FIRST Quantum Information Processing Project
Summer School 2011

15 August 2011 Kyoto

Quantum Simulation of Hubbard Model Using Ultracold Atoms in an Optical Lattice

Kyoto University

Y. Takahashi

Introduction

Undergraduate : Kyoto University, Faculty of Science
Graduate : Kyoto University, Graduate School of Science
Degree : Kyoto University
Anomalous Behavior of Raman Heterodyne Signal in Pr^{3+}:LaF_3
Employment :
Kyoto University,
Research Associate :Atoms in Superfluid Helium
Lecturer :Photo-excited triplet DNP
Associate Professor :Laser Cooling
Professor

Introduction

Research Interest:

Quantum Information Science Using Cold Atoms

Quantum Simulation (of Hubbard Model)

Spin Squeezing by QND Measurement

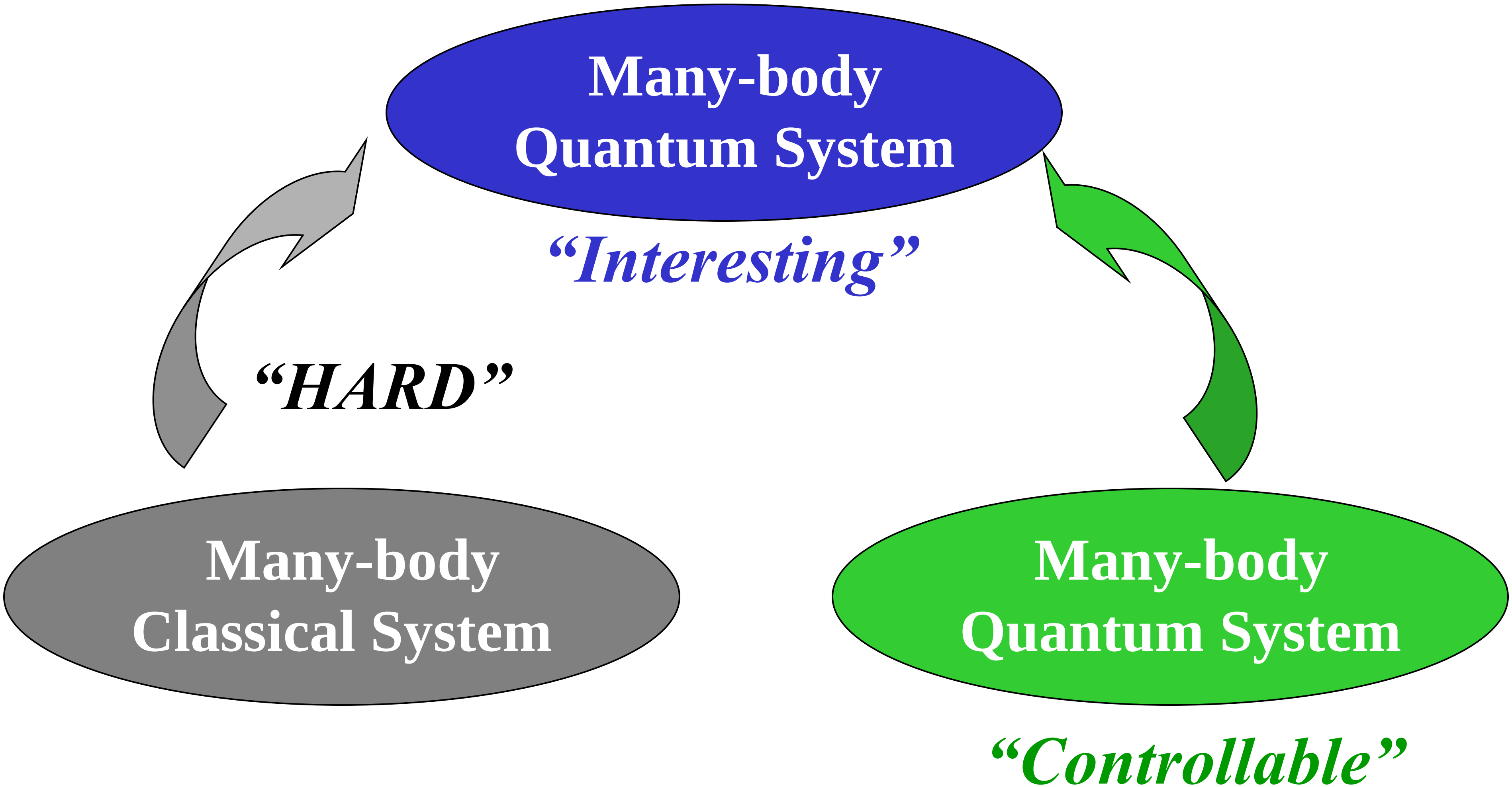
Fundamental Physics Using Cold Atoms:

(Searching for Permanent Electric Dipole Moment)

Test of Newton Gravity:

$$V = -G\frac{M_1 M_2}{r}(1+\alpha\exp(-\frac{r}{\lambda}))$$

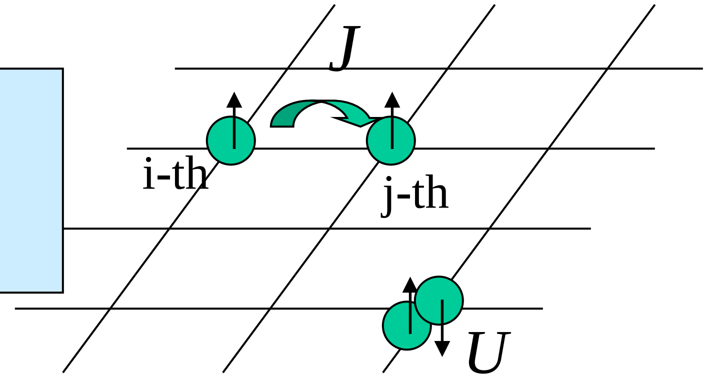
Quantum Simulation



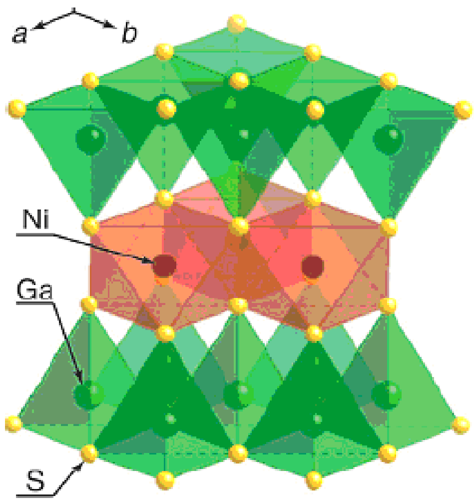
Quantum Simulation

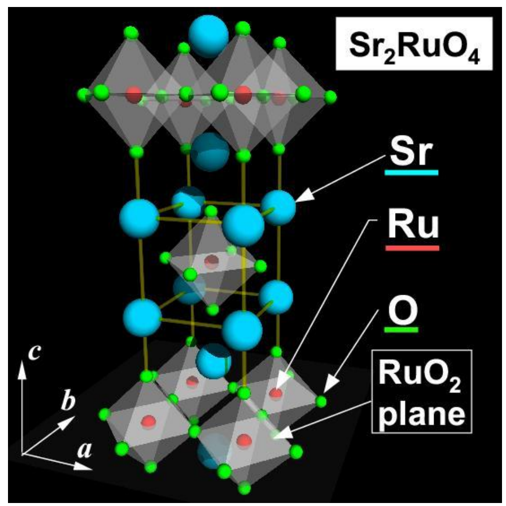
Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Magnetism, Superconductivity

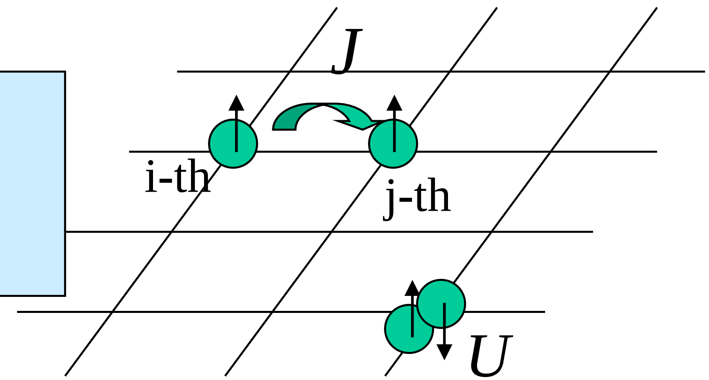




Quantum Simulation

Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



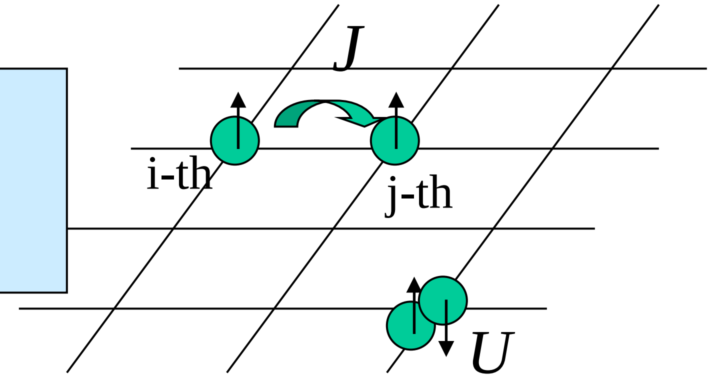
→ Numerical Calculation

DMFT(動的平均場)
Gutzwiller
QMC(量子モンテカルロ)
DMRG(密度行列繰り込み群)
Exact Diagonalization (厳密対角化)

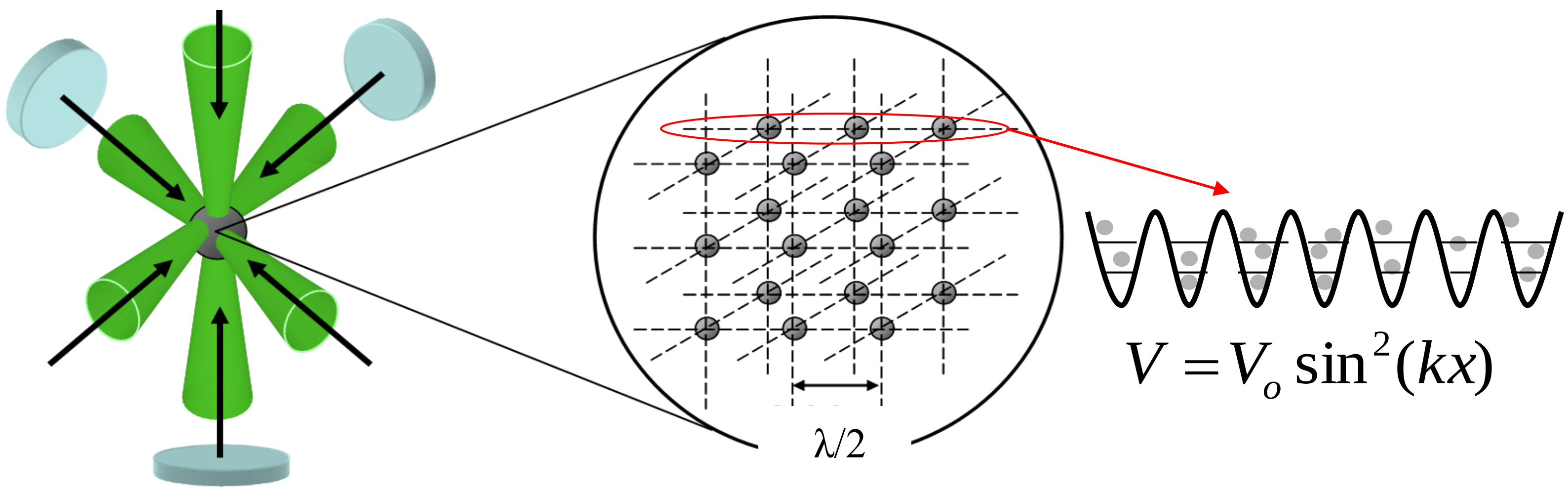
Quantum Simulation

Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Cold Atoms in Optical Lattice



Outline

Atom Manipulation Technique

Laser Cooling and Trapping

Optical Lattice

Tuning Interatomic Interaction

Bose-Hubbard Model

Superfluid-Mott Insulator Transition

Quantum Gas Microscope

Fermi-Hubbard Model

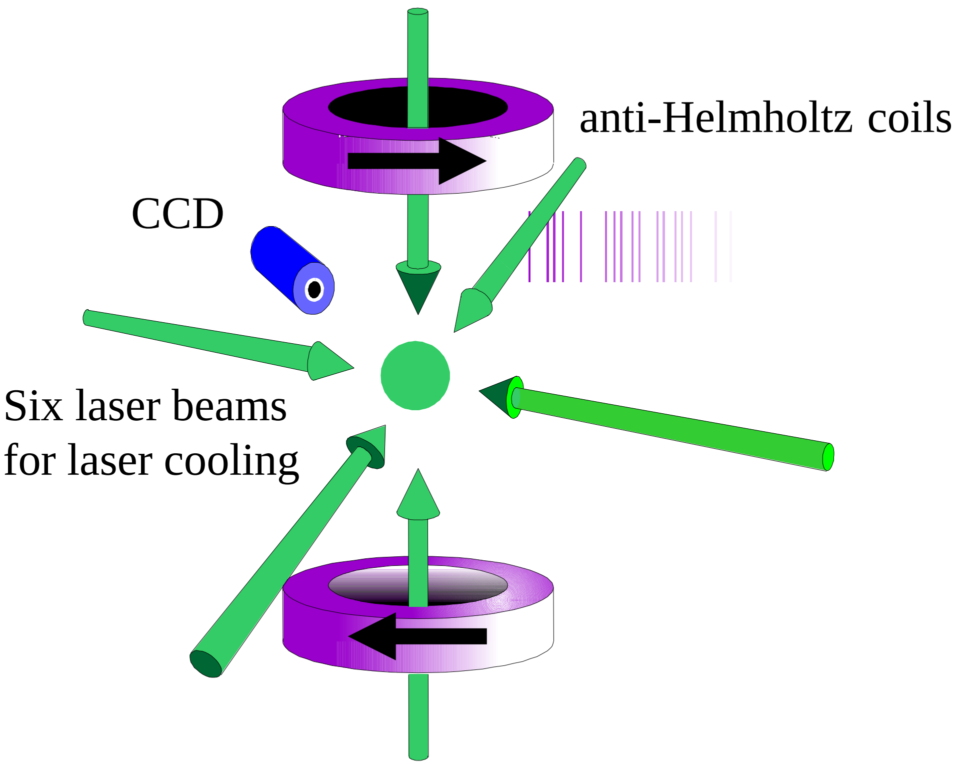
SU(2) & SU(6) Mott insulator

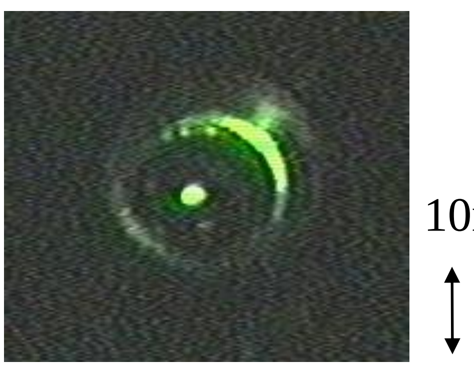
Pomeranchuk cooling

Bose-Fermi Hubbard Model

Dual Mott insulators

Laser Cooling and Trapping





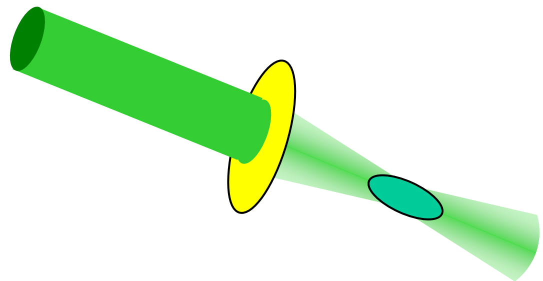
10mm

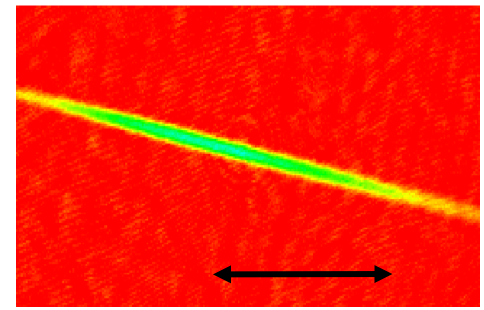
- Number: 10^7
- Density: $10^{11}/cm^3$
- Temperature: 10μK

"Magneto-optical Trap"

"optical trap"

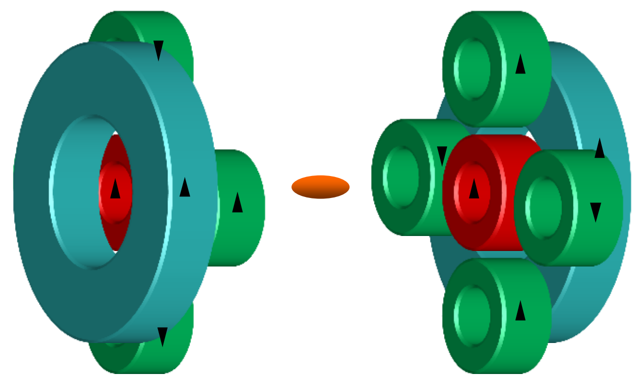
$$V_{\text{int}} = -p \cdot E \qquad U_{pot}(r) = -\frac{\chi E(r)^2}{2}$$





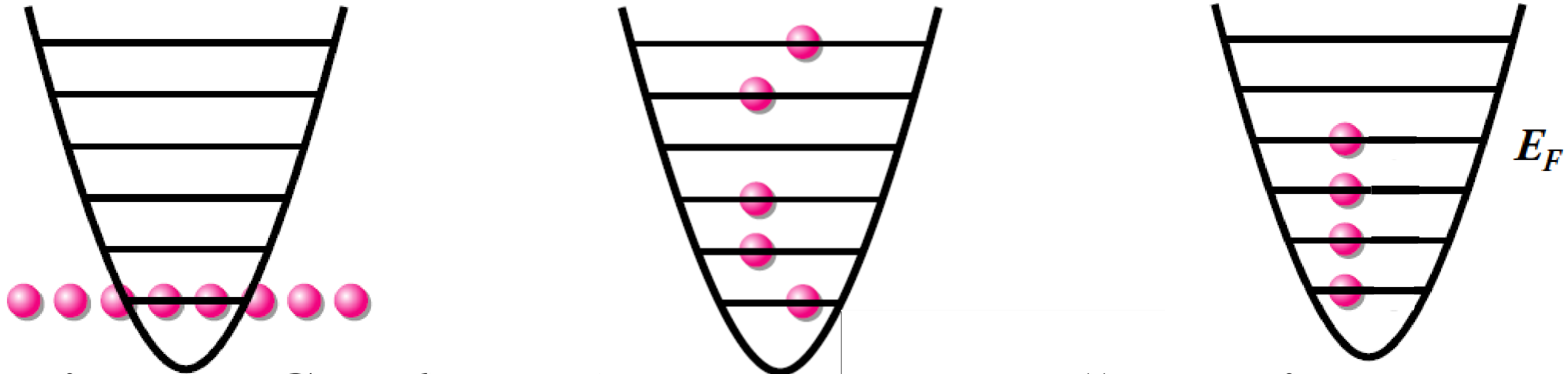
500μm

"magnetic trap" $V_{\text{int}} = -\mu \cdot B$



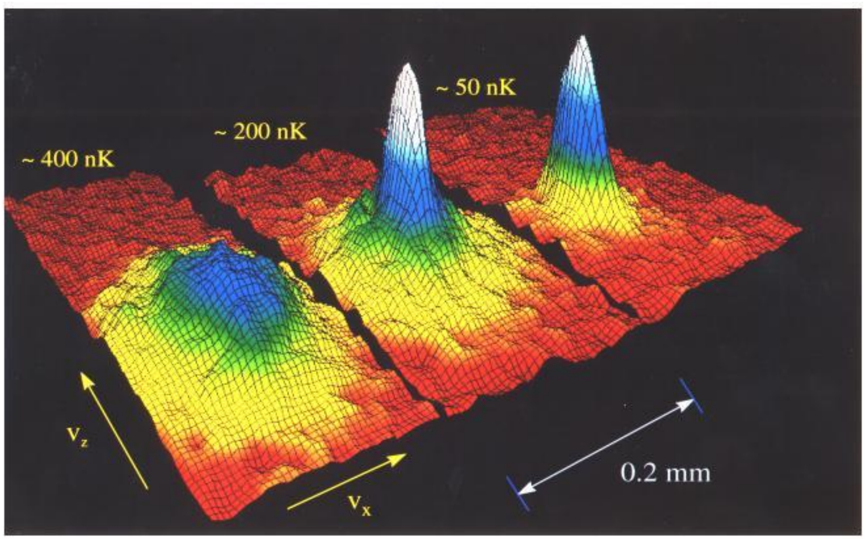
Atomic Gases Reach the Quantum Degenerate Regime

"Boson versus Fermion"

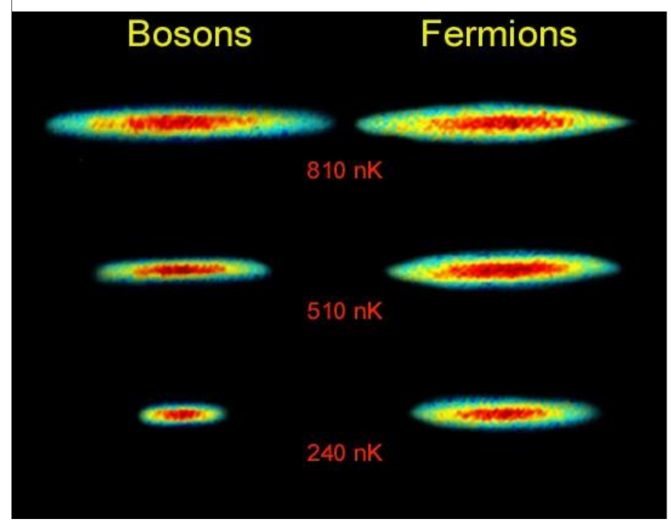


"Bose-Einsten Condensation" ***"Fermi Degeneracy"***

^{87}Rb



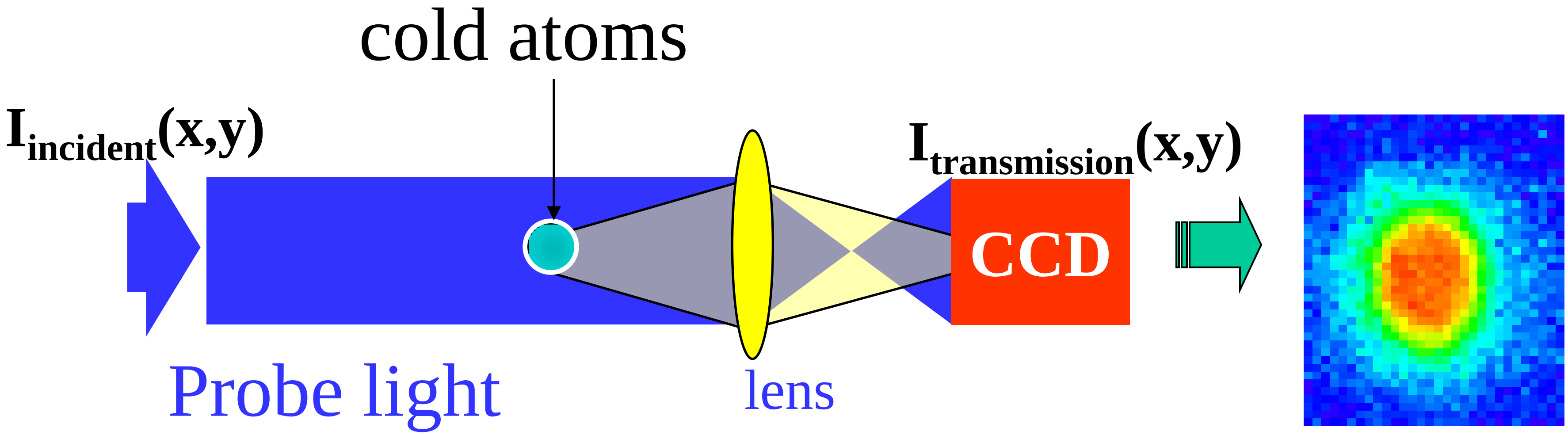
^{6}Li and ^{7}Li



Momentum Distribution
[E. Cornell et al, (1995)]

Spatial Distribution
[R. Hulet et al, (2000)]

Optical Absorption Imaging of Atoms



- *In-Situ* Image: $\longrightarrow$ Reflect "**density**" distribution in a trap

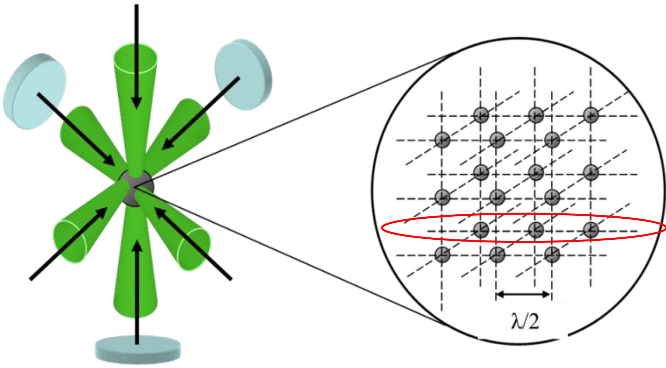
- Time-of-Flight Image: $\longrightarrow$ Reflect "**momentum**" distribution in a trap

  t=0 release atoms from a trap

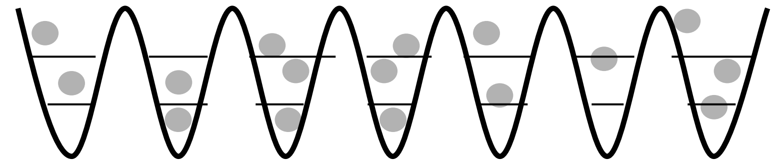
  $t=t_{TOF}$ observe atom density distribution

$$x = p / M \cdot t_{TOF}$$

Optical Lattice

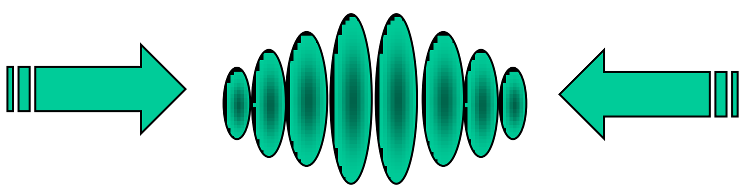


$$V_o(x) = V_o \sin^2(k_L x)$$

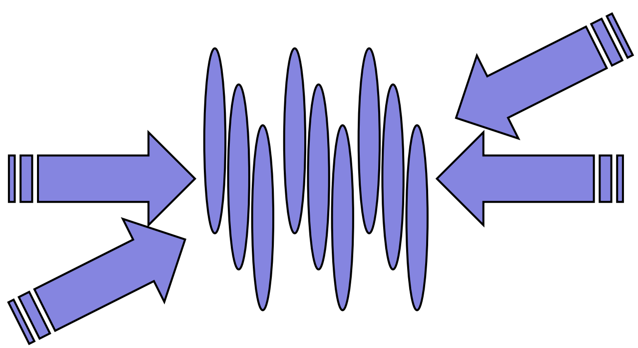


$$V_o(\mathbf{x}) = \sum_{j=1}^{3} V_{oj} \sin^2(k_L x_j) = V_o \sum_{j=1}^{3} \sin^2(k_L x_j)$$

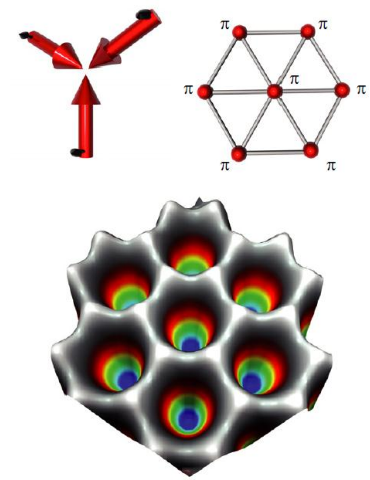
$$\boxed{E_R = \frac{(\hbar k_L)^2}{2m}, s = \frac{V_0}{E_R}}$$



2D gas
(pancake)



1D gas
(tube)

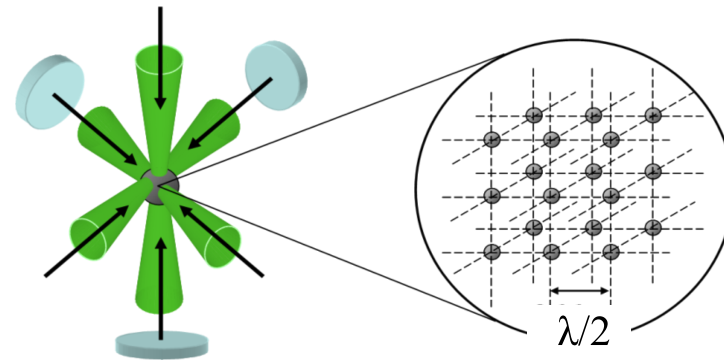


[C. Becker *et al.*,
New J. Phys. **12** 065025(2010)]

Quantum Simulation of Hubbard Model using "Cold Atoms in Optical Lattice"

[D. Jaksch *et al.*, PRL, **81** , 3108(1998)]

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



$$J = E_R (2/\sqrt{\pi}) s^{3/4} \exp(-2\sqrt{s}) \qquad U = E_R a_s k_L \sqrt{8/\pi}\, s^{3/4}$$

$s \equiv V_o / E_R$, $E_R \equiv (\hbar k_L)^2 / 2m$, a_s : scattering length

Controllable Parameters

hopping between lattice sites	: J	lattice potential	: V_0
On-site interaction	: U	Feshbach Resonance	: a_s
filling factor (e- or h-doping)	: n	atom density	: n

Various geometry

Feshbach Resonance: ability to tune an inter-atomic interaction

Collision is in Quantum Regime

It is described by s-wave scattering length a_s

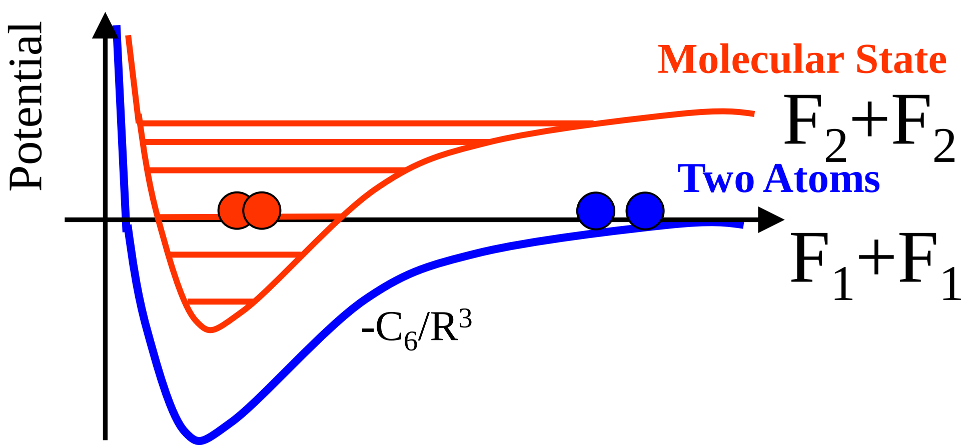
$$a_s = -\delta_l / k$$

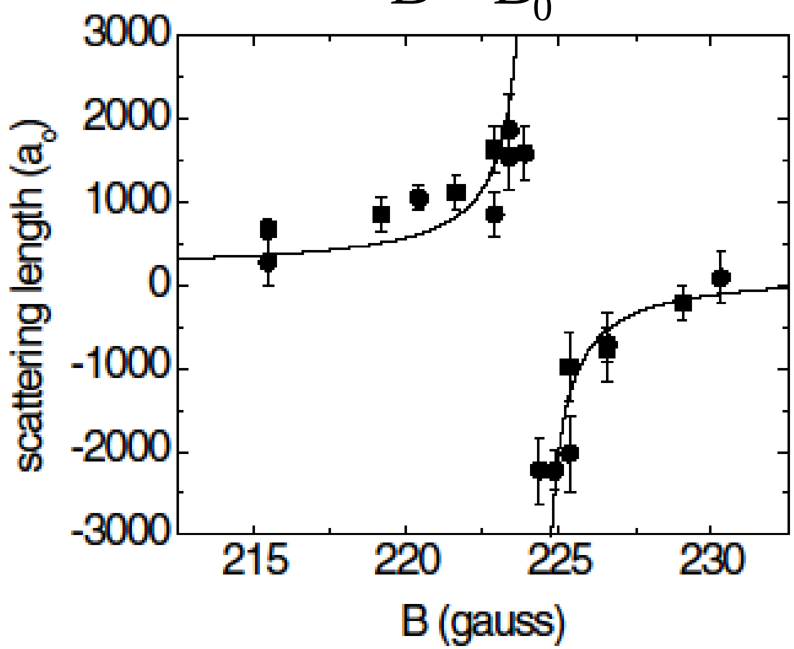
$$\sigma_0 = 4\pi |f_0|^2 = 4\pi |a_s|^2$$

Coupling between "**Open Channel**" and "**Closed Channel**"

→ Control of Interaction(a_s)

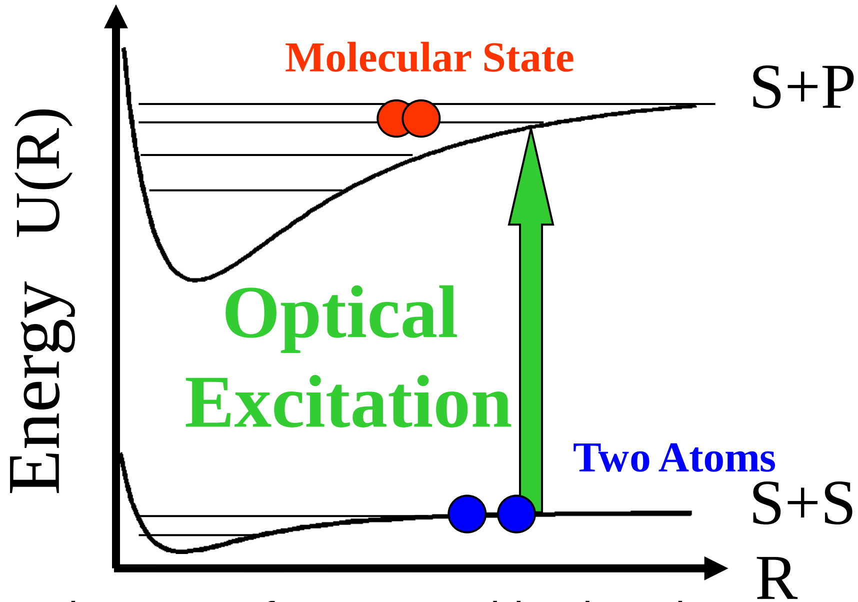
$$a_s(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$





[C. Regal and D. Jin, PRL**90**, 230404(2003)]

Optical Feshbach Resonance



Advantages for Intercombination Lines

R. Ciurylo, *et al.* *Phys. Rev. A* **70**. 062710 (2004)

$$S_{00} = \frac{\Delta - i\Gamma_S/2 + i\gamma/2}{\Delta + i\Gamma_S/2 + i\gamma/2}$$

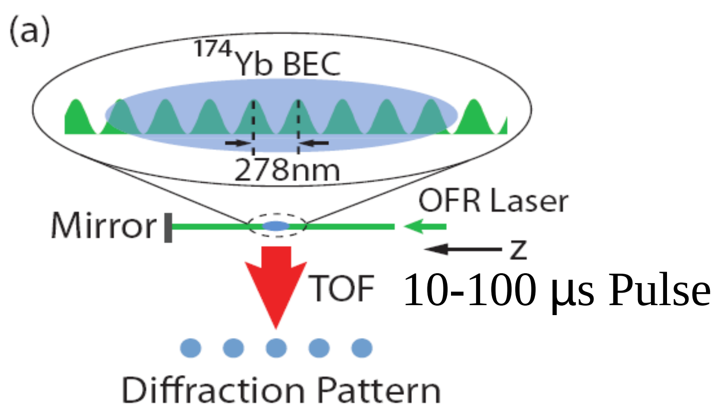
$\Gamma_S \propto \left|\langle b | V_{las} | f \rangle\right|^2$

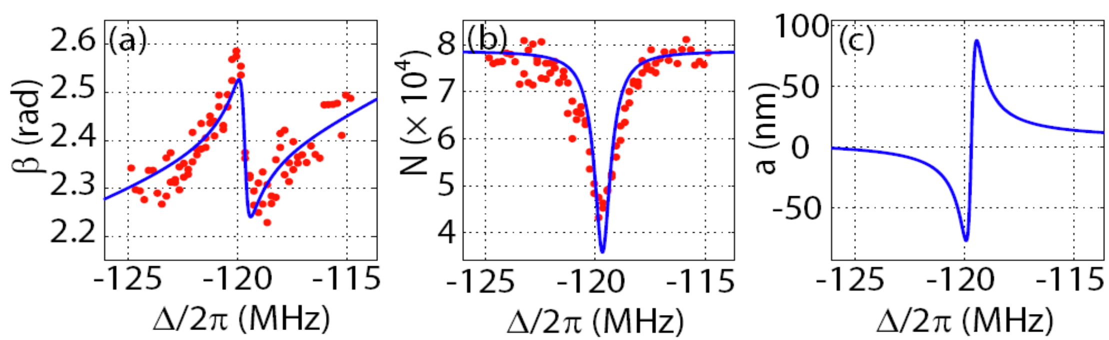
γ :spontaneous decay rate

Δ :detuning from the PA resonance

[J. Bohn and P. Julienne PRA(1999)]

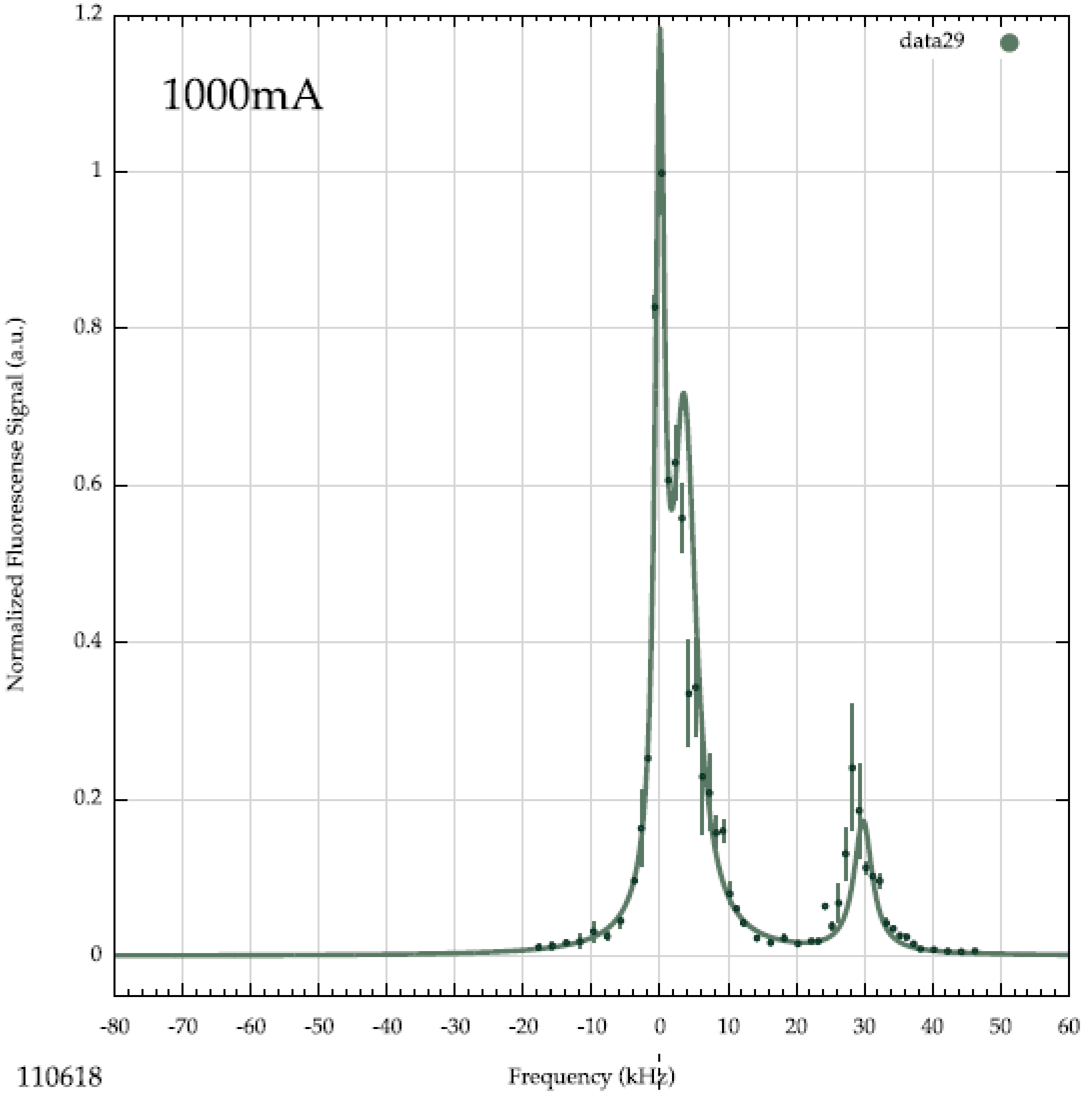
Nanometer-scale Spatial Modulation





[R. Yamazaki *et al* ., PRL**105**, 050405 (2010)]

Tuning of Scattering Length via *Non*-Feshbach Resonance



$^3P_2(m=+2)+^1S_0$

(B_0=200 mG ~ 1000 mG)

"Formation of bound state in a **Purely Long-Range Molecule**"

$$\Delta f \propto a_{eg} - a_{gg}$$

Ref.
A. Derevianko *et al.*, PRL**90**, 063002 (2003).
V. Kokoouline *et al.*, PRL**90**, 253201 (2003).

Analytical Expression of Scattering Length

$$V(R) \underset{R\to\infty}{\approx} -\frac{C_6}{R^6} \longrightarrow a_s = \bar{a}_s \times \left[1 - \tan(\phi - \frac{\pi}{8})\right]$$

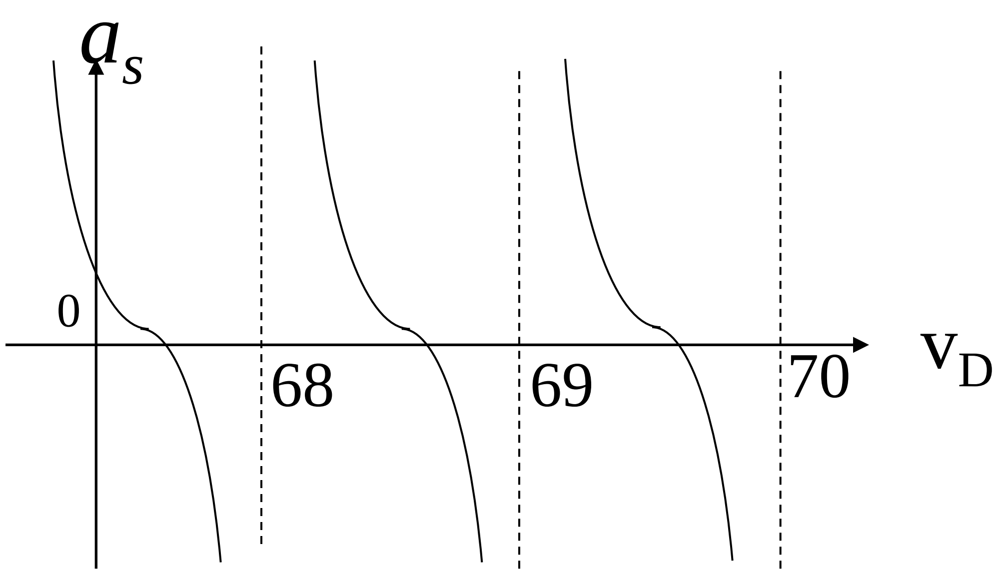
[Gribakin & Flambaum PRA, **48** 546(1993)]

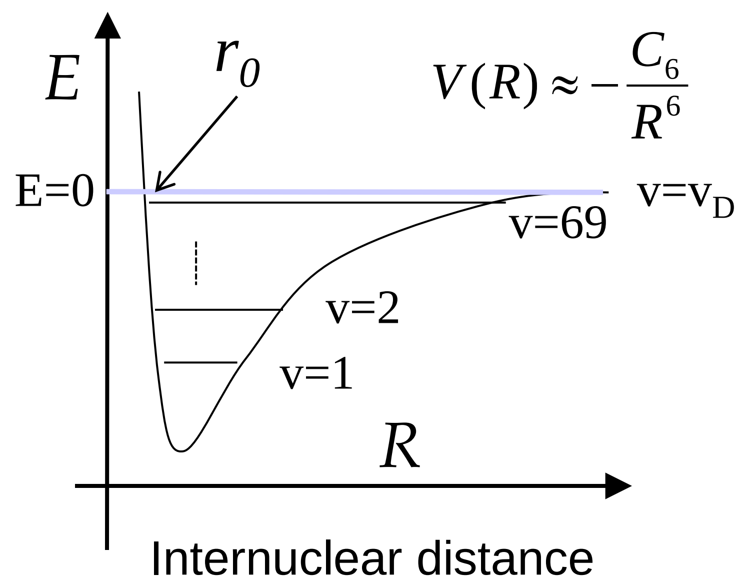
$$\bar{a}_s = \cos(\frac{\pi}{4})\left(\frac{\sqrt{2\mu C_6}}{4\hbar}\right)^{1/2}\left[\frac{\Gamma(3/4)}{\Gamma(5/4)}\right]$$

$$\phi = \frac{1}{\hbar}\int_{r_0}^{\infty}\sqrt{-2\mu V(R)}\,dR$$

Reduced mass

$$\phi - \frac{\pi}{8} = \pi(v_D + \frac{1}{2}) \longrightarrow a_s = \bar{a}_s \times \left[1 - \tan(\pi(v_D + \frac{1}{2}))\right]$$

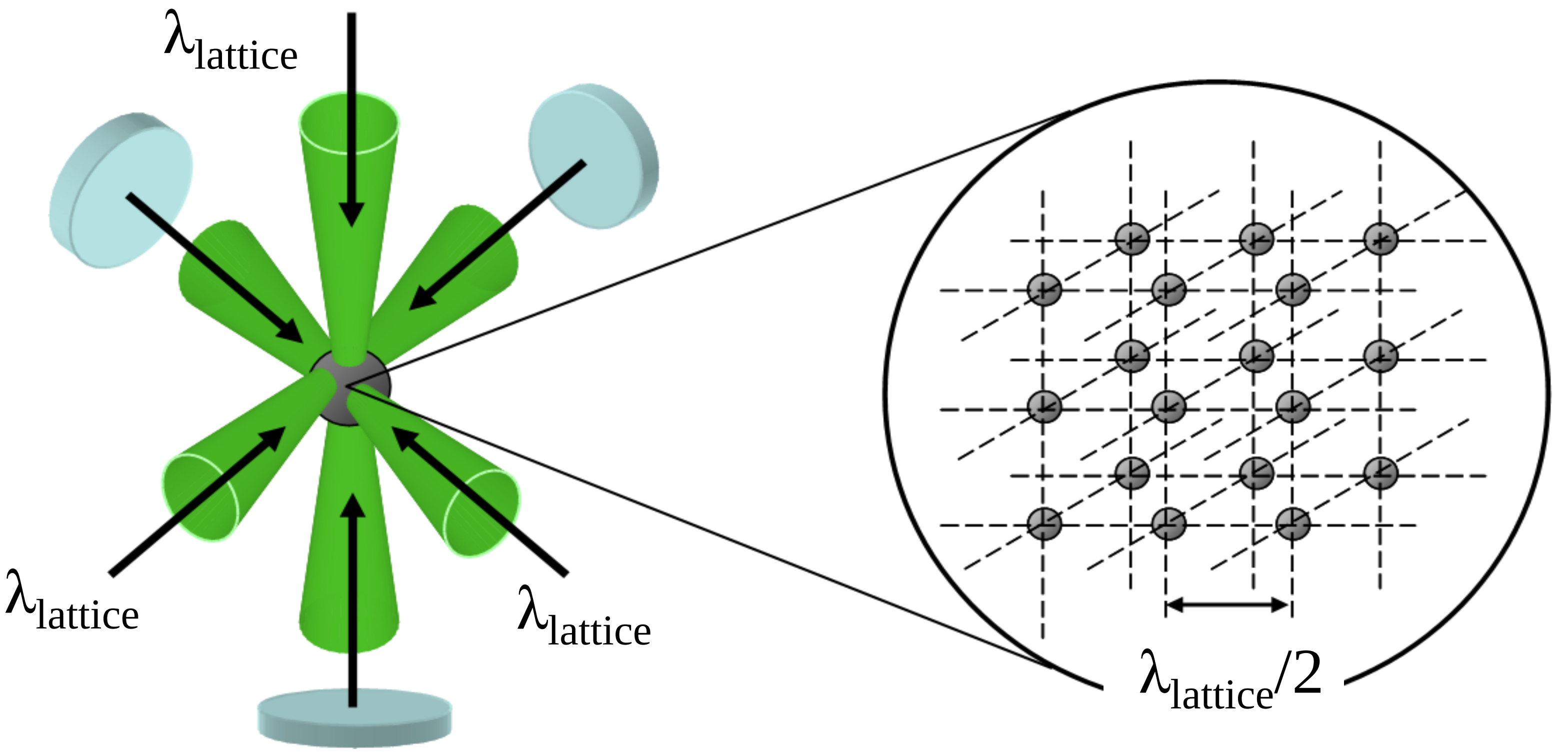




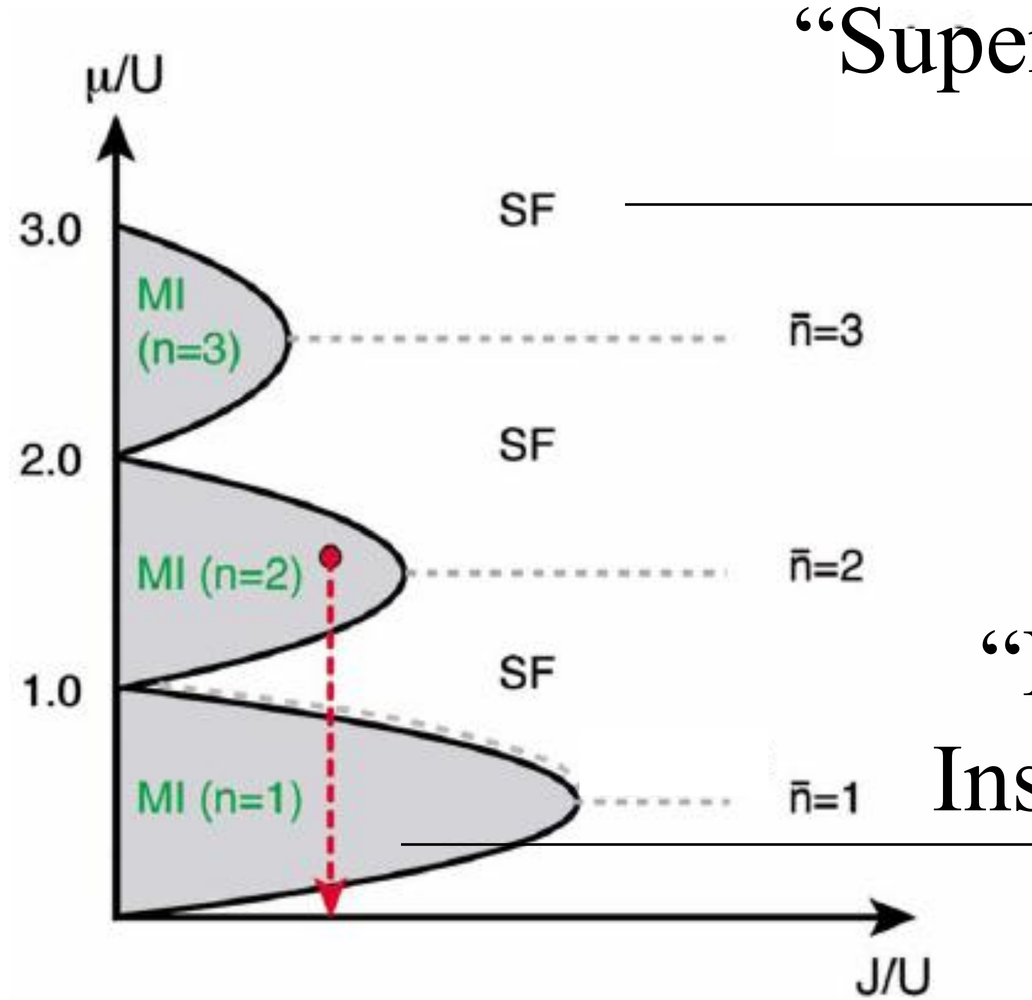
Bosons in a 3D optical lattice

$$H = -J\sum_{\langle i,j\rangle} a_i^{+} a_j + \frac{U}{2}\sum_i n_i(n_i - 1) + \sum_i \varepsilon_i n_i$$

"Bose-Hubbard Model"

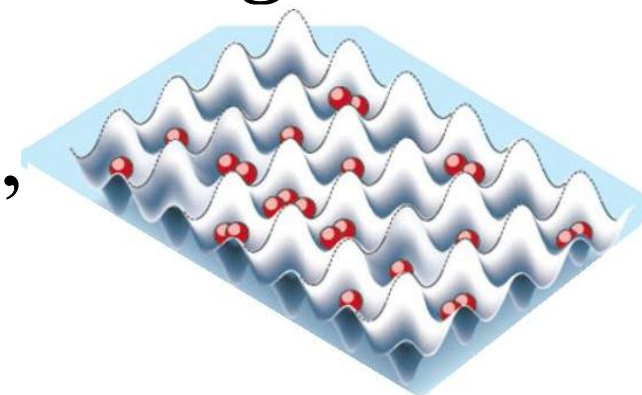


Phase Diagram of Repulsively Interacting Bosons



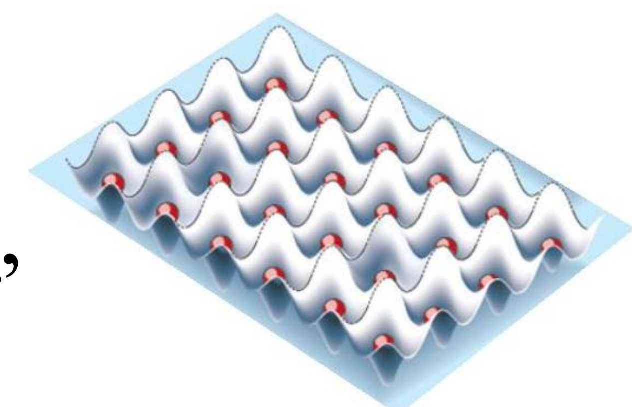
[RMP**80**,885(2008)]

"Superfluid"



$$\left|\Psi_{SF}\right\rangle \propto \left(\frac{1}{\sqrt{M}}\sum_{i=1}^{M} a_i^{+}\right)^{N} \left|0\right\rangle$$

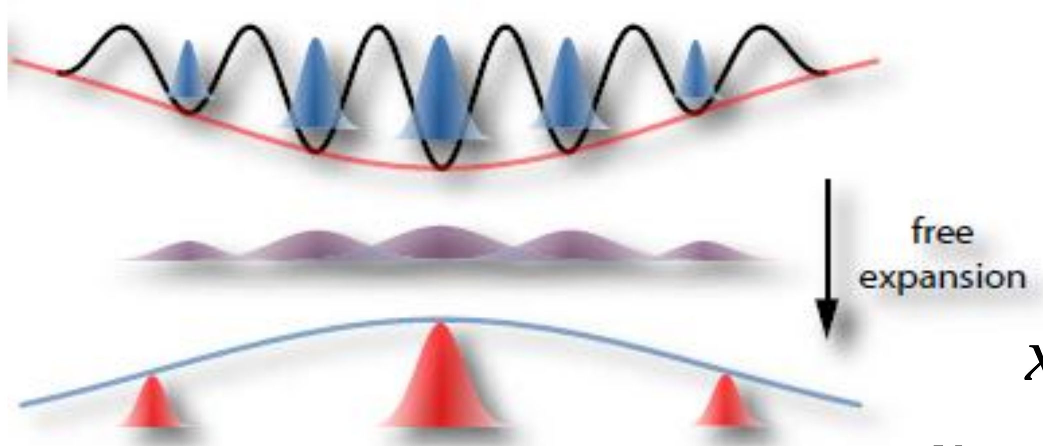
"Mott Insulator"



$$\left|\Psi_{MI}\right\rangle \propto \prod_{i=1}^{M}\left(a_i^{+}\right)^{n} \left|0\right\rangle$$

Interference Fringe : the direct signature of the phase coherence

"Sudden Release"



t_{TOF}

$x \leftrightarrow \hbar k$

$x = (\hbar k / M) t_{TOF}$

$$n(k) \propto \left|\tilde{w}(k)\right|^2 G(k)$$

Fourier Transform of the Wannier function

$$G(k) = \sum_{R,R'} \exp(ik \cdot (R - R')) \left\langle \hat{a}_R^+ \hat{a}_{R'} \right\rangle$$

no long-range order: $\left\langle \hat{a}_R^+ \hat{a}_{R'} \right\rangle = \delta_{R,R'} \rightarrow G(k) = N$

uniform long-range order: $\left\langle \hat{a}_R^+ \hat{a}_{R'} \right\rangle = 1 \rightarrow G(k) = \dfrac{\sin^2(kdN/2)}{\sin^2(kd/2)}$

peaks at $\pm 2n\hbar k_L$ (n=0,1,2...)

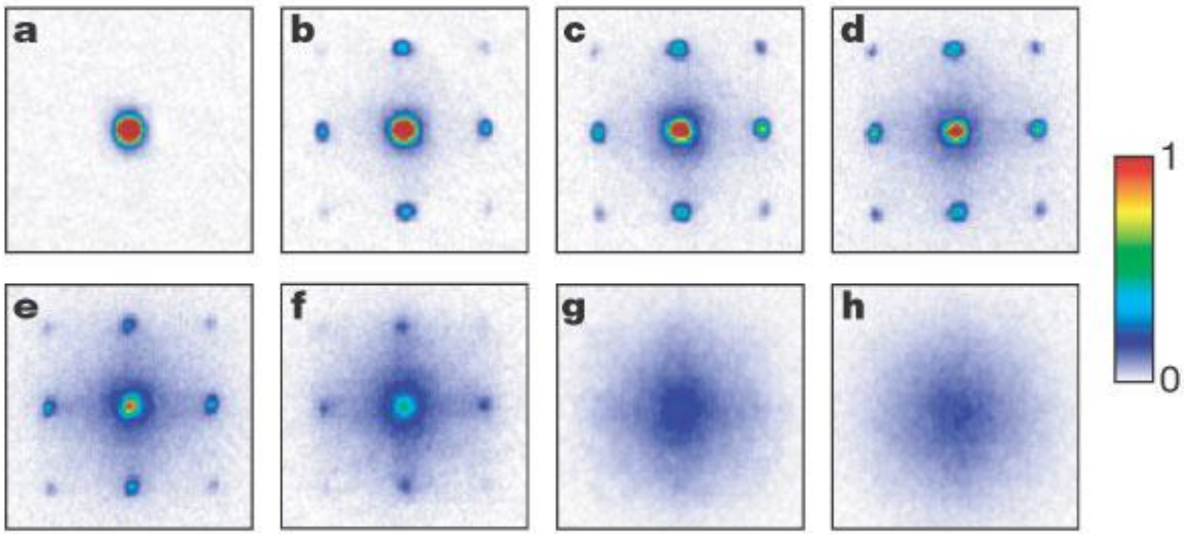
Bose-Hubbard Model:

"Superfluid - Mott-insulator Transition"

[M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415,39 (2002)]

No lattice $V_0 / E_R = 3$ 7 10

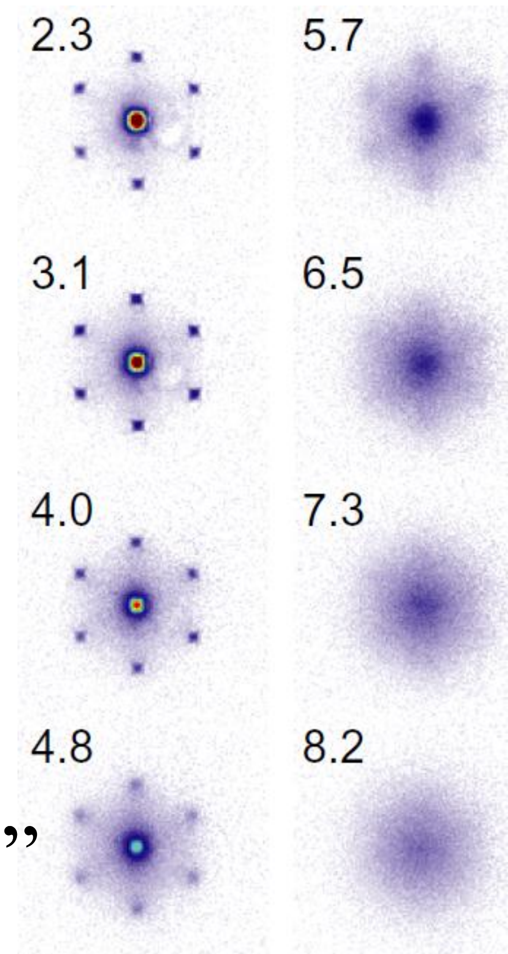
^{87}Rb



13 14 16 20

"cubic lattice"

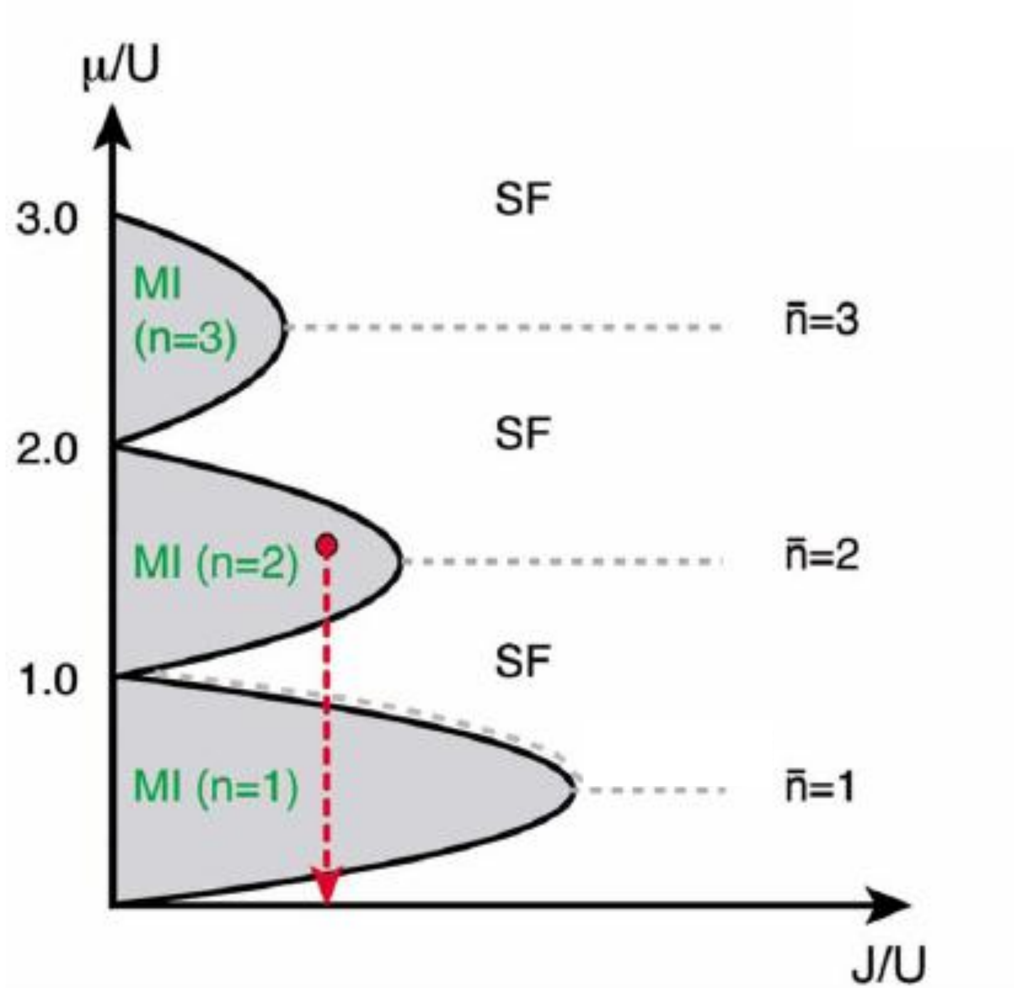
$s=$



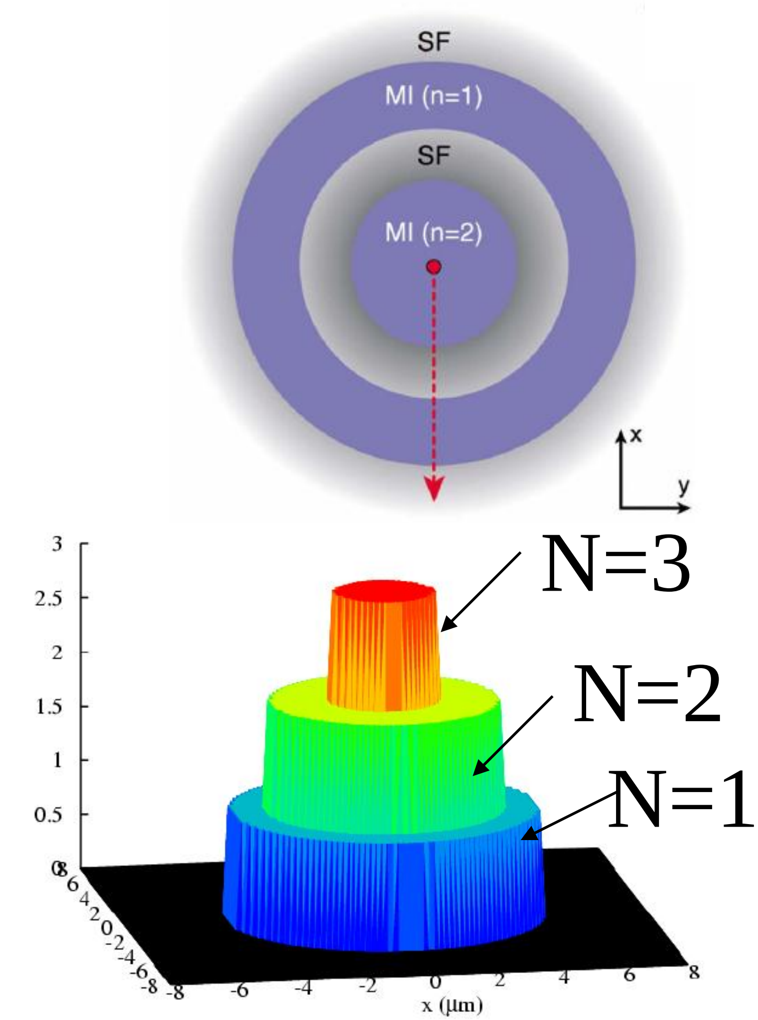
"triangular lattice"

[C. Becker *et al.*, New J. Phys. **12** 065025(2010)]

Phase Diagram of Repulsively Interacting Bosons



[RMP**80**,885(2008)]



Shell Structure of Mott States

High-Resolution RF Spectroscopy: Observation of Mott Shell Structure

[G. K. Campbell et al., Science 313, 649 (2006)]

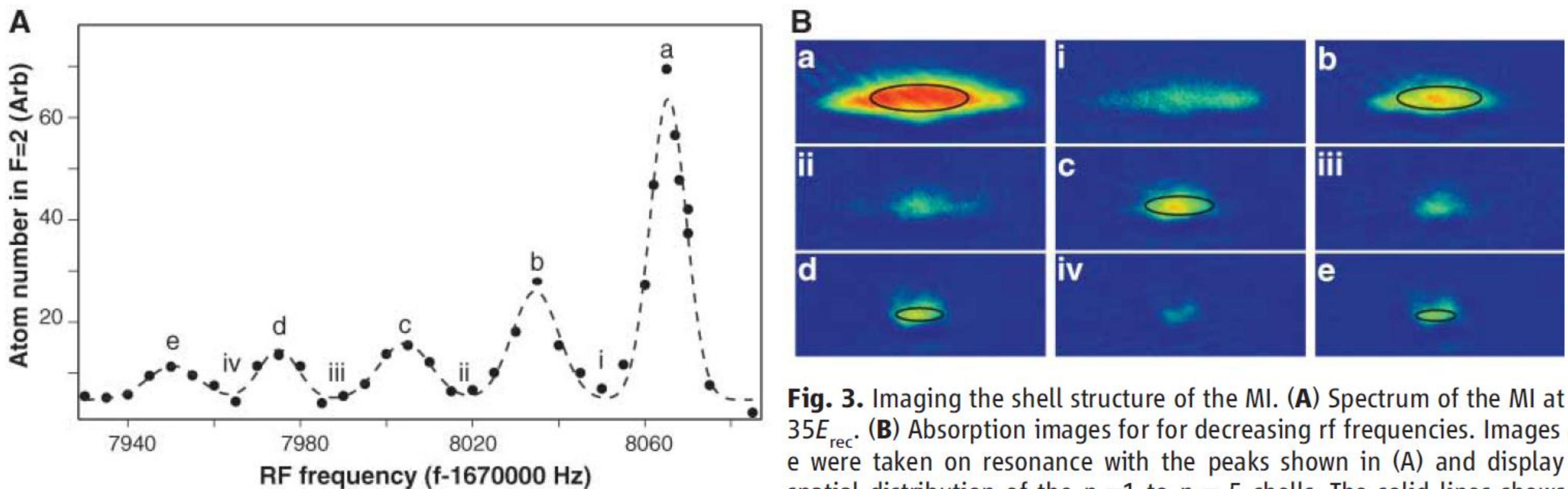
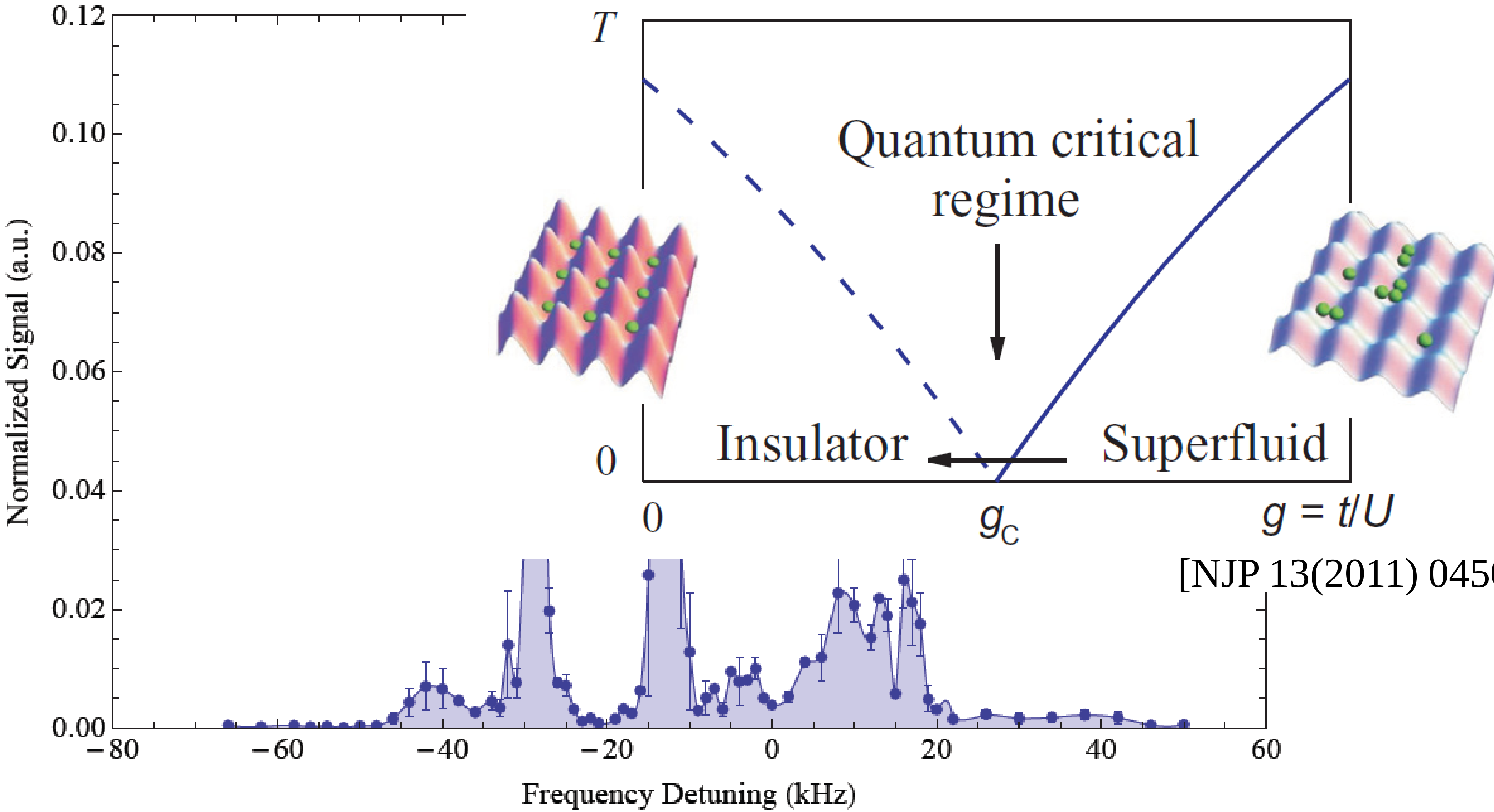


Fig. 3. Imaging the shell structure of the MI. (**A**) Spectrum of the MI at V = $35E_{rec}$. (**B**) Absorption images for for decreasing rf frequencies. Images a to e were taken on resonance with the peaks shown in (A) and display the spatial distribution of the n =1 to n = 5 shells. The solid lines shows the predicted contours of the shells. Absorption images taken for rf frequencies between the peaks (images i to iv) show a much smaller signal. The field of view was 185 μm by 80 μm.

$$h\nu_n = \frac{U}{a_{11}}(a_{12} - a_{11})(n-1)$$

Superfluid-Mott Insulator Transition

$0\ E_R \longrightarrow 15\ E_R$

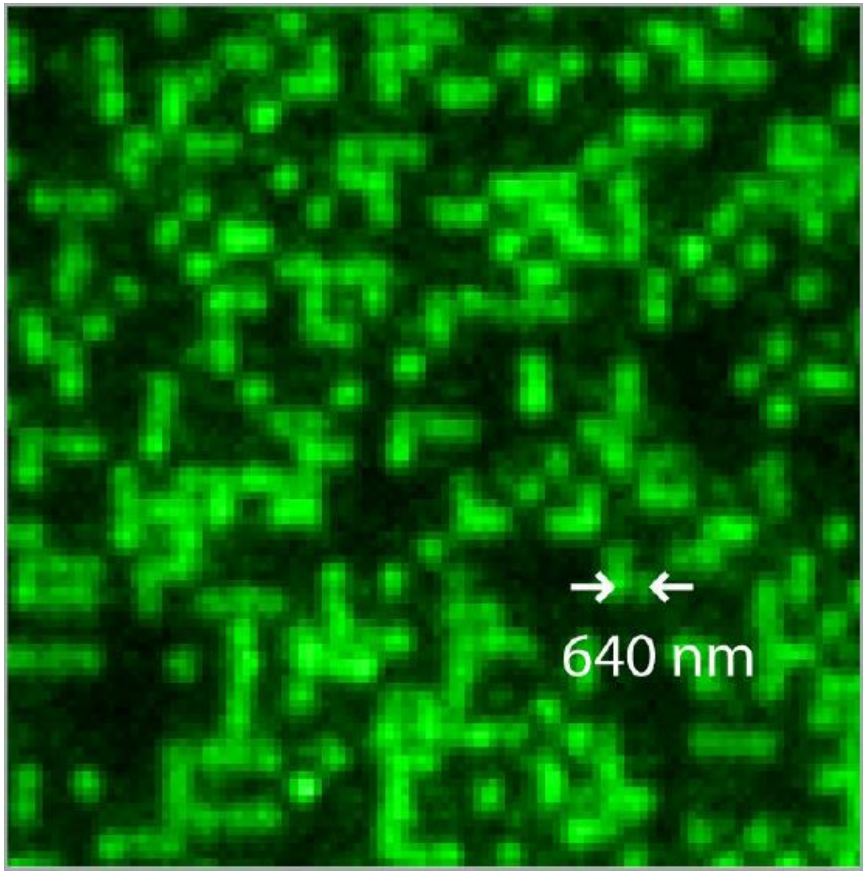


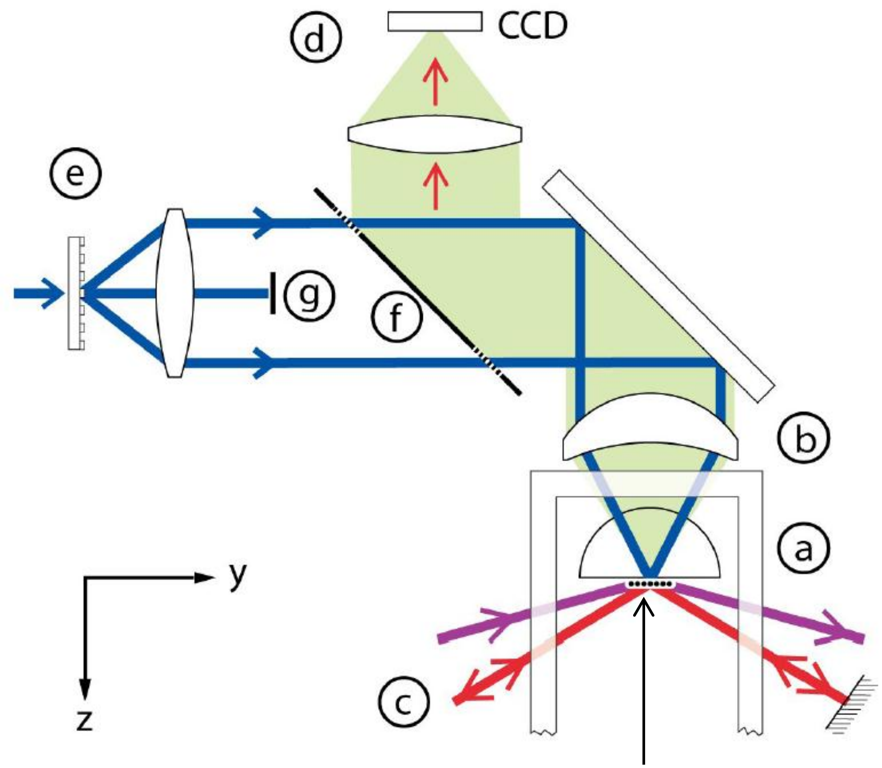
[NJP 13(2011) 045011]

New Technique: Single Site Observation

[WS. Bakr, I. Gillen, A. Peng, S. Folling, and M. Greiner, Nature 462(426), 74-77(2009)]

Fluorescence Imaging

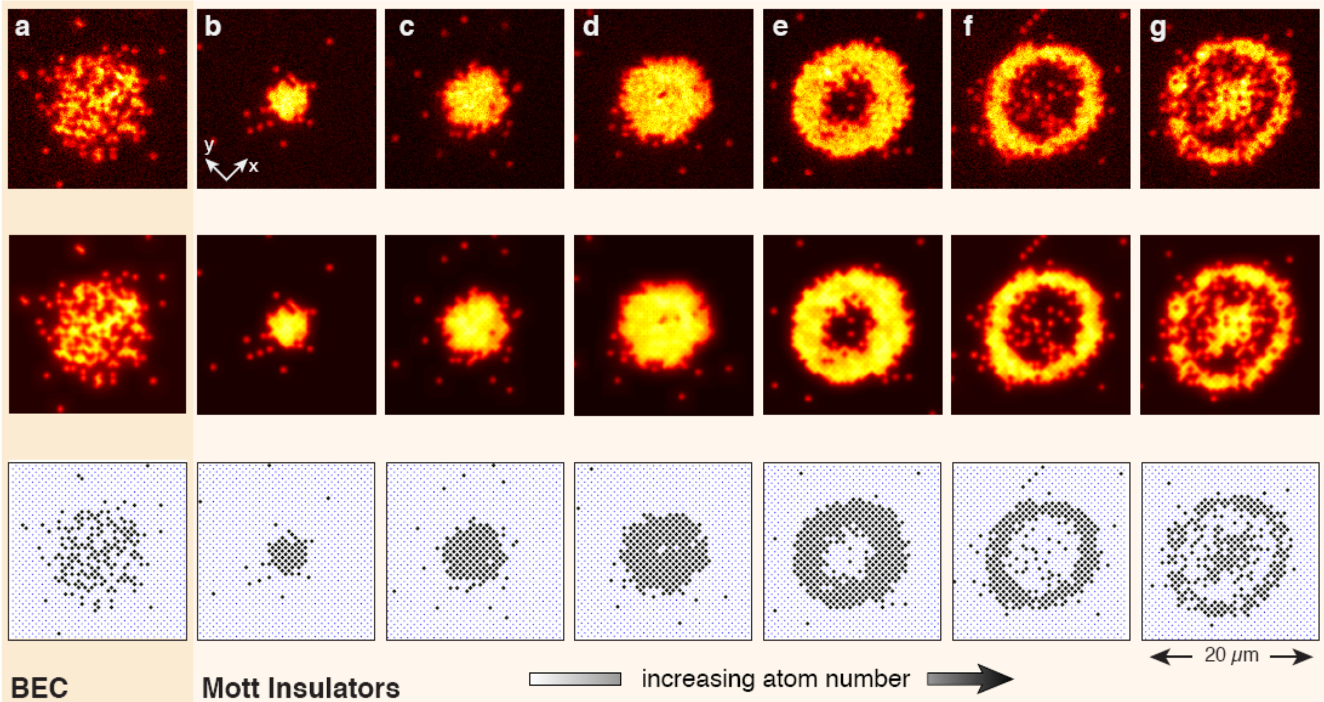


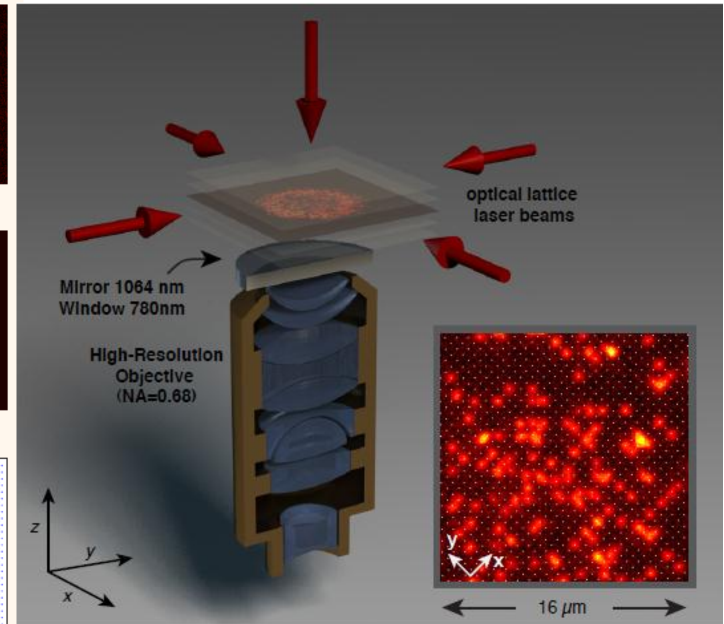


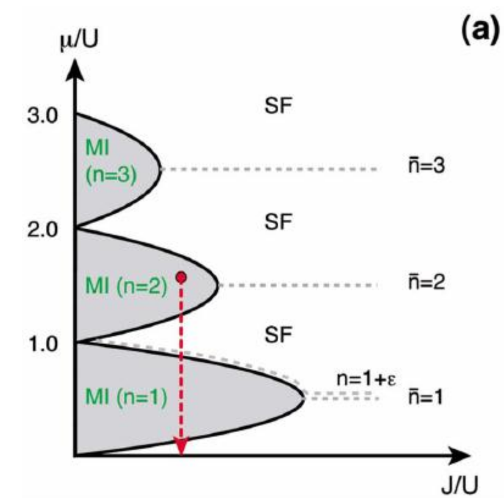
^{87}Rb

Single Site Resolved Detection of MI

[J. F. Sherson, et al., Nature 467, 68–72 (2010).]

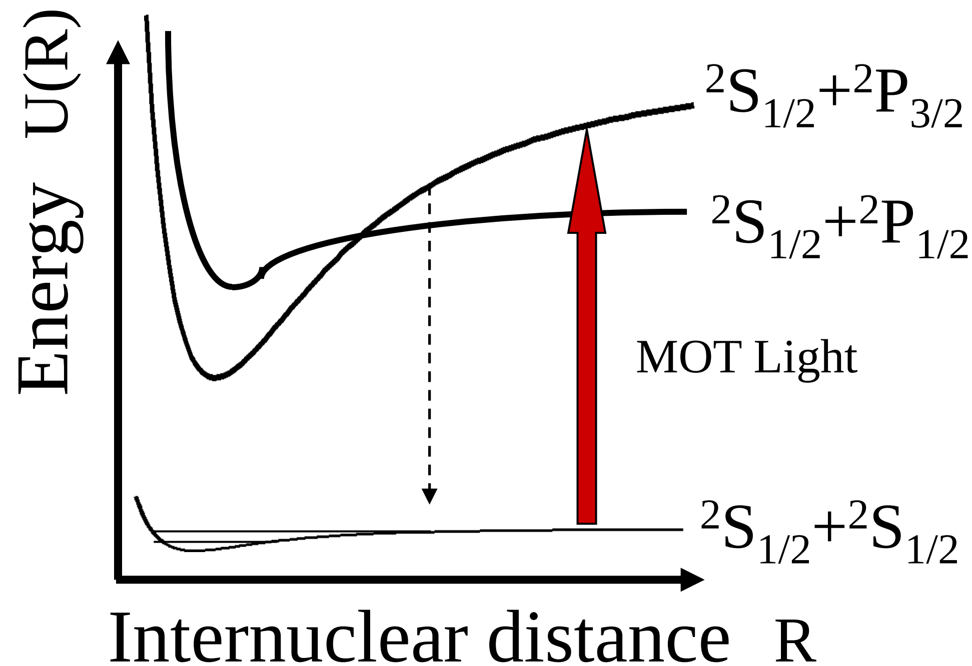






IG. 2: **High resolution fluorescence images of a BEC and Mott insulators.** Top row: Experimentally obtained images f a BEC (a) and Mott insulators for increasing particle numbers (b-g) in the zero-tunneling limit. Middle row: Numerically econstructed atom distribution on the lattice. The images were convolved with the point-spread function of our imaging ystem for comparison with the original images. Bottom row: Reconstructed atom number distribution. Each circle indicates single atom, the points mark the lattice sites.

Light-Assisted Collision



1) Fine-structure changing collision

$$^2S_{1/2}+^2P_{3/2} \longrightarrow \; ^2S_{1/2}+^2P_{1/2} + \text{K.E.}$$

2) Radiative Escape

$$^2S_{1/2}+^2P_{3/2} \longrightarrow \; ^2S_{1/2}+^2S_{1/2} + \text{K.E.}$$

Single Site Resolved Detection of MI

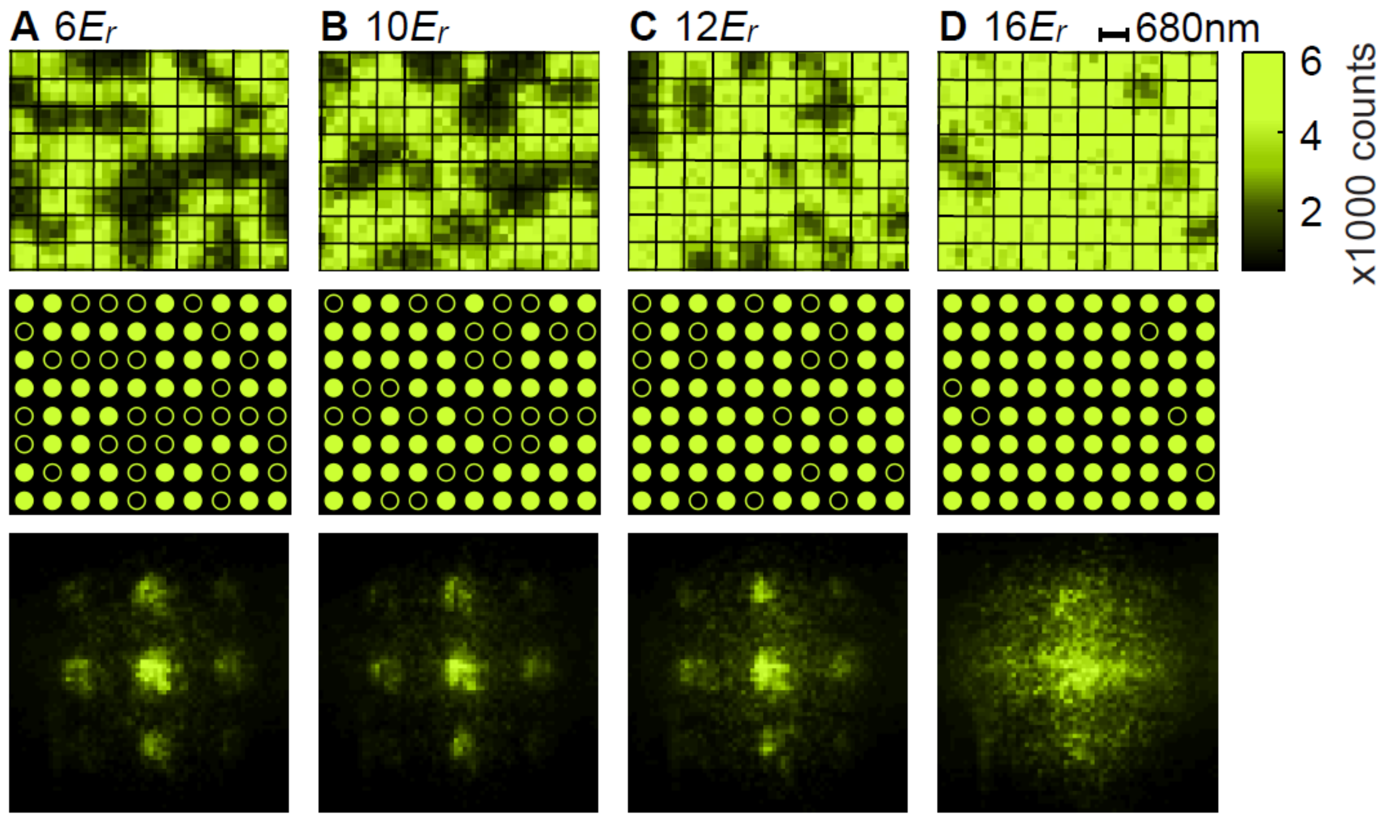
[WS Bakr, et al., Science 329, 547–550 (2010)]

SF ⟷ MI

In Situ-image

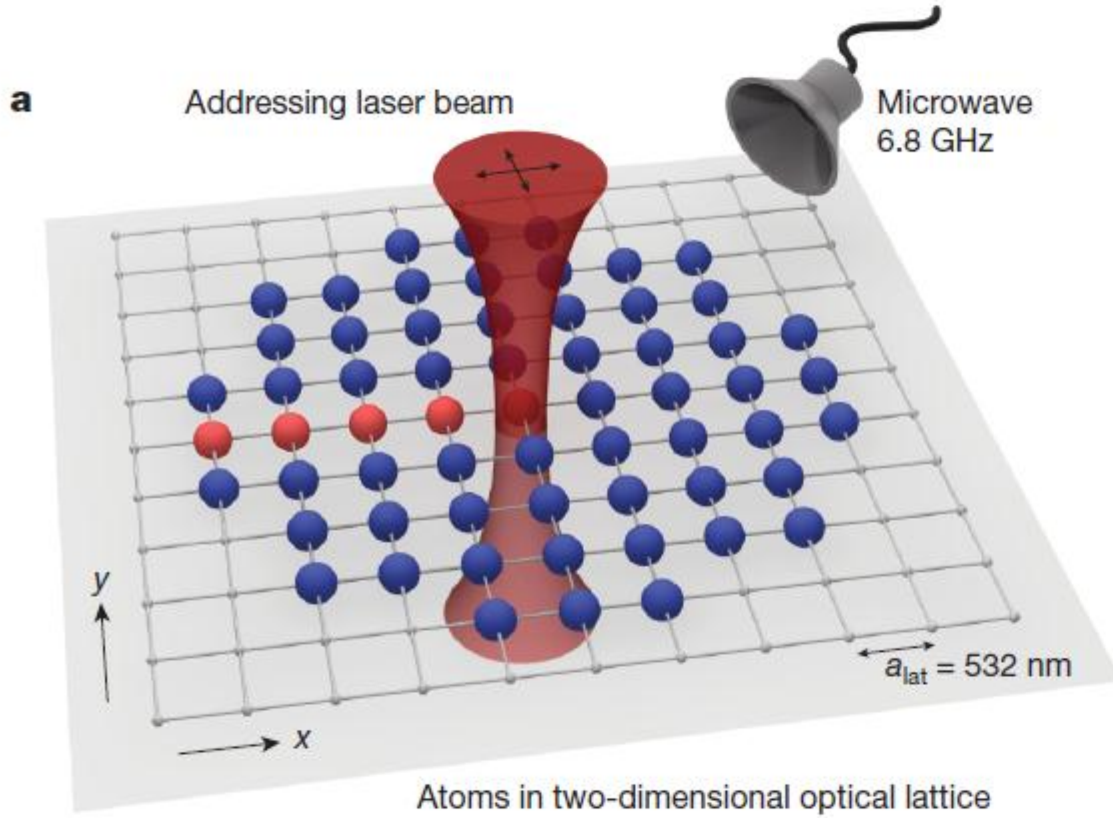
after analysis

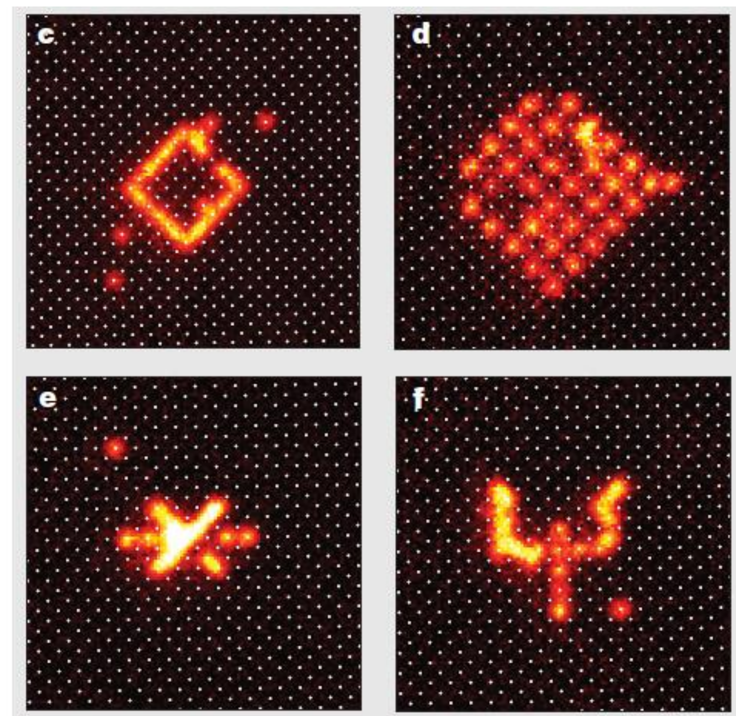
TOF-image



New Technique: Single Site Manipulation

[C. Ewitenberg *et al*, Nature 471, 319(2011)]



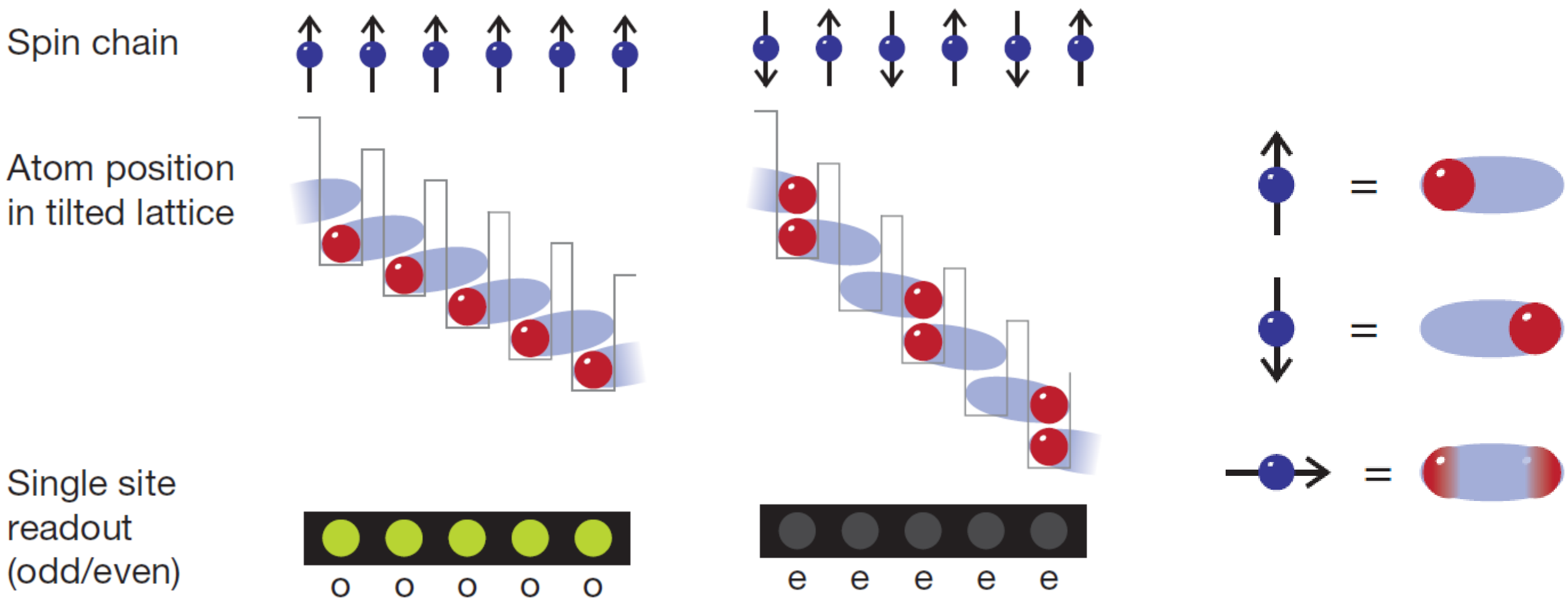


“quantum magnetism” in a 1D tilted lattice

[J. Simon, *et al.*, Nature, **472,** 307(2011)]

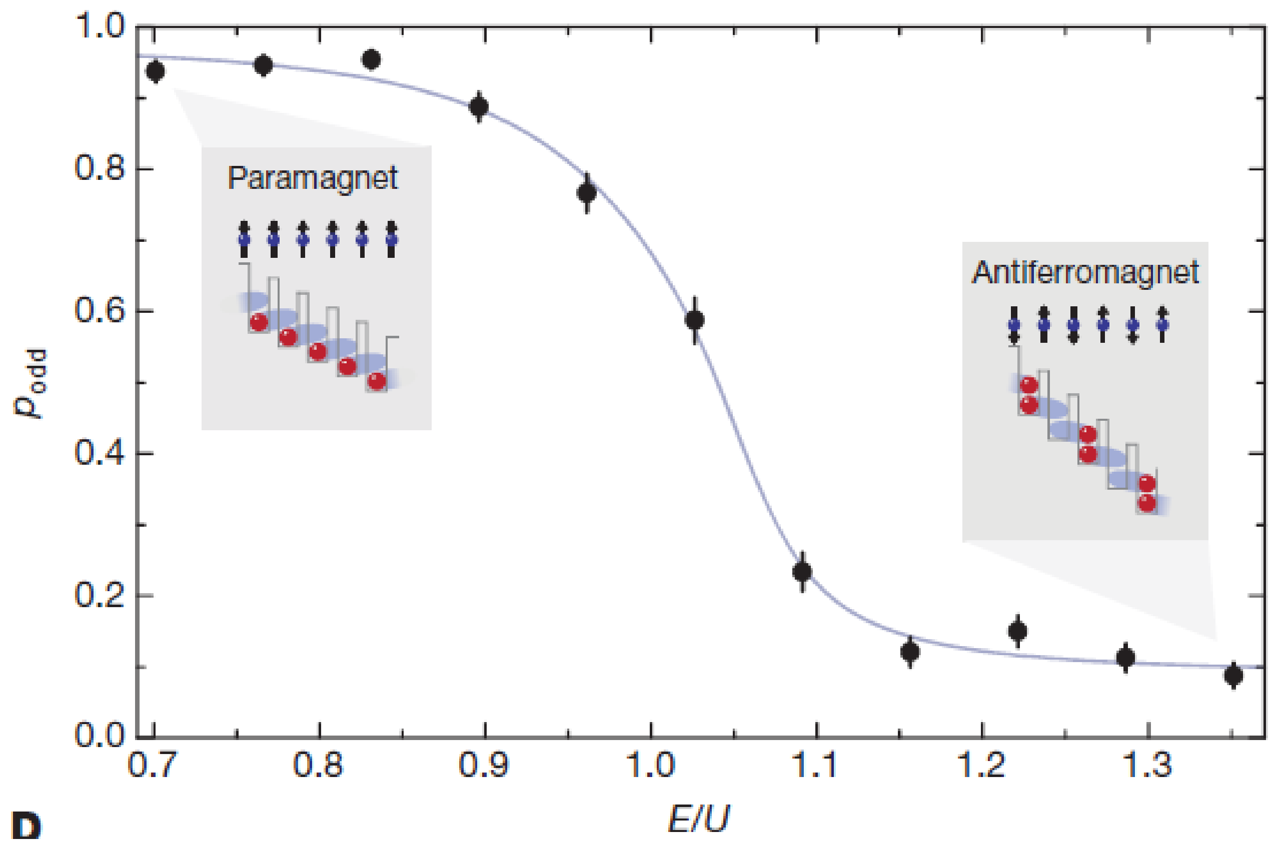
$$\boxed{H = J\sum_i S_z^i S_z^{i+1} - h_z^i S_z^i - h_x^i S_x^i}$$

$$(h_z, h_x) = \left(1-\tilde{\Delta}, 2^{3/2}\tilde{t}\right) \qquad \tilde{\Delta} = \Delta/J = (E-U)/J \qquad \tilde{t} = t/J$$



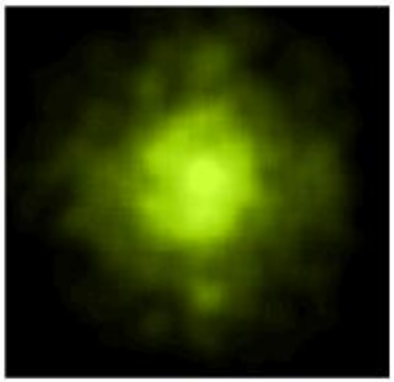
“quantum magnetism” in a 1D tilted lattice

[J. Simon, *et al.*, Nature, **472,** 307(2011)]

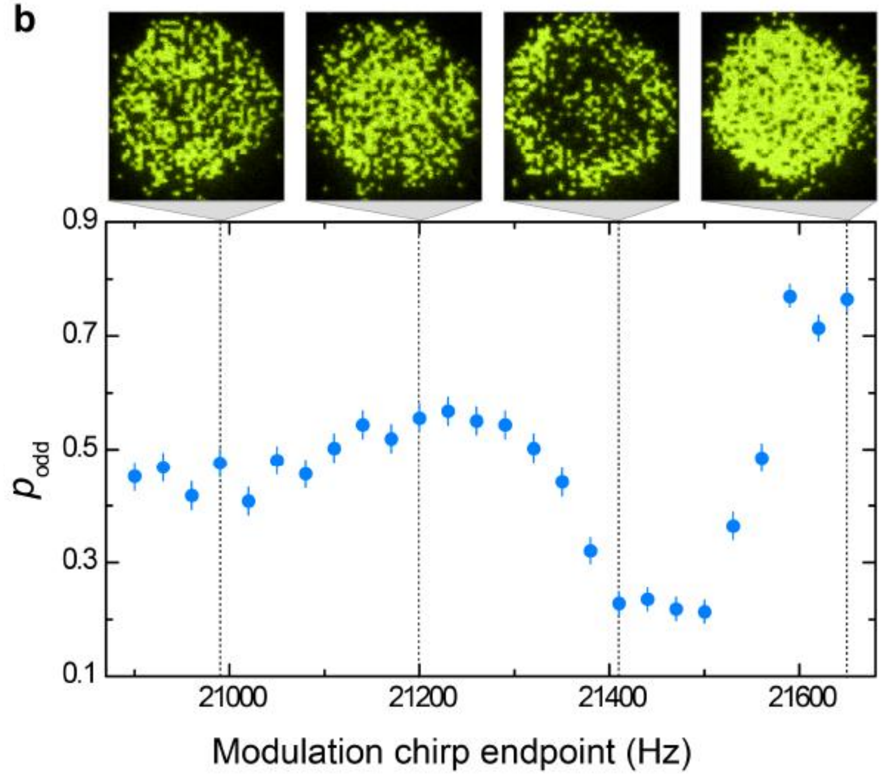


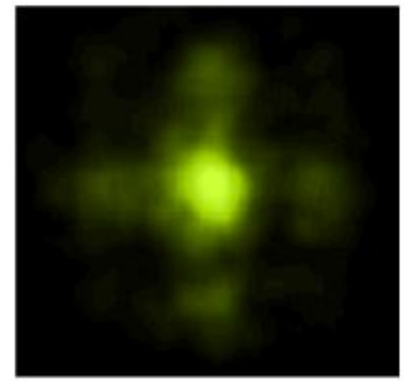
Manipulation of Mott Shell / Filter Cooling (Maxwell Demon)

[arXiv:1105.5834v1, W. S. Bakr, *et al.*,]

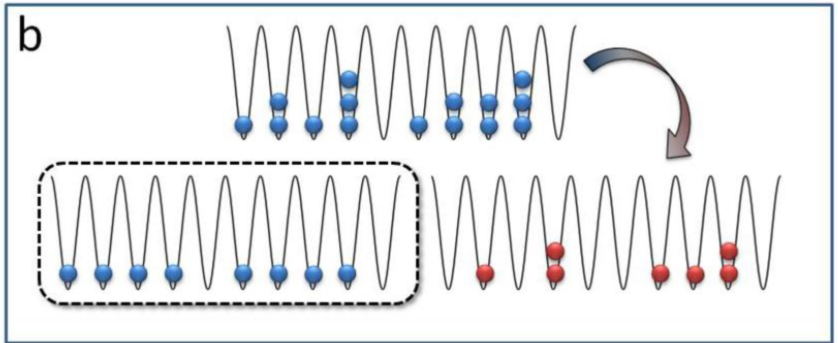


Dephased cloud





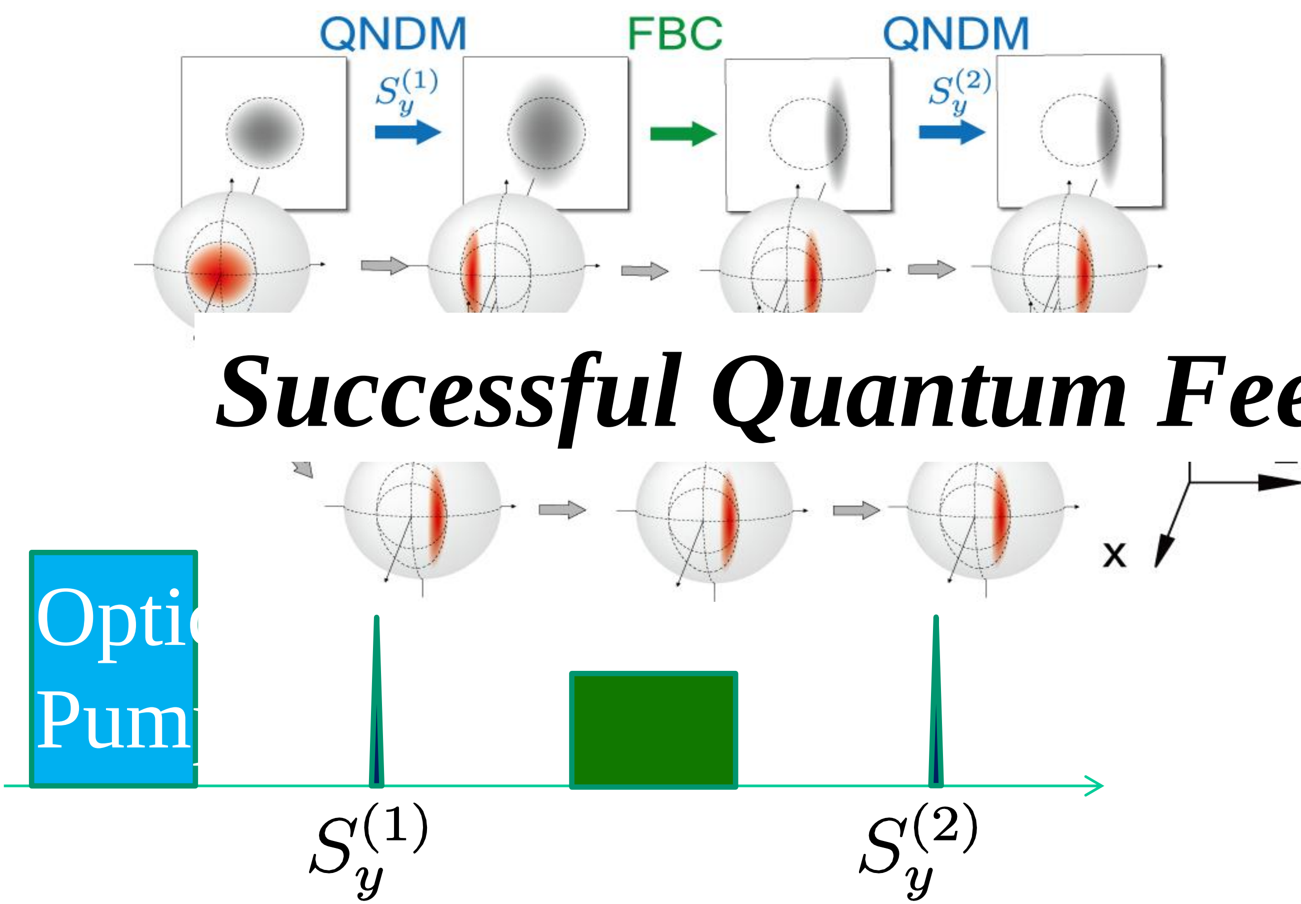
Recooled superfluid



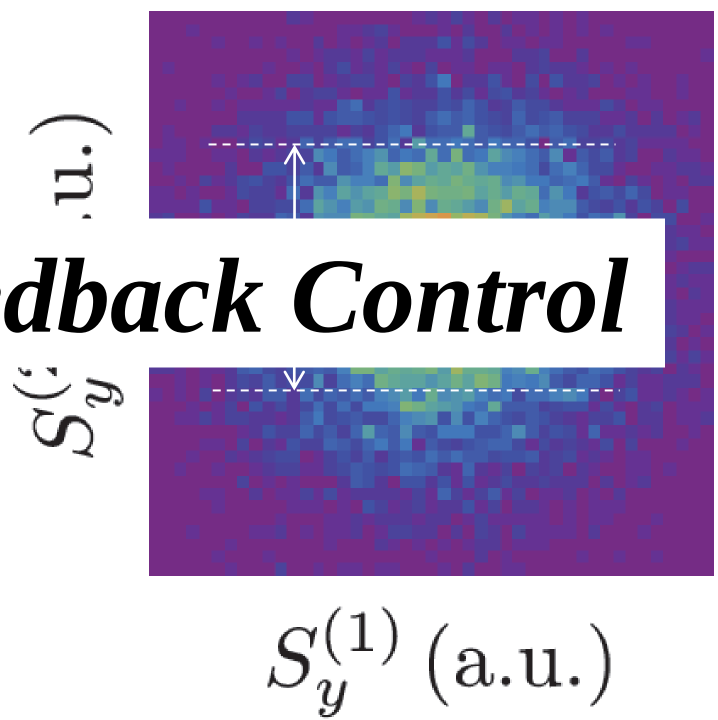
[D. C. McKay and B. DeMarco, Rep. Prog. Phys. 74, 054401 (2011).]

Implementing Quantum Feedback Control (quantum Maxwell Demon)

[R. Inoue *et al.*, in preparation]



Joint probability distribution



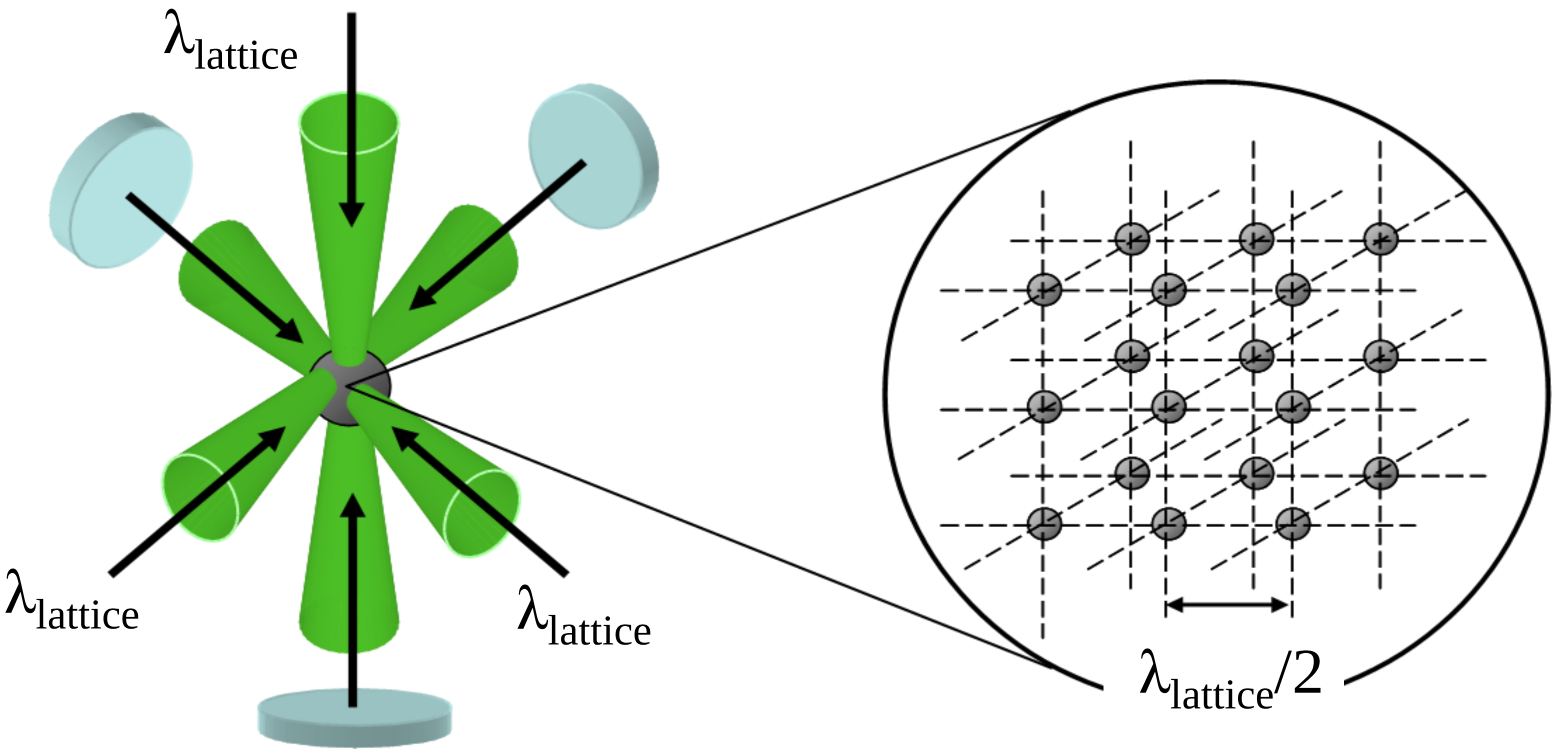
Successful Quantum Feedback Control

$(\Delta y)^2 \longrightarrow$ **-1.4dB Reduction**

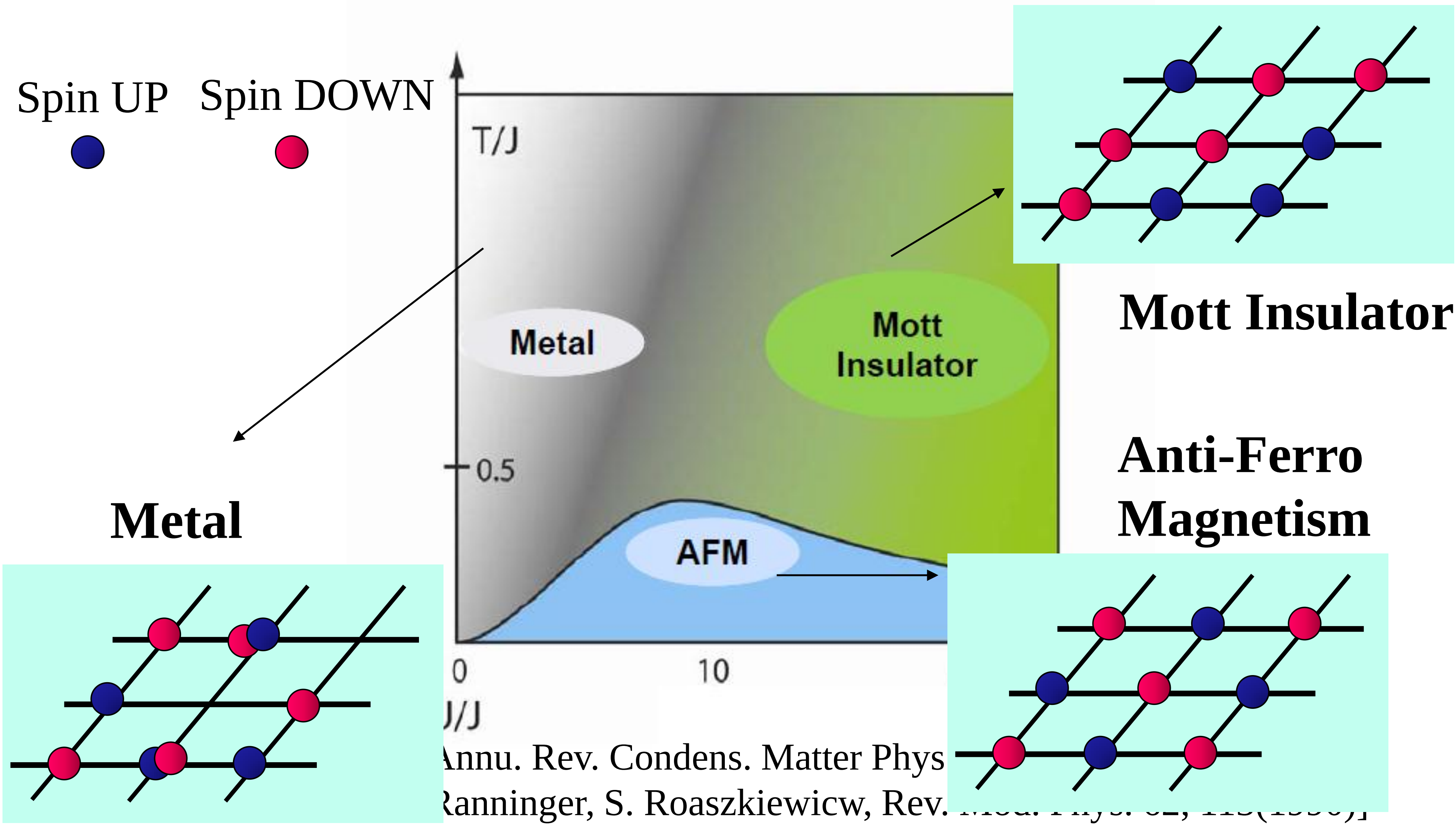
Fermions in a 3D optical lattice

$$H = -J \sum_{<i,j>} c_i^+ c_j + U \sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_i \varepsilon_i n_i$$

"Fermi-Hubbard Model"

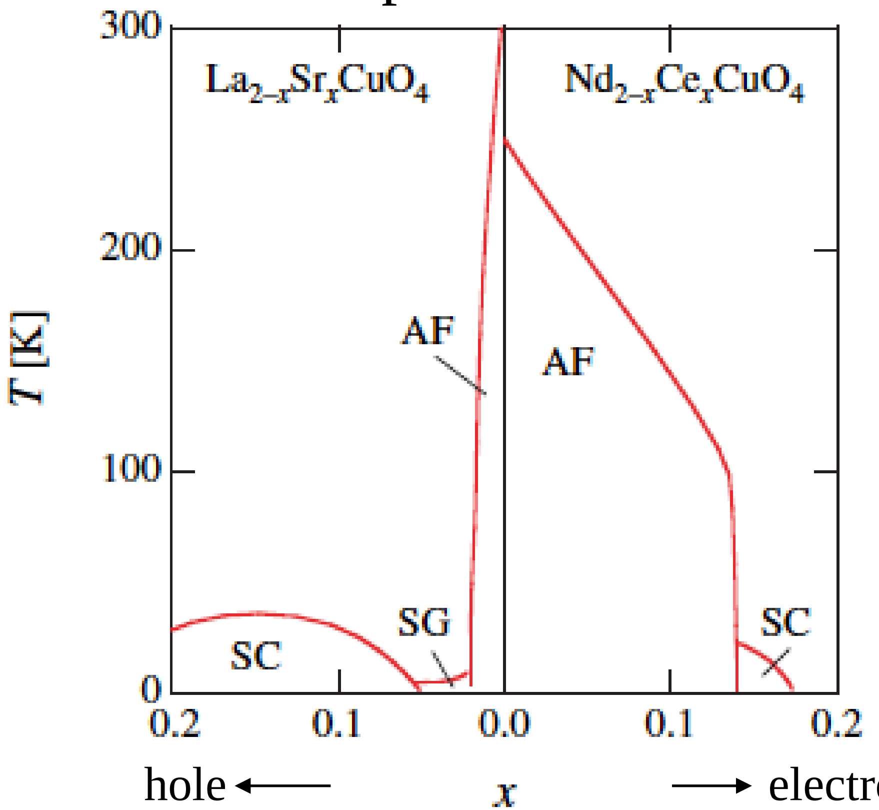


Phase Diagram of Repulsive Fermi-Hubbard Model



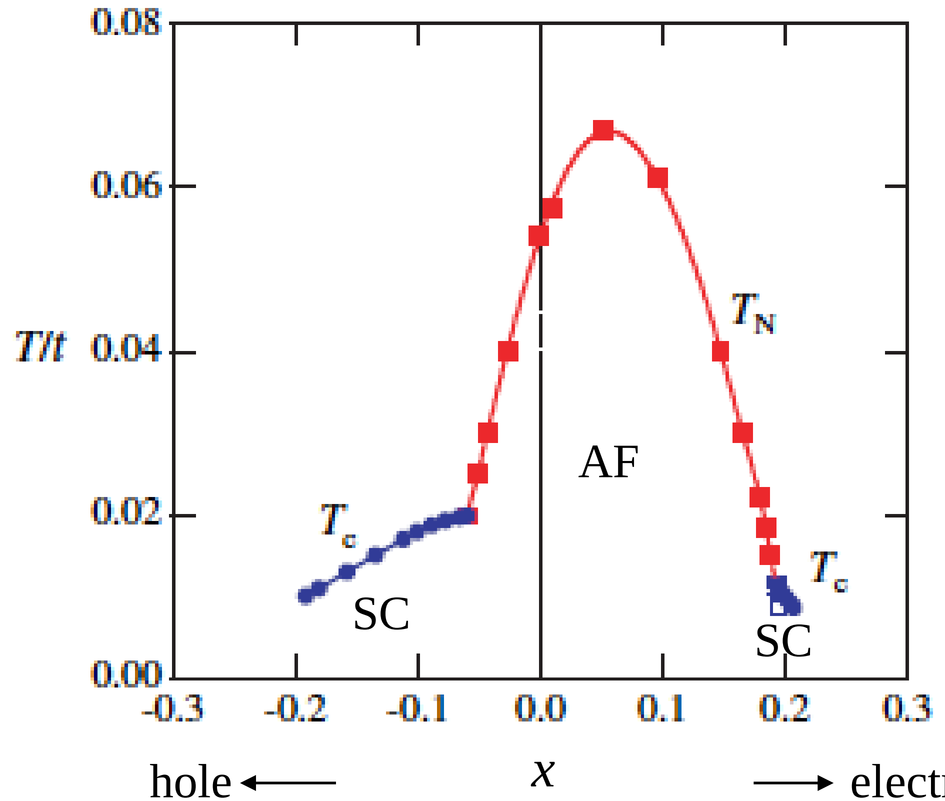
Phase Diagram of High-T_c Cuprate Superconductor

experiment



hole ⟵ ⟶ electron
(carrier doping)

theory



hole ⟵ ⟶ electron
(carrier doping)

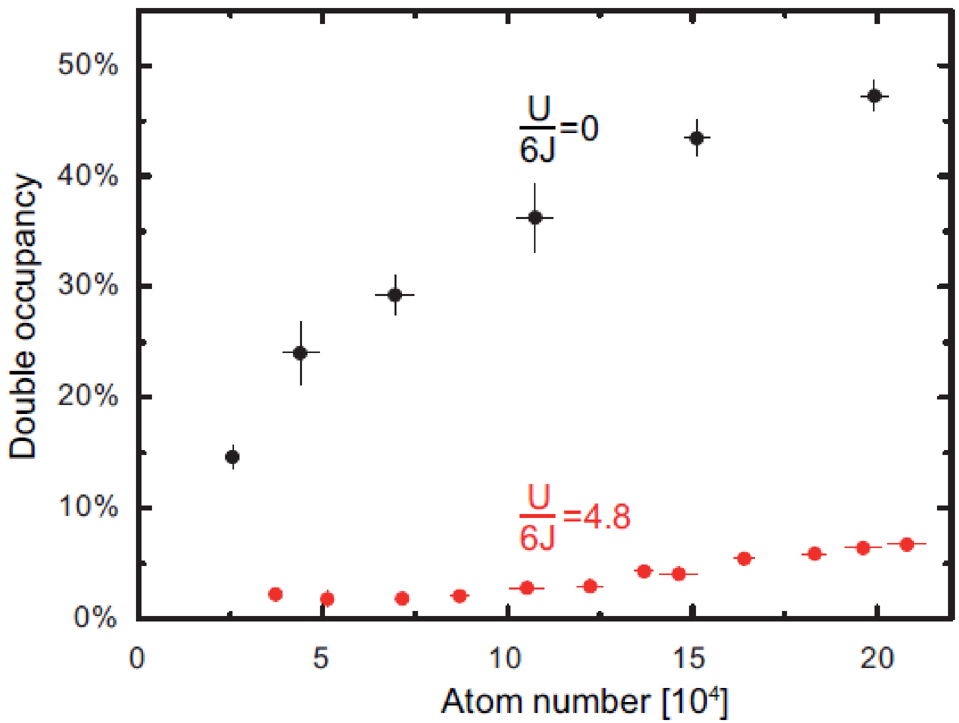
[in T. Moriya and K. Ueda, Rep. Prog.Phys.66(2003)1299]

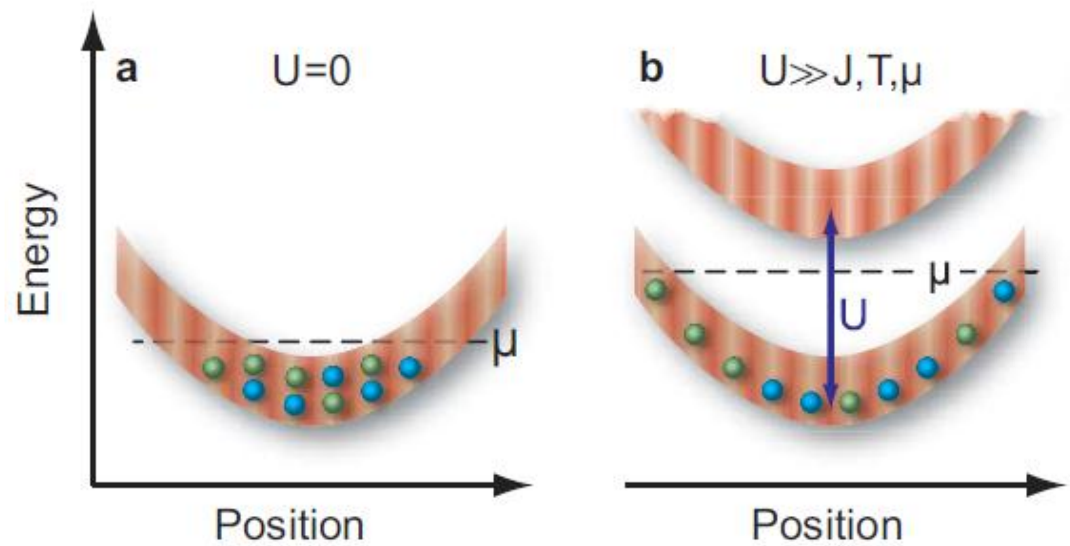
There is controversy in the under-dope region

Current Status of
Quantum Simulation of Fermi Hubbard Model:
"Formation of (paramagnetic) Mott insulator"

"A Mott insulator of ^{40}K atoms (2-component) "

[R. Jördens *et al.*, Nature **455**, 204 (2008)] [U. Schneider, *et al.*, Science **322**,1520(2008)]

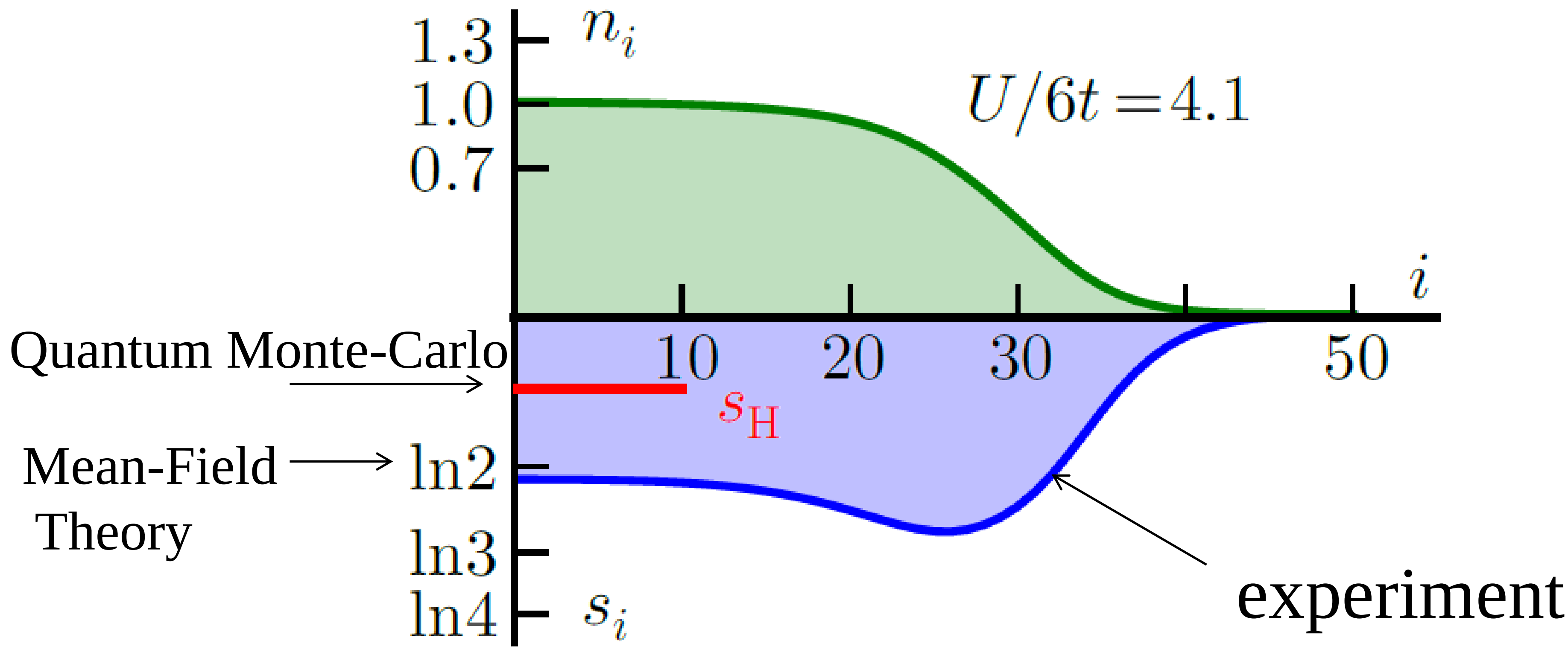




Current Status of
Quantum Simulation of Fermi Hubbard Model:
"Formation of (paramagnetic) Mott insulator"

[R. Jördens *et al.*, PRL **104**, 180401 (2010)]

^{40}K atoms (2-component)



Current Status of
Quantum Simulation of Fermi Hubbard Model:
"Formation of (paramagnetic) Mott insulator"

[S. Taie *et al*,]

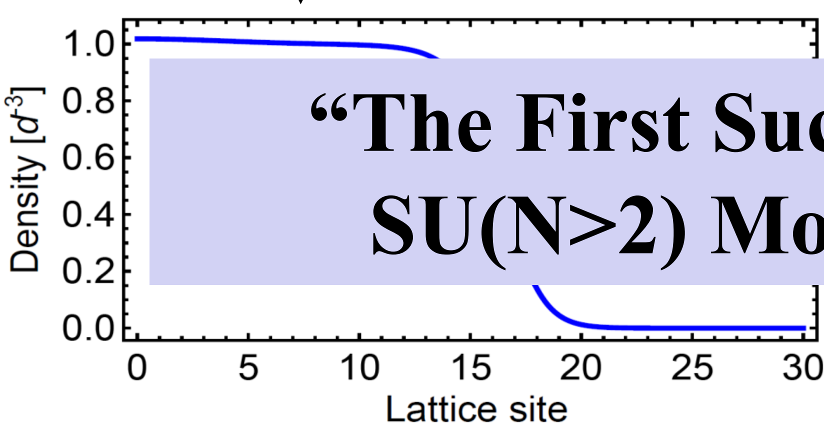
^{173}Yb atom (6-component)

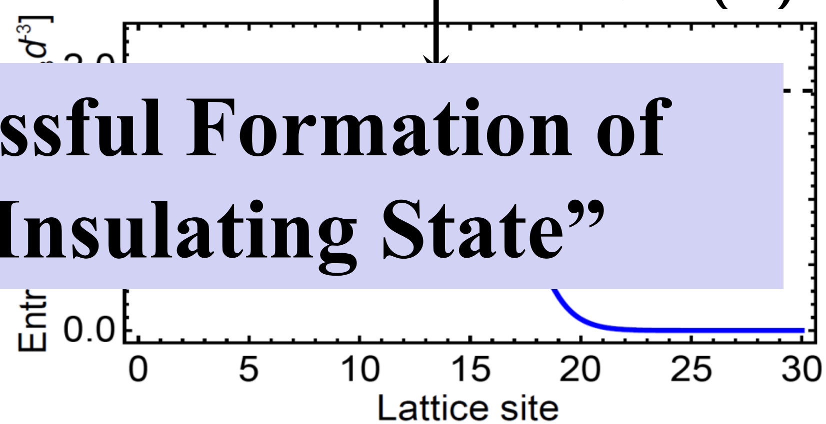
$T_{lattice}$=5.1t= 16 nK U/t=62.4

Mott Plateau (n=1)

Minimum: s = 1.81

cf. ln(6) = 1.79





"The First Successful Formation of SU(N>2) Mott Insulating State"

SU(6) Fermion (^{173}Yb)

^{173}Yb: +5/2 +3/2 +1/2 -1/2 -3/2 -5/2

"origin of spin degrees of freedom is "*nuclear spin*"

$$H_{\text{int}} = \frac{4\pi\hbar^2 a_s}{M}\delta(\vec{r}_1 - \vec{r}_2)$$ **SU(6) system**

Physics of large-spin Fermi gas:

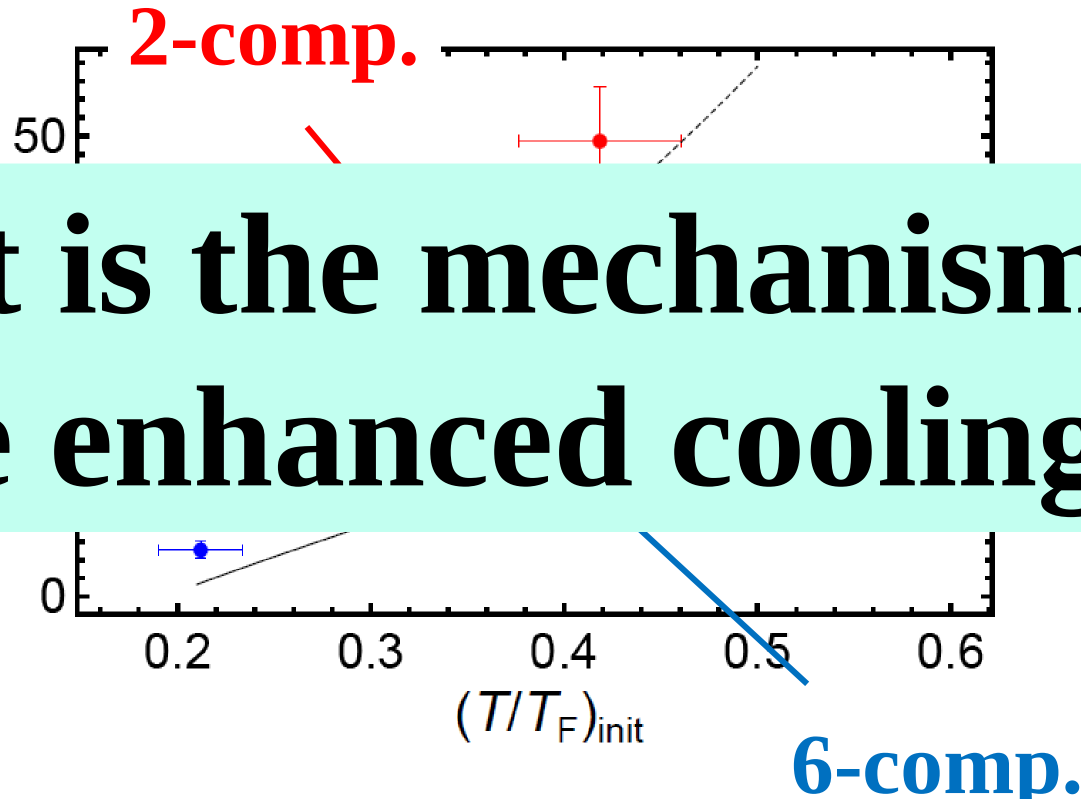
C. Wu *et al.*, PRL**91**, 186402(2003); C. Wu, MPL.B**20**, 1707(2006);
C. Wu, PRL**95**, 266404(2005), etc

E. Szirmai and J. Solyom, PRB**71**, 205108(2005)
K. Buchta, et al., PRB**75**, 155108(2007)

M. A. Cazalilla, *et al.*, N. J. Phys**11**, 103033(2009)
M. Hermele *et al.*,PRL **103**, 135301(2009)
A. V. Gorshkov, *et al.*, Nat. Phys. **6**, 289(2010)
, etc

Atomic Pomeranchuk Cooling

[^{173}Yb atoms in optical lattice; Taie *et al*,]



What is the mechanism of the enhanced cooling ?

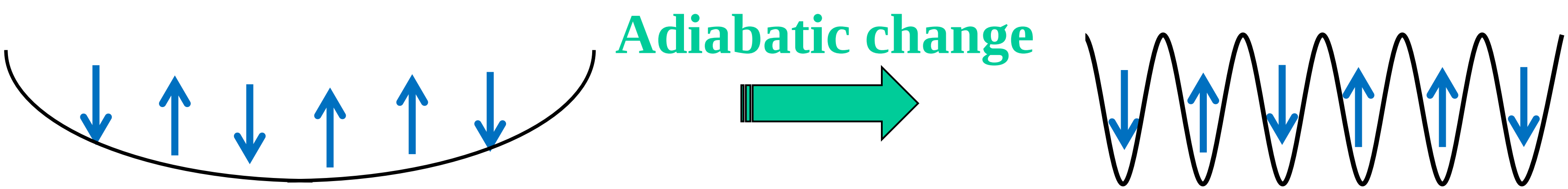
Spin Degrees of Freedom *is Cool*

Pomeranchuk Cooling [Pomeranchuk, (1950)]

→ Discovery of Superfluid ^{3}He by Osheroff, Lee, Richardson

Initial state: Spin *de*polarized and also with *degeneracy*:

Final state: Spin *de*polarized and also with *localization*



$s \sim k_B \pi^2 T/T_F$

liquid ^{3}He   atoms in a trap

$s \sim k_B \ln(N)$

solid ^{3}He   atoms in Mott Insulator

"entropy flows from **motional** degrees of freedom to **spin**, which results in the low temperature"

→ "Pomeranchuk Cooling of an Atomic Gas"

Apply to **MIXTURE** of **2-spin**-component-system and **6-spin**-component system

Spin Degrees of Freedom *is Cool*

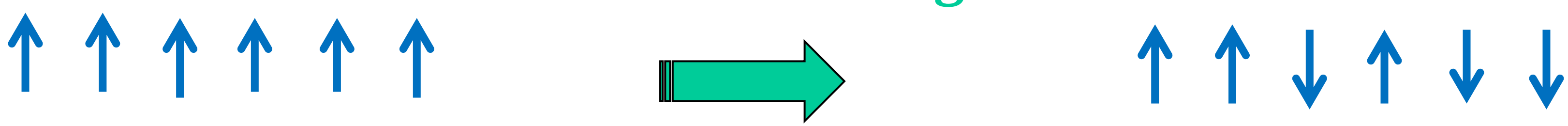
Demagnetization Cooling

[W. J. De Haas, *et al.*, (1934)]

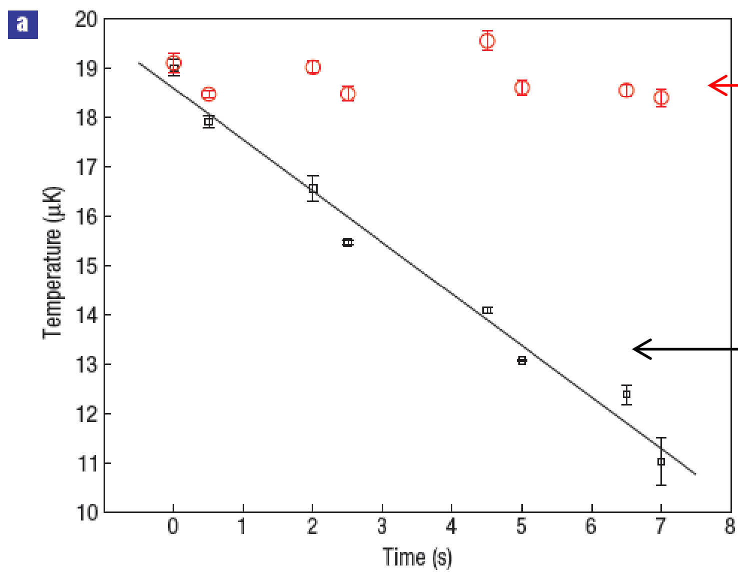
Initial state: Spin-polarized:

Final state: Spin-depolarized:

Adiabatic change



"entropy flows from **motional** degrees of freedom to **spin**, which results in the cooling of the system"



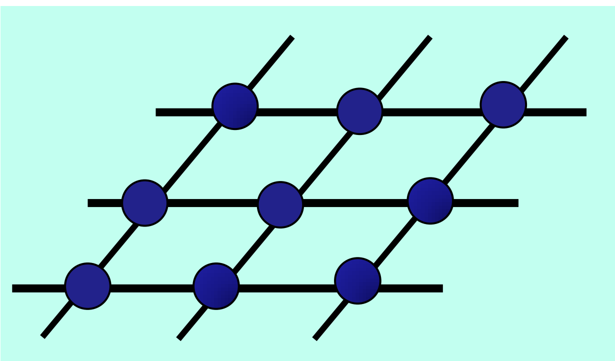
kept at high filed(1G)

kept at low filed(50mG) and Optical Pumping

[M. Fattori, *et al* .,Nat. Physics**2**, 765(2006)]

Quantum Magnetism via Quantum Feedback ?

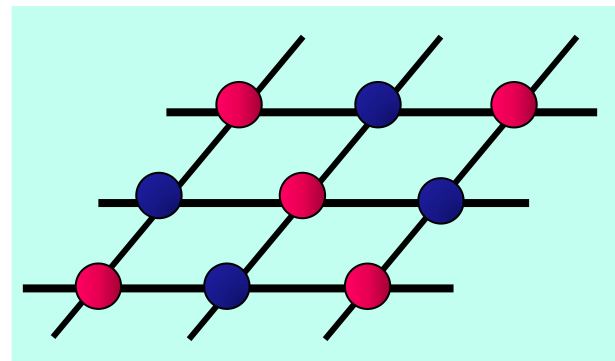
Band Insulator



Cooling

$$s < k_B \ln(N)$$

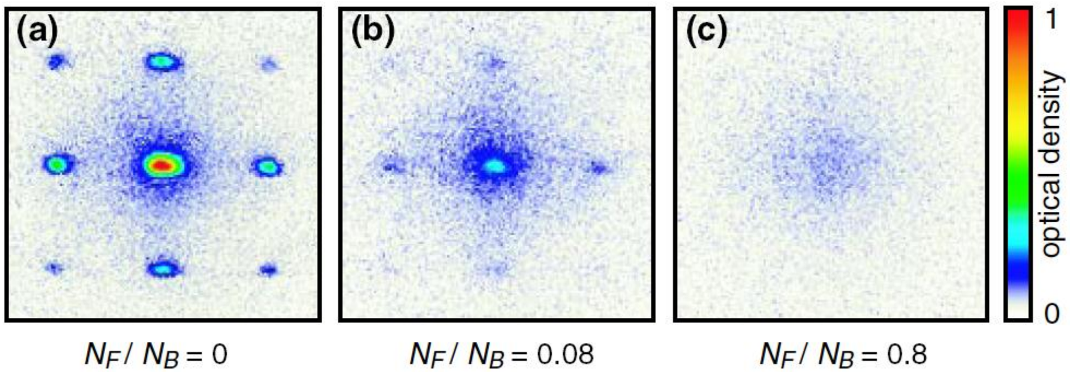
Anti-Ferro Magnetic Order



(Measurement & Feedback) Control
With Single Atom Level

Bose-Fermi Mixture in a 3D optical lattice

Superfluidity of Boson affected by Fermion:



“ ^{40}K(Fermion)-^{87}Rb(Boson)”

[K. Günter, et al, PRL**96**, 180402 (2006)]

[S. Ospelkaus, et al, PRL**96**, 180403 (2006)]

[Th. Best, *et al*, PRL**102**, 030408 (2008)]

Dual Mott Insulating Regime of Boson and Fermion:

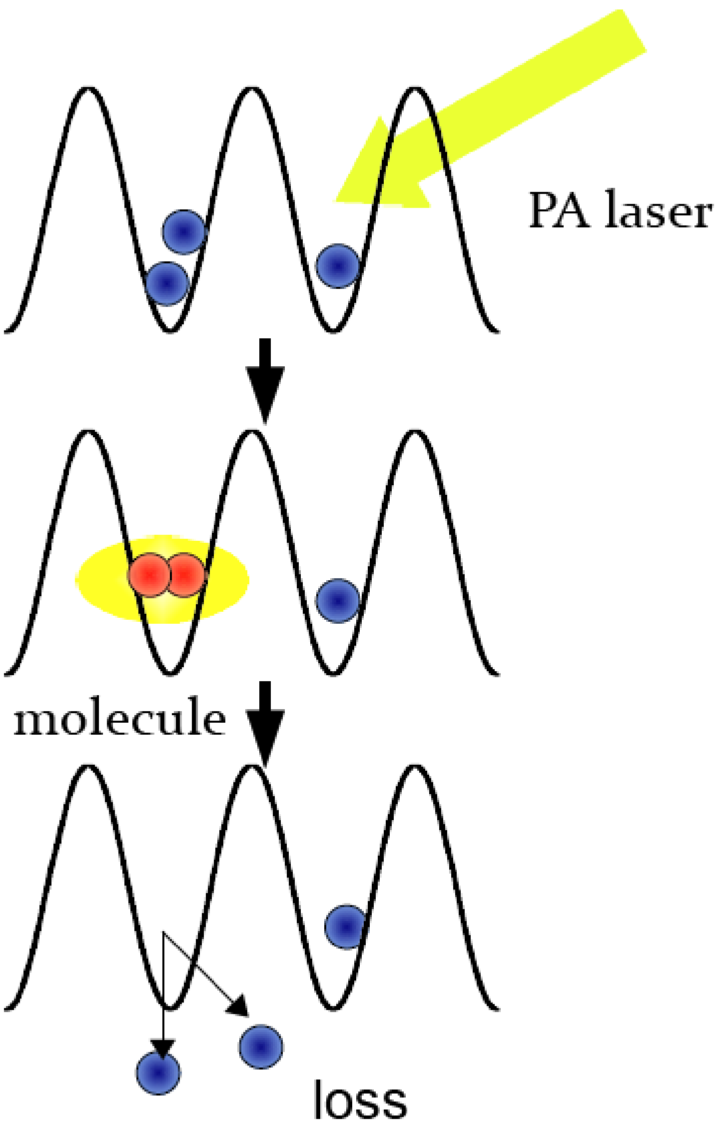
$$J \ll k_B T < U_{BB} < |U_{BF}| < U_{FF}$$

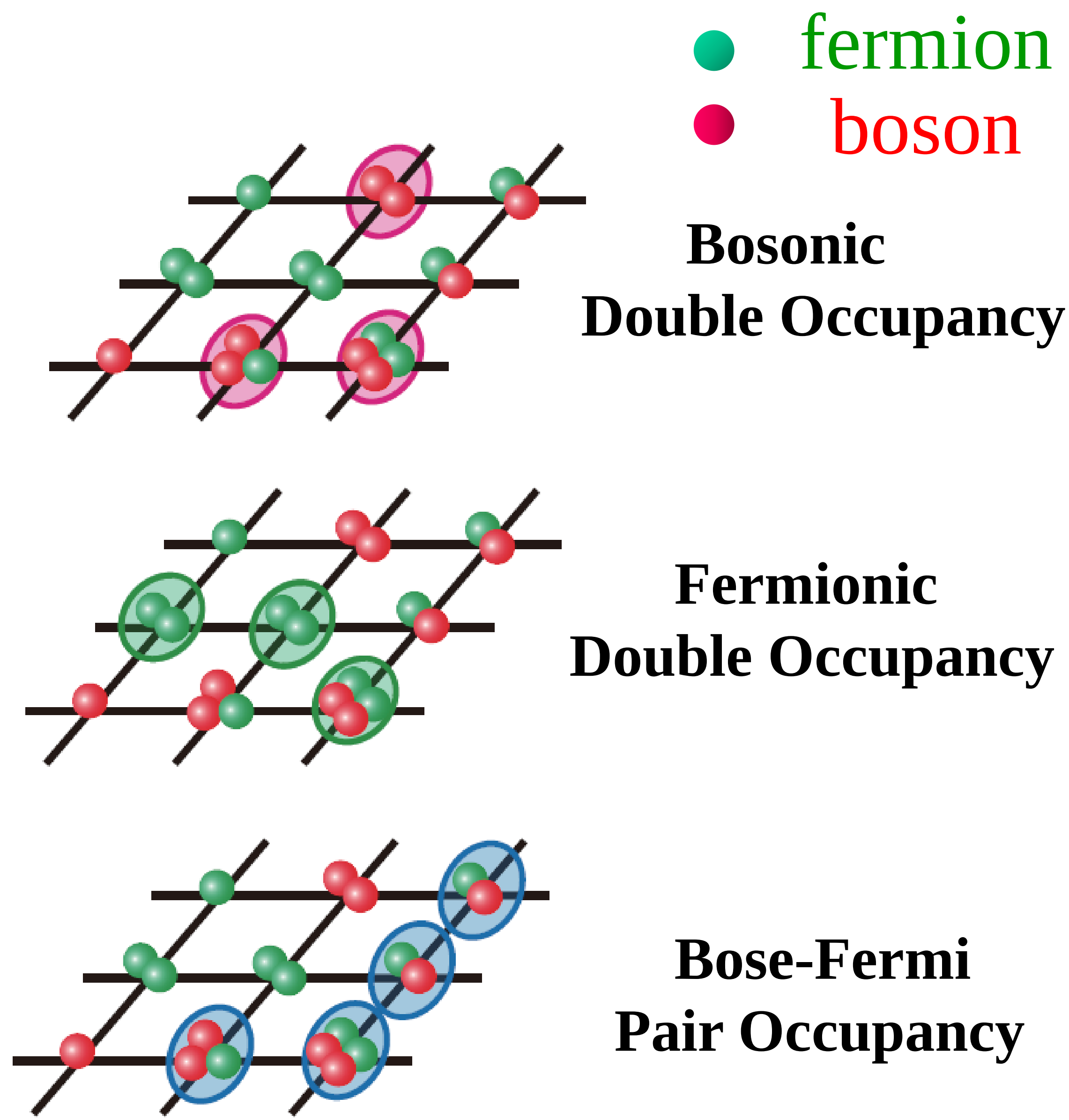
“ ^{173}Yb(Fermion)-^{174}Yb(Boson)”

“ ^{173}Yb(Fermion)-^{170}Yb(Boson)”

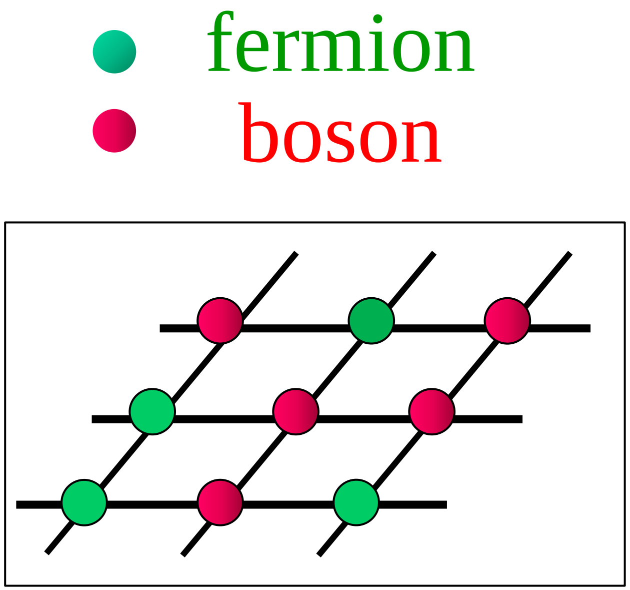
[Sugawa, S. *et al. Nature Phys.* **7,** 642–648 (2011)]

Measurement of Site Occupancy by Photoassociation

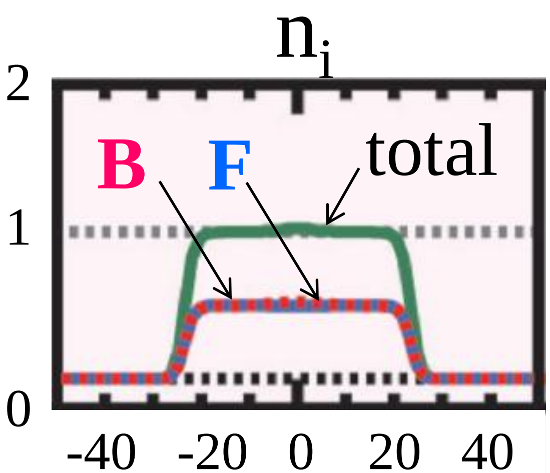


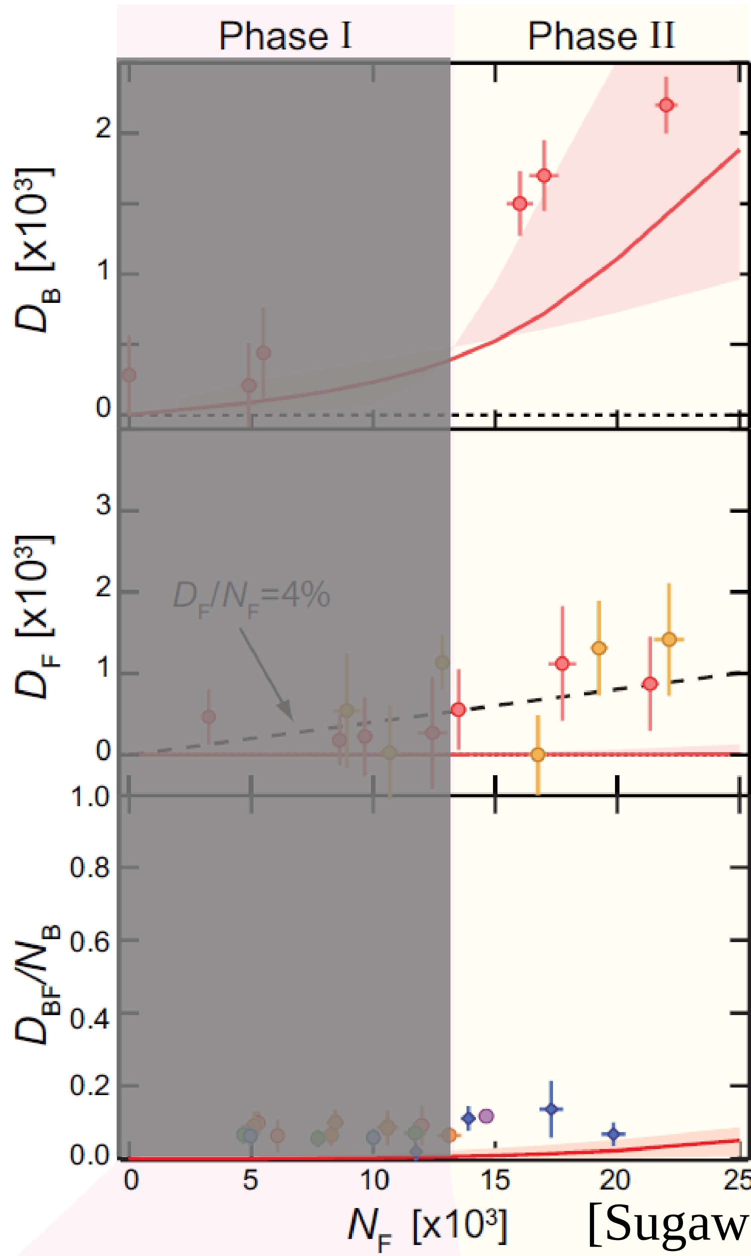


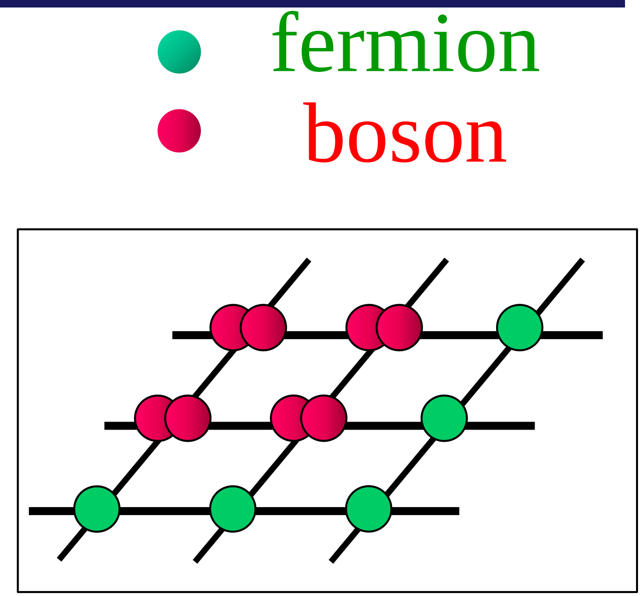
Repulsively Interacting Bose-Fermi Mott Insulators



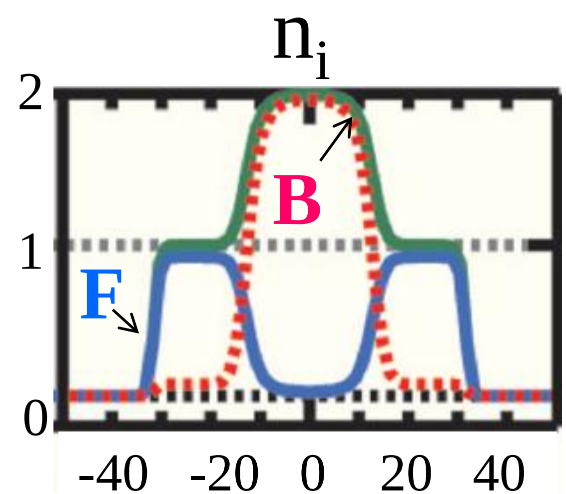
“Mixed Mott Insulator”







“Phase Separation”



[Sugawa *et al.* *NP.* **7**, 642–648 (2011)]

Summary

Quantum Simulation of Hubbard Model Using Optical Lattice

Tuning Interatomic Interaction:

magnetic-, optical-, non-, Feshbach resonance

Superfluid-Mott Insulator Transition

matter-wave interference, spectroscopy

Quantum Gas Microscope

SF-Mott insulator transition, Single-site manipulation,

"quantum magnetism", entropy reduction by Maxwell demon

Fermi Mott Insulator

SU(2) & SU(6) Mott insulator, Pomeranchuk cooling

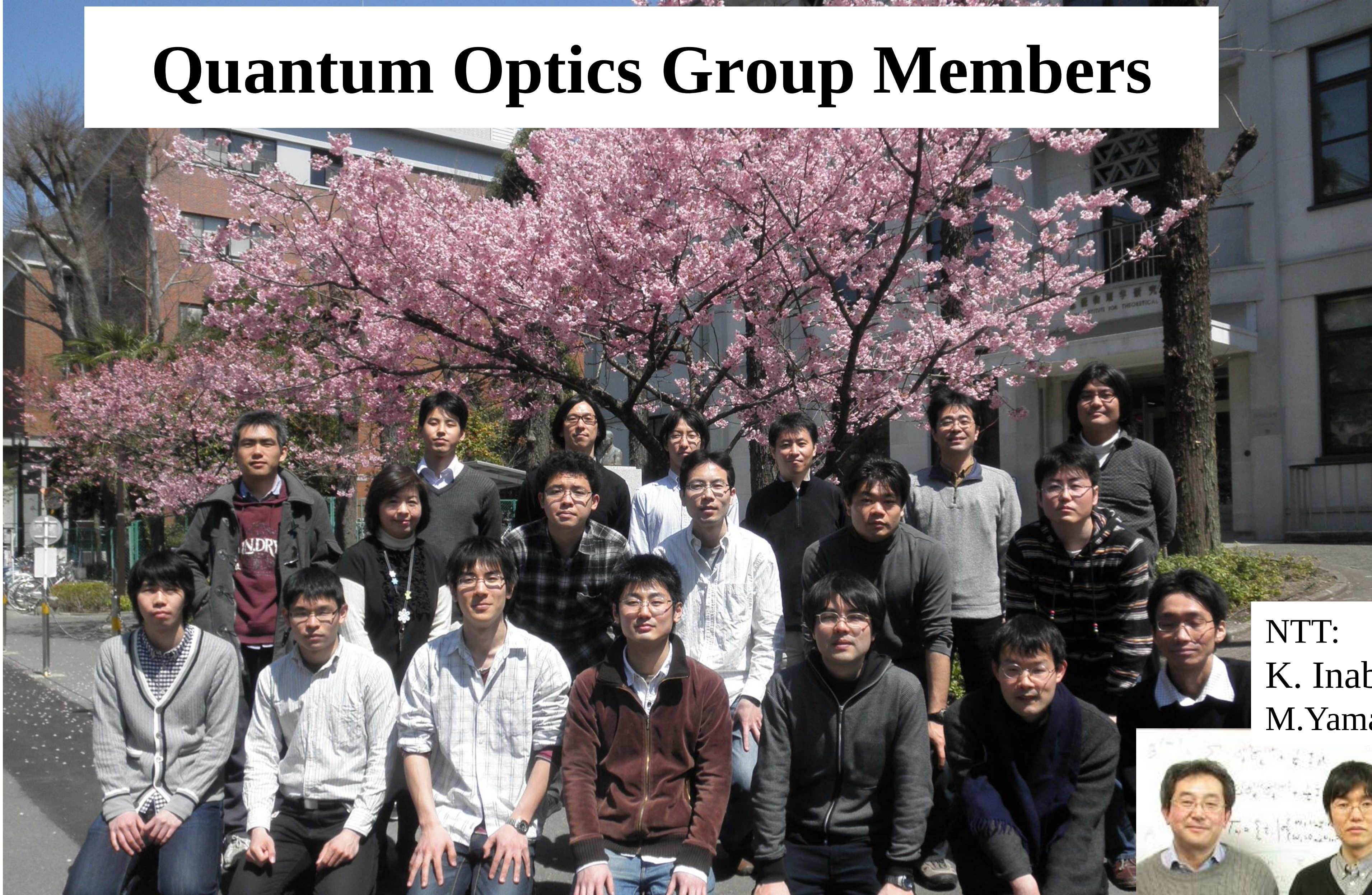
Strongly Interacting Bose-Fermi Mott Insulators

mixed Mott insulator, phase separation, composite particle

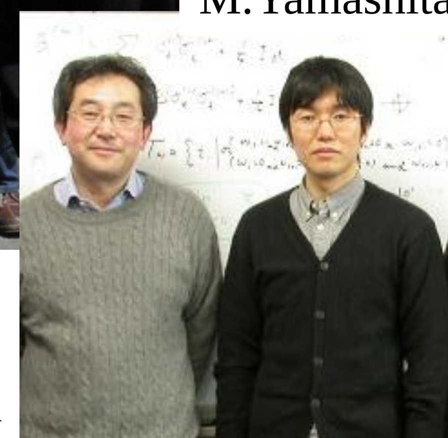
Artificial gauge potentials for neutral atoms

[J. Dalibard, et al., arXiv:1008.5378v1]

Quantum Optics Group Members



NTT:
K. Inaba
M.Yamashita



Ben Li, Y. Nakamura, R. Yamazaki, S. Sugawa, YT, Y. Takasu, R. Inoue,
H. Shimizu, S. Nakajima, S. Uetake, Y. Yoshikawa, H. Hara, (S. Kato, I. Takahashi)
H. Konishi, Y. Kikuchi, H. Yamada, R. Yamamoto, S. Taie, R. Namiki, K. Shibata

Thank you very much for attention

16 August Mount Daimonji at Kyoto