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Surface Code Quantum Error Correction

Rodney Van Meter, Keio University http://aqua.sfc.wide.ad.jp/ FIRST Project Summer School @ Kyoto U. 2011 Aug 16 (all the good slides and animations are by Shota Nagayama, Austin Fowler and Clare Horsman)



2

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Simplest Example ~Encoding logical qubit~



Simplest Example ~Encoding logical qubit~ measured





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Surface Code Strengths







 Simple, 2-D or 3-D nearest-neighbor-only operation (physical feasibility high!)







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 - Supporting classical processing achievable



5

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Scalability: fault-tolerance

- Trade-off between resources and threshold
- Thresholds
 - –unlimited range, unlimited qubits: ~ 10⁻²
 - Knill, quant-ph/0410199
 - –unlimited range, many qubits: ~ 10⁻³–10⁻⁴
 - Steane, Phys. Rev. A 68, 042322 (2003)
 - -2D lattice, nearest neighbor: ~ 10⁻⁵
 - Svore, QIC 7, 297 (2007)
 - -bilinear nearest neighbor: ~ 10⁻⁶
 - Stephens, QIC 8, 330 (2008)
 - –linear nearest neighbor: ~ 10⁻⁸
 - Stephens, in preparation





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Surface Code Drawbacks







Non-Clifford group operations difficult







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- Direct calculation of residual error rates difficult due to many error chains; determined via simulation







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- Non-Clifford group operations difficult
- Direct calculation of residual error rates difficult due to many error chains; determined via simulation
- Almost uniform set of operations across whole device, but not quite!
- Extremely difficult to explain to classical computer engineers!









Outline

- Stabilizers
- Surface Code Operation
- Theory of the Surface Code
- Advanced topics
- (system architecture reserved for next lecture)







$$\begin{array}{l} |0\rangle \equiv \begin{pmatrix} 1\\0 \end{pmatrix} \equiv +1 \text{ eigenstate of } Z = \begin{pmatrix} 1&0\\0&-1 \end{pmatrix} \\ |+\rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1\\1 \end{pmatrix} \equiv +1 \text{ eigenstate of } X = \begin{pmatrix} 0&1\\1&0 \end{pmatrix} \end{array}$$

 $\bullet~n~$ qubits, ~n~ independent commuting stabilizers $\Rightarrow~$ unique state





Stabilizers

- *n-k* stabilizers on set of *n* qubits leaves *k* degrees
 of freedom, can encode *k* logical qubits
- Measure set of stabilizers to get error syndrome
- See Gottesman PhD. thesis, and the set of notes by Clare Horsman in download materials

Introduction to stabilizer theory

June 16, 2011

1 Stabilizer definition

In quantum mechanics, a state is given by a vector, and an operator is given by a matrix. The state of N qubits is a 1×2^N vector, and an operation on the state is a $2^N \times 2^N$ matrix.

For example, a state of 1 qubit could be

 $|+\rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array} \right)$

and an operation on it could be

 $X = \sigma_x = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)$

In linear algebra, if for a matrix M there is a vector ${\bf V}$ and a scalar v such that

M.V = v.V

then V is an eigenvector of M and v is the associated eigenvalue.

The vector V can be the eigenvector of more than one matrix. The vector V can be fully defined by the set $\{M, v\}$ of matrices that V is an eigenvector of.

The stabilizers of a state $|\psi\rangle$ are the operators $\frac{1}{v}$. M where $|\psi\rangle$ is an eigenvector of M with eigenvalue v.

We see that

$$X|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1\\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1\\ 1 \end{pmatrix} = |+\rangle$$



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Operations

- Lattice cycle
- Detecting & correcting errors
- Holes as logical qubits
- Single-qubit X and Z gates
- Moving holes
- Braiding for CNOT (Primal & dual lattice)
- Non-Clifford gates using singular qubits
- "Singular factories"
- Measuring a logical qubit



Surface Code



Surface Code

edges of two holes =logical qubit

data qubit
phase-flip checker
bit-flip checker



Surface Code



Entangling the Lattice



Entangling the Lattice





Measuring Z stabilizers







Full lattice cycle



Full lattice cycle



Error syndromes



Memory Error I



Memory Error I



Memory Error I



<u>Memory Error 2</u>



<u>Memory Error 2</u>


<u>Memory Error 2</u>



Measurement Error I



Measurement Error I







Gate Error I



Gate Error I



Measurement Error





Gate Error 2







memory error



gate error





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- Introduce degree of freedom by not measuring stabilizer (first, measure out any data qubits in the *interior* of the hole in X basis, to disentangle)
- Only one degree of freedom associated with arbitrary size hole
- Here, 24 lattice qubits, 9 Z stabilizers, 16 X stabilizers (but one *not independent*) = 24 independent stabilizers
- n.b.: Holes referred to as "defects" in most papers, I reserve that term for physically defective qubits



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31



Aqua : Advancing Quantum Architecture Larger holes



- Only one degree of freedom associated with arbitrary size hole
- Must correct three-term X stabilizers (green)
- Initialize to $|0_L
 angle$







Logical X and Z gates





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Logical X and Z gates

 Degree of freedom can be operated on by chains and rings of single-qubit X and Z on lattice qubits





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- Chain of X flips connecting hole to boundary gives X gate on logical qubit



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Aqua : Advancing Quantum Architecture Logical X and Z gates



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Logical X and Z gates





- Chain of X flips connecting hole to boundary gives X gate on logical qubit
- Ring of Z flips *around* hole gives logical Z gate
- n.b.: This can be *any* ring around the hole or *any* chain connecting hole to lattice edge



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Pairing up holes



- Better to use a pair of defects to represent one qubit
 - No need to connect operators to boundary
 - Easier to move qubit around computer
- Independent adjustment of X/Z error correction strength
- Terminology: smooth qubit
 - Logical X gate chain connects the holes
 - Logical Z gate circles one of the holes





Rough qubits (Dual lattice)



- Can make a different type of qubit
- X_L and Z_L reversed
- Primal and dual lattice



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 First, expand hole using measurements and stop measuring stabilizers







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- First, expand hole using measurements and stop measuring stabilizers
- Fix up w/ bit flips (takes multiple rounds, depending on distance)





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Moving holes

- First, expand hole using measurements and stop measuring stabilizers
- Fix up w/ bit flips (takes multiple rounds, depending on distance)
- Shrink hole to new location

















Initializing logical qubits



- Start with region of |+
 angle
- Smooth qubit in ±1 eigenstate of X_L
- Measure Z stabilizers



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Braiding smooth pair with rough pair gives CNOT

(WHY? Stay tuned...)



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Braiding: 2D+T Picture



} dual qubit.
(Rough holes)

} primal qubit.
(Smooth holes)

Time



picture from Raussendorf et al., NJP 9, 2007


• Circuit is equivalent to CNOT followed by $(Z \otimes Z)^{M_X}$ followed by $(X_t)^{M_Z}$



Circuit can be represented as a braiding of defects



Equivalent more compact braiding





45





- ullet We can prepare states $|0_L
 angle$, $|1_L
 angle$, $|+_L
 angle$ and $|-_L
 angle$
- \bullet We can perform gates X_L , Z_L , M_X , M_Z and CNOT
- Not universal without the following:





46



Aqua : Advancing Quantum Architecture State injection





- Set a single qubit to desired state $\alpha |0\rangle + \beta |1\rangle$ (M_Z then rotate)
- Increase size of defects using X measurements
- Separate defects by measuring and correcting ${\cal Z}$ stabilizes
- Procedure is not fault-tolerant, restart if errors detected early
- Logical state will not be perfect lpha|0
 angle+eta|1
 angle, however...



State distillation: "Singular factories"

- Two very special states exist $|Y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ and $|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/4}|1\rangle)$
- These states can be "distilled"
- Imperfect $|Y\rangle$ and $|A\rangle$ states approach perfect versions exponentially quickly
- Probability of success asymptotically close to 1, though some byproduct operators needed











Universality

Adding these states:

$$\begin{split} |Y\rangle &:= (|0\rangle + i|1\rangle)/\sqrt{2} \\ |A\rangle &:= (|0\rangle + e^{i\pi/4}|1\rangle)/\sqrt{2} \\ \text{gives us probabilistic, heralded universality} \end{split}$$

- Logical Toffoli gate consumes 7 $|Y\rangle$ and 4.5 $|A\rangle$ states (average)
- Distillation of enough states for one Toffoli gate ~1800 singular qubits + braidings (certain assumptions, not detailed here)







Measuring a Qubit



- Measure region in Z (or X) basis
- Every ring around either defect odd parity, state is 1
- Majority vote when not all same







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Quantum





Theory

- Why braiding works
- Code distance
- Estimating logical error rate









CNOT: moving defects

- Start with surface $|\Psi\rangle$ satisfying $Z_L|\Psi\rangle=|\Psi\rangle$ and $Z_{\rm face}$
- Measure center qubit in the X basis to produce $|\Psi'
 angle$
- Surface $|\Psi'\rangle$ satisfies $Z_L Z_{face} |\Psi'\rangle = |\Psi'\rangle$ and $\pm X |\Psi'\rangle = |\Psi'\rangle$



$\mathsf{WDECNOT}: X \otimes I \to X \otimes X \overset{\mathsf{ture}}{\boxtimes} \mathsf{I}$

- If $M|\psi\rangle = |\psi\rangle$, then $U|\psi\rangle = UMU^{\dagger}U|\psi\rangle \Rightarrow M \to UMU^{\dagger}$
- CNOT manipulates stabilizers in the following manner:
- Need to show we can do this by braiding defects





- Mappings $I \otimes X \to I \otimes X$ and $Z \otimes I \to Z \otimes I$ also easy to show
- Defects can be interacted over arbitrary distances in almost constant time
- Still need CNOT between two smooth qubits

57



• Circuit is equivalent to CNOT followed by $(Z \otimes Z)^{M_X}$ followed by $(X_t)^{M_Z}$



Circuit can be represented as a braiding of defects



Equivalent more compact braiding





58



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Error correction

• All error correction based on XZ = -ZX, eg:

 $Z_1 Z_2 Z_3 Z_4 |\psi\rangle = |\psi\rangle$ $Z_1 Z_2 Z_3 Z_4 X_1 |\psi\rangle = -X_1 Z_1 Z_2 Z_3 Z_4 |\psi\rangle = -X_1 |\psi\rangle$





Need to carefully handle false syndromes and error chains 59



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 Record time and position of changed syndromes



- Match closest pairs
- Apply corrective operations to spacelike edges
- Works *very* well, threshold error rate ~1%



Error Chains and Logical Gates

- Learn only endpoints of error chains
- Guess a path
- Usually (hopefully) results in meaningless loop
- Complete error chain (natural or mis-correct) results in logical X or Z gate





61





What is error correction?





 To protect against errors, encode a few logical states in larger physical Hilbert space







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- Physical errors take the system out of the code space
 - → Goal of correction is to get back into code space





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 Some measure of "distance" from a legitimate code word (logical state)

→ Usually move to "closest" logical state





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- More than halfway, you mis-correct
 - → Mis-correct equivalent to accidentally executing a logical gate



62











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- Most threshold analyses assume gate, memory, measurement errors same probability
- Generally must *beat* threshold by 1-2 orders of magnitude

















 $P_{logical gate error rate} = P_{physical gate error rate}^{7}$





Exponential Suppression

- Classical simulation of small lattices
- Error rate

 $\epsilon_{\rm top} \sim \exp(-\kappa(p)l)$

p = phy err rate
 K ~ 0.9
 I = min err chain length



number of non-trivial error cycles of length \boldsymbol{l}

- Still a polynomial correction needed
- Raussendorf et al., NJP 9, 2007






Threshold



- Gate = Memory = Meas errors = 0.75%
- Raussendorf et al., NJP 9, 2007

72





Aqua : Advancing Quantum Architecture Advanced Topics

- 3-D version, talk to Simon
- Defective lattice
- Planar code
- Surface code communications
- Uses in distributed quantum computation



Defective lattice

Current estimate is that yield of 90% halves the effective code distance.

Nagayama et al., in preparation



Planar Code

75

- Use one whole surface per logical qubit instead of holes in surface
- Gates done by merging and splitting surfaces

Horsman et al., in preparation

• Useful for small-scale experiments





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Key References

- clearest explanation: Fowler *et al.*, PRA 80, 052312 (2009) (source of many of the figures)
- detailed paper: Raussendorf *et al.*, NJP 9, 199 (2007)
- cryptic seminal paper: Raussendorf and Harrington, PRL 98, 190504 (2007)
- Surface code communication: Fowler *et al.*, PRL 104, 180503 (2010)
- Defects in the surface code: Stace *et al.*, PRL 102, 200501 (2009)

78



Surface code 量子誤り訂正に関するチュートリアル・ワークショップ



<u>http://aqua.sfc.wide.ad.jp/Publications.html</u>





Preview: What kind of system can run surface code effectively?

- Billions of qubits
- GHz physical gates
- Millisecond memory lifetimes
- Error rate ~0.1%
- ~1 year to factor 2048-bit number
- Stay tuned...