超伝導量子ビット研究の進展

中村 泰信

東京大学先端科学技術研究センター　　　（2012年4月～）

東京大学大学院工学系研究科物理工学専攻（2012年1月～）

東京大学ナノ量子情報エレクトロニクス研究機構

理化学研究所基幹研究所

THE UNIVERSITY OF TOKYO　RIKEN　NanoQuine QUantum INformation Electronics

ジョセフソンパラメトリック増幅器

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THE UNIVERSITY OF TOKYO

RIKEN

NanoQuine
QUantum INformation Electronics

Acknowledgements

NEC/RIKEN
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Alexander Baust (WMI-TMU)

自己紹介

1989~92	銅酸化物高温超伝導体の電子・熱輸送現象	東大物工 内田研
1992~	メソスコピック系の物理・単一電子トランジスタ	NEC基礎研
1996頃~	超伝導回路における量子コヒーレンス	
1998頃~	超伝導電荷量子ビット	
2001-2002	超伝導磁束量子ビット	TU Delft
2002-	超伝導量子ビット回路	NEC＆理研
	超伝導量子回路中のマイクロ波量子光学	
2012-	ハイブリッド量子系の制御	東大先端研＆物工

趣味: サッカー

新研究室@先端研（東大駒場Ⅱキャンパス）

http://www.qc.rcast.u-tokyo.ac.jp

最近のお気に入り

https://www.chikumashobo.co.jp/product/9784480093950/

その他文献：
「量子力学入門I-IV」, 科学 Jun-Sep (1963).
「量子雑音」, 日本物理学会誌 22, 824 (1967).

“Information theory of quantum-mechanical channel”
Advances in Communication Systems
Vol.1 (Academic Press, New York, 1965) p.227.

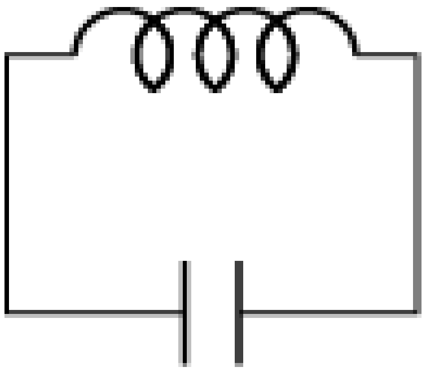
高橋 秀俊
タカハシ ヒデトシ
1915-85年。東京生まれ。東京大学物理学科卒業。同大名誉教授。
のち慶応義塾大学工学部教授。強誘電体、システム理論、制御理論、
パラメトロン計算機などで成果をあげた。さまざまな分野に通用する
普遍的・独創的な発想で研究を牽引。ロゲルギスト主宰者でもあった。
著書に『電磁気学』（裳華房）、『線形集中定数系論』『線形分布定数
系論』（ともに岩波書店）などがある。

Outline

- Josephson junction and superconducting qubits
- Parametric amplification
- Josephson parametric amplifier (JPA)
- Applications of JPA

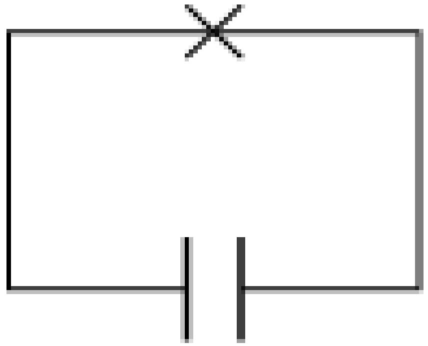
Superconducting qubit – nonlinear resonator

LC resonator

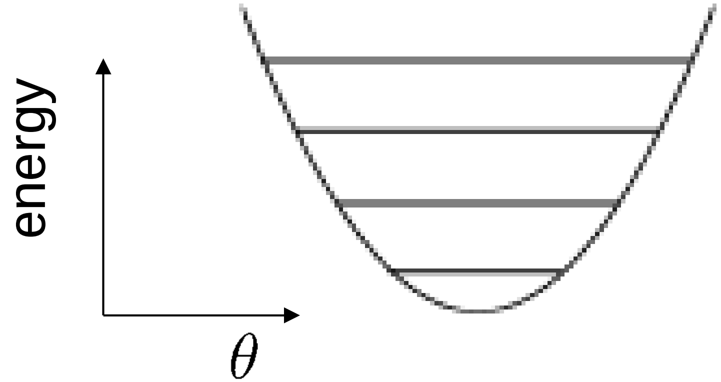


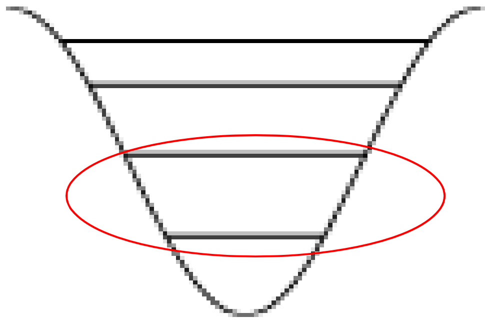
Josephson junction resonator

Josephson junction = nonlinear inductor



anharmonicity ⇒ effective two-level system



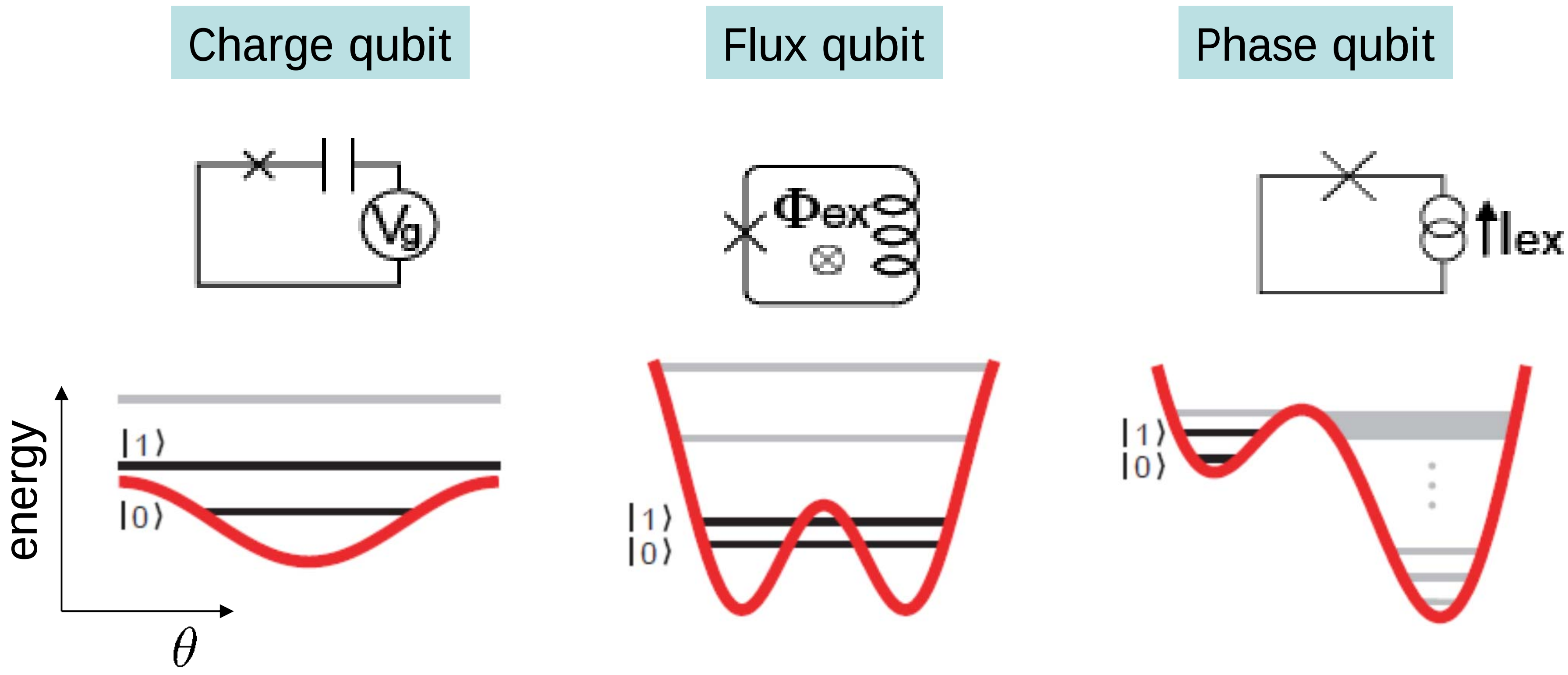


inductive energy = confinement potential

charging energy = kinetic energy ⇒ quantized states

Superconducting qubits – artificial atoms in electric circuit

small ⟵ E_J/E_C ⟶ large



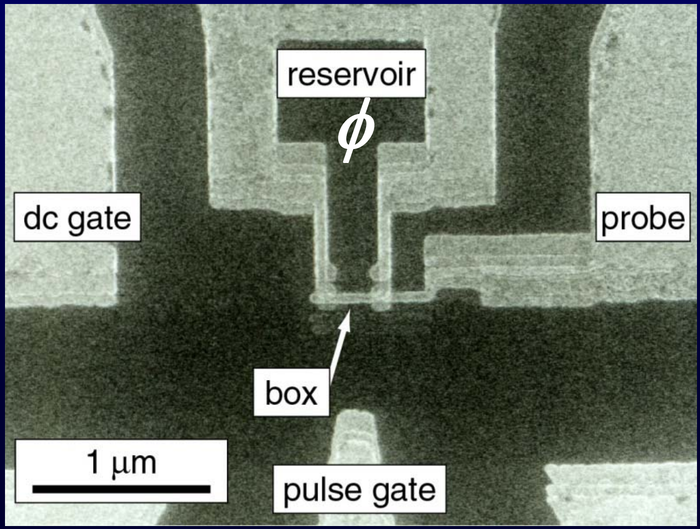
Josephson energy E_J = confinement potential
charging energy E_C = kinetic energy ⇒ quantized states

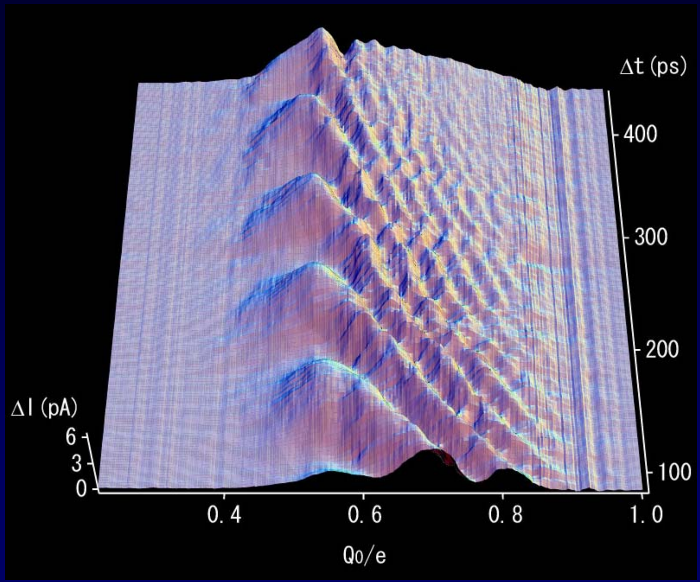
typical qubit energy $E_{01} \sim 10\,\mathrm{GHz} \sim 0.5\,\mathrm{K}$

typical experimental temperature $T \sim 0.02\,\mathrm{K}$

Superconducting qubits

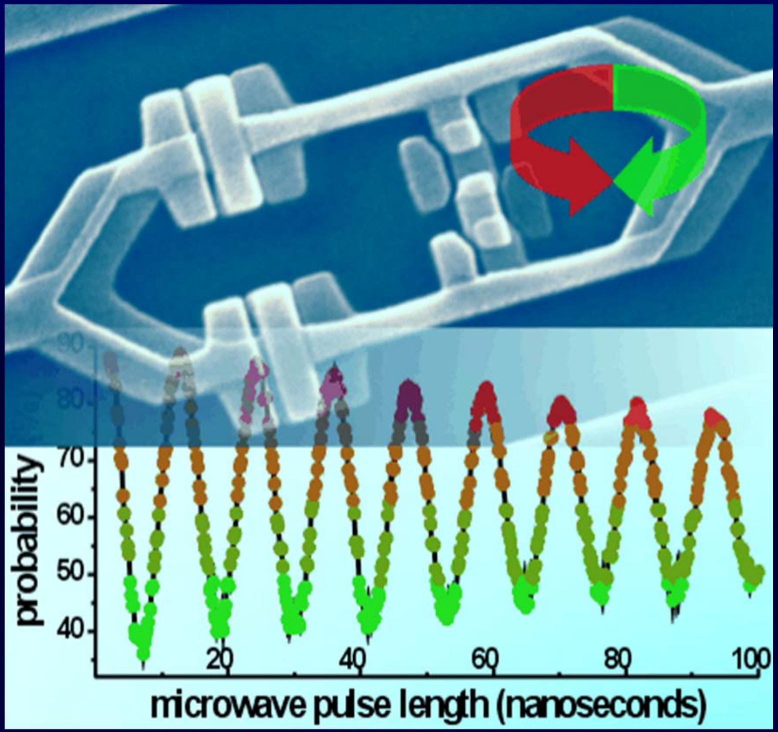
Charge qubit





Nakamura, Pashkin, Tsai, Nature (1999)

Flux qubit

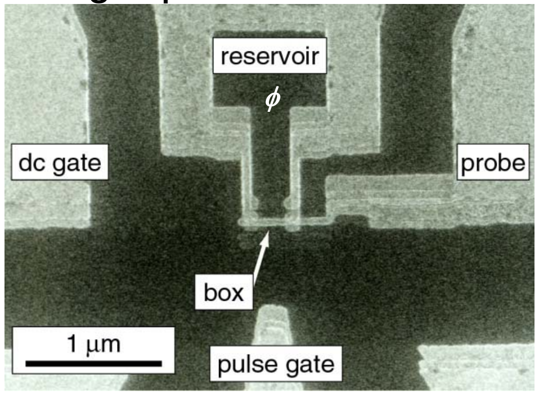


Chiorescu, Nakamura, Harmans, Mooij, Science (2003)

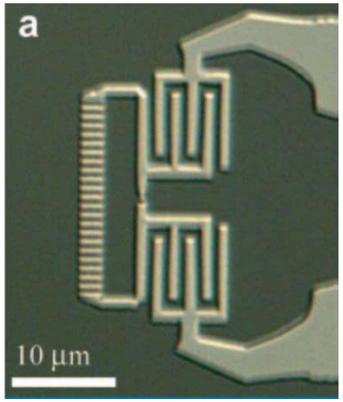
- Artificial two-level system in electric circuits
- Coherent control of quantum states in macroscopic systems

Superconducting qubits – macroscopic artificial atom in circuits

charge qubit/NEC $E_J/E_C \sim 0.3$

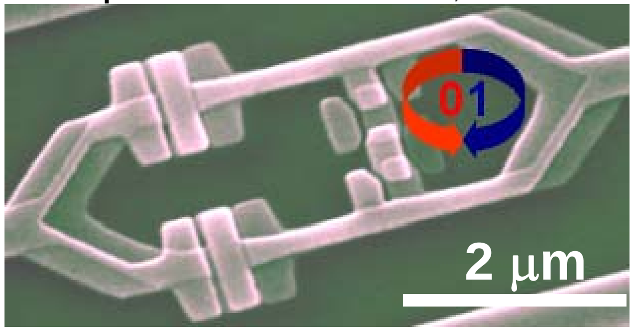


"fluxonium"/Yale

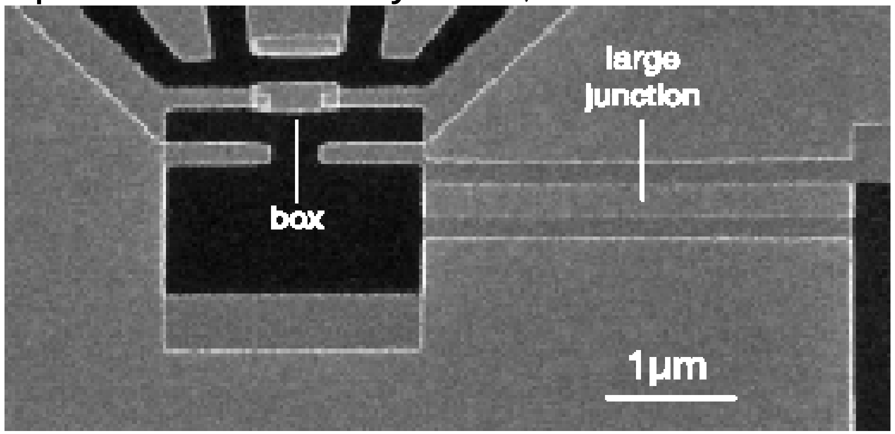


$E_J/E_C \sim 3$

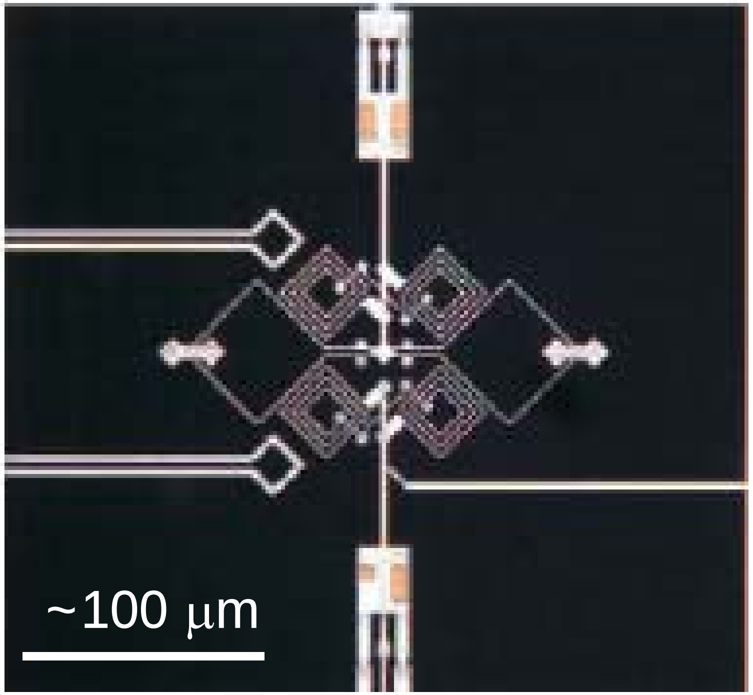
flux qubit/Delft $E_J/E_C \sim 40$



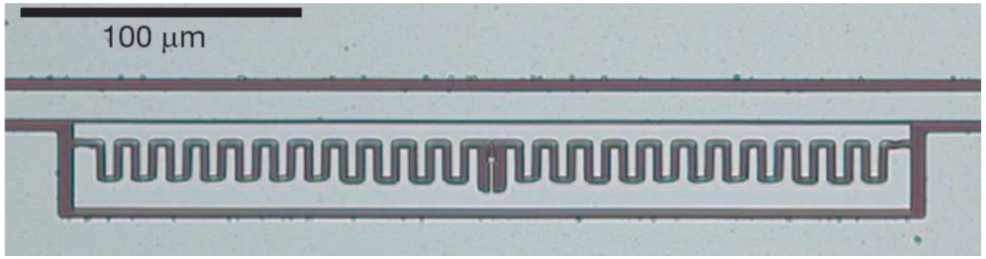
"quantronium"/Saclay $E_J/E_C \sim 5$



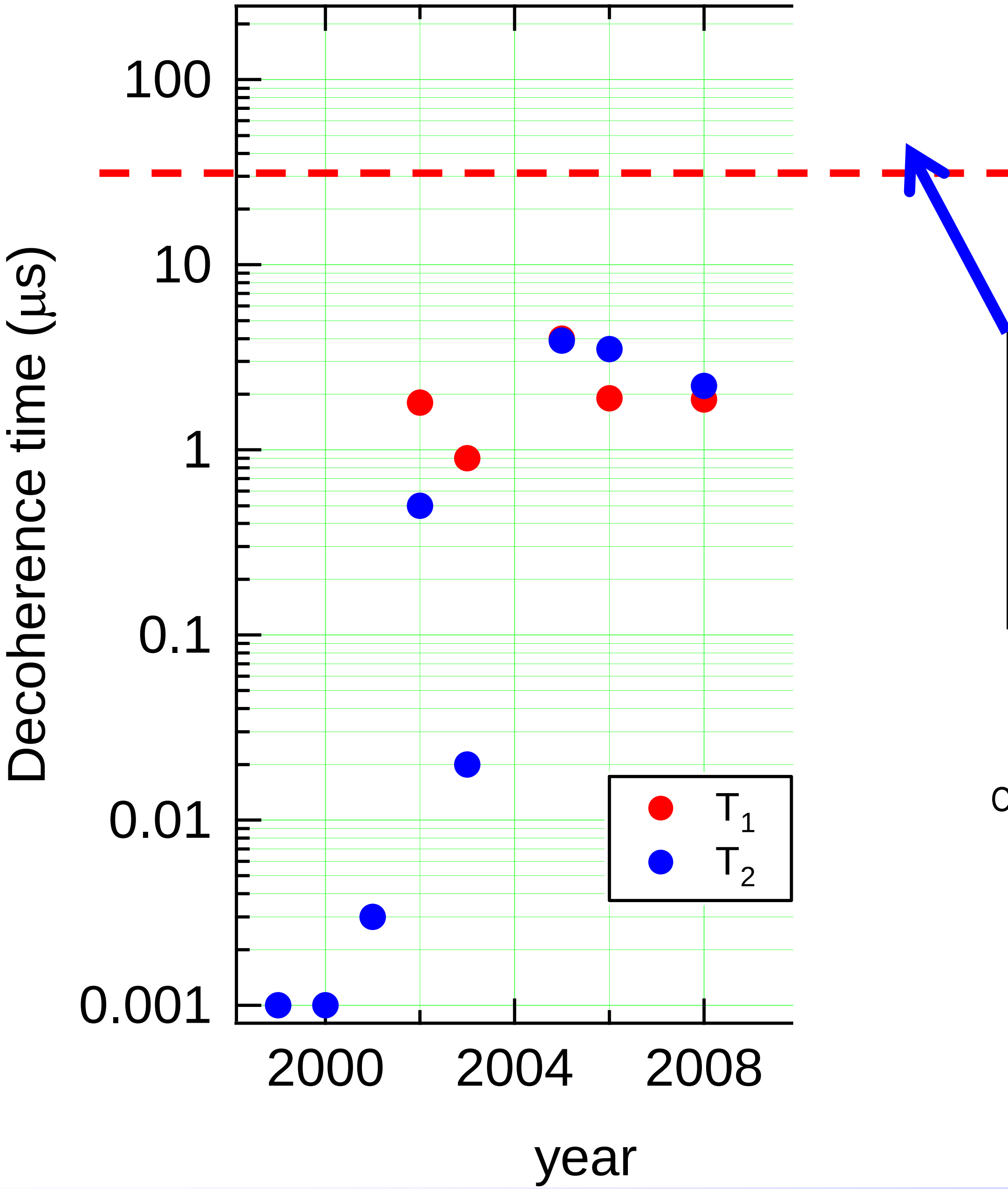
phase qubit/NIST/UCSB $E_J/E_C \sim 10^4$

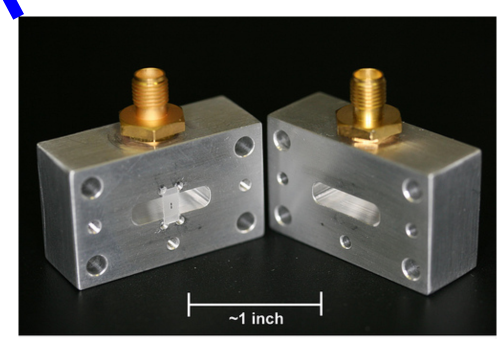


"transmon"/Yale $E_J/E_C \sim 50$



Decoherence time of superconducting qubits





3D cavity @IBM

T_1~70 μs, T_2*~96 μs

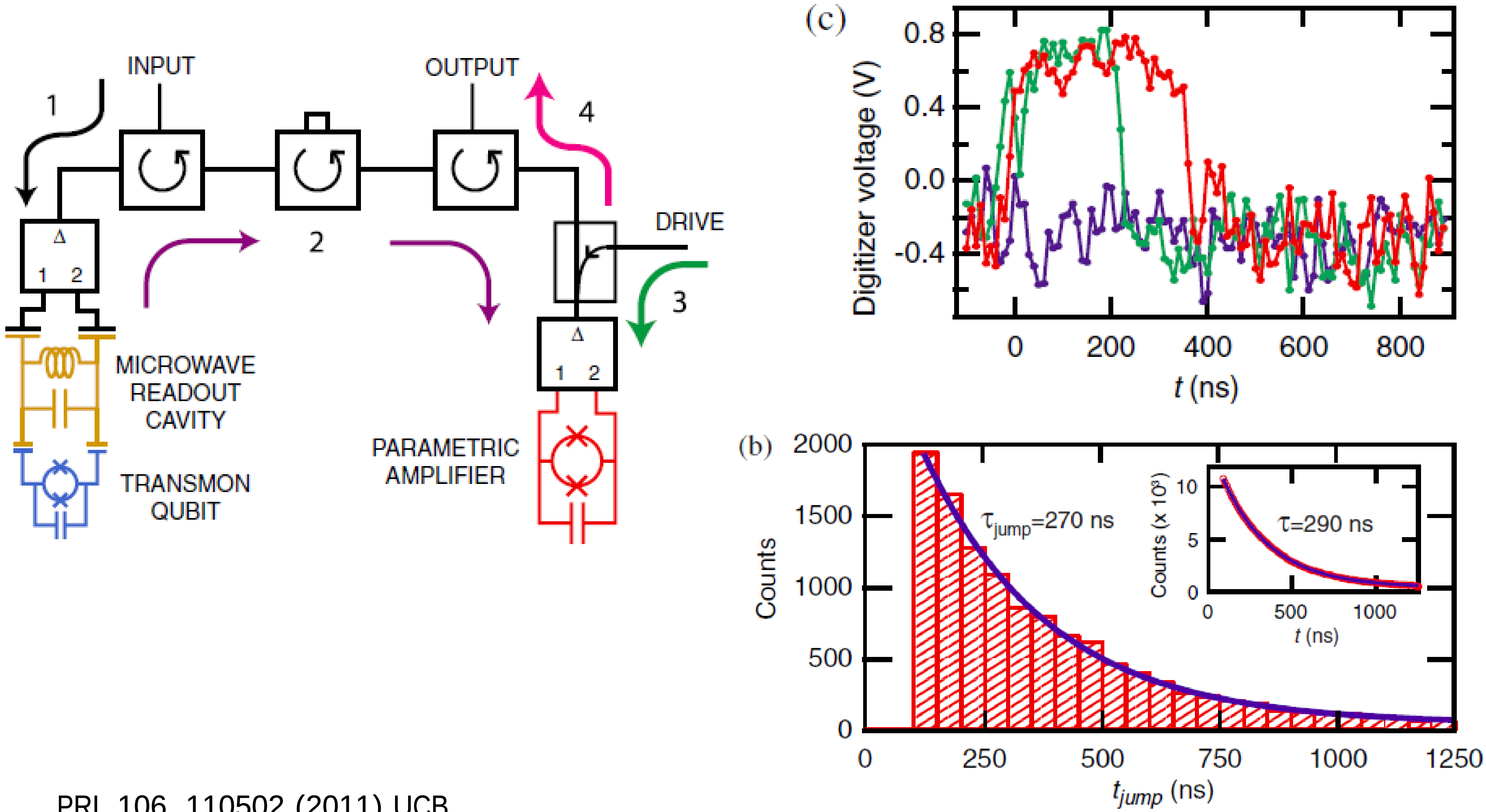
C. Rigetti et al. arXiv:1202.5533

State-of-the-art in superconducting qubit experiments

- Long coherence time in 3D cavity
  - High-fidelity gate and measurement (Yale, IBM)
- Quantum error correction (Yale)
- Shor algorithm in 4 qubits + 5 resonators (UCSB)
- Observation of quantum jumps (UCB)
- Quantum feedback control (UCB, Delft)
- Generation and measurement of nonclassical itinerant microwave
  - Single-photon source (Yale, ETH)
  - Single-photon detector (Wisconsin)
  - Squeezed-state generator (JILA, Yale, ETH)
  - Tomography of itinerant microwave field (JILA, Yale, ETH)
- Hybrid quantum system
  - Tool for control and measurement of other quantum systems
    - spin ensembles (Yale, Saclay, NTT, Chalmers, Wien)
    - nanomechanics (JILA, UCSB, Aalto, EPSF)
- Adiabatic quantum computing (D-Wave)

Observation of Quantum Jumps in a Superconducting Artificial Atom

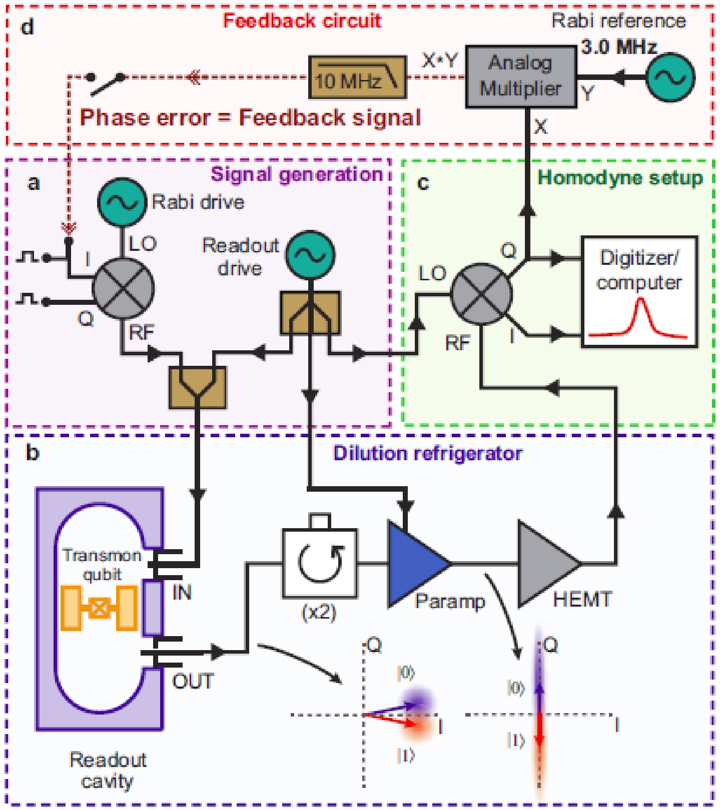
R. Vijay, D. H. Slichter, and I. Siddiqi

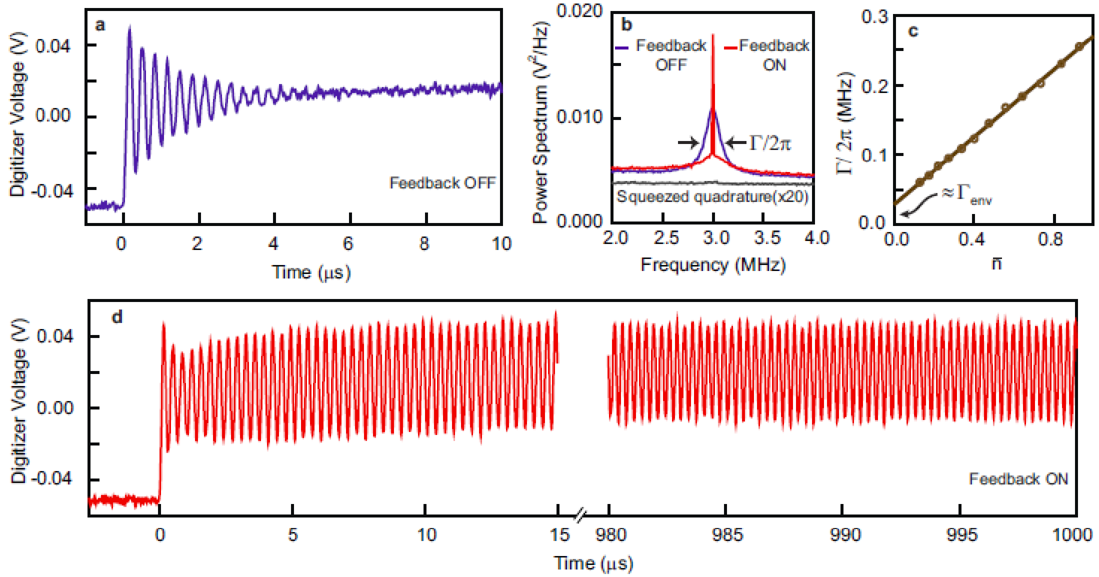


PRL 106, 110502 (2011) UCB

Quantum feedback control of a superconducting qubit: Persistent Rabi oscillations

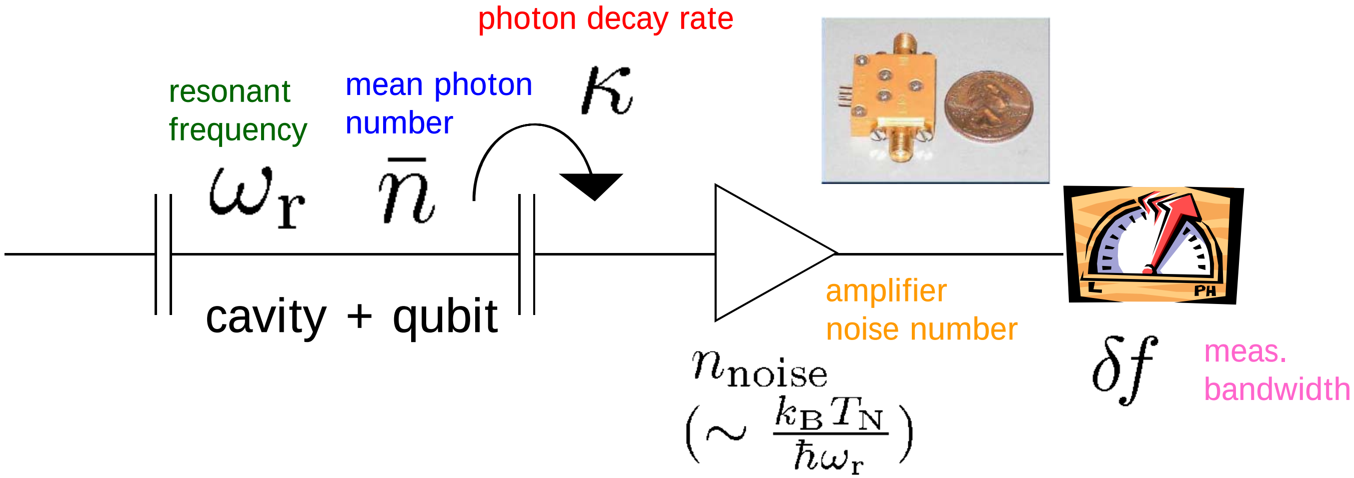
R. Vijay[1], C. Macklin[1], D. H. Slichter[1], S. J. Weber[1], K. W. Murch[1], R. Naik[1],
A. N. Korotkov[2], I. Siddiqi[1]





arXiv:1205.5591 (UCB)

Signal-to-noise ratio in qubit dispersive readout



$$SNR \sim \sqrt{\frac{\bar{n}\hbar\omega_r\kappa}{n_{noise}\hbar\omega_r\delta f}} = \sqrt{\frac{\bar{n}\kappa}{n_{noise}\delta f}}$$

n_{noise} for best commercial HEMT amplifier : 10 ~ 20

n: < ~10 required for avoiding backaction to qubit

δf: > ~10 MHz limited by qubit lifetime

κ: < ~10 MHz required for enhancing qubit lifetime (Purcell effect)

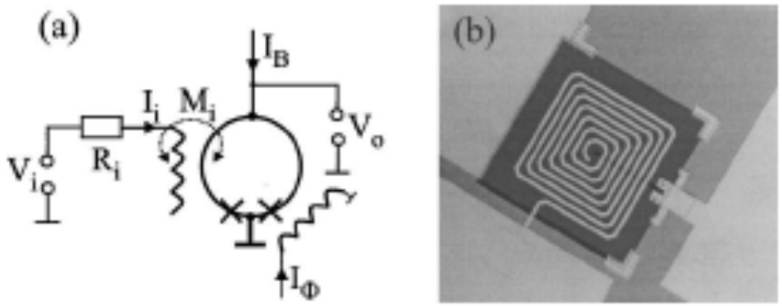
➡ < 1.0 !

To achieve single-shot measurement,

better amplifier needed!!

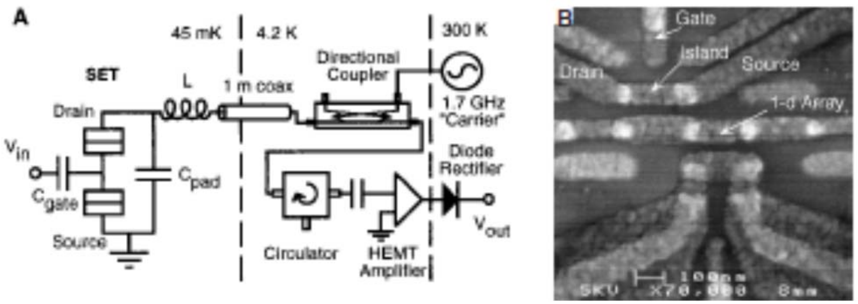
Low noise amplifiers at GHz range

SQUID amplifier



Muck 2003

rf-SET



Schoelkopf 1998

parametric amplifier

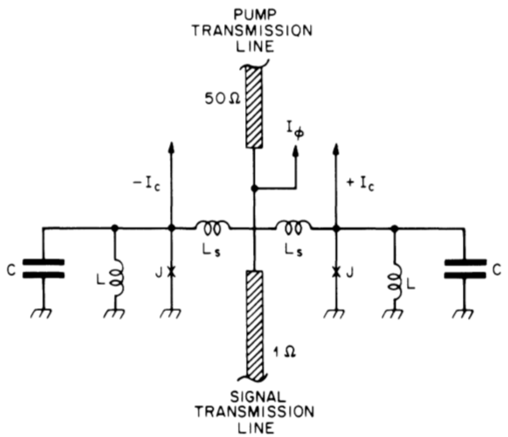
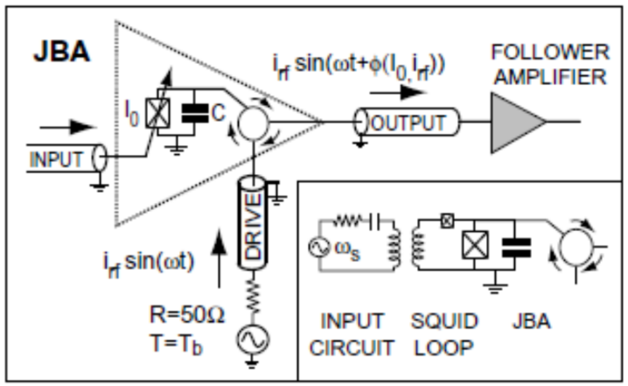


FIG. 1. A Josephson-parametric amplifier. See text for details.

Yurke 1988

Josephson bifurcation amplifier

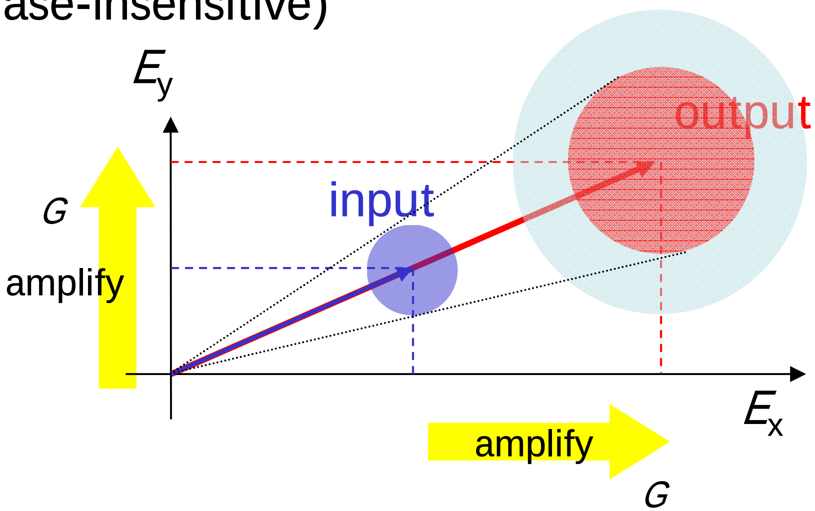


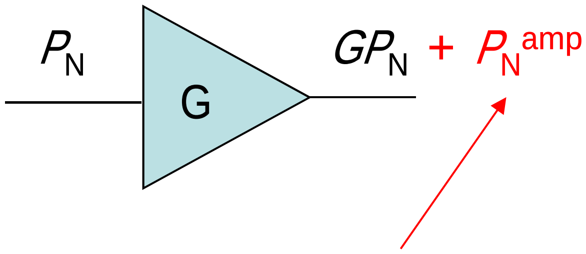
Siddiqi 2004

Quantum limit in amplifiers

H. Haus and J.A. Mullen (1962)
H. Takahashi (1965)
C. M. Caves (1982)

Phase-preserving amplifier
(phase-insensitive)





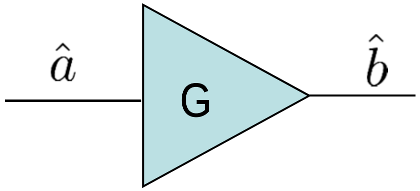
noise added by amplifier
(noise temperature)
cannot be zero

$P_N^{amp} > G\,\delta f\,(\hbar\omega\,/2)$
$\Rightarrow$ Standard quantum limit

Standard quantum limit of phase-preserving amplifier

Canonical input and output modes

$$[\hat{a}, \hat{a}^\dagger] = 1 \qquad [\hat{b}, \hat{b}^\dagger] = 1$$



Amplification with photon-number gain G

$$\hat{b} = \sqrt{G}\hat{a} + \hat{\mathcal{F}} \qquad \hat{b}^\dagger = \sqrt{G}\hat{a}^\dagger + \hat{\mathcal{F}}^\dagger$$

Noise added by amplifier

$$[\hat{\mathcal{F}}, \hat{\mathcal{F}}^\dagger] = 1 - G$$

Required for maintaining the commutation relations of b

$$(\Delta a)^2 \equiv \frac{1}{2}\left\langle \{\hat{a}, \hat{a}^\dagger\} \right\rangle - |\langle \hat{a} \rangle|^2$$

Symmetrized noise

$$\begin{aligned}(\Delta b)^2 &= G(\Delta a)^2 + \frac{1}{2}\left\langle \{\hat{\mathcal{F}}, \hat{\mathcal{F}}^\dagger\} \right\rangle \\ &\geq G(\Delta a)^2 + \frac{1}{2}\left|\left\langle [\hat{\mathcal{F}}, \hat{\mathcal{F}}^\dagger] \right\rangle\right| \\ &\geq G(\Delta a)^2 + \frac{|G-1|}{2}\end{aligned}$$

For G>>1

$$\frac{(\Delta b)^2}{G} \geq (\Delta a)^2 + \frac{1}{2}$$

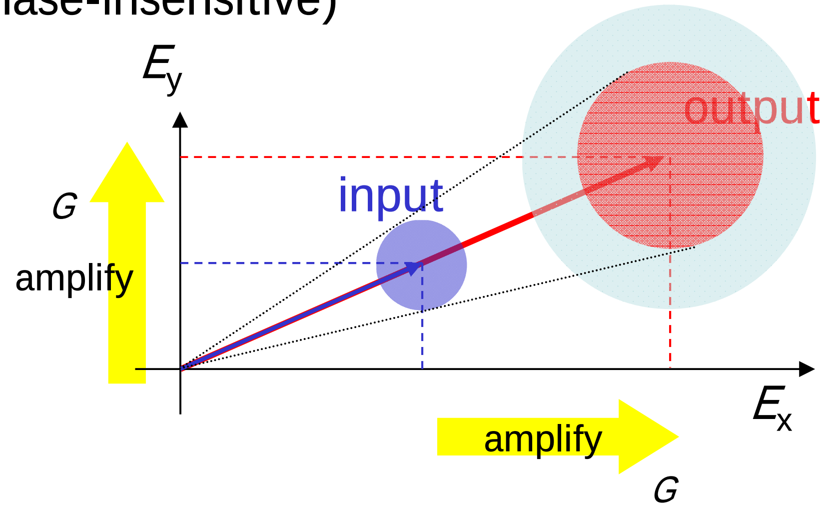
Added quantum noise

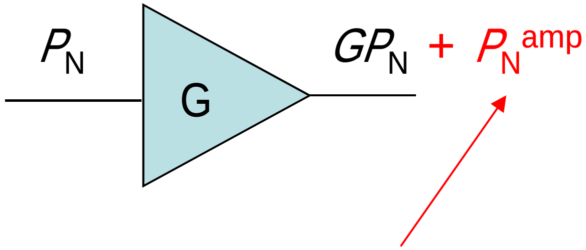
A.A. Clerk et al. arXiv:0810.4729

Quantum limit in amplifiers

H. Haus and J.A. Mullen (1962)
H. Takahashi (1965)
C. M. Caves (1982)

Phase-preserving amplifier
(phase-insensitive)



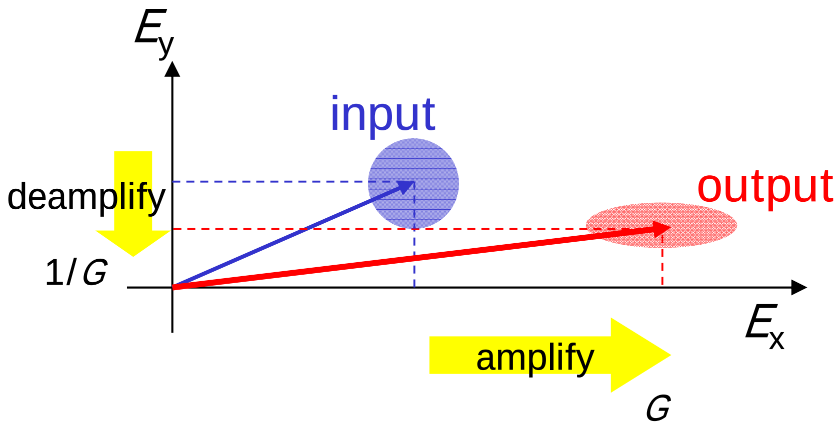


noise added by amplifier
(noise temperature)
cannot be zero

$P_N^{amp} > G\,\delta f\,(\hbar\omega/2)$

$\Rightarrow$ Standard quantum limit

Phase-nonpreserving amplifier
(phase-sensitive)

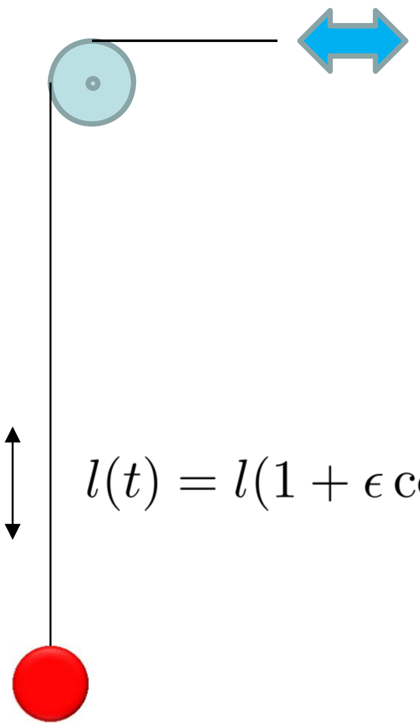


No additional noise required
(noiseless amplification)
But, at the price of the information
on one of the two quadratures.

Can beat the standard quantum limit!

Parametric excitation of pendulum

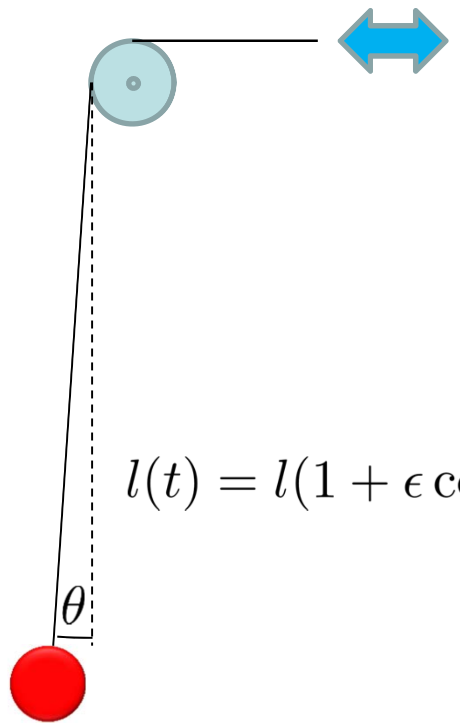
initial condition: $\theta=0$



$$l(t) = l(1 + \epsilon \cos \omega t)$$

no excitation

initial condition: $\theta \neq 0$



$$l(t) = l(1 + \epsilon \cos \omega t)$$

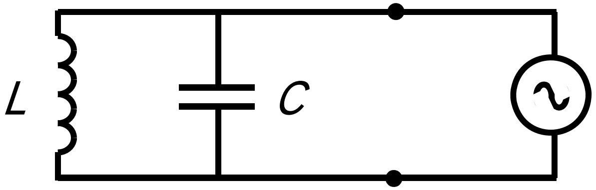
$$\ddot{\theta} = -\frac{g}{l}(1 - \epsilon \cos \omega t)\theta$$

for parametric excitation $\omega \simeq 2\sqrt{\frac{g}{l}}$

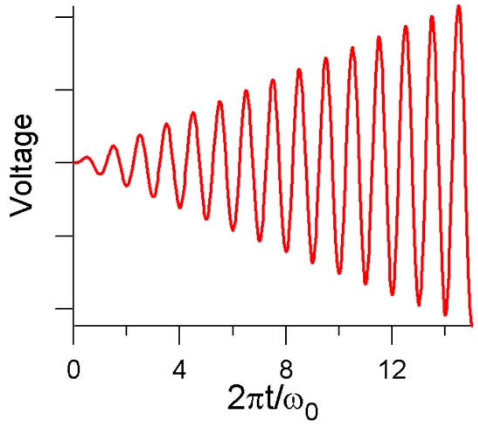
Phase sensitive

Parametric excitation of LC resonator

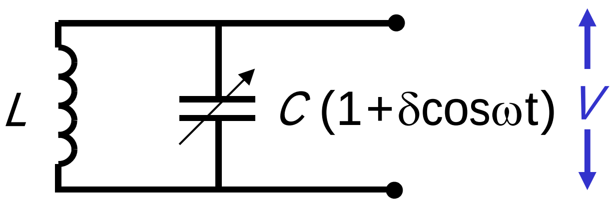
External driving



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

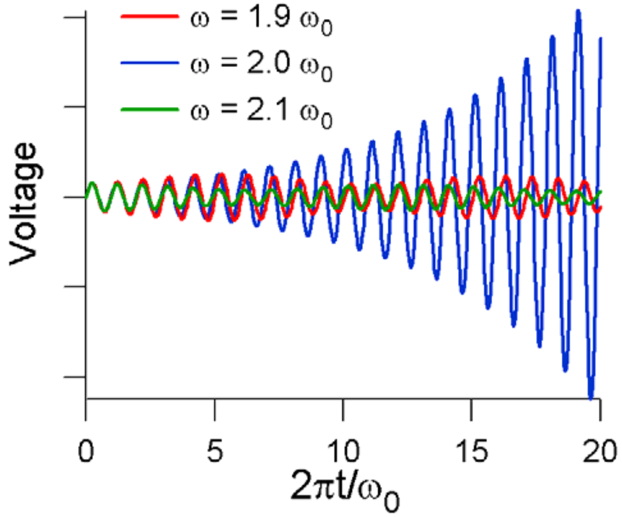


Parametric excitation



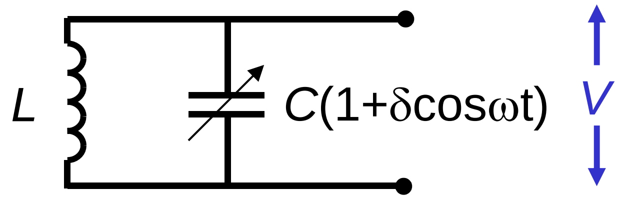
time-varying reactance

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



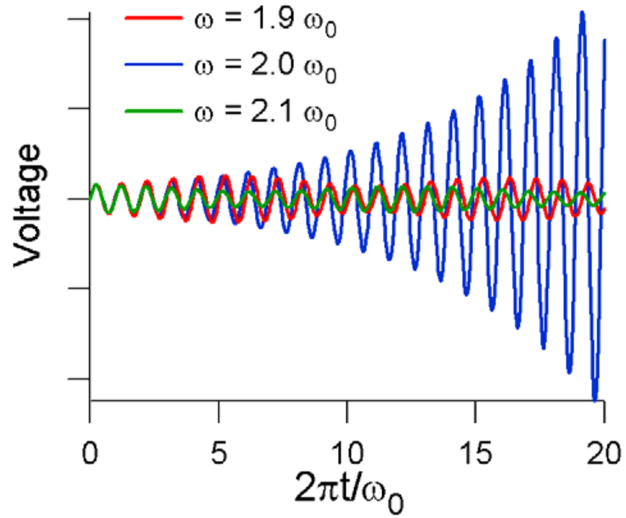
Parametric amplification with nonlinear element

Parametric excitation

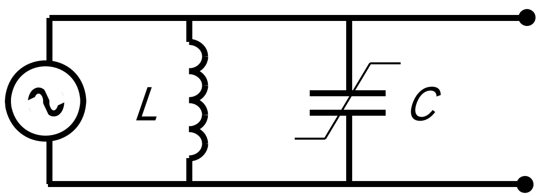


variable reactance

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Non-linear reactance



$$Q = C_1 V + C_2 V^2$$

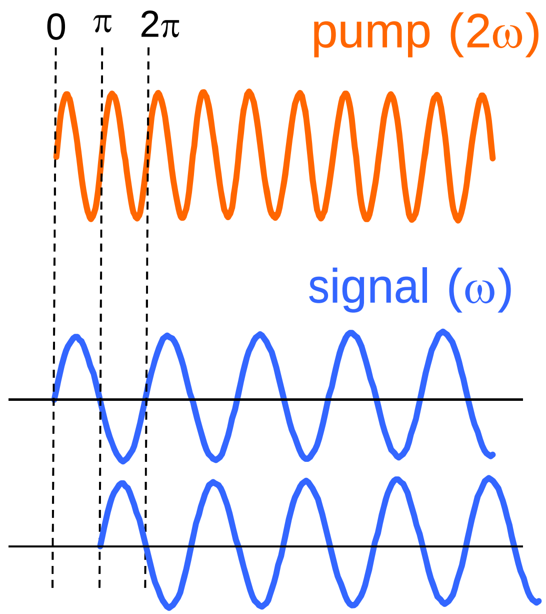
$$C_{\text{eff}} = Q/V = C_1 + C_2 V$$

$$V = V_0 \cos 2\omega_0 t$$

Parametrically driven damped oscillator

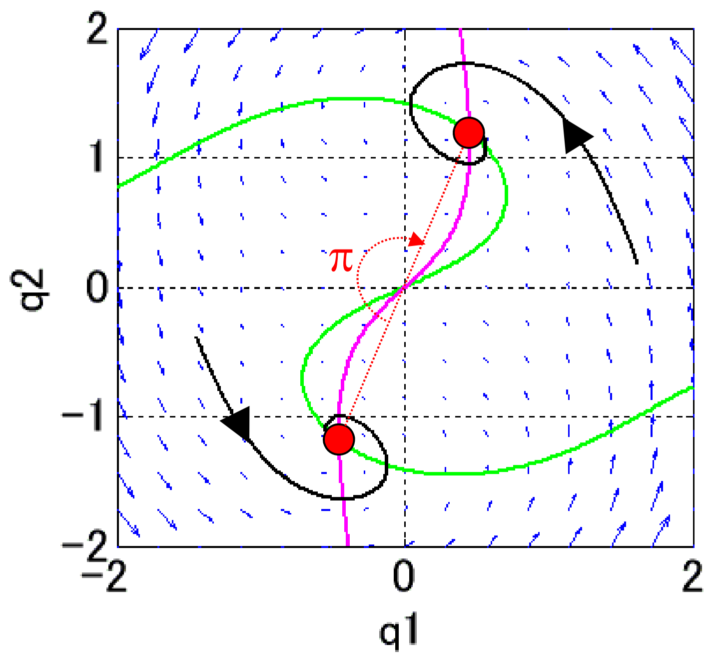
$$\frac{d^2q}{dt^2} + 2\Gamma\frac{dq}{dt} + \omega_0 q + \gamma q^3 + qF\cos\omega_F t = \xi(t)$$

damping nonlinearity drive noise



$\Gamma > F/2\omega_F$ Stable at origin

$\Gamma < F/2\omega_F$ Two stable points at q≠0 separated by phase π



M. I. Dykman et al. PRE 57, 5202 (1998) MSU.

Parametron

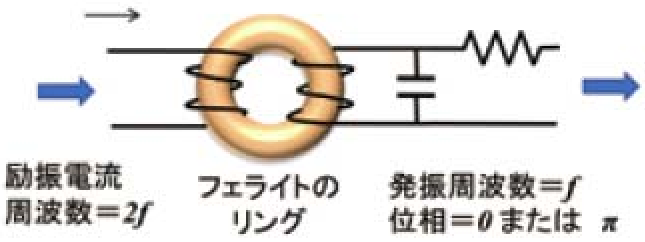


図1:パラメトロンの原理

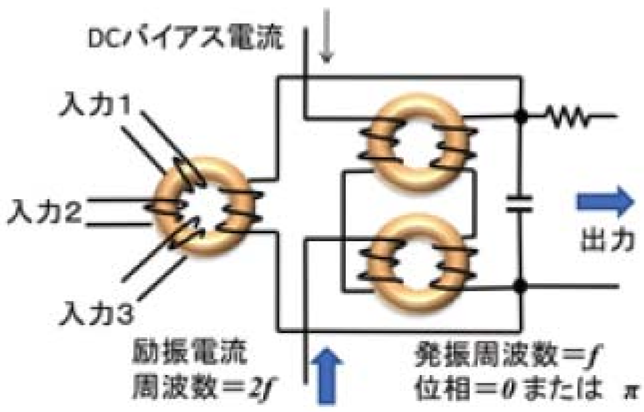
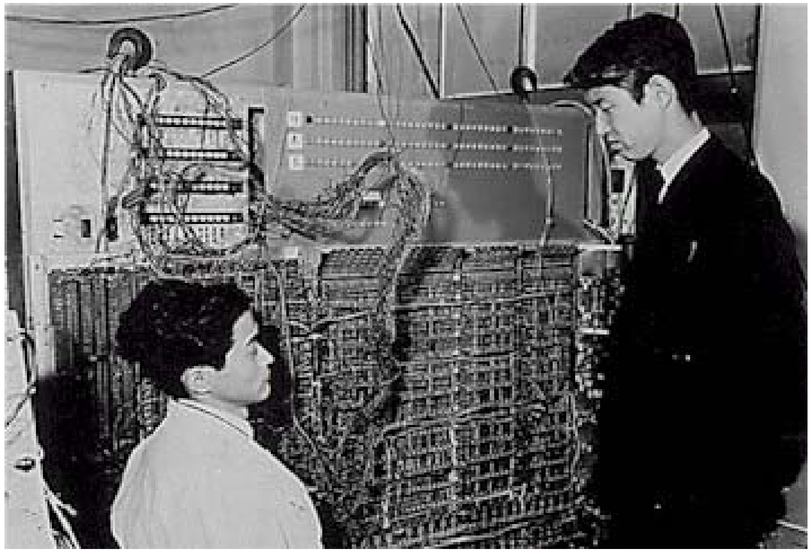


図2:3入力多数決パラメトロン素子

Parametron computer (1958)

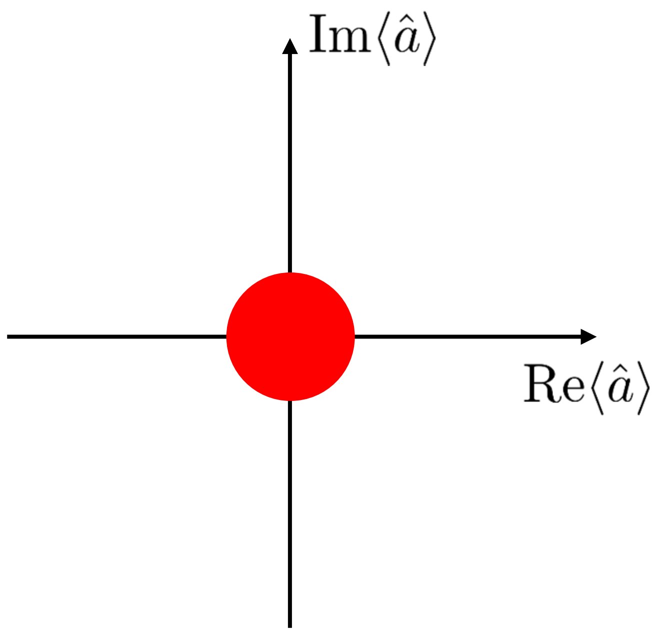


E. Goto and H. Takahashi in front of PC-1

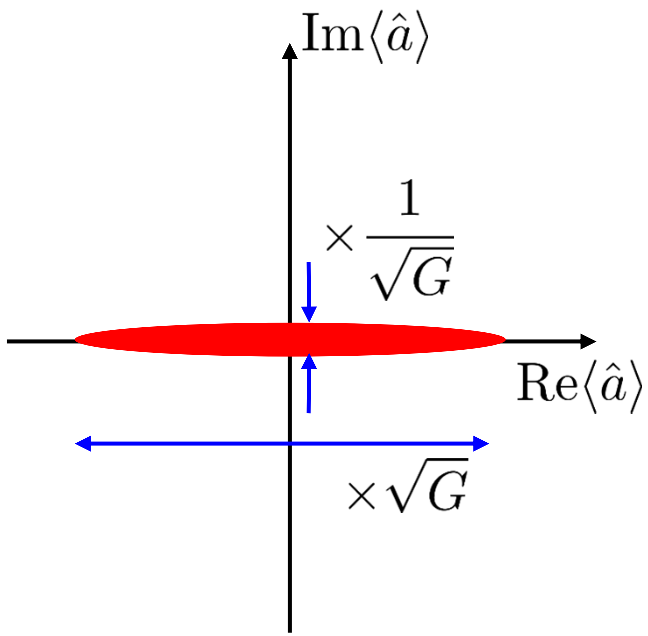
www.s.u-tokyo.ac.jp/ja/story/newsletter/treasure/05.html

Parametrically driven quantum oscillator

Amplify only one quadrature
De-amplify the other

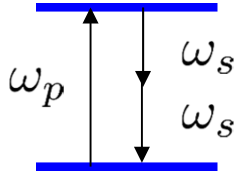


Vacuum fluctuation



Squeezed vacuum

Optical parametric amplifiers



Nonlinear optical crystals

Generation of Squeezed States by Parametric Down Conversion

Ling-An Wu, H. J. Kimble, J. L. Hall,[(a)] and Huifa Wu

Department of Physics, University of Texas at Austin, Austin, Texas 78712

(Received 11 September 1986)

Squeezed states of the electromagnetic field are generated by degenerate parametric down conversion in an optical cavity. Noise reductions greater than 50% relative to the vacuum noise level are observed in a balanced homodyne detector. A quantitative comparison with theory suggests that the observed squeezing results from a field that in the absence of linear attenuation would be squeezed by greater then tenfold.

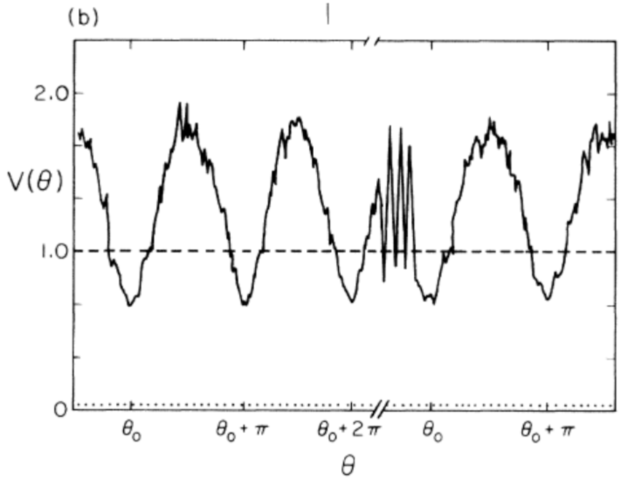


FIG. 1. (a) Phase plot of the uncertainties in the quadrature amplitudes of the electric field. The solid line represents the variance $v^2(\theta)$ of the field $\hat{X}(\theta) = \hat{X}_+ \cos\theta + \hat{X}_- \sin\theta$ as a function of θ for a squeezed state; the dashed line is for the vacuum state. (b) Measurement of the phase dependence of the quantum fluctuations in a squeezed state produced by degenerate parametric down conversion. The plot corresponds roughly to

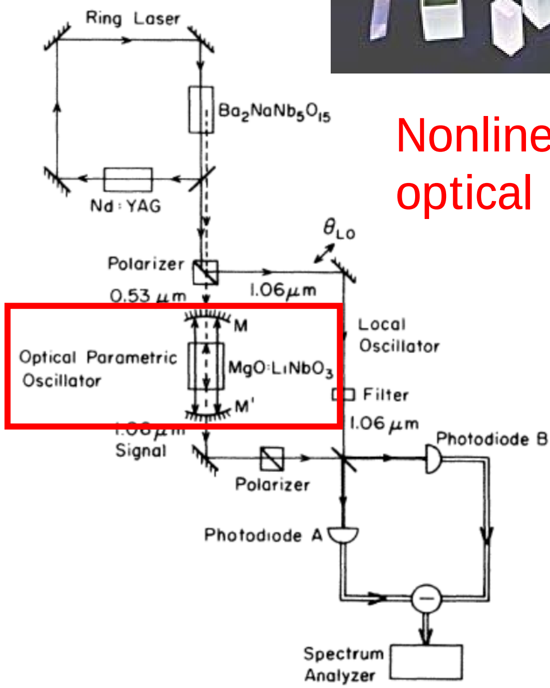


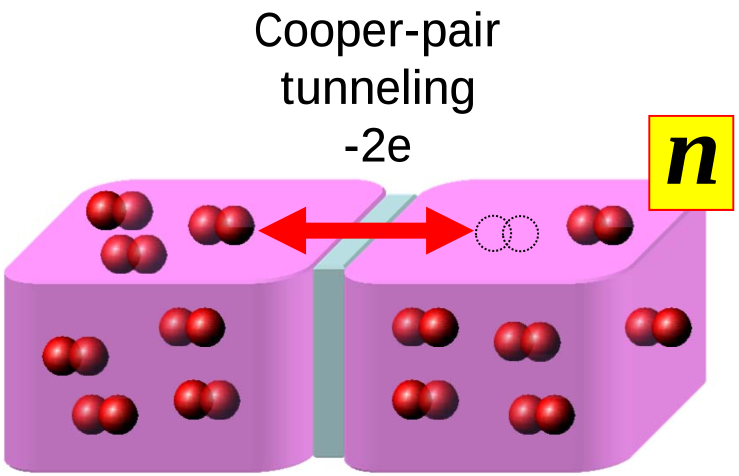
FIG. 2. Diagram of the principal elements of the apparatus for squeezed-state generation by degenerate parametric down conversion.

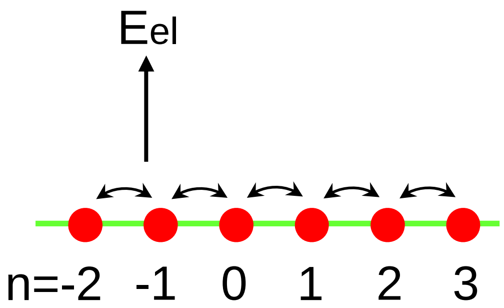
Phys. Rev. Lett. **57**, 2520 (1986)

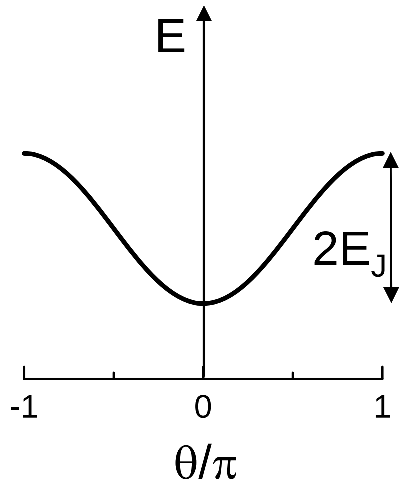
Josephson effect

B. D. Josephson 1962

number ***n*** ⇔ phase difference ***θ*** $[n, \theta] = -i$







$$H = -\frac{E_J}{2}\sum_n \{|n\rangle\langle n+1| + |n+1\rangle\langle n|\} = -\int_0^{2\pi} d\theta E_J \cos\theta |\theta\rangle\langle\theta|$$

Tight-binding model in 1d lattice ⇒ Bloch band

$$|\theta\rangle = \sum_n e^{in\theta}|n\rangle$$

Josephson junction – non-dissipative nonlinear inductor

Hamiltonian

$$\hat{H}_J = -E_J \cos\hat{\theta} - 2e\hat{n}\hat{V}$$

Josephson relations

$$\begin{cases} \hat{I} = -2e\dfrac{d\hat{n}}{dt} = -\dfrac{2e}{i\hbar}[\hat{n},\hat{H}] = -\dfrac{2e}{\hbar}\dfrac{\partial\hat{H}}{\partial\hat{\theta}} = I_c\sin\hat{\theta} \\ \dfrac{d\hat{\theta}}{dt} = \dfrac{1}{i\hbar}[\hat{\theta},\hat{H}] = -\dfrac{1}{\hbar}\dfrac{\partial\hat{H}}{\partial\hat{n}} = \dfrac{2e\hat{V}}{\hbar} = \dfrac{2\pi}{\Phi_0}\hat{V} \end{cases}$$

$$I_c \equiv \frac{2eE_J}{\hbar} = \frac{2\pi}{\Phi_0}E_J$$

$$\Phi_0 \equiv \frac{h}{2e} \qquad \hat{\theta} = 2\pi\frac{\hat{\Phi}}{\Phi_0}$$

Josephson inductance

$$\hat{H}_J = -E_J\cos\hat{\theta} \simeq \frac{\hat{\Phi}^2}{2L_J} = \left(\frac{\Phi_0}{2\pi}\right)^2\frac{\hat{\theta}^2}{2L_J}$$

$$L_J = \left(\frac{\partial^2\hat{H}}{\partial\hat{\Phi}^2}\right)^{-1} = \frac{\Phi_0}{2\pi I_c\cos\hat{\theta}}$$

Current driven Josephson parametric amplifiers

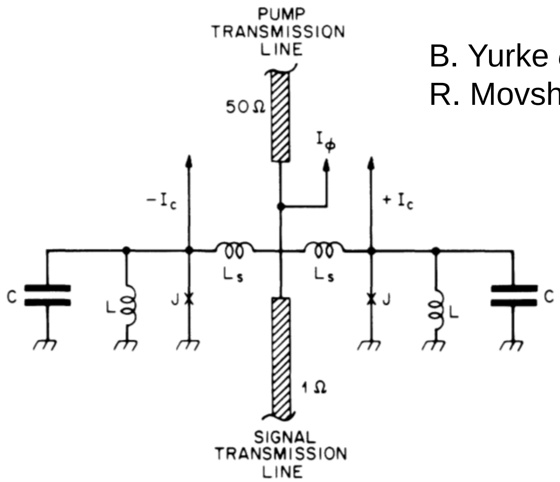
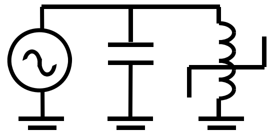


FIG. 1. A Josephson-parametric amplifier. See text for details.

B. Yurke *et al.*, PRL **60** 764 (1988),
R. Movshovich *et al.*, PRL **65** 1419 (1990)

$$L_J = L_{J0}\left[1 + \frac{1}{2}\left(\frac{I}{I_c}\right)^2 + \frac{3}{8}\left(\frac{I}{I_c}\right)^4 + \cdots\right]$$



Current drive at $\omega_p = \omega_s$
for modulating L_J

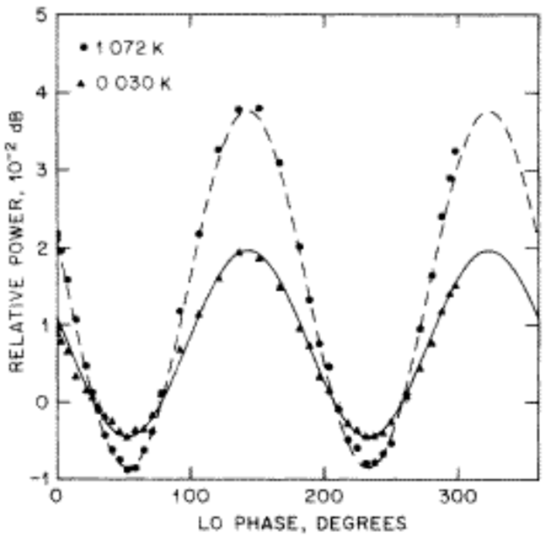
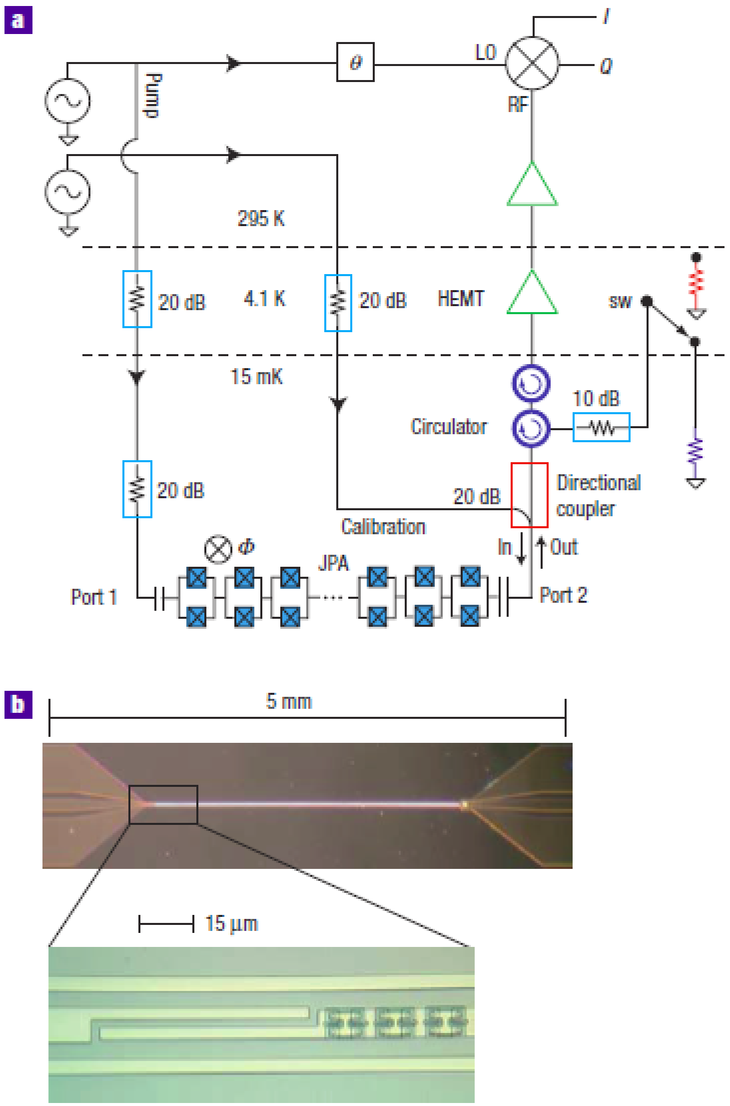


FIG. 2. Spectrum-analyzer noise power referenced to the pump-off noise floor as a function of the local-oscillator phase.

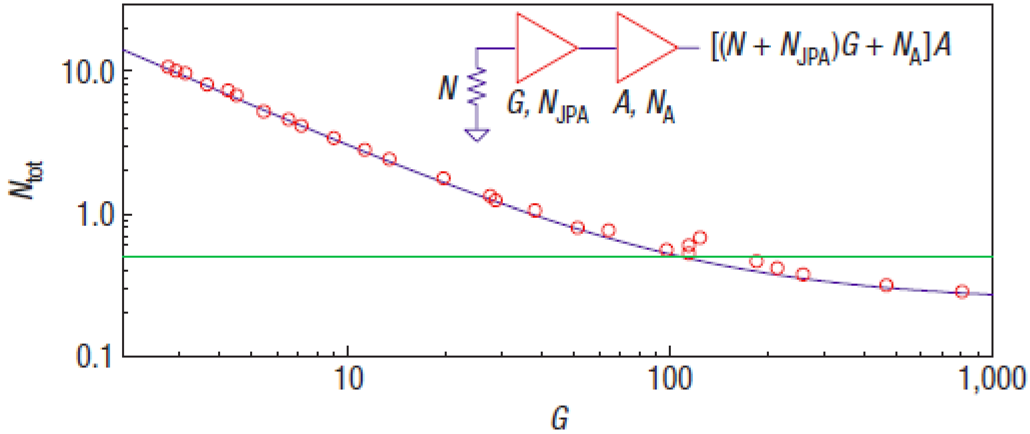
Recently,
E. A. Tholen *et al.*, APL **90**, 253509 (2007)
M. A. Castellanos-Beltran *et al.*, APL **91**, 083509 (2007)
T. Yamamoto *et al.*, APL **93**, 042510 (2008)
N. Bergeal *et al.*, Nature **465**, 64 (2010)
M. A. Castellanos-Beltran *et al.*, Nature Physics **4**, 929 (2008)

Current driven Josephson parametric amplifiers



Amplification and squeezing of quantum noise with a tunable Josephson metamaterial

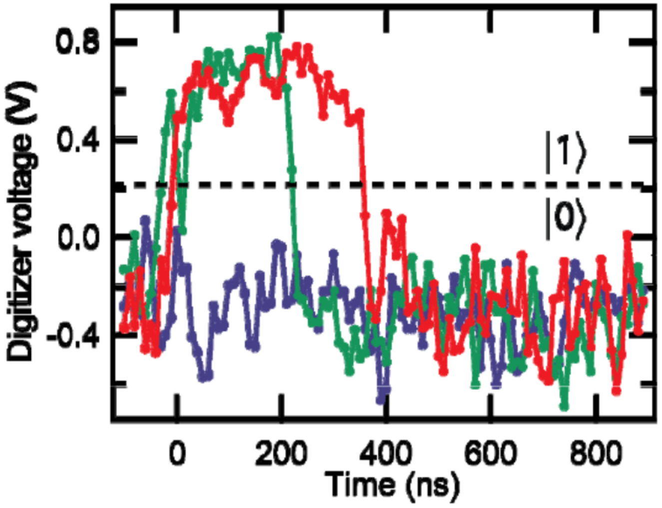
M. A. CASTELLANOS-BELTRAN[1]*, K. D. IRWIN[2], G. C. HILTON[2], L. R. VALE[2] AND K. W. LEHNERT[1]



Nature Phys. 4, 928 (2008) JILA

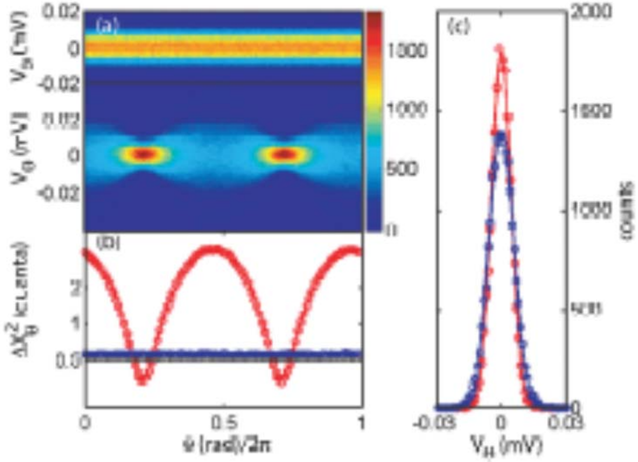
Applications of JPA

Observation of quantum jumps

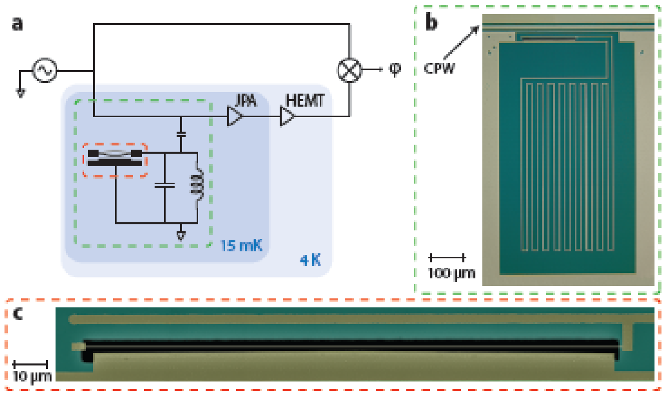


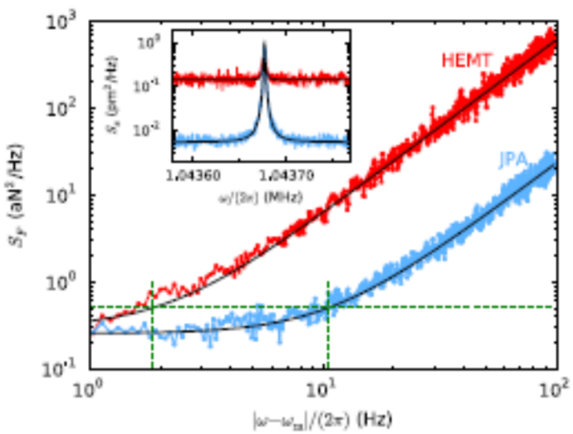
R. Vijay *et al.*,
PRL **106** 110502 (2011)

Generation and detection of squeezed microwave



F. Mallet *et al.*,
PRL **106** 220502 (2011)





Detection of nanomechanical resonator motion

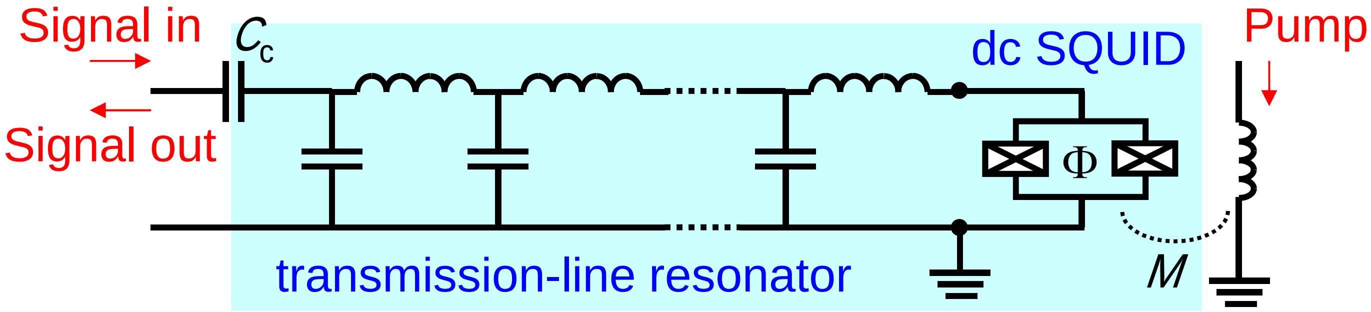
J. D. Teufel *et al.*,
Nature Nanotech. **4** 820 (2009)

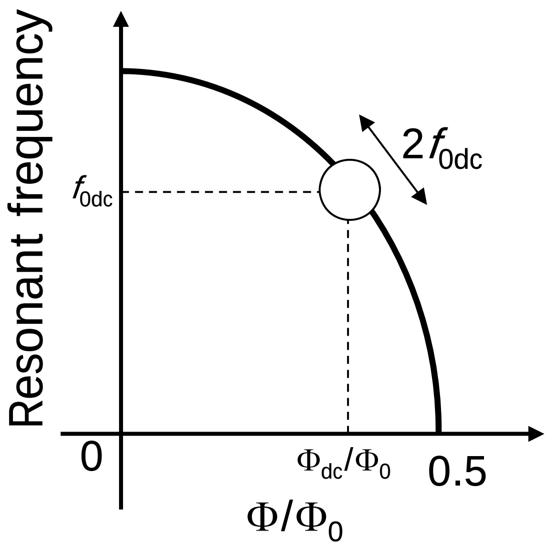
Flux-driven parametric amplifiers

T. Yamamoto *et al.*, APL **93**, 042510 (2008)

also T. Ojanen, PRB **75**, 184508 (2007)

- SQUID-terminated CPW resonator





Tuneable resonant frequency

Advantages:

- Band center tunable
- Signal well isolated from the pump

(frequency: twice different, leakage: small)

M. Wallquist, PRB **74**, 224506 (2006)

Hamiltonian

$$\mathcal{H} = \hbar\Omega_0\left[a^\dagger a + \epsilon\cos(\alpha\Omega_0 t)(a+a^\dagger)^2\right]$$

parametrically modulated harmonic oscillator $\alpha \sim 2$

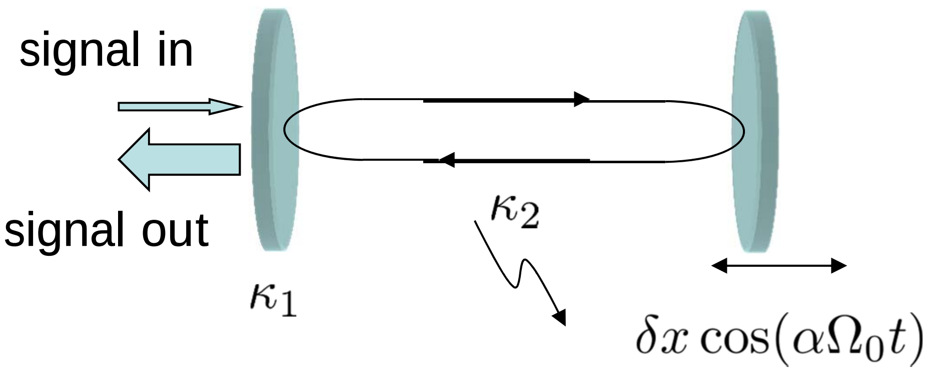
$$+ \int d\omega\left[\hbar\omega b(\omega)^\dagger b(\omega) + i\hbar\sqrt{\frac{\kappa_1}{2\pi}}\left(a^\dagger b(\omega) - b(\omega)^\dagger a\right)\right]$$

signal port

$$+ \int d\omega\left[\hbar\omega c(\omega)^\dagger c(\omega) + i\hbar\sqrt{\frac{\kappa_2}{2\pi}}\left(a^\dagger c(\omega) - c(\omega)^\dagger a\right)\right]$$

loss port

$$\omega_r = \Omega_0[1 + \delta\cos(\alpha\Omega_0 t)]$$

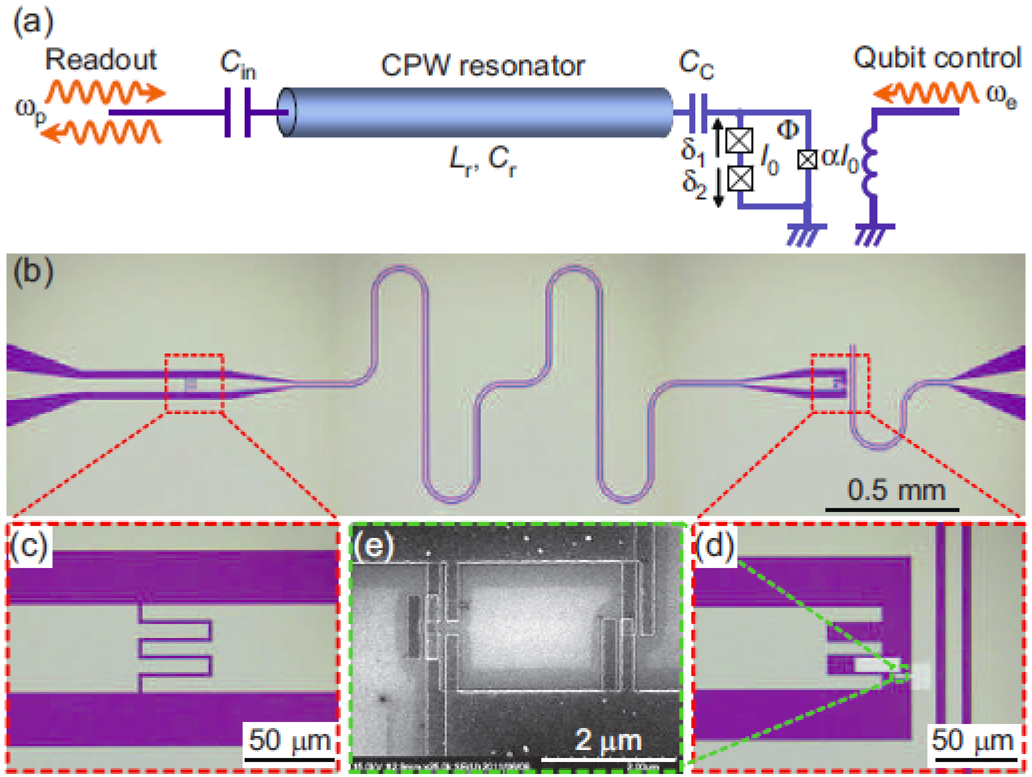


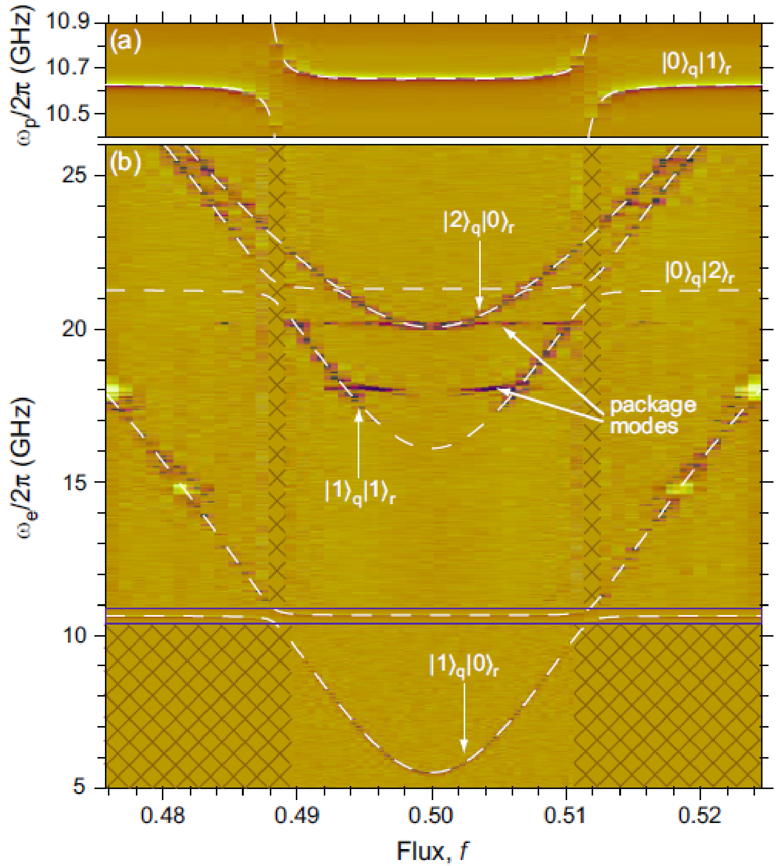
$$\kappa_1 = \frac{\Omega_0}{Q_{\mathrm{ext}}}$$

$$\kappa_2 = \frac{\Omega_0}{Q_{\mathrm{int}}}$$

Opto-mechanical analogue

Flux qubit capacitively coupled to a resonator





Capacitive coupling

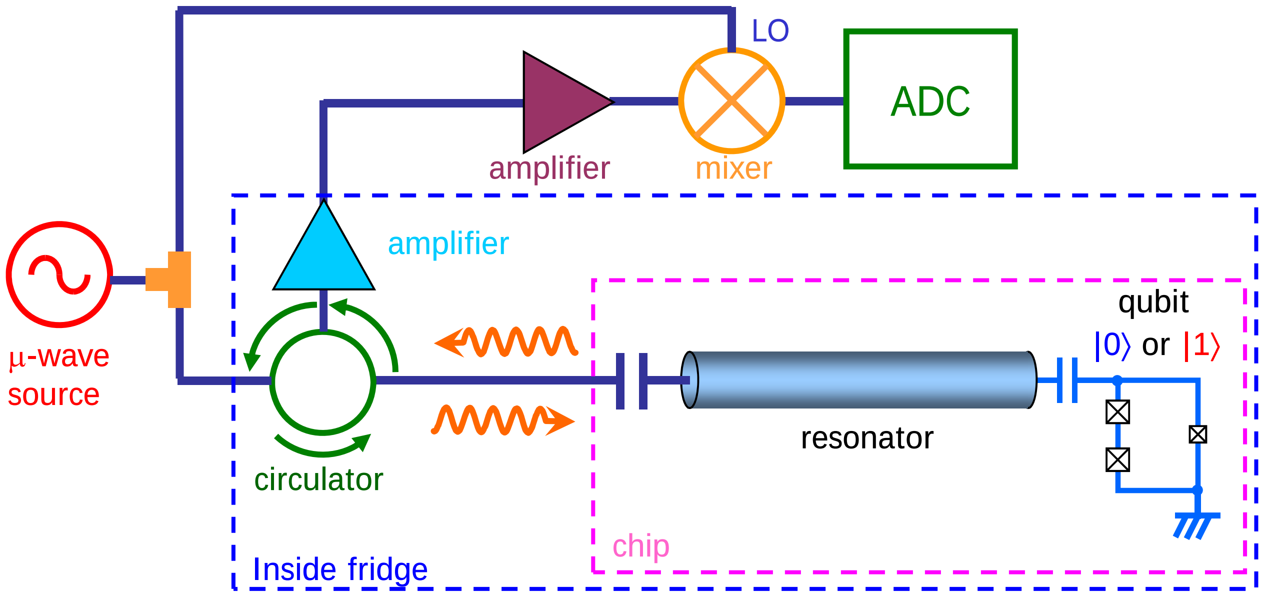
$$H = \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\omega_q\sigma_z + \hbar g(a + a^\dagger)\sigma_x$$

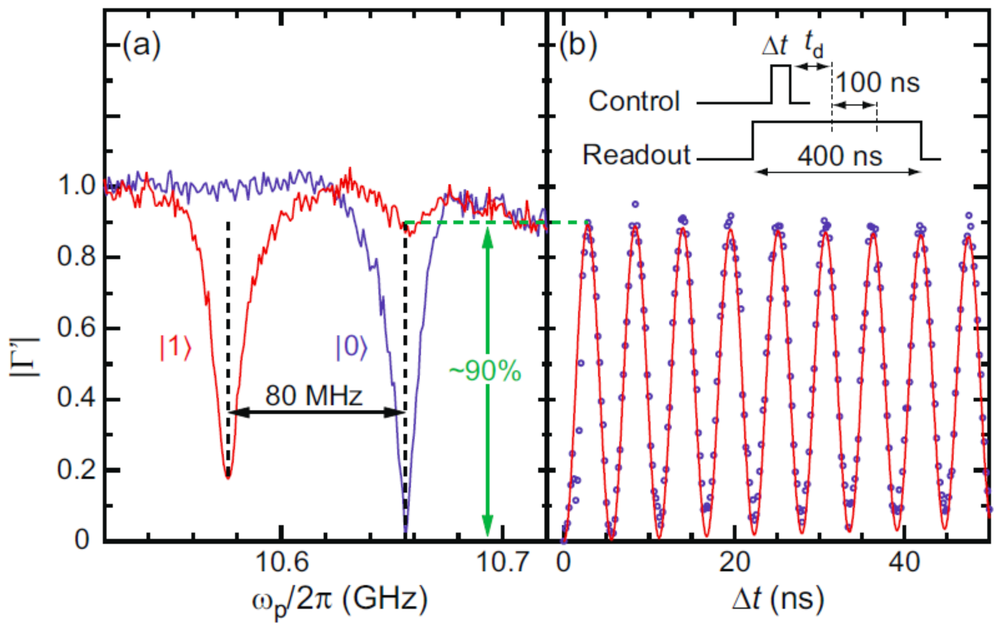
Rotating wave approximation ⇒ Jaynes-Cummings model

$$H_{\rm JC} = \hbar\omega_r a^\dagger a + \frac{1}{2}\hbar\omega_q\sigma_z + \hbar g(a\sigma_+ + a^\dagger\sigma_-)$$

Generalized JC model analysis
K. Inomata et al. arXiv:1207.6825

Dispersive readout of a flux qubit





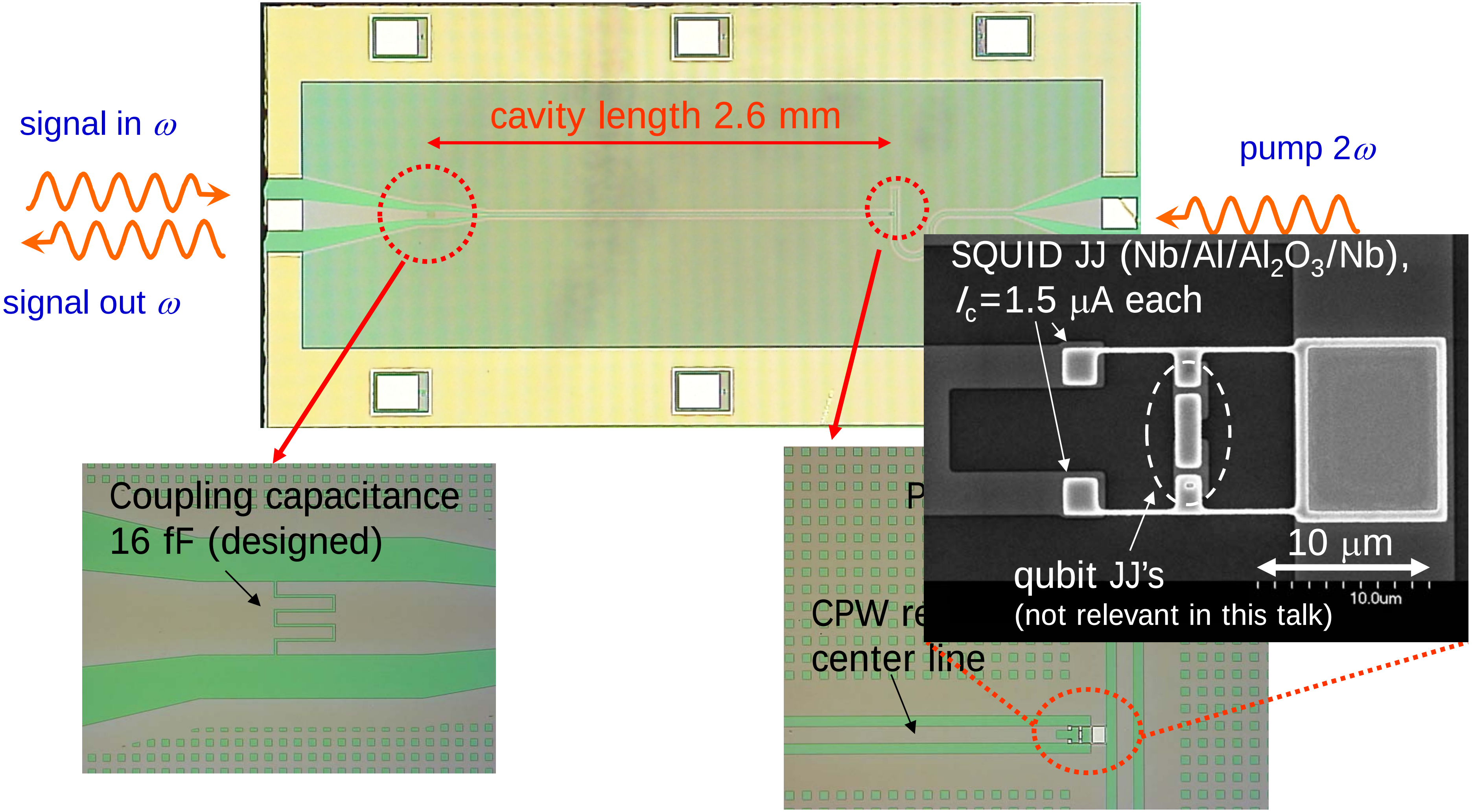
Dispersive limit $g \ll \Delta$

$$H_{\mathrm{JC}} \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_q + \frac{g^2}{\Delta}\right)\sigma_z$$

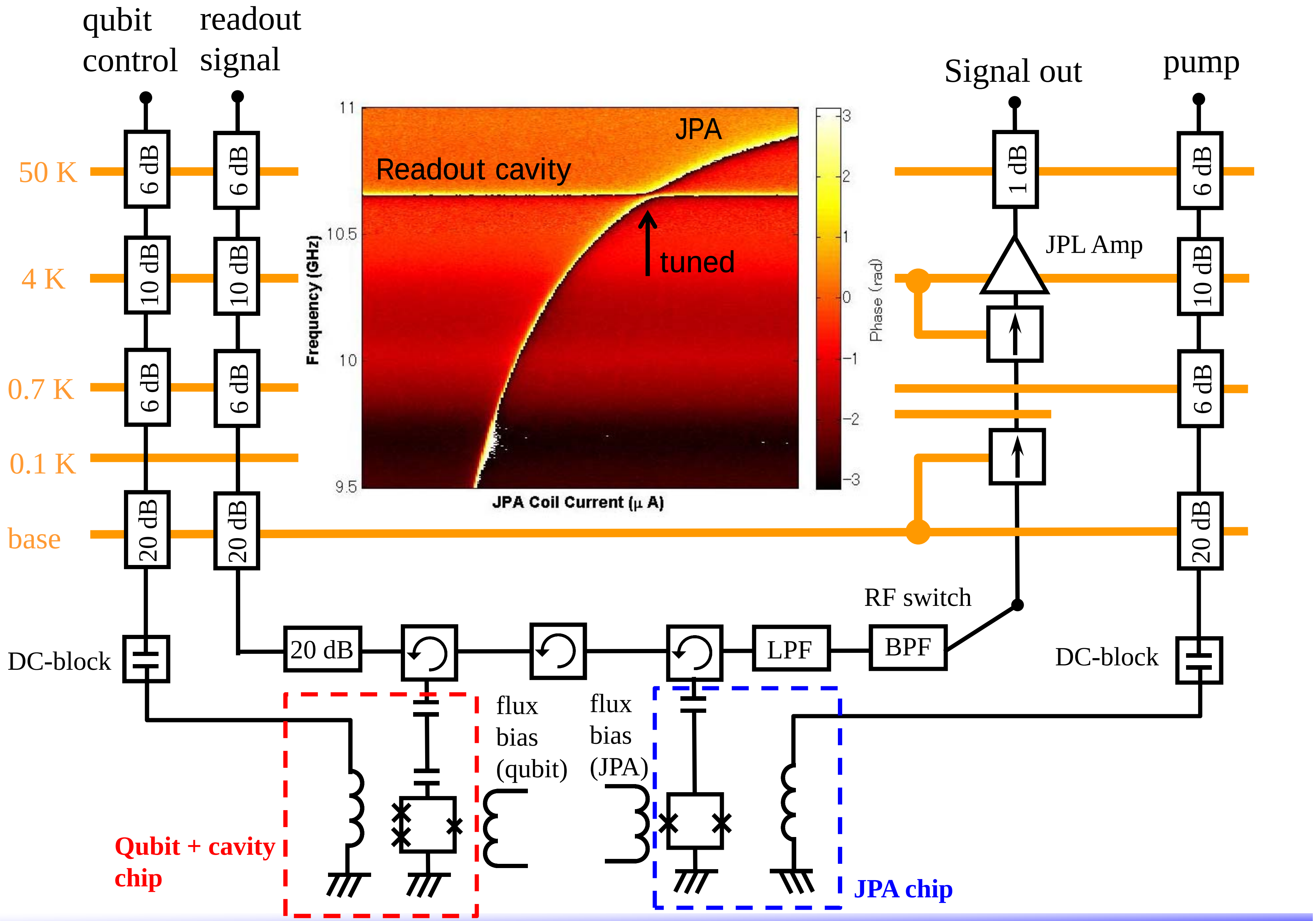
high-contrast
Rabi oscillations
(60000 times average)

Flux driven Josephson parametric amplifier

MIT-LL Deep-Submicron Process for Nb
Nb: 150 nm thick

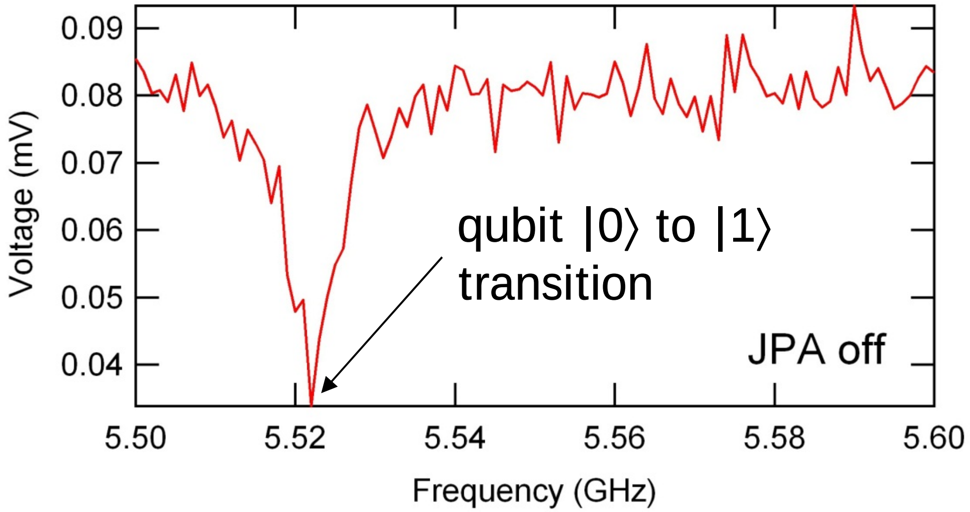


Measurement setup

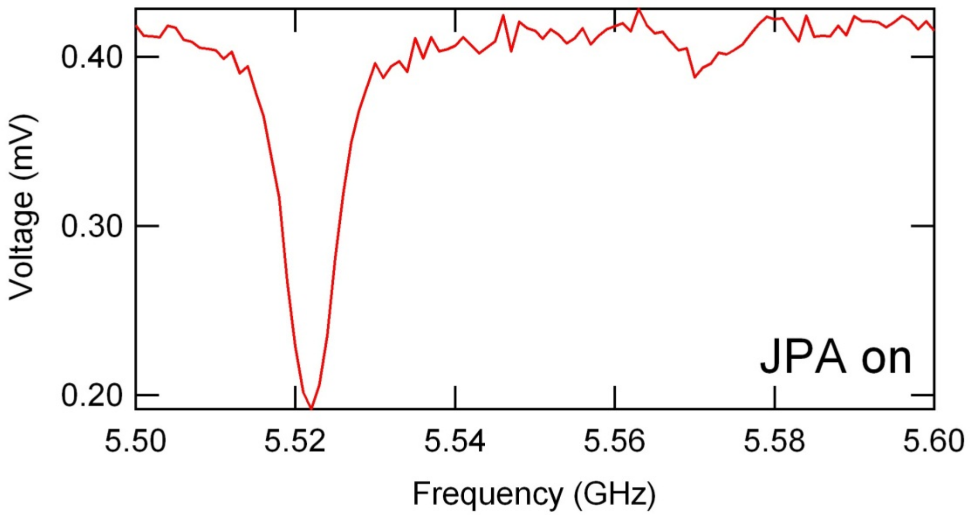


Improvement of signal-to-noise ratio

qubit spectroscopy
(ensemble meas.)



Average photon number in cavity
$\bar{n} \sim 0.02$



JPA operated in non-degerate mode with a gain of 14 dB

12 dB improvement!!

But, additional 17 dB is needed to achieve single-shot readout.

degenerate mode
+larger signal power

Summary

- Josephson junctions
  - Small dissipation
  - Large nonlinearity
  - Useful in qubits, amplifiers, detectors, etc.

- Josephson parametric amplifier
  - Current “de facto standard” tool for quantum measurement in microwave domain
  - Beyond standard quantum limit
  - Current driven and flux driven
  - Phase preserving and phase non-preserving

- Applications of JPA
  - Vacuum squeezing
  - Noiseless amplification
  - Quantum state tomography
  - Quantum feedback control etc.