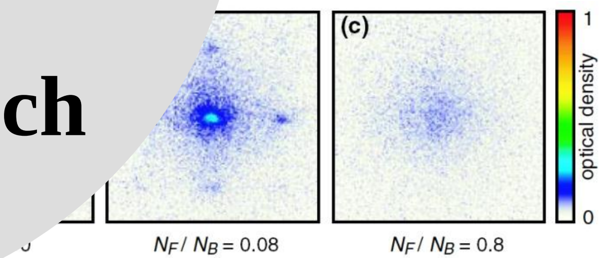
^{40}K

Bose-Fermi-Hubba

[K. Günter, et al, PRL

[S. Ospelkaus, et al, PR

[Th. Best, *et al*, PRL**102**, 0



^{87}Rb + ^{40}K

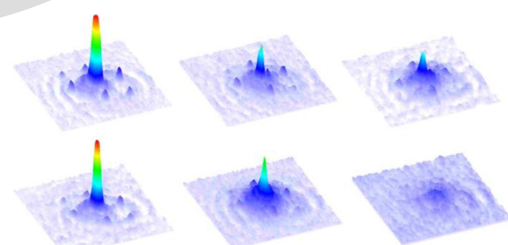
two-electron atom

Yb Atoms

Our Approach

Bose-Bose-Hubbard Model:

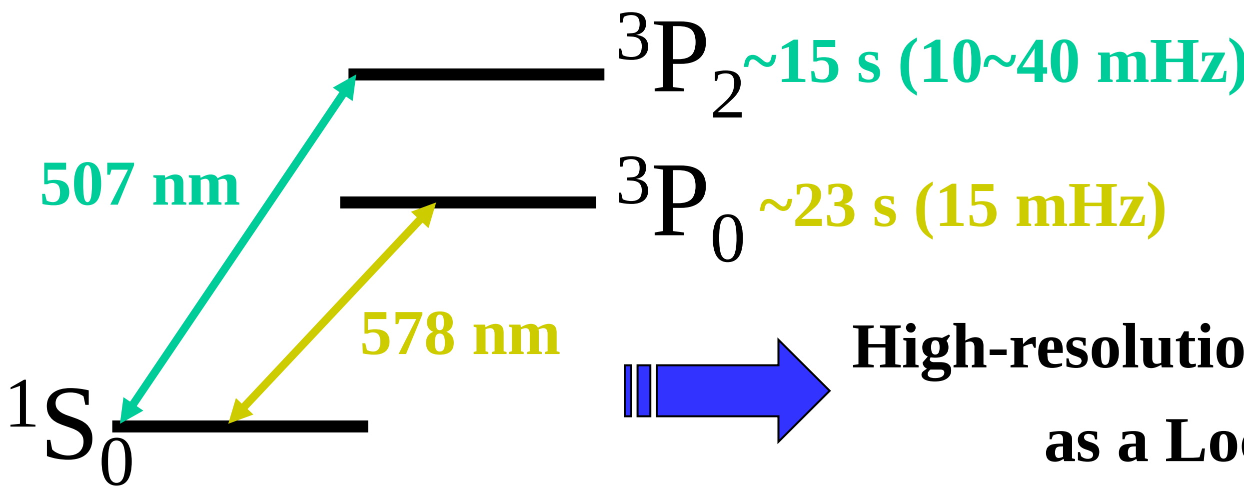
[J. Catani, et al, PRA**77**, 011603(R) (2008)]



$^{87}Rb + ^{41}K$

Unique Features of Ytterbium Atoms

<u>Long-lived metastable state</u>
<u>/Ultra-narrow Optical Transitions</u>

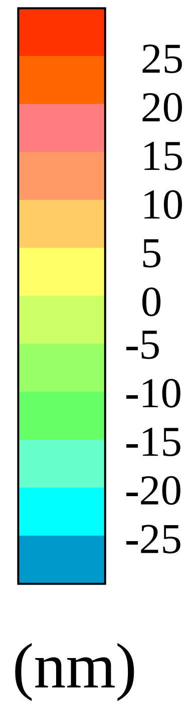


High-resolution laser spectroscopy as a Local Probe

Another Useful Orbital States with Different Characters

Unique Features of Ytterbium Atoms: *Rich Variety of Isotopes*

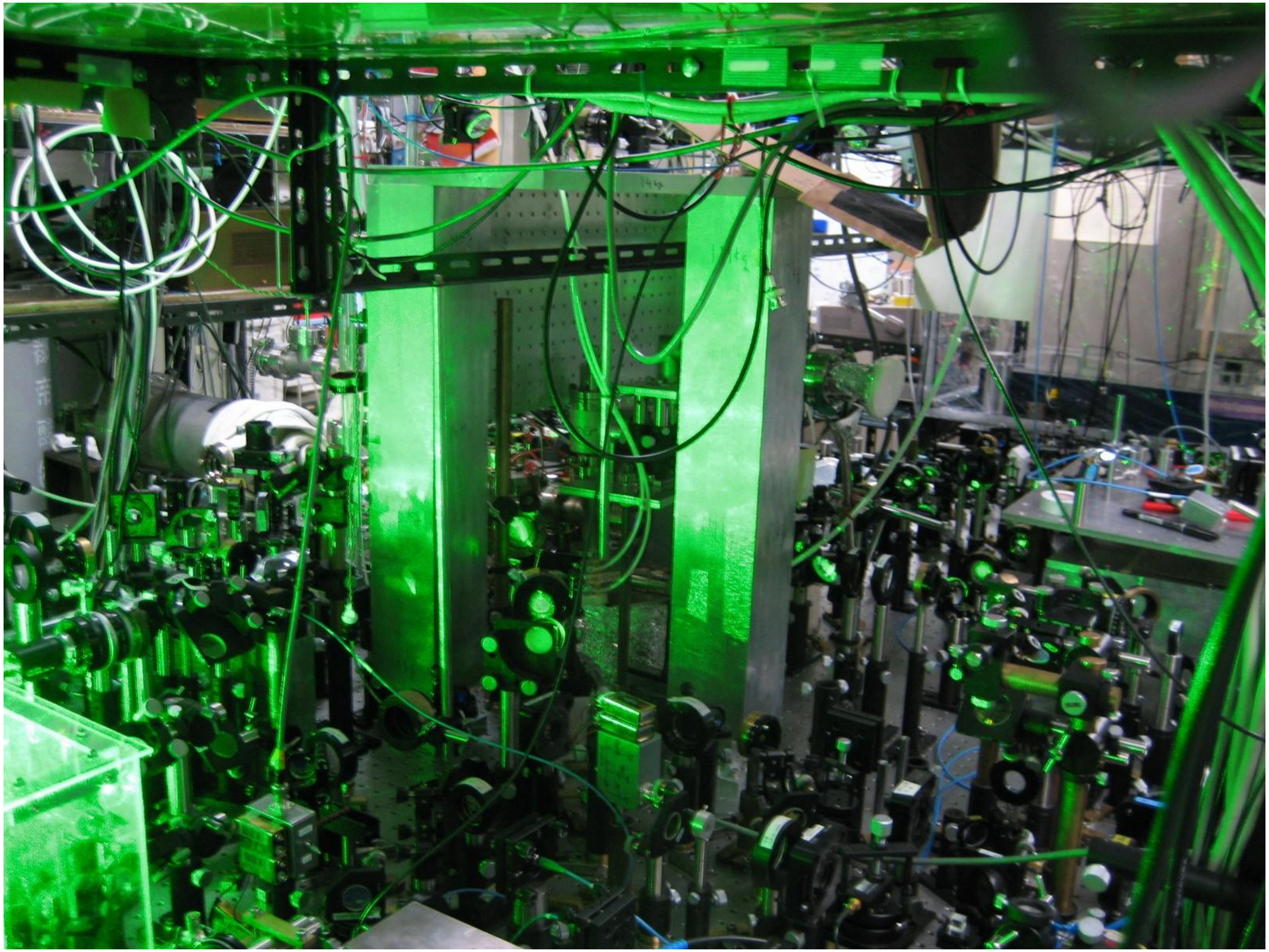
		0.13%	3.05%	14.3%	21.9%	16.2%	31.8%	12.7%
	Mass number	168	170	171	172	173	174	176
B	168							
B	170							
F	171							
B	172							
F	173							
B	174							
B	176							

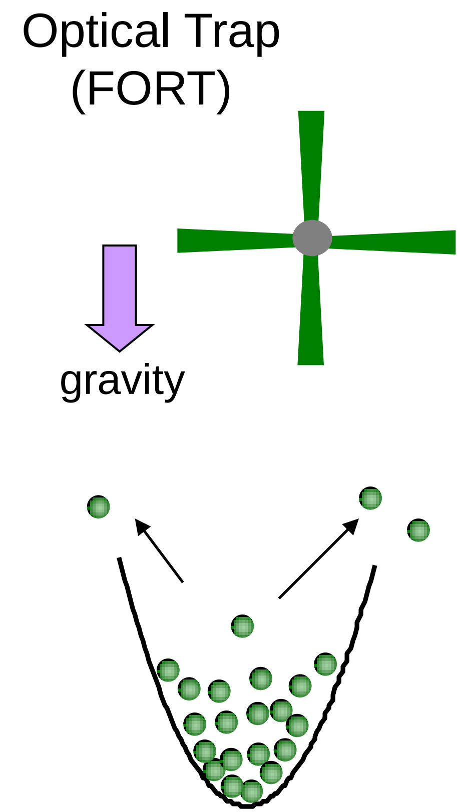


[M. Kitagawa, *et al*, PRA**77**, 012719 (2008)]

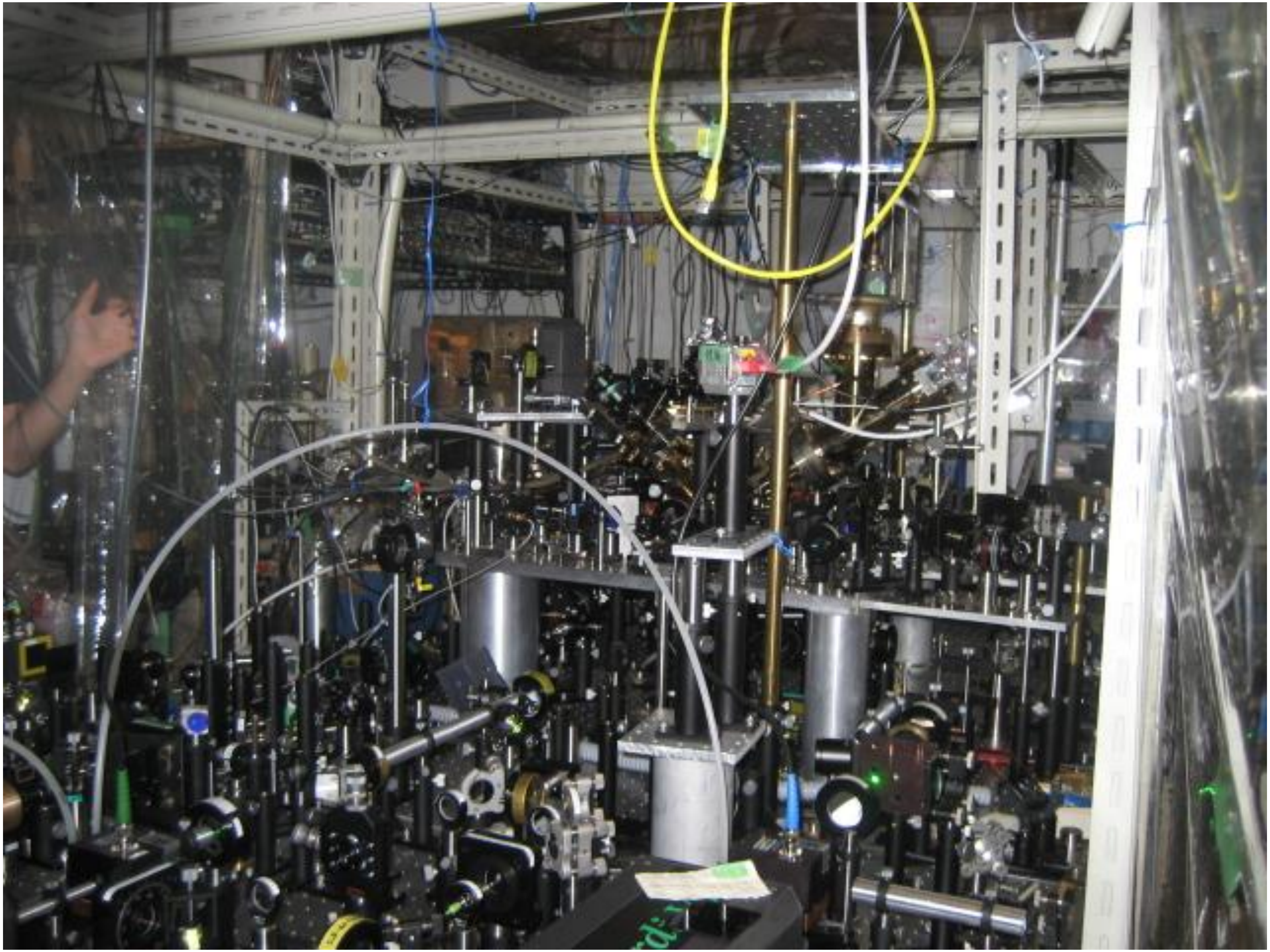
Collaboration with R. Ciurylo, P. Naidon, P. Julienne

Preparation of Quantum Degenerate Gases





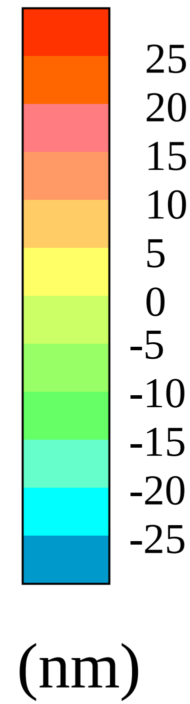
Current Experimental Setup



Unique Features of Ytterbium Atoms:
Rich Variety of Isotopes

Scattering Length

		0.13%	3.05%	14.3%	21.9%	16.2%	31.8%	12.7%
	Mass number	168	170	171	172	173	174	176
B	168	**13**						
B	170	**6.2**	**3.4**					
F	171	**4.7**	**1.9**	**-0.2**				
B	172	**3.4**	**-0.1**	**-4.5**	**-32**			
F	173	**2.0**	**-4.3**	**-31**	**22**	**11**		
B	174	**0.1**	**-27**	**23**	**11**	**7.3**	**5.6**	
B	176	**-19**	**11**	**7.5**	**5.6**	**4.2**	**2.9**	**-1.3**



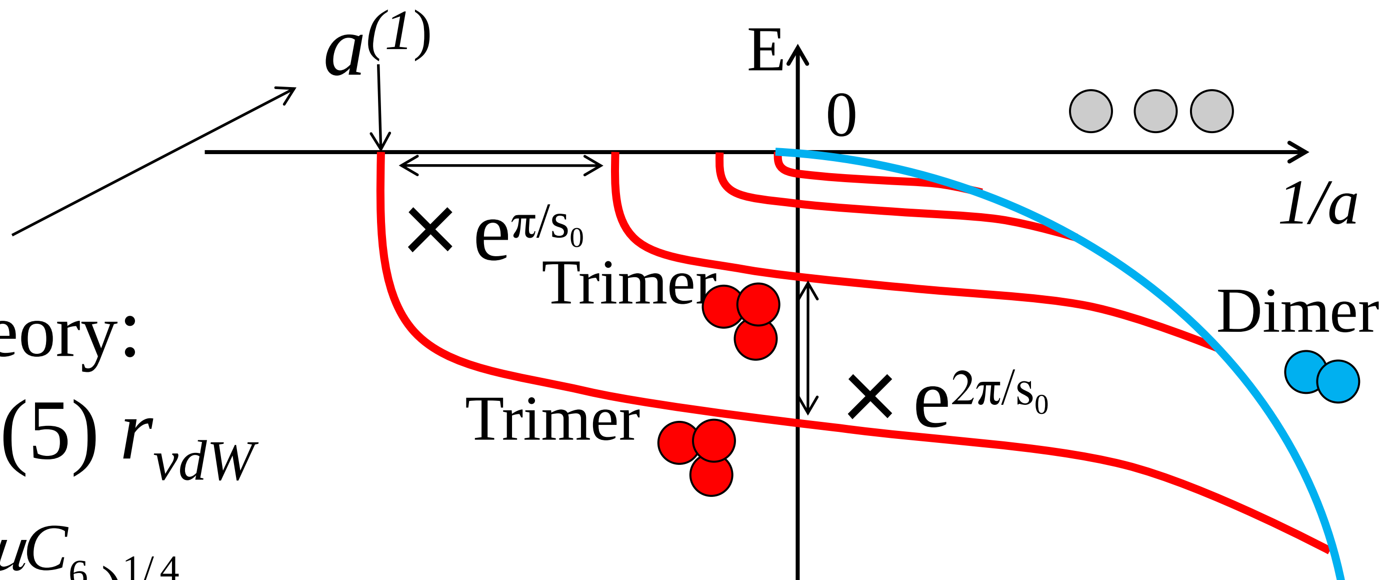
[M. Kitagawa, *et al*, PRA**77**, 012719 (2008)]

Collaboration with R. Ciurylo, P. Naidon, P. Julienne

^{172}Yb: No BEC ! No Fun ?

"Energy Spectrum of Universal Efimov Trimer "

[E. Braaten and H.-W. Hammer, Annals of Phys. 322, (2007) 120]



Recent theory:

$a^{(1)}$=-9.0(5) r_{vdW}

$$r_{vdW} = \frac{1}{2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{1/4}$$

Identical Boson: $e^{\pi/s_0} \simeq 22.7$

^{172}Yb : a = -32 nm = -7.6 r_{vdW}

"Naturally Prepared Universal Efimov Trimer Resonance"

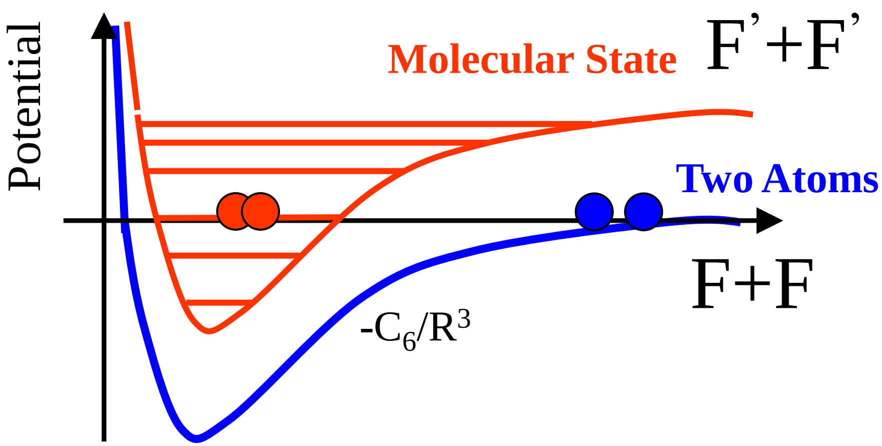
How to Control U

Magnetic Feshbach Resonance

Coupling between "**Open Channel**" and "**Closed Channel**"

→ Control of Interaction(a_s)

$$a_s(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



$M_{total}=M_1+M_2+m_l$: conserved

$l_{open}=l_{closed}$, $l_{open}\neq l_{closed}$ if $V_{ss}\neq 0$

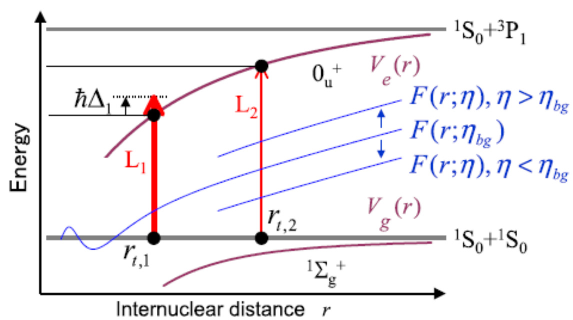
How to Control *U* for Yb

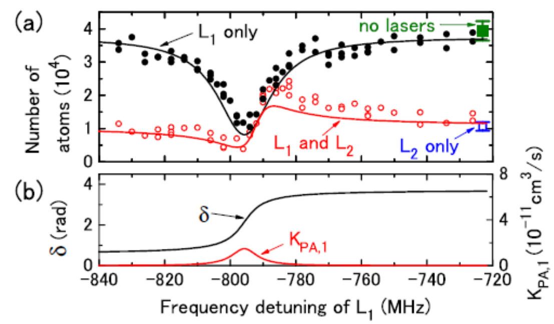
Optical Feshbach Resonance for Yb atoms ($^1S_0 + ^1S_0$)

PRL,101, 203201(2008)

"Optical Feshbach Resonance Using the Intercombination Transition"

K. Enomoto, K. Kasa, M. Kitagawa, and Y. Takahashi

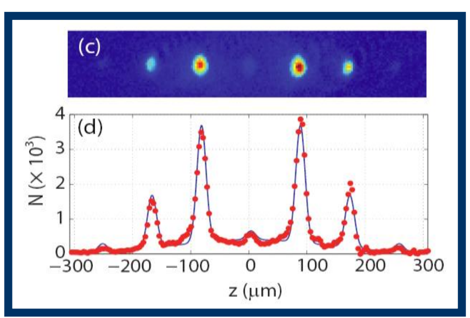


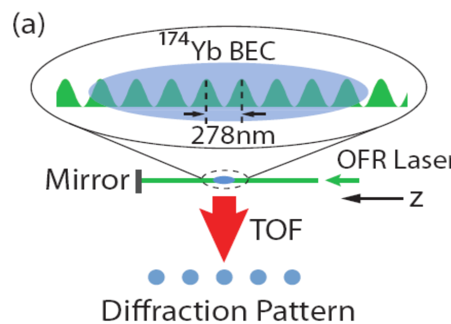


PRL,105, 050405(2010)

"Submicron Spatial Modulation of an Interatomic Interaction in a BEC"

Rekishu Yamazaki, Shintaro Taie, Seiji Sugawa, Yoshiro Takahashi

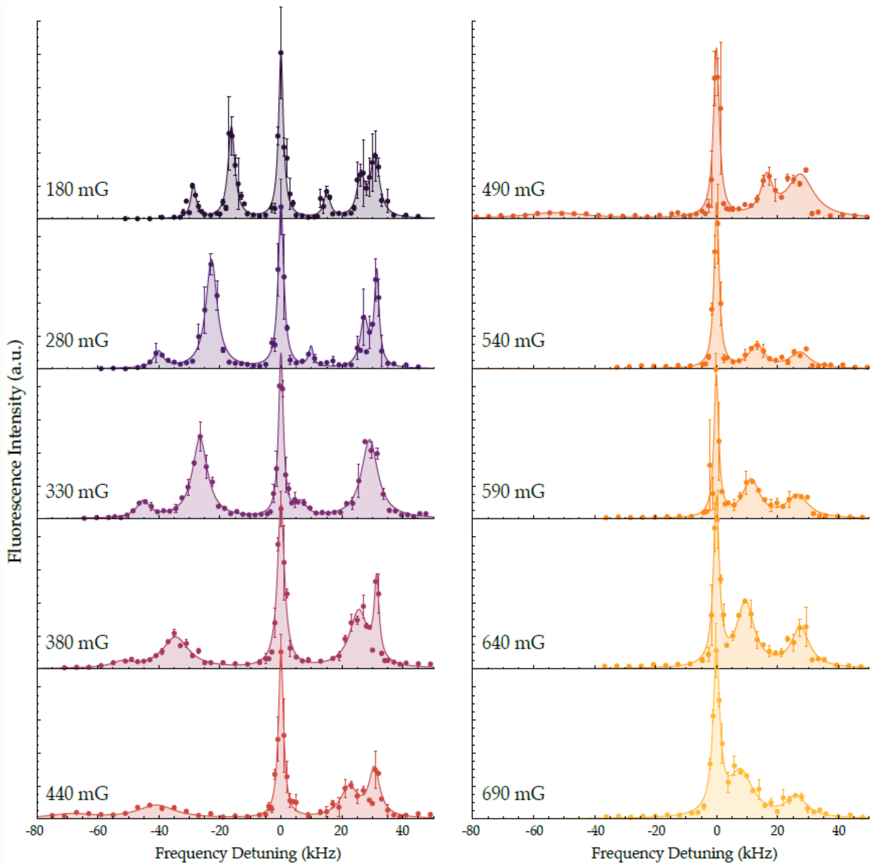


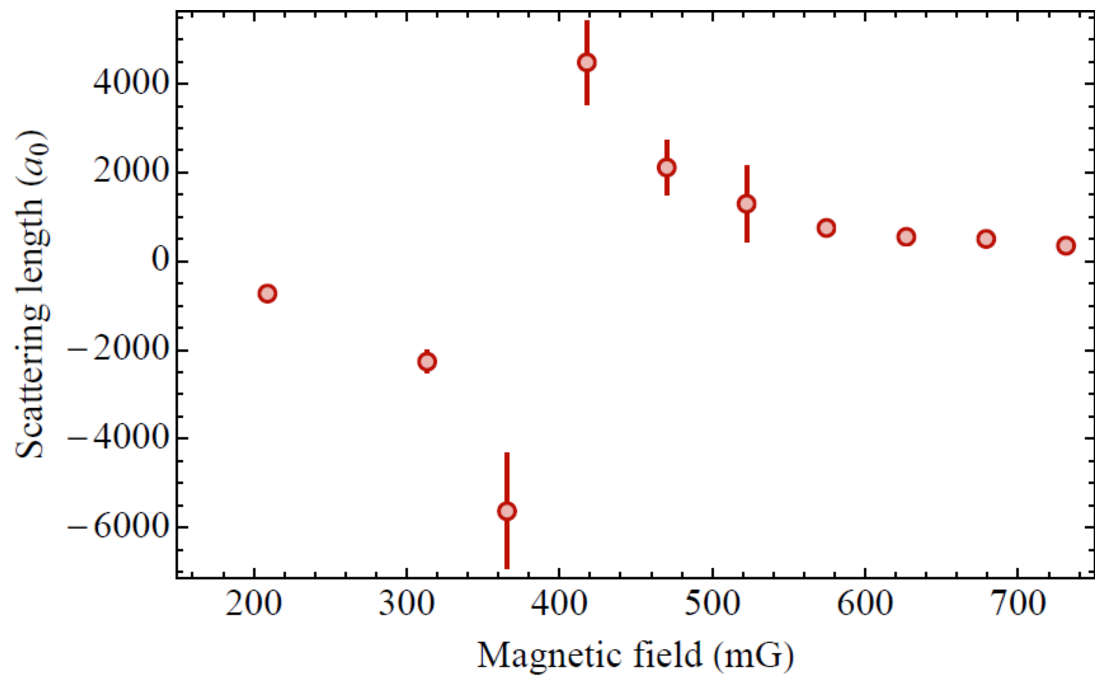


How to Control *U* for Yb

Magnetic Feshbach Resonance for Yb atoms ($^1S_0 + {}^3P_2$)

“$^3P_2(m=+2)$”: ^{174}Yb

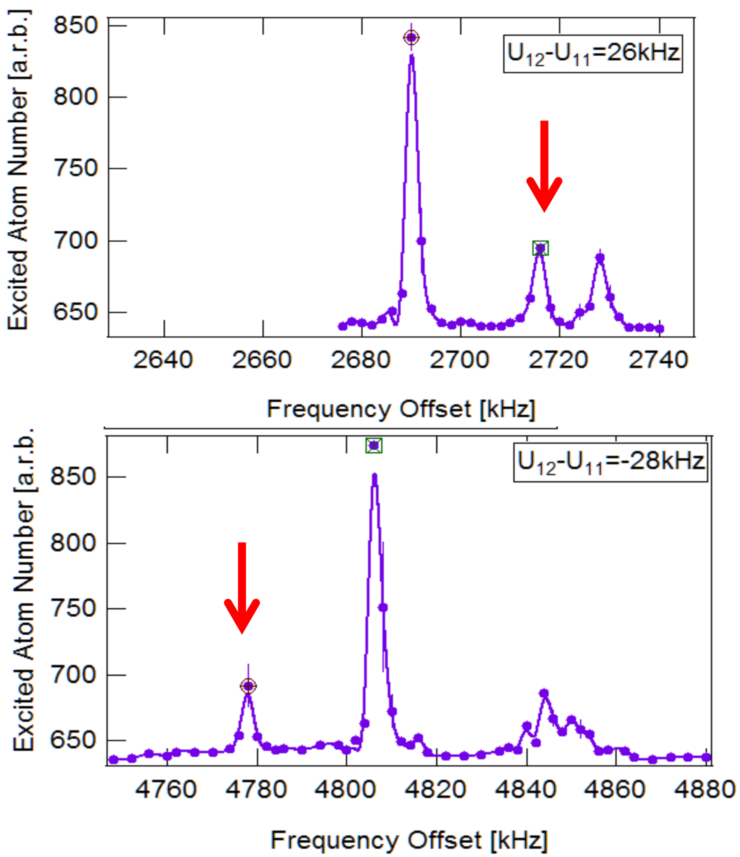




How to Control *U* for Yb

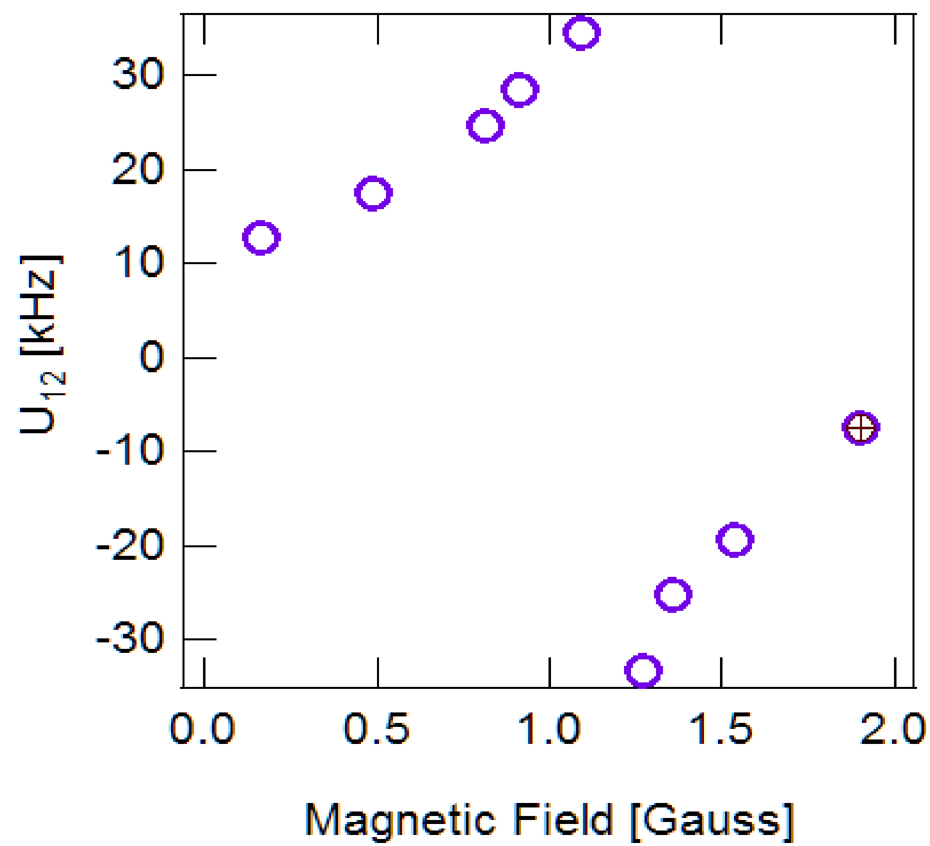
Magnetic Feshbach Resonance for Yb atoms ($^1S_0$+$^3P_2$)

“$^3P_2$(m=-2)”: ^{170}Yb



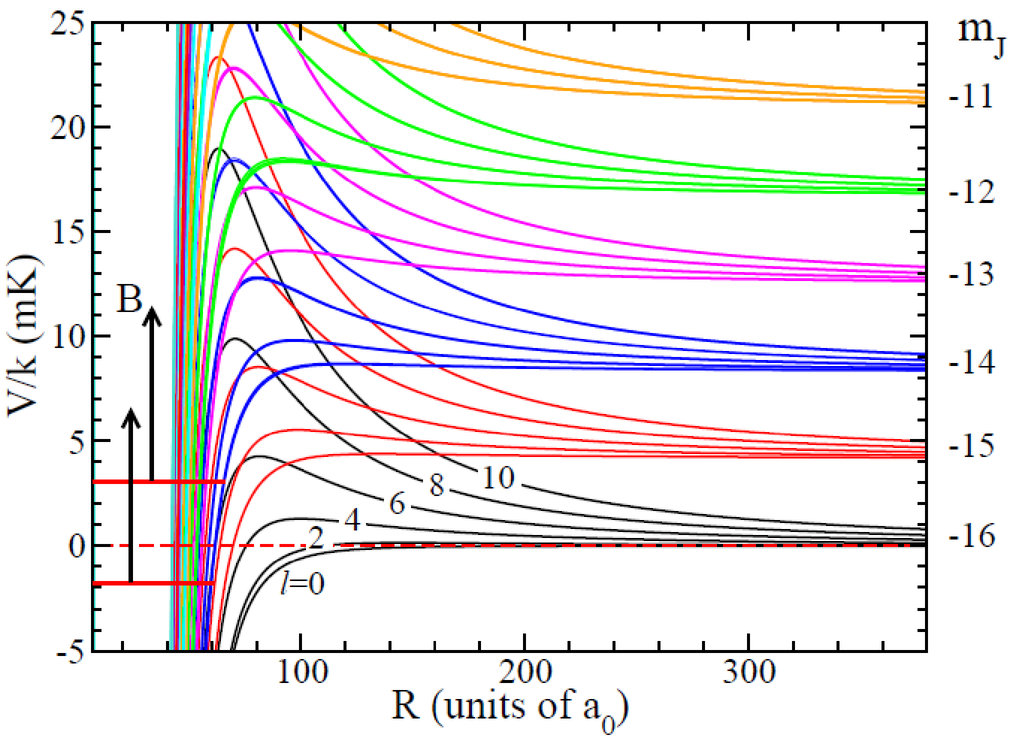
3d lattice, V=25E_R

U_{11}=2.6 kHz @ 25E_R



Anisotropy-induced Feshbach Resonance

" Anisotropy induced Feshbach resonances in a quantum dipolar gas of magnetic atoms"
A. Petrov, E. Tiesinga, and S. Kotochigova arXiv:1203.4172v1



$M_{total}=M_1+M_2+m_l$: conserved

$l_{open}=l_{closed}$, $l_{open}\neq l_{closed}$ if $V_{ss}\neq 0$

Anisotropic electrostatic interaction induces coupling between different partial waves

Anisotropic Interaction in $^1S_0 + ^3P_2$

[R. Krems and A. Dalgarno, PRA **68**, 013406 (2003)]

$$V_{ES} = \sum_{\lambda=0,2} \frac{4\pi}{2\lambda+1} V_\lambda(R) \sum_{m_\lambda} Y^*_{\lambda m_\lambda}(\hat{R}) Y_{\lambda m_\lambda}(\hat{r})$$

$\hat{r}$: "electronic coordinates"

$V_\lambda(R)$, $\hat{R}$: "inter-atomic separation"

$$\longrightarrow \langle l m_l j(LS) m_j | V_{ES} | j'(LS) m_j' l' m_l' \rangle$$

$$= \sum_{\lambda=0,2} V_\lambda \sum_{m_\lambda} (-1)^{S+j+j'+\lambda+m_\lambda-m_l-m_j}$$

$$\times[(2L+1)(2L+1)(2j+1)(2j'+1)$$

$$\times(2l+1)(2l'+1)]^{1/2} \begin{Bmatrix} L & j & S \\ j' & L & \lambda \end{Bmatrix}$$

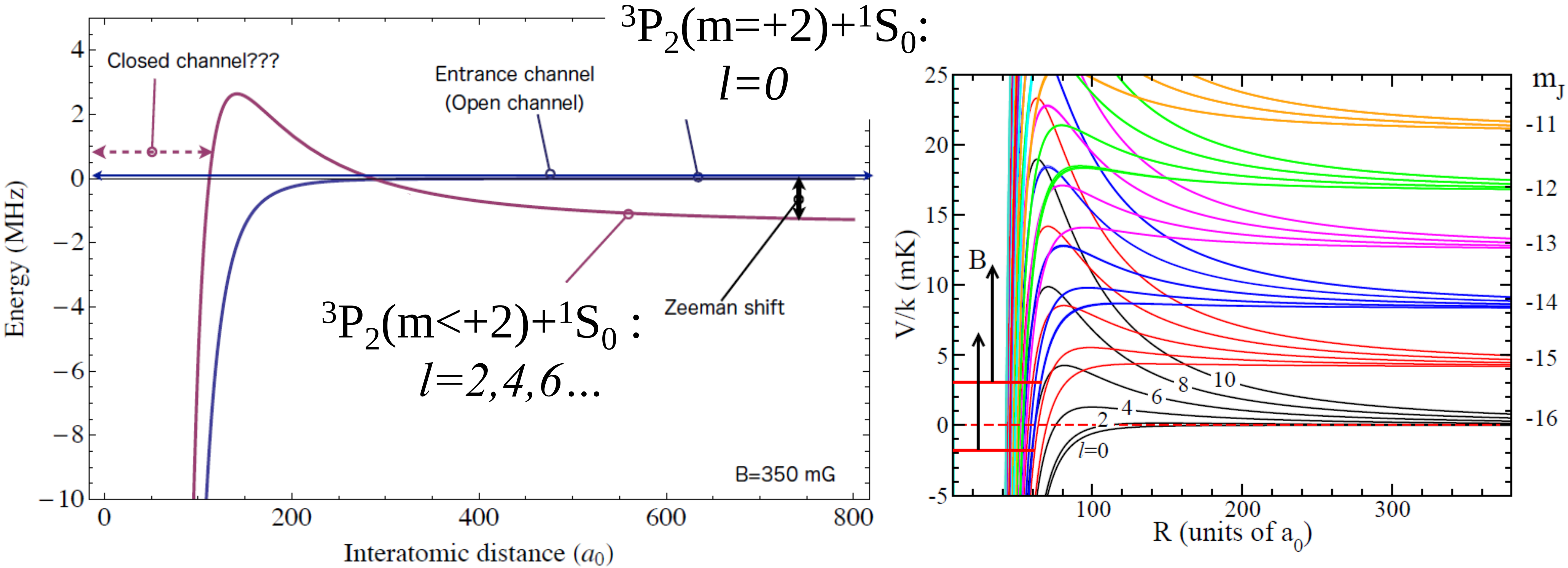
$$\times \begin{pmatrix} j & \lambda & j' \\ -m_j & m_\lambda & m_j' \end{pmatrix} \begin{pmatrix} l & \lambda & l' \\ -m_l & -m_\lambda & m_l' \end{pmatrix}$$

$$\times \begin{pmatrix} L & \lambda & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix},$$

$$V_{\lambda=0} = (V_\Sigma + 2V_\Pi)/3,$$

$$V_{\lambda=2} = 5(V_\Sigma - V_\Pi)/3.$$

Combination of Feshbach Resonance & Shape Resonance in the presence of ***Anisotropy***



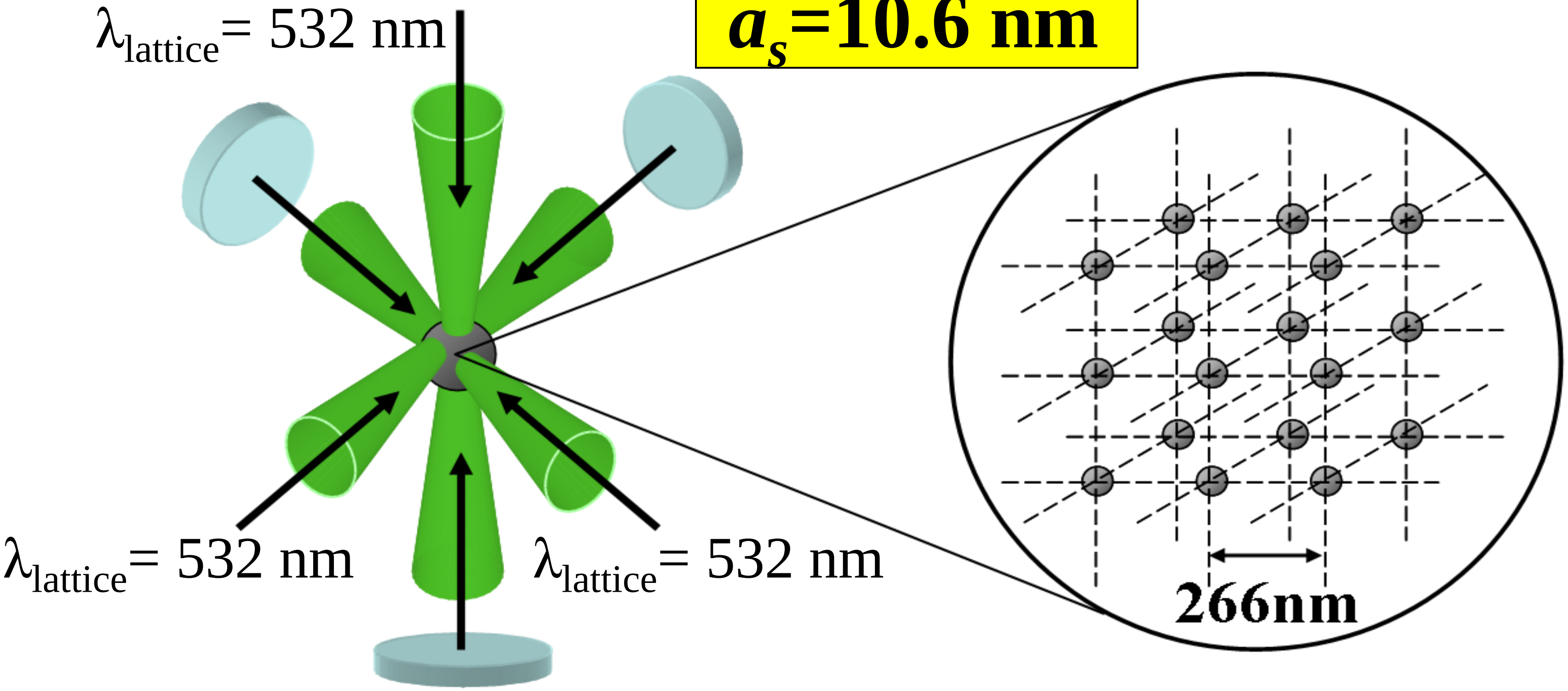
We are now searching for Feshbach Resonance for Fermions

Fermion ^{173}Yb in a 3D optical lattice

$$H = -t_F \sum_{<i,j>} c_i^{+} c_j + U_{FF} \sum_{i, m_F \neq m_F'} n_{m_F,i} n_{m_F',i}$$

^{173}Yb

$\boldsymbol{a_s}$**=10.6 nm**



SU(6) Fermion (^{173}Yb)

^{173}Yb: +5/2 +3/2 +1/2 -1/2 -3/2 -5/2

"origin of spin degrees of freedom is "*nuclear spin*"

$$H_{\text{int}} = \frac{4\pi\hbar^2 a_s}{M}\delta(\vec{r}_1 - \vec{r}_2)$$ **SU(6) system**

M. A. Cazalilla, *et al.*, N. J. Phys **11**, 103033(2009)
A. V. Gorshkov, *et al.*, Nat. Phys. **6**, 289(2010) , etc

"Experimental realization is very difficult in solid state system"

Nuclear spin permutation operators: $S_n^m \equiv c_n^+ c_m = |n\rangle\langle m|$

SU(N) algebra : $\left[S_n^m, S_q^p\right] = \delta_{mq} S_n^p - \delta_{pn} S_q^m$

SU(N) symmetry: $\left[H, S_n^m\right] = 0$

SU(N) Hubbard → Mott Insulator → Heisenberg model: $H = \frac{2t^2}{U}\sum_{\langle i,j\rangle m,n} S_n^m(i) S_m^n(j)$
$(U >> t)$

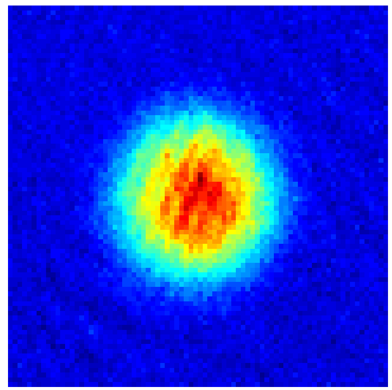
Spin Selective Detection of SU(6) Fermion

[S. Taie *et al* ., PRL**105**, 190401(2010)]

^{173}Yb:SU(6)

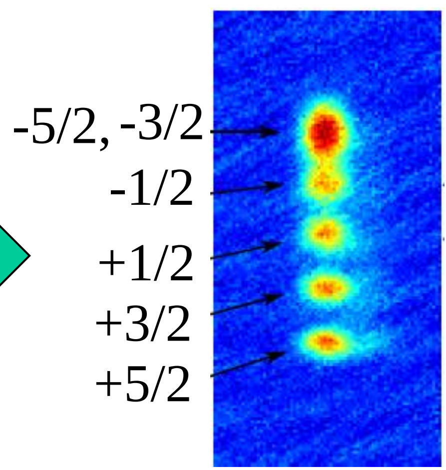
+5/2 +3/2 +1/2 -1/2 -3/2 -5/2

Conventional TOF-image →



$T/T_F = 0.14$

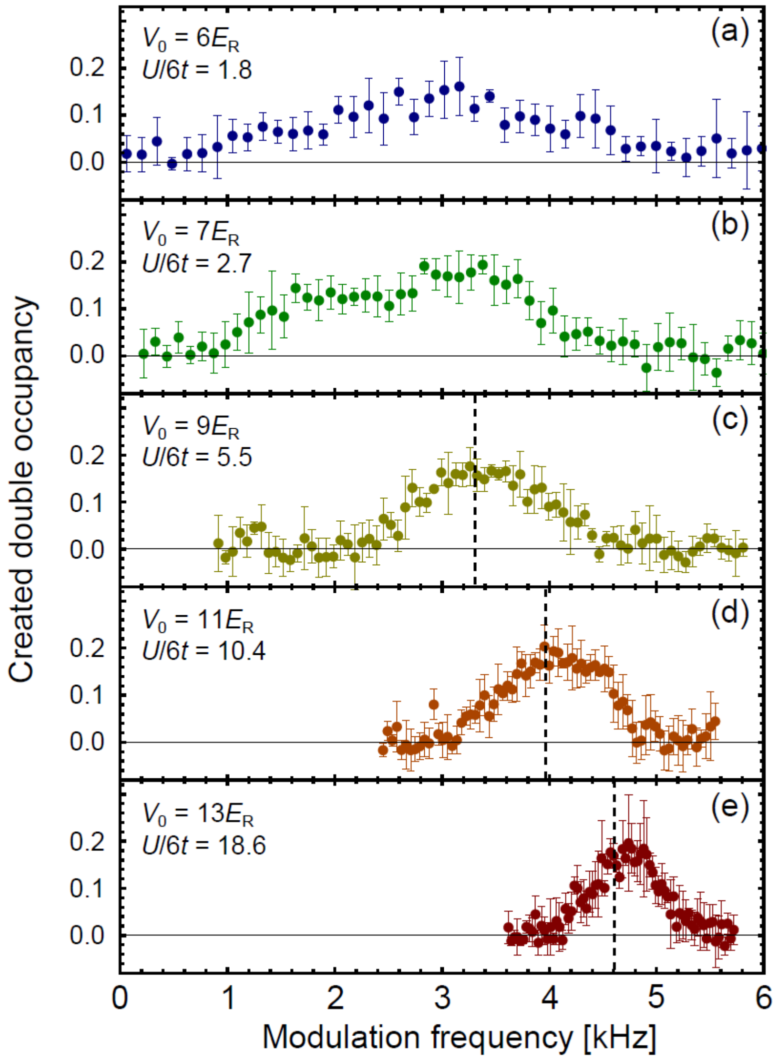
Optical **Stern-Gerlach Spin-Separation** →

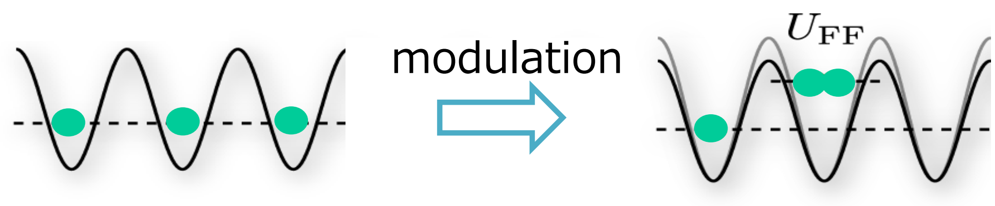


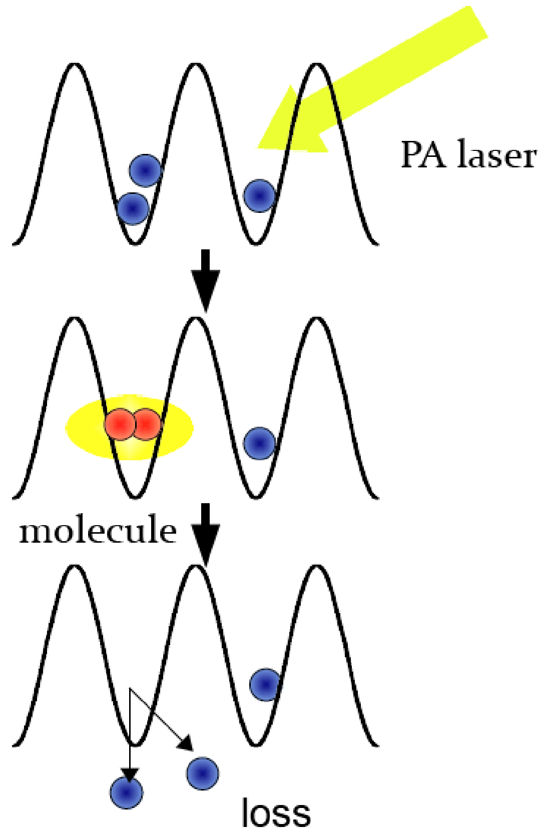
"Formation of SU(6) Mott insulator"

[S. Taie *et al*,]

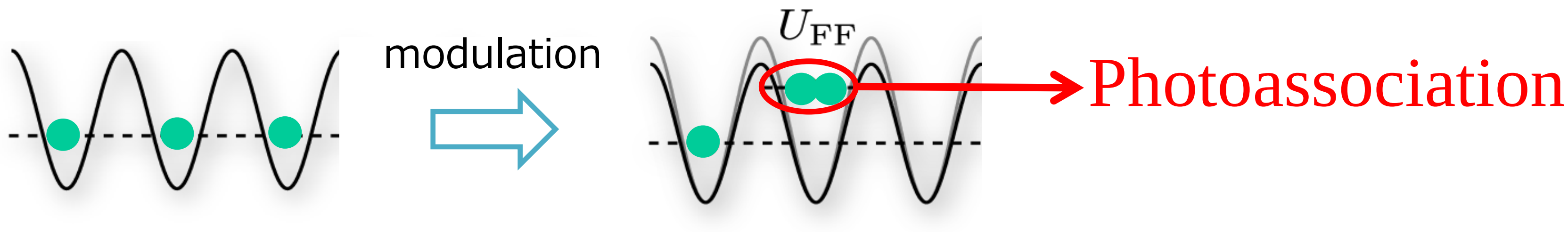
Excitation (Mott) Gap







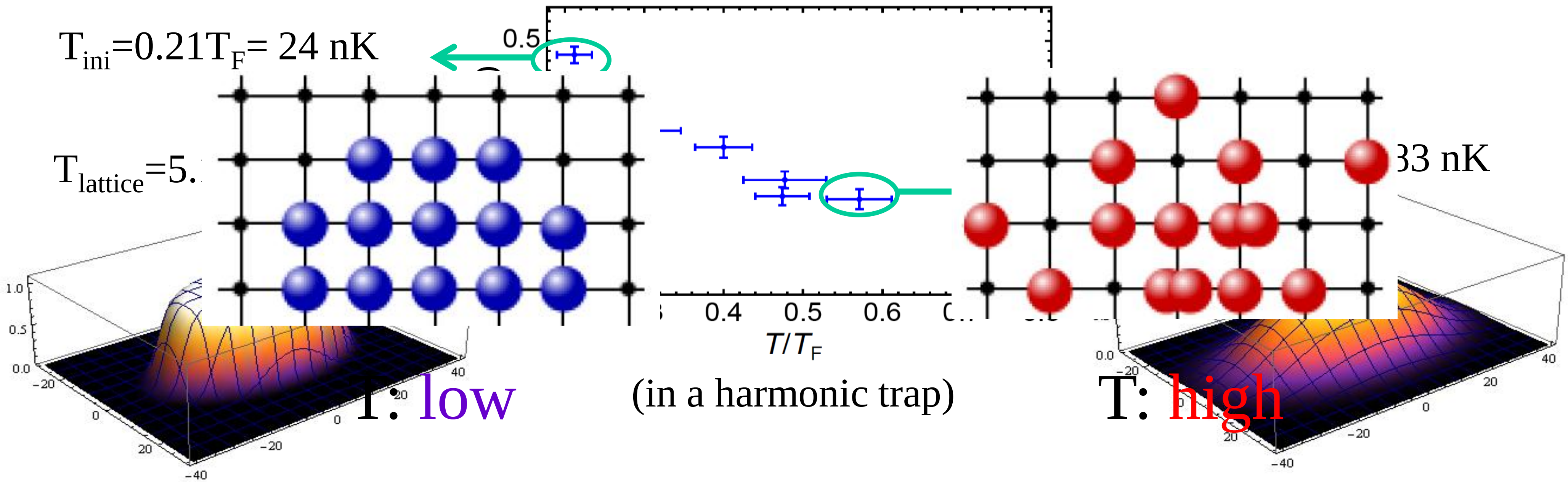
Doublon Production Rate Measurement by lattice modulation



"doublon production rate Γ is a sensitive probe of $T_{lattice}$"

[D. Greif *et al* ., PRL**106**, 145302 (2011)]

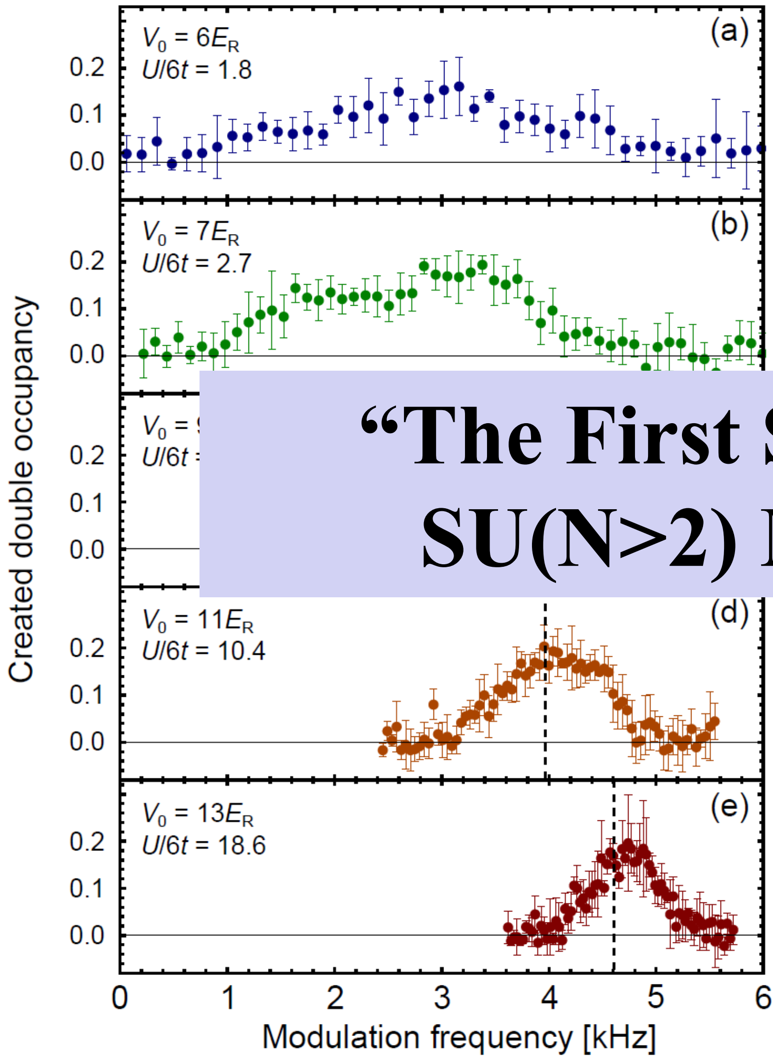
N=1.9× 10^4, $11E_R$, 18%pp mod. U/J=62.4



“Formation of SU(6) Mott insulator”

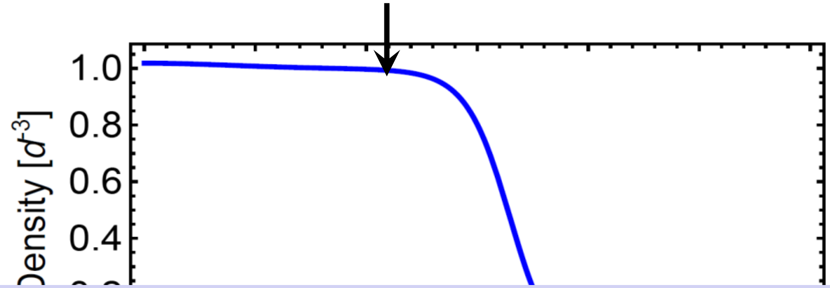
[S. Taie *et al*,]

Excitation (Mott) Gap



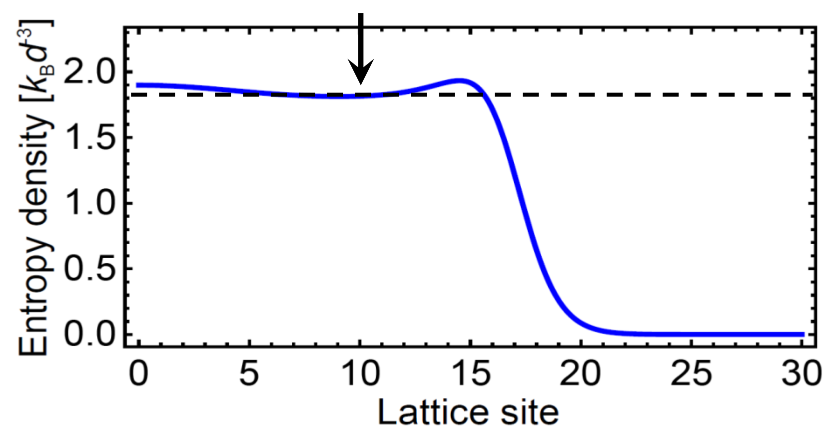
$T_{lattice}$=5.1t= 16 nK U/t=62.4

Mott Plateau (n=1)



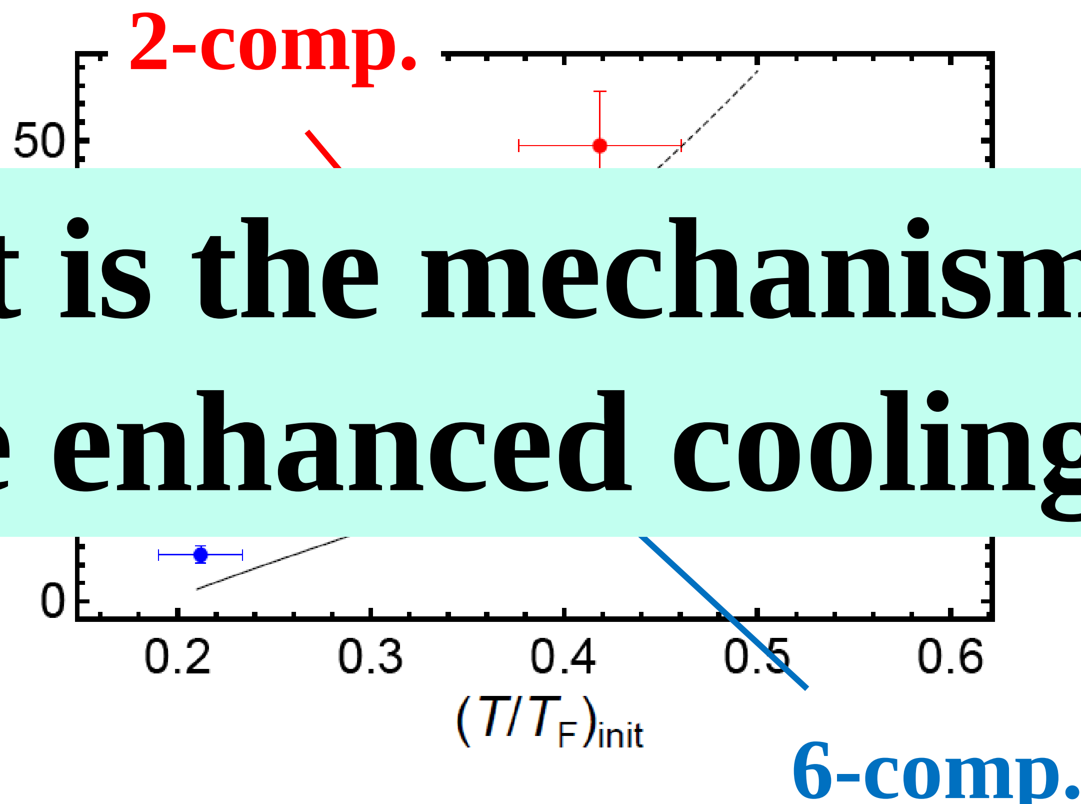
“The First Successful Formation of SU(N>2) Mott Insulating State”

Minimum: s = 1.81 cf. ln(6) = 1.79



Atomic Pomeranchuk Cooling

[^{173}Yb atoms in optical lattice; Taie *et al*,]



What is the mechanism of the enhanced cooling ?

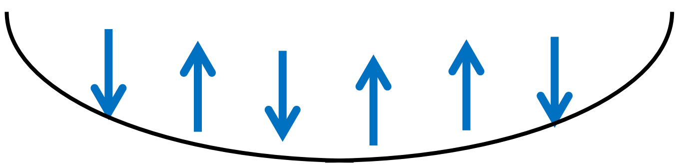
Pomeranchuk Cooling

Pomeranchuk Cooling [Pomeranchuk, (1950)]

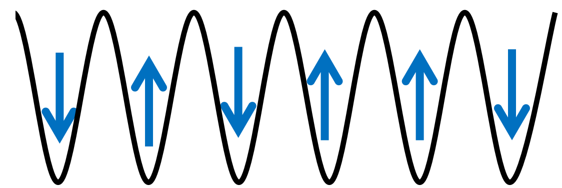
⟶ Discovery of Superfluid ^{3}He by Osheroff, Lee, Richardson

Initial state: Spin *de*polarized and also with *degeneracy*:

Final state: Spin *de*polarized and also with *localization*



Adiabatic change



$s \sim k_B \pi^2 T/T_F$

liquid ^{3}He atoms in a trap

$s \sim k_B \ln(N)$

solid ^{3}He atoms in Mott Insulator

"entropy flows from **motional** degrees of freedom to **spin**, which results in the low temperature"

⟶ "Pomeranchuk Cooling of an Atomic Gas"

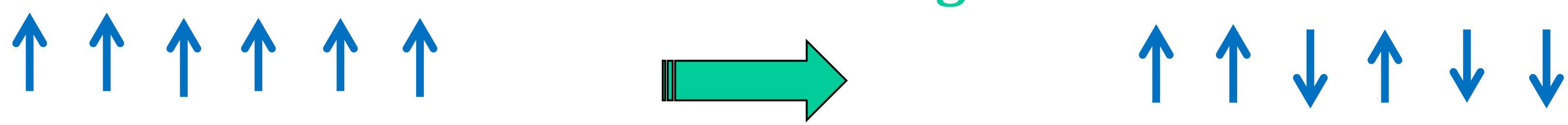
Spin Degrees of Freedom *is Cool*

Demagnetization Cooling

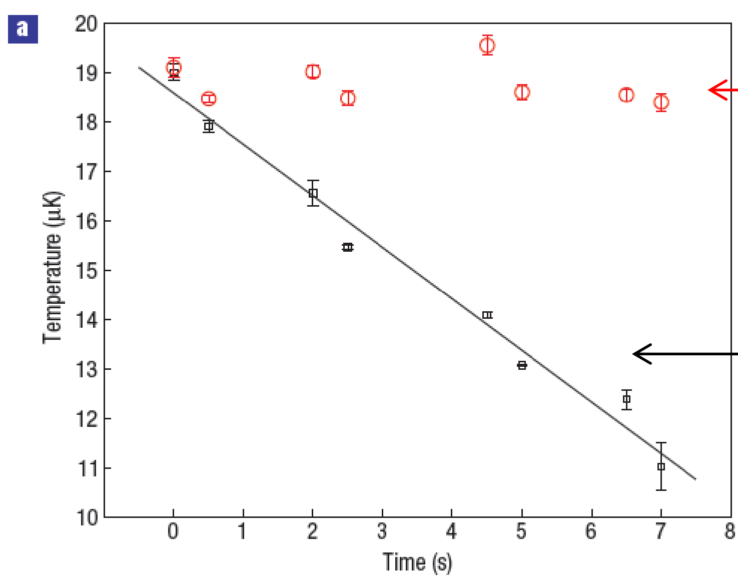
[W. J. De Haas, *et al*., (1934)]

Initial state: Spin-polarized: **Final state: Spin-depolarized:**

Adiabatic change



"entropy flows from **motional** degrees of freedom to **spin**, which results in the cooling of the system"



kept at high filed(1G)

kept at low filed(50mG) and Optical Pumping

[M. Fattori, *et al* .,Nat. Physics**2**, 765(2006)]

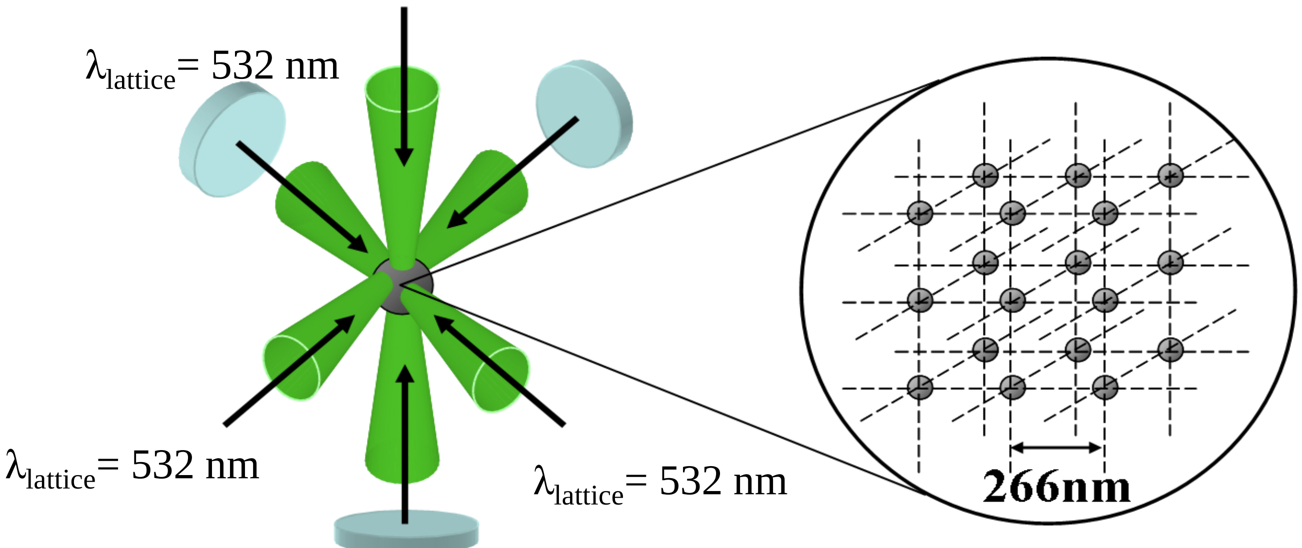
Bose-Fermi Mixture in a 3D optical lattice

- Repulsive Interaction: $a_{BF} = +7.3$ nm

^{174}Yb(Boson) + ^{173}Yb(Fermion):
$a_{BB} = +5.6$ nm $a_{FF} = +10.6$ nm

- Attractive Interaction: $a_{BF} = -4.3$ nm

^{170}Yb(Boson) + ^{173}Yb(Fermion):
$a_{BB} = +3.4$ nm $a_{FF} = +10.6$ nm

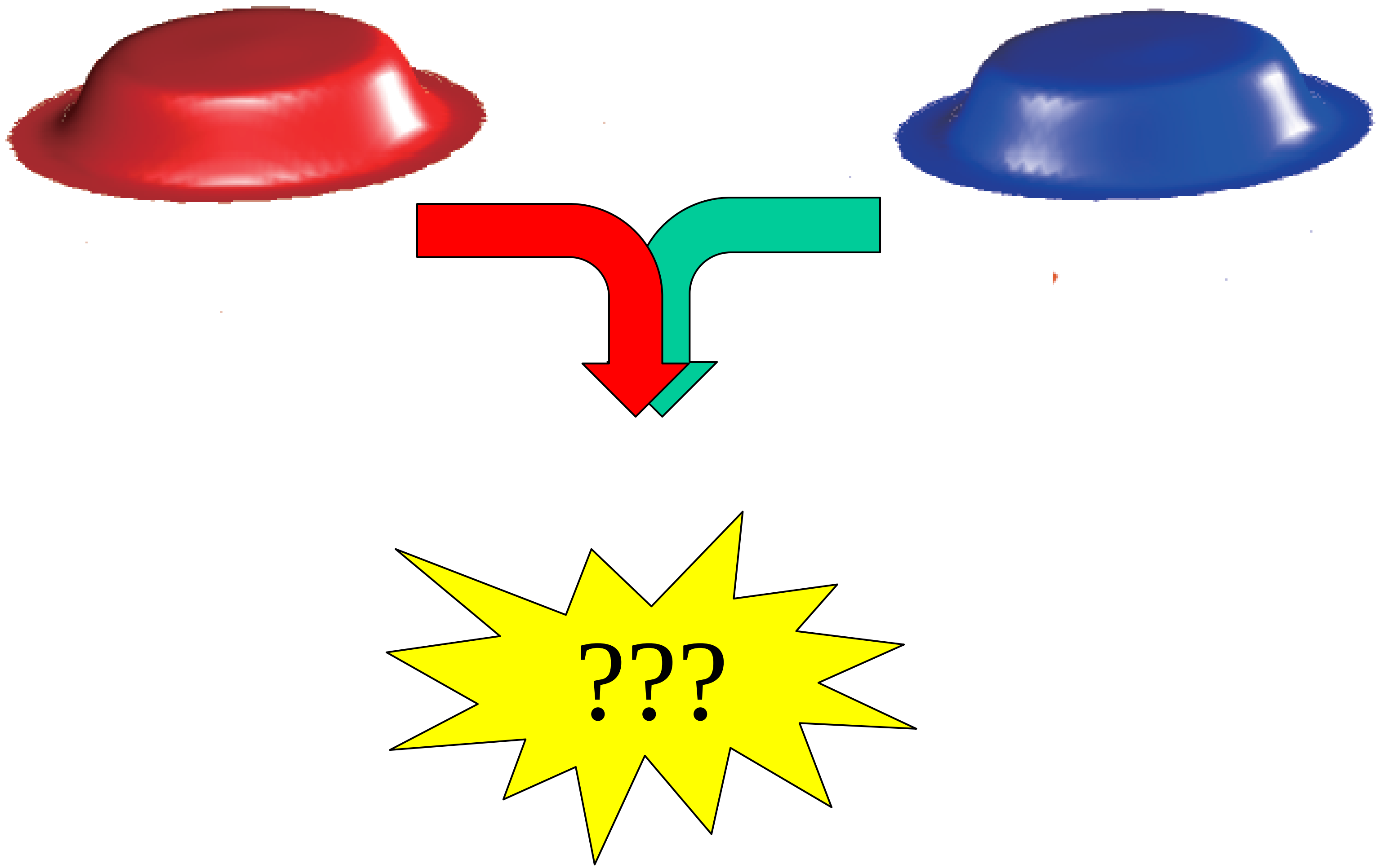


$$V_B \sim V_F$$
$$\omega_B \sim \omega_F$$
$$t_B \sim t_F$$
$$\Delta z_B \sim \Delta z_F$$

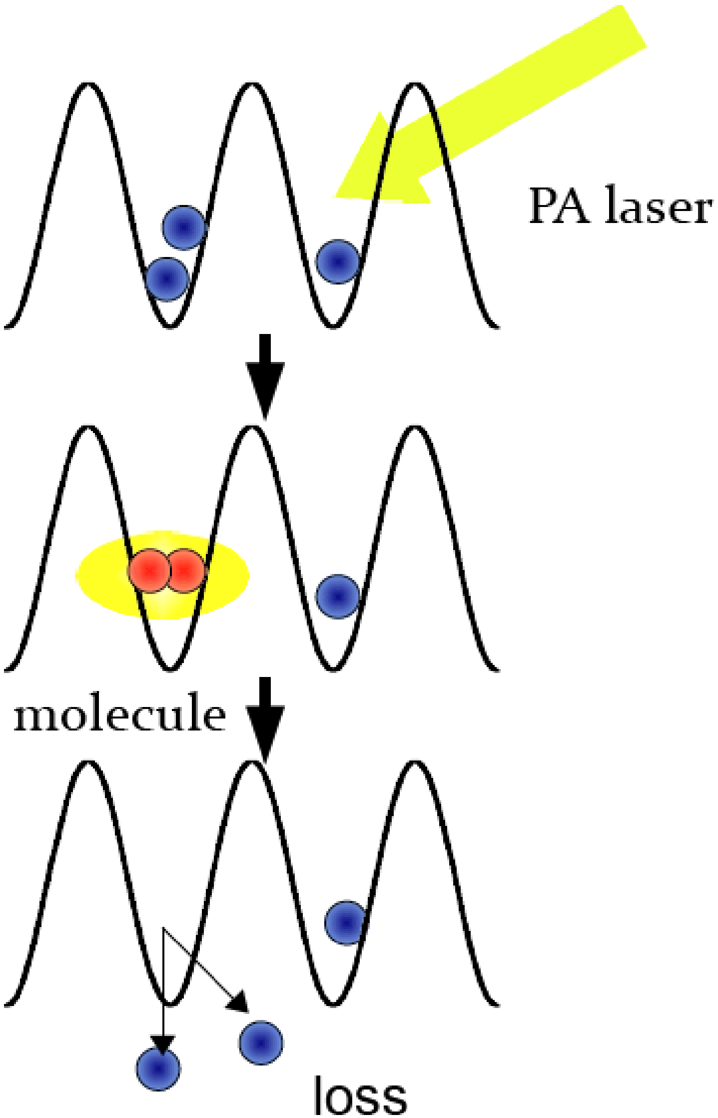
Strongly Interacting Two Different Mott Insulators

[S. Sugawa, K. Inaba, *et al.*, *Nature Phys.* **7,** 642–648 (2011)]

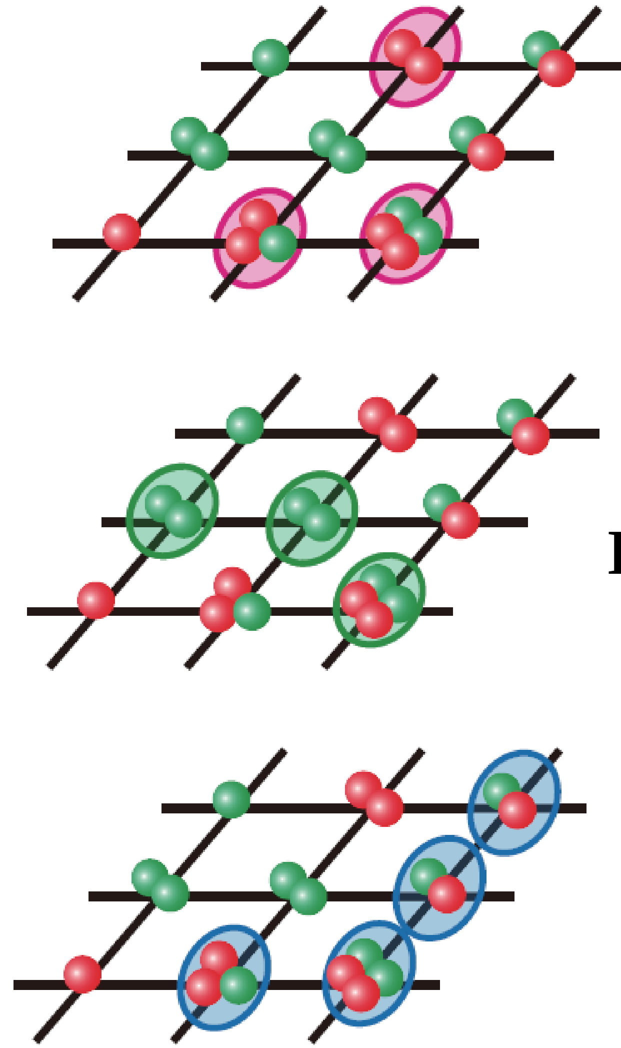
Bosonic Mott insulator　　　Fermionic Mott Insulator



Measurement of Site Occupancy by Photoassociation



fermion
boson



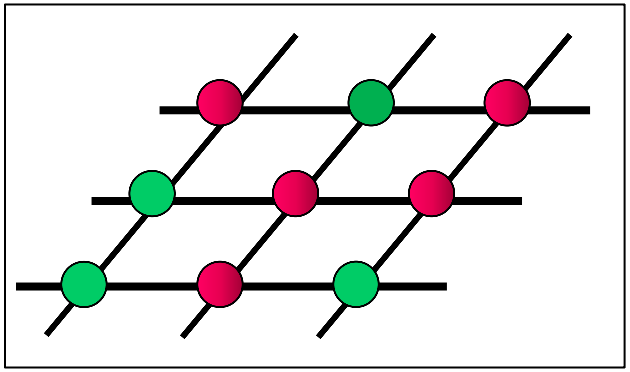
Bosonic Double Occupancy

Fermionic Double Occupancy

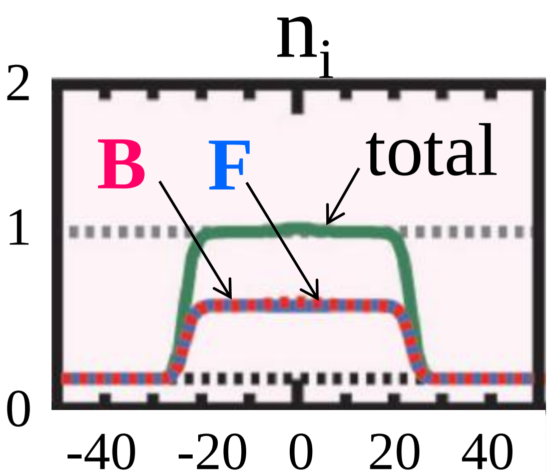
Bose-Fermi Pair Occupancy

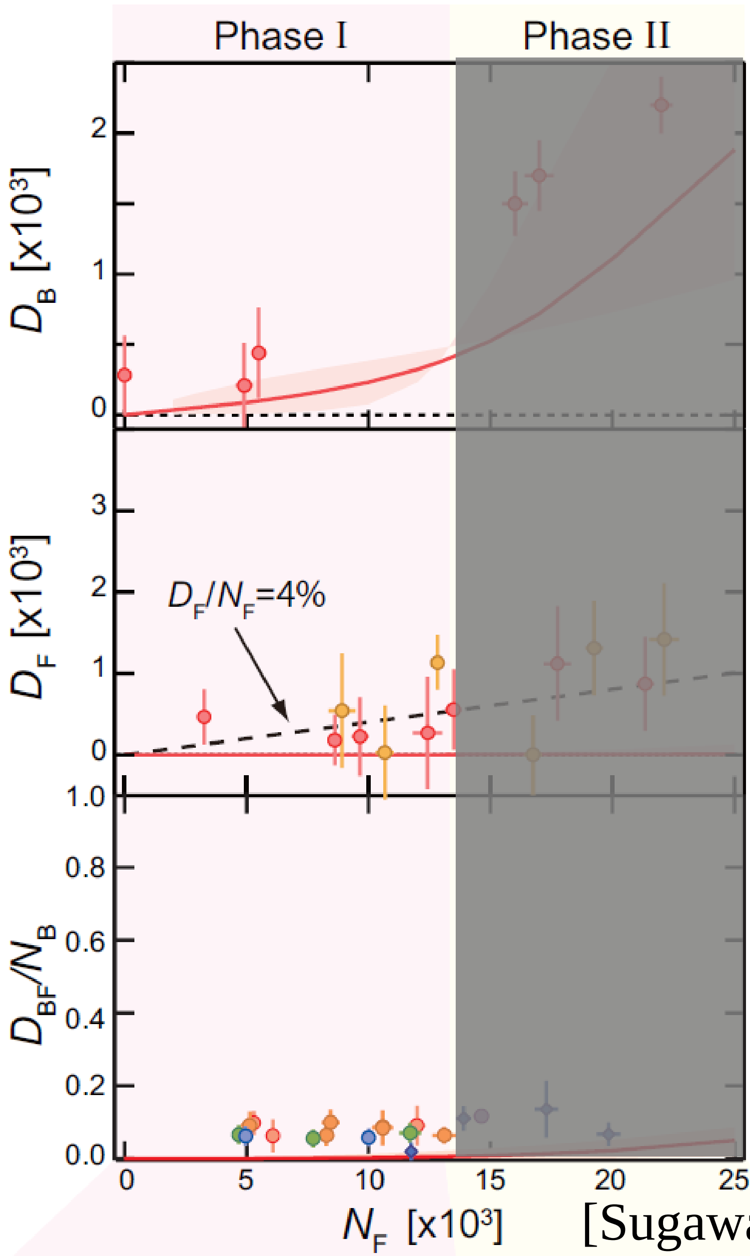
Repulsively Interacting Bose-Fermi Mott Insulators

- fermion
- boson



“Mixed Mott Insulator”

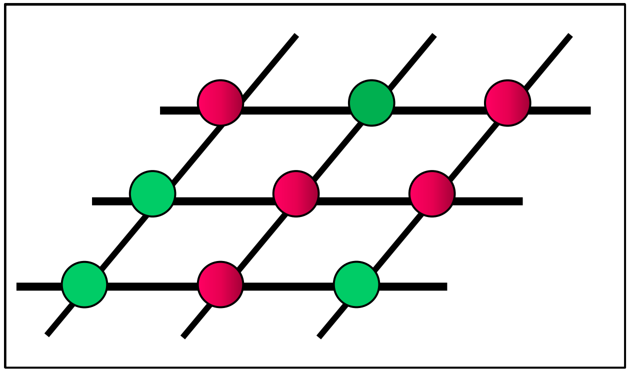




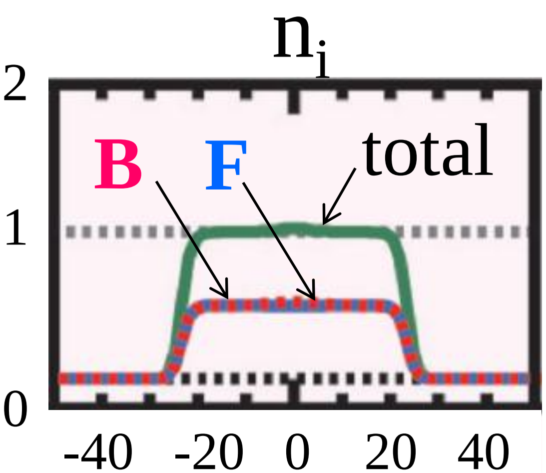
[Sugawa *et al.* *NP.* **7,** 642–648 (2011)]

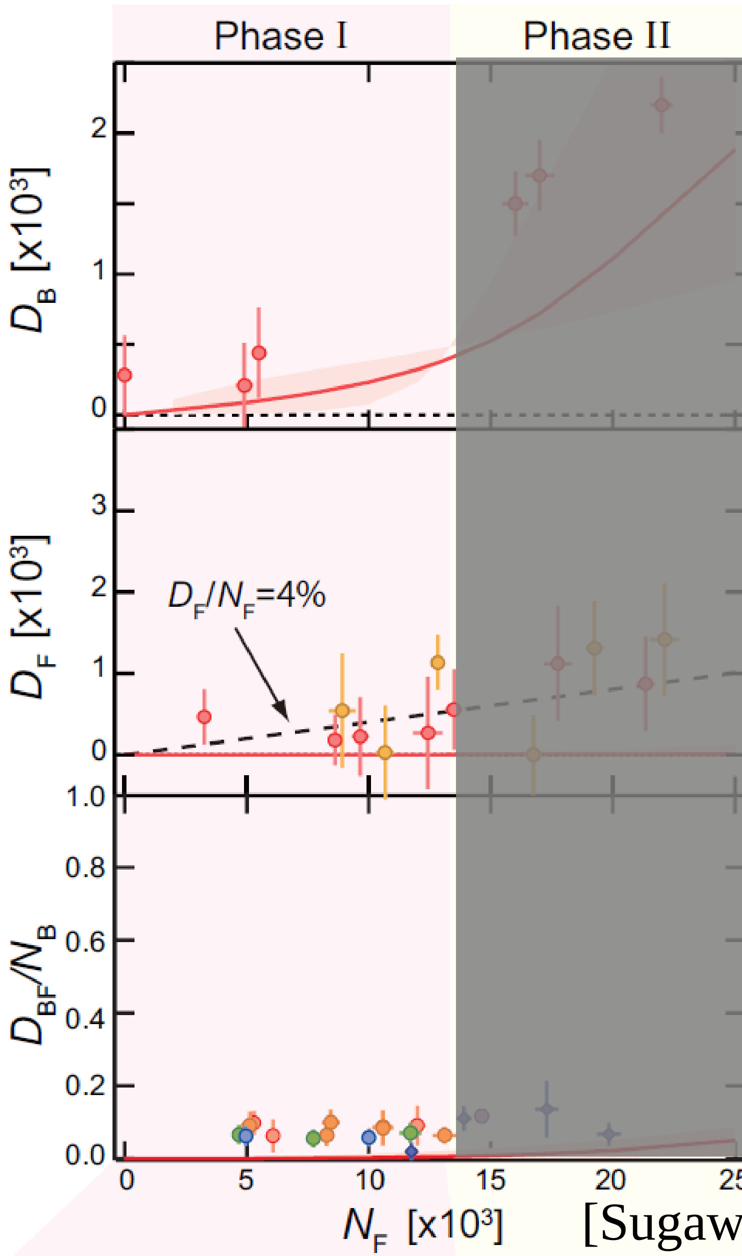
Repulsively Interacting Bose-Fermi Mott Insulators

- fermion
- boson



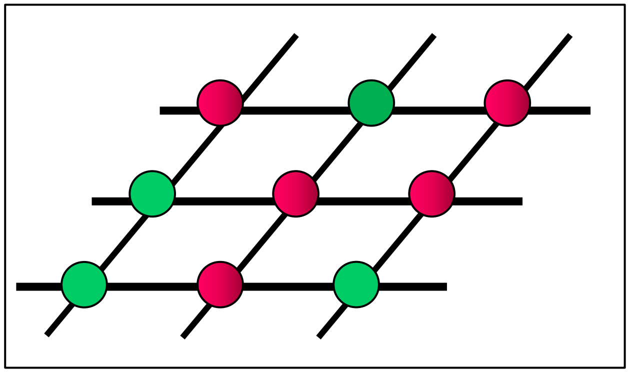
"Mixed Mott Insulator"



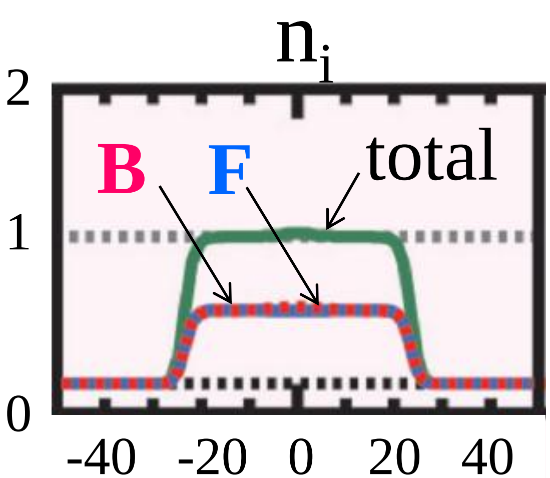


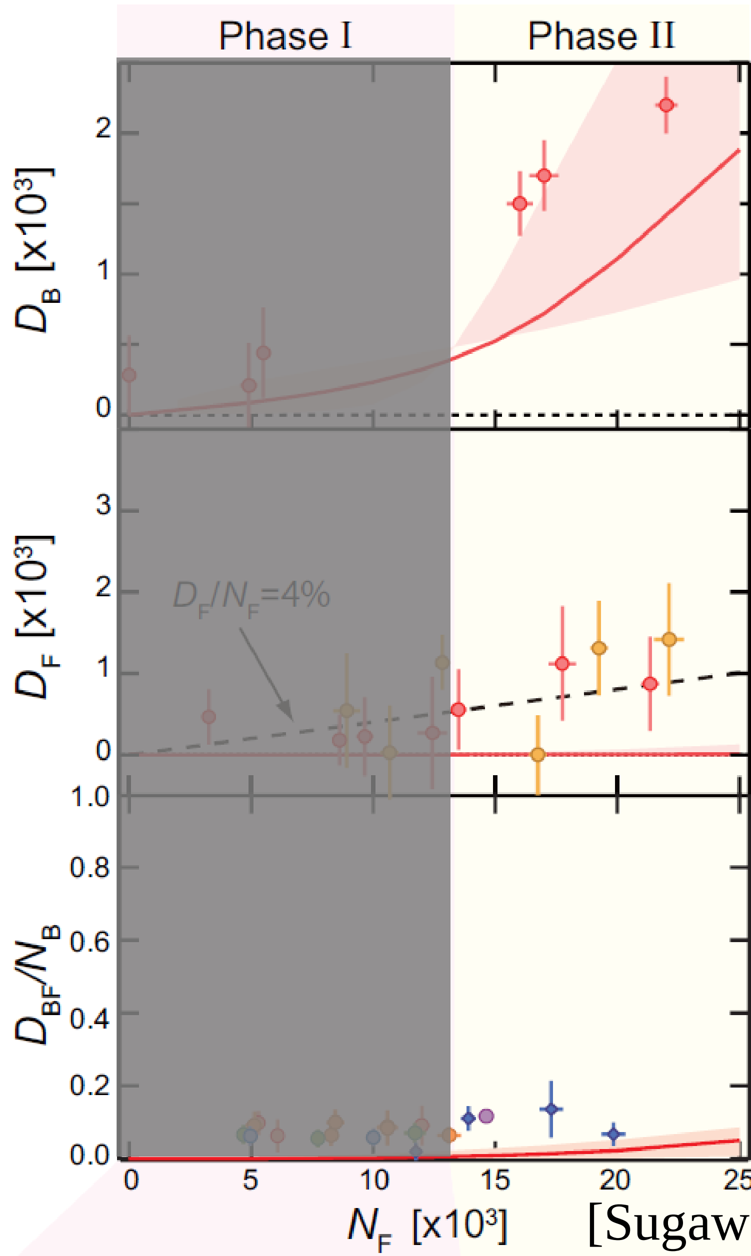
[Sugawa *et al*. *NP*. **7**, 642–648 (2011)]

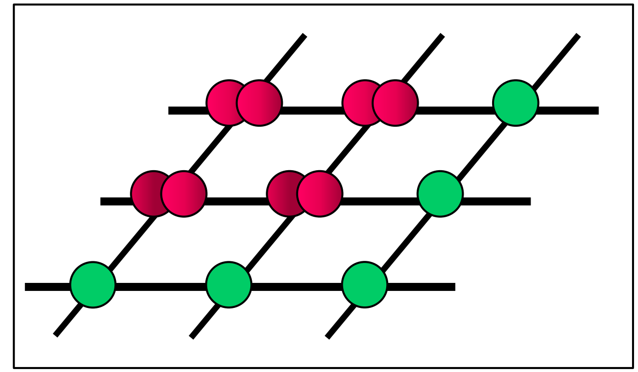
Repulsively Interacting Bose-Fermi Mott Insulators



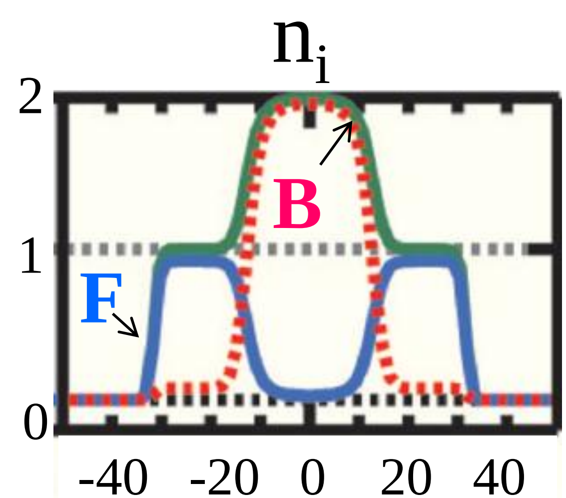
“Mixed Mott Insulator”



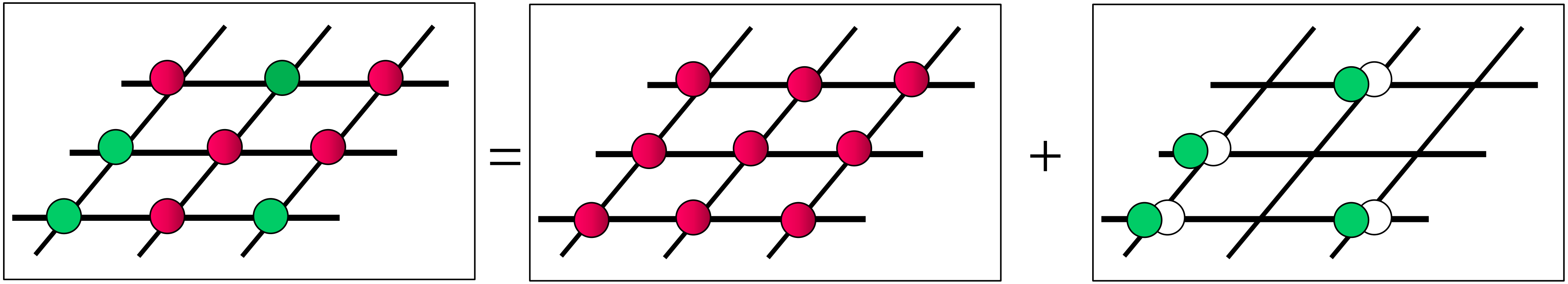




“Phase Separation”



[Sugawa *et al.* *NP.* **7**, 642–648 (2011)] arXiv:1205.4026v1 Ehud Altman, Eugene Demler, Achim Rosch



"Mixed Mott Insulator" = "Bosonic Mott Insulator" + "composite fermion" of hole(○) & fermion(●)

● fermion

● boson

$|\,\Omega\,\rangle$: Reference vacuum

$$c_i^\dagger\,|\,\Omega\,\rangle = f_i^\dagger b_i\,|\,\Omega\,\rangle$$

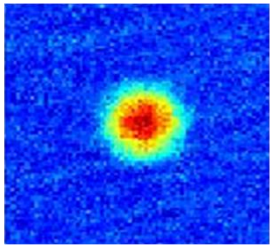
$$H = -t_{\rm eff}\sum_{\langle ij\rangle}(c_i^\dagger c_j + {\rm H.c.}) + V_{\rm eff}\sum_{\langle ij\rangle} n_i n_j$$

Anderson Hubbard Model with Li-Yb Mixture

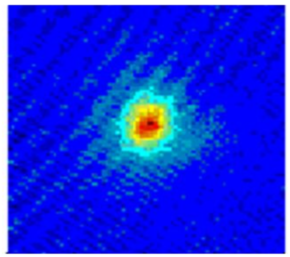
Poster by Dr. Shuta Nakajima

Fermion(^{6}Li)-Boson(^{174}Yb)

^{6}Li



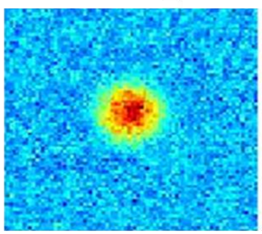
^{174}Yb



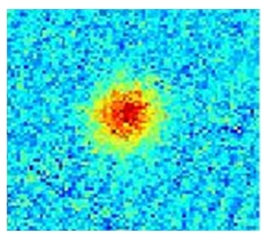
$T/T_F = 0.08 \pm 0.01$

Fermion(^{6}Li)-Fermion(^{173}Yb)

^{6}Li



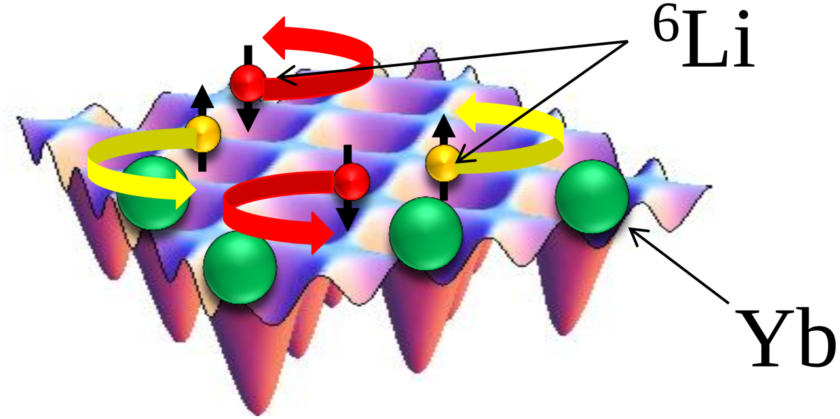
^{173}Yb



$T/T_F = 0.07 \pm 0.02$

[H. Hara *et al.* , PRL **106**, 205304, (2011)]

$$M_{^{174}Yb} / M_{^6Li} \cong 29$$



[D. Semmler, K. Byczuk, and W. Hofstetter, PRB **81**, 115111(2010)]

Li($^2S_{1/2}$)-Yb($^1S_0$)

$|a_{6Li\text{-}Yb}| \sim 1$ nm

Feshbach Resonance: $\Delta < 1$ mG

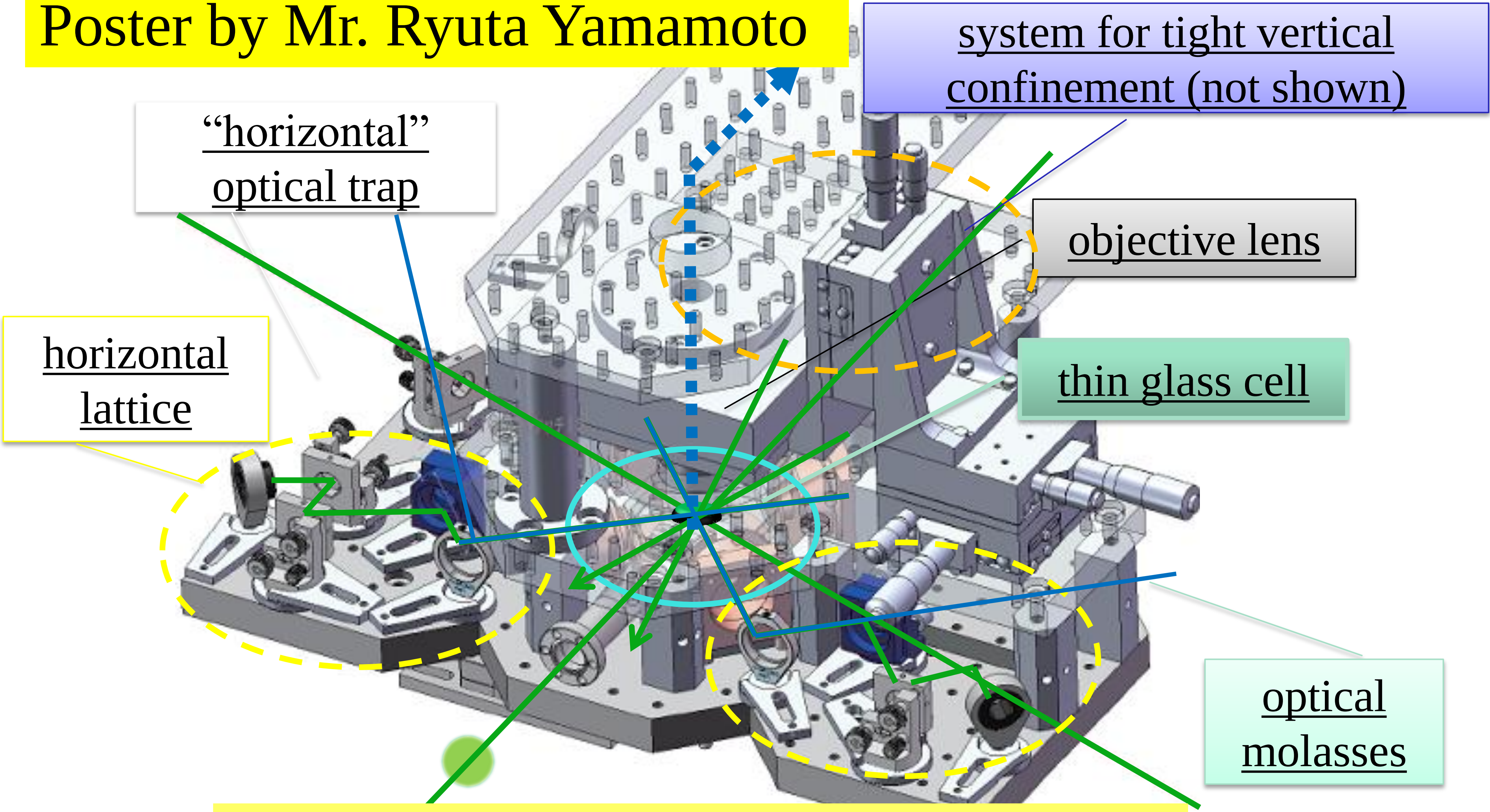
[D. A. Brue and J. M. Hutson, PRL**108**, 043201 (2012)]

Li($^2S_{1/2}$)-Yb($^3P_2$)

Anisotropy-induced Feshbach Resonance

Developing Yb Quantum Gas Microscope

Poster by Mr. Ryuta Yamamoto



Boson, Fermion, Bose-Fermi Mixture

Summary

Atom Manipulation Technique

Various Optical Lattices (square, honeycomb, kagome, Lieb)

Feshbach Resonance (optical/magnetic, isotropic/anisotropic)

Bose-Hubbard Model

Superfluid-Mott Insulator Transition

matter-wave interference

lattice-modulation spectroscopy

RF/Optical Spectroscopy

Quantum Gas Microscope

Fermi-Hubbard Model

SU(2) & SU(6) Mott insulator(Pomeranchuk cooling)

Bose-Bose/Bose-Fermi Hubbard Model

Anderson-Hubbard model

Dual Mott insulators

Thank you very much for attention

16 August   Mount Daimonji at Kyoto