準熱平衡状態にある電子正孔系の理論

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目次

Optical Science of Dynamically Correlated Electrons

自己紹介

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経歴

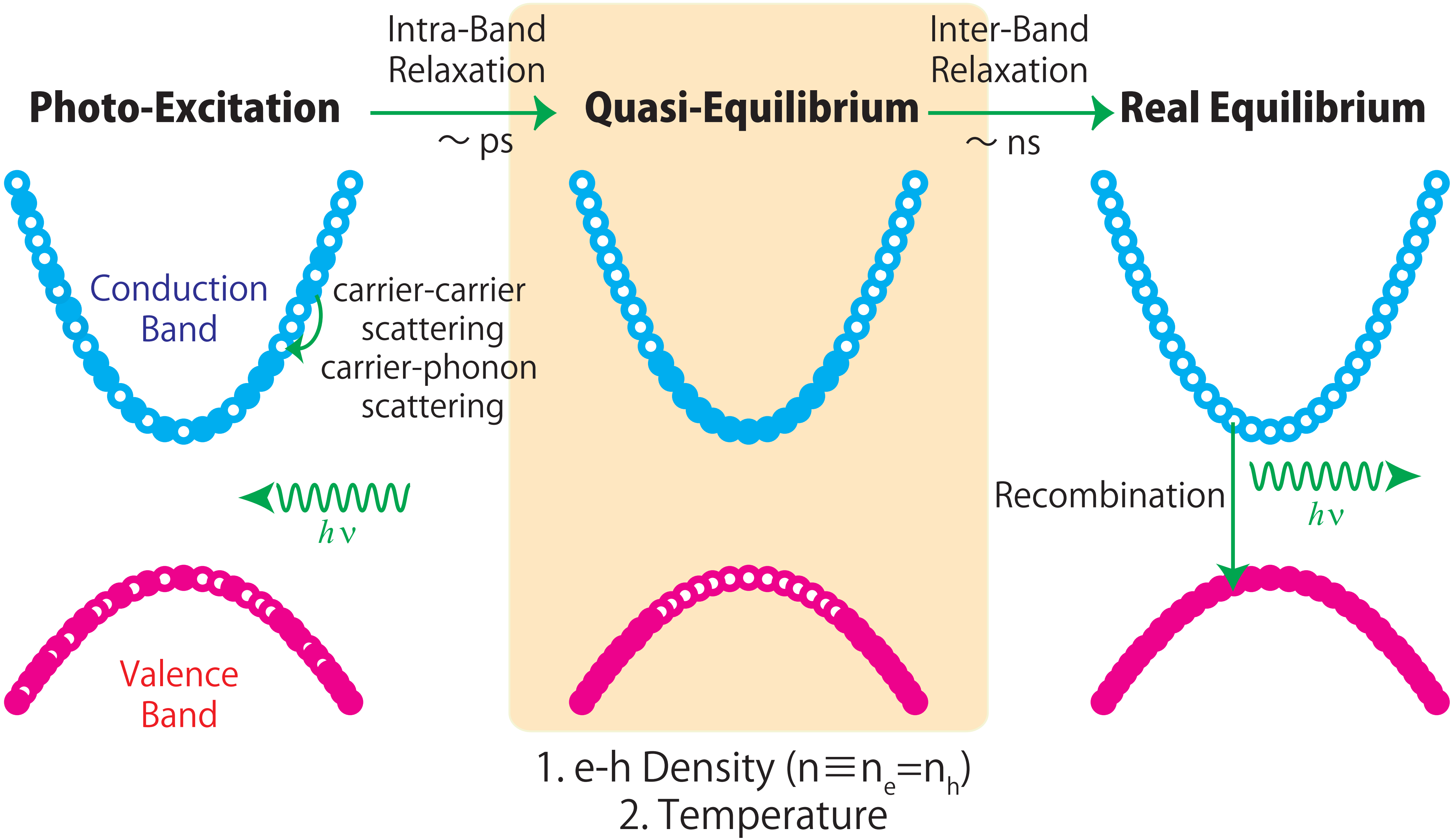
物性研(安藤恒也研)で Ph.D ⇒ PD 渡り鳥 ⇒ 現職

研究の興味

半導体(特に低次元系)における相互作用効果
特にそれがどのように光学応答に現れるか？

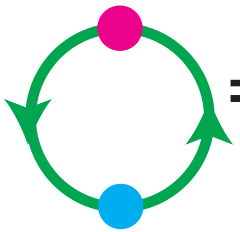
例） 分数量子ホール系の光学応答
カーボンナノチューブ・グラフェンの光学応答
電子正孔系の物理

Concept of Quasi-Equilibruium



Exciton

Exciton (Bound state of 1e and 1h) ⇒ **Analog of H atom**



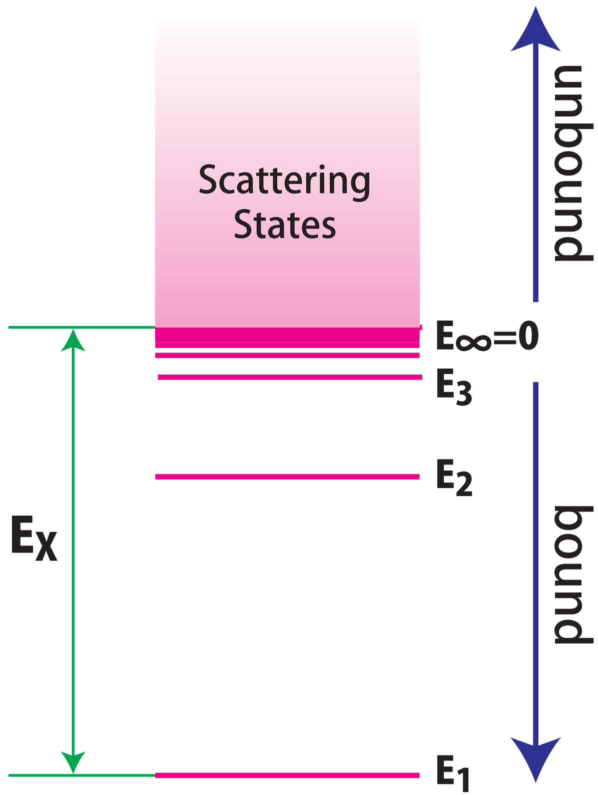
Relative motion between an electron and a hole
⇒ Bound state induced by the attractive Coulomb interaction

"quasi-Boson"

Exciton Bohr radius： $a_B = \dfrac{\hbar^2 \epsilon}{m_r e^2}$

Exciton energy levels： $E_n = -E_X \dfrac{1}{n^2} \quad (n = 1, 2, 3, \cdots)$

Exciton binding energy: $E_X = \dfrac{e^4 m_r}{2\epsilon^2 \hbar^2}$



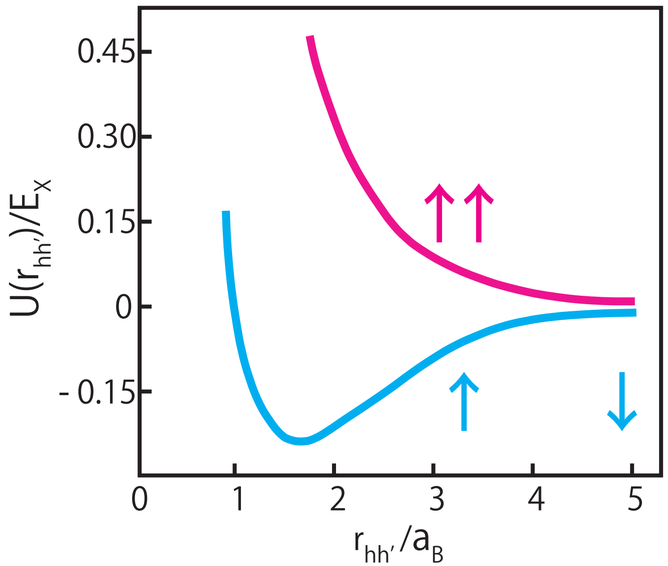
Exciton in semiconductors v.s. H atom

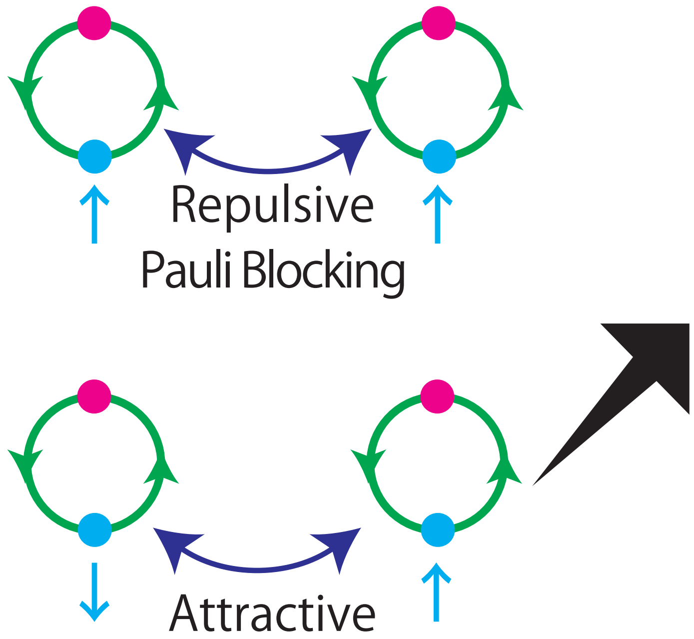
		Binding Energy	Bohr radius
Reduced mass ～×1/10 **Dielectric const.～×10**	⇒	**～×1/1000** **～10meV**	**～×100** **～10nm**

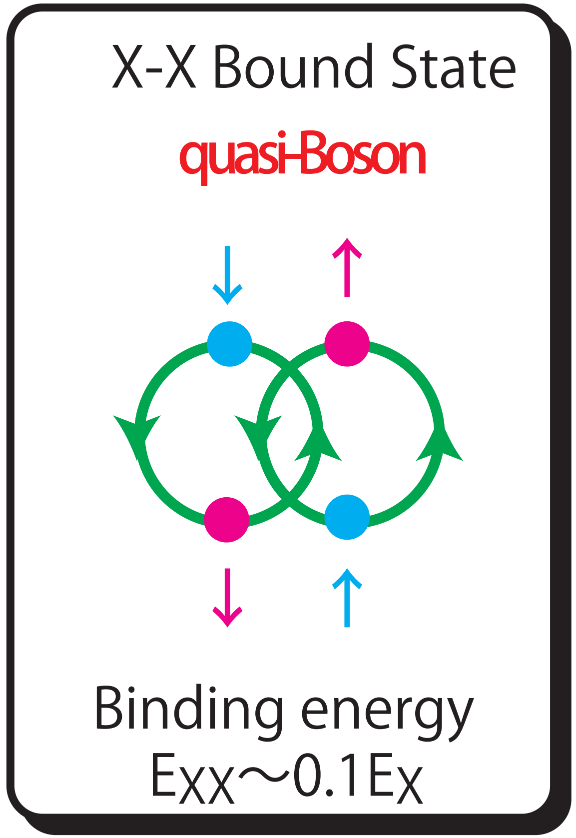
Biexciton

Biexciton ⇒ Analog of H_2 molecule

Exciton-Exciton Interaction Potential (Heavy hole mass limit)







Two electrons and two holes form spin-singlets
→ Orbital wave function without node

Energy Scales of Electron-Hole Systems

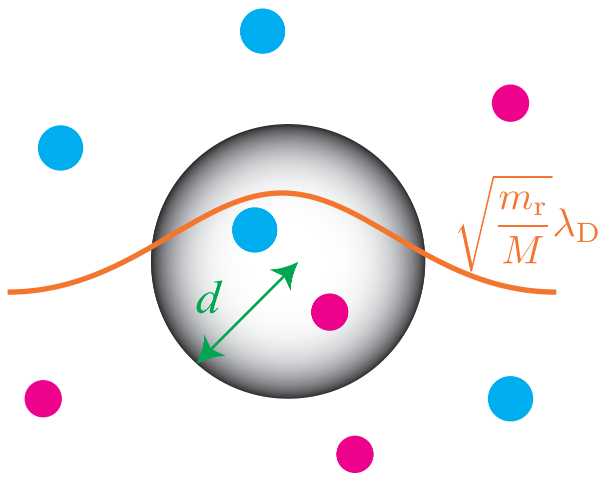
Kinetic energy per e-h pair

Quantum Regime
$\lambda_D/d \gg 1$

$$K = \frac{p_F^2}{2m_e} + \frac{p_F^2}{2m_h} \sim \frac{(\hbar/d)^2}{m_r} \propto d^{-2}$$

Classical Regime
$\lambda_D/d \ll 1$

$$K \sim \frac{3}{2}k_B T \propto d^0$$



Interaction energy per e-h pair

$$U = \frac{e^2}{\epsilon d} \propto d^{-1}$$

Exciton Bohr radius $a_B = \dfrac{\epsilon \hbar^2}{m_r e^2}$

Mean inter-particle distance $d = \left(\dfrac{3}{4\pi n}\right)^{1/3}$

Thermal de Broglie length $\lambda_D = \dfrac{h}{\sqrt{2\pi m_r k_B T}}$

Landau length $\ell = \dfrac{e^2}{\epsilon k_B T}$

Coupling Strength U/K

Quantum Regime
$\lambda_D/d \gg 1$

$$r_s = \frac{d}{a_B}$$

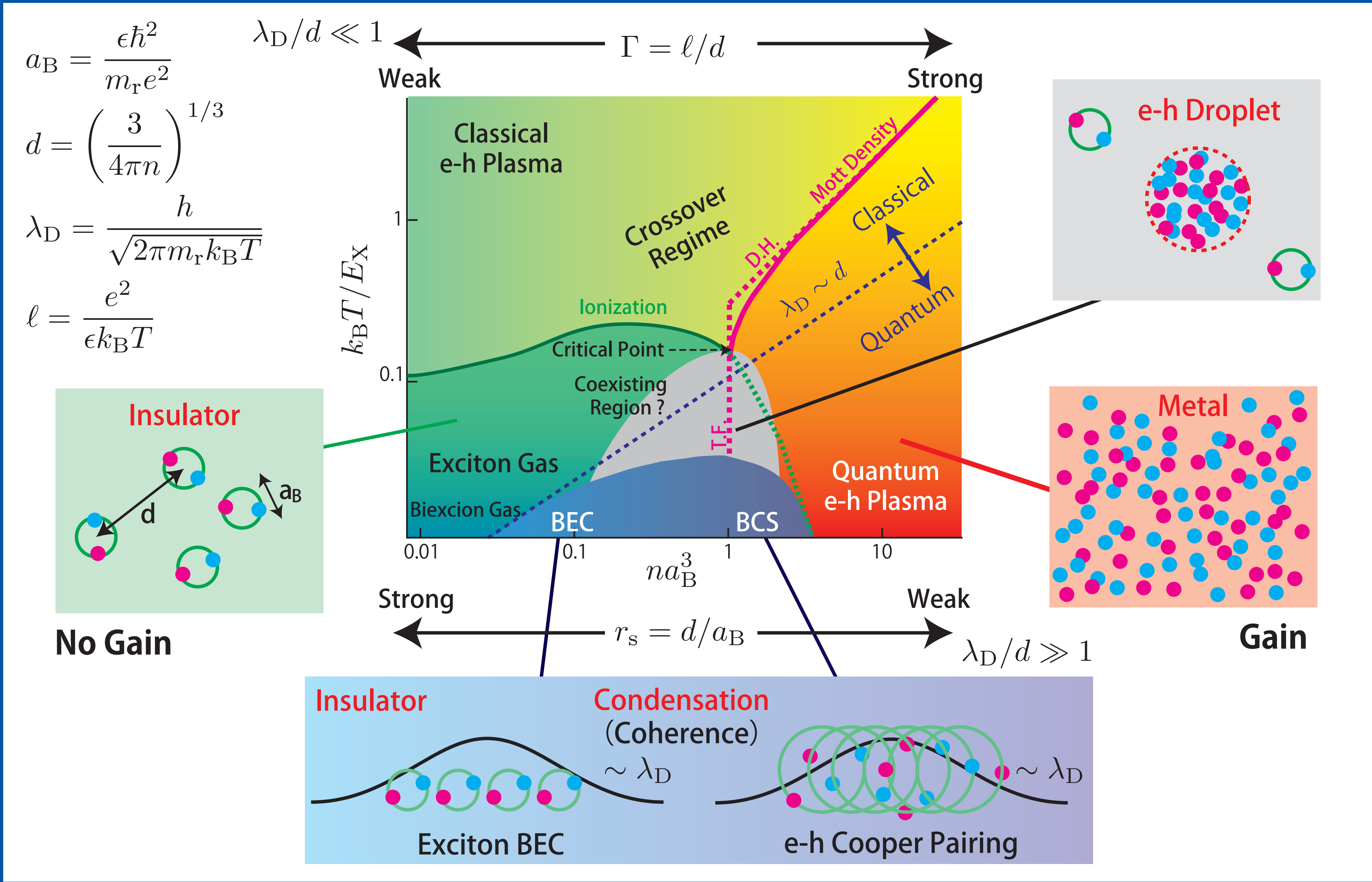
Low $n \Rightarrow$ Strong
High $n \Rightarrow$ Weak

Classical Regime
$\lambda_D/d \ll 1$

$$\Gamma = \frac{\ell}{d}$$

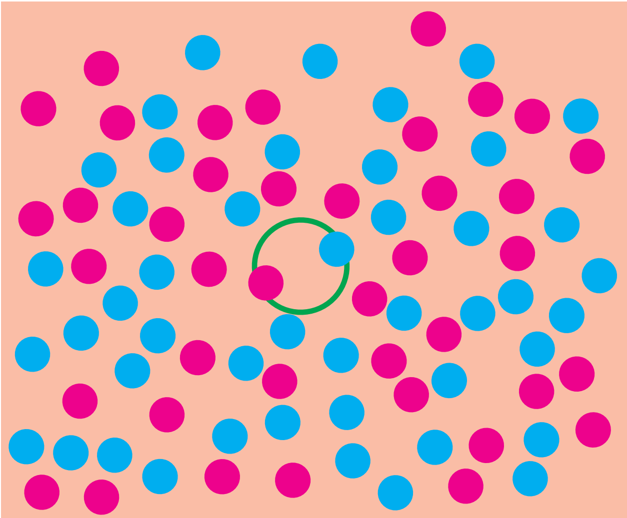
Low $n \Rightarrow$ Weak
High $n \Rightarrow$ Strong

Phase Diagram of 3D e-h Systems (Schematic)



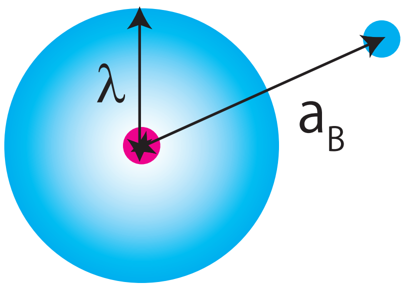
Exciton-Mott Crossover (Mott Density)

Is a single exciton embedded in the e-h Fermi liquid is stable or not ?

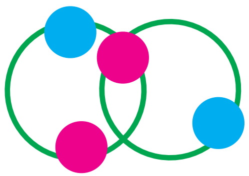


1. Band gap renormalization (BGR)
   Self-energy corrections.

2. Screening

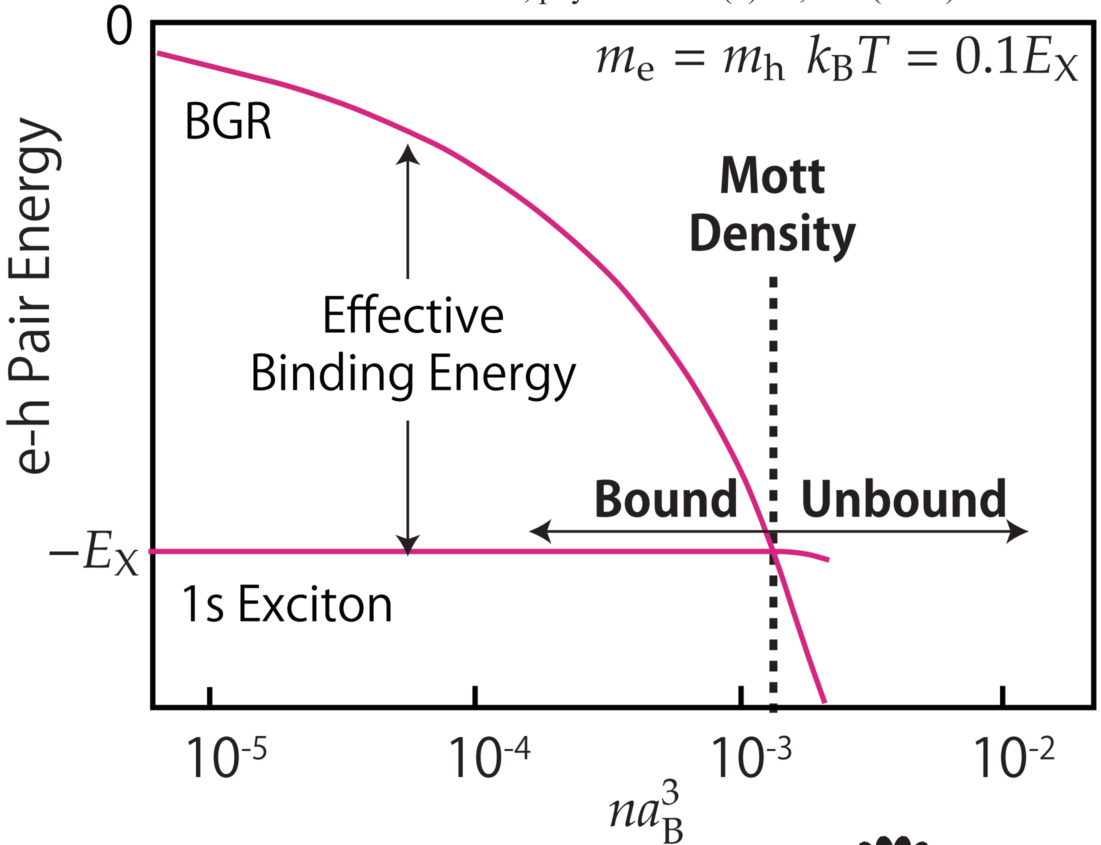


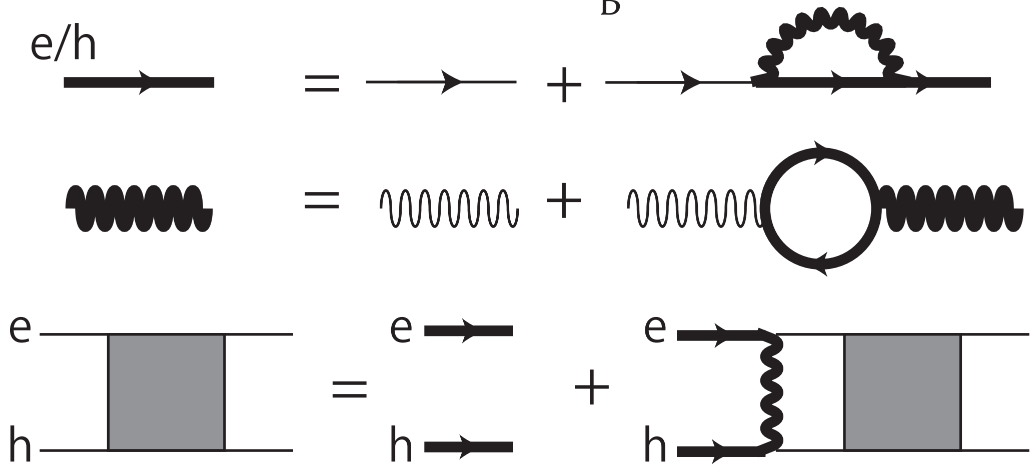
3. Pauli blocking



Fermionic Nature

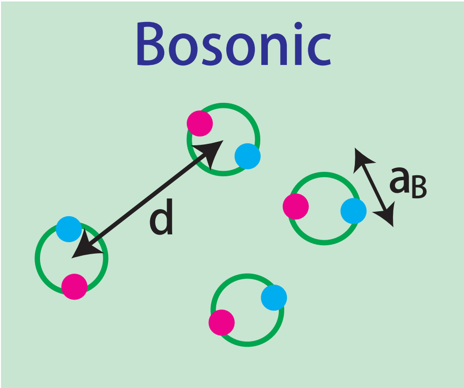
Zimmermann et al, phys. stat. sol (b) **90**, 175 (1978).



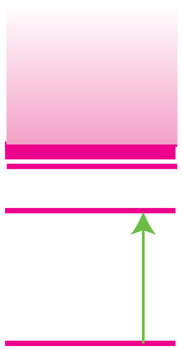


Exciton Mott Crossover & Absorption/Gain

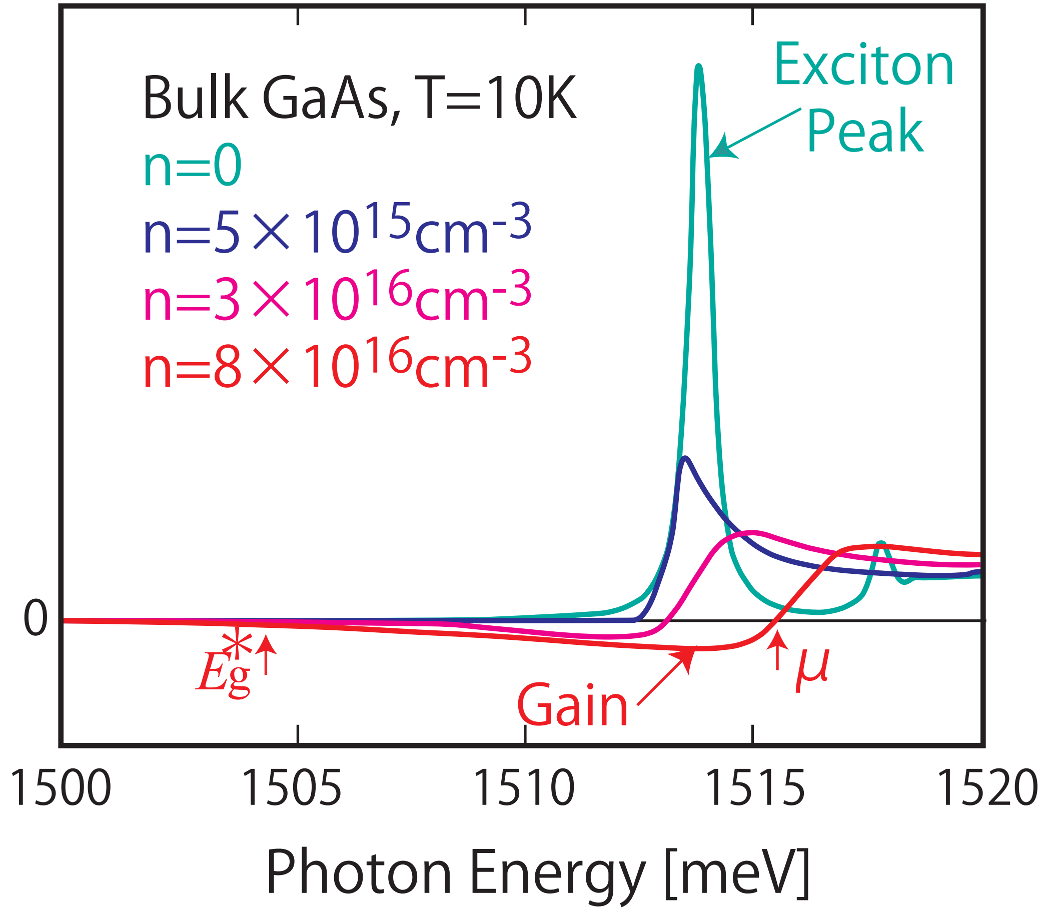
Low Density



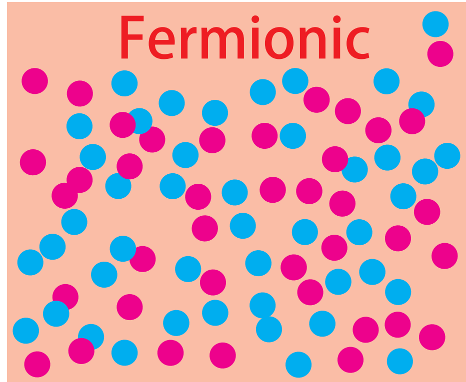
~ nearly free bosons
(1s excitons).

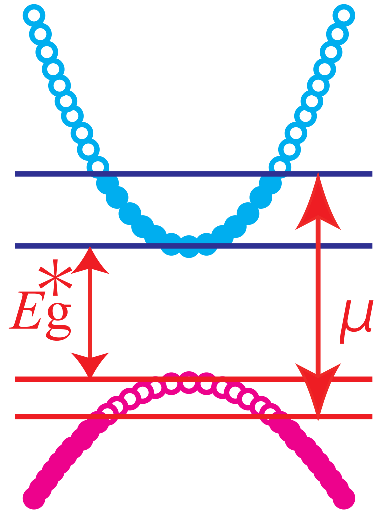


Schmitt-Rink et al, Z. Phys.B **47**, 13 (1982).
Haug and Schmitt-Rink, Prog. Quant. Electr. **9**, 3 (1984).



High Density





Population Inversion

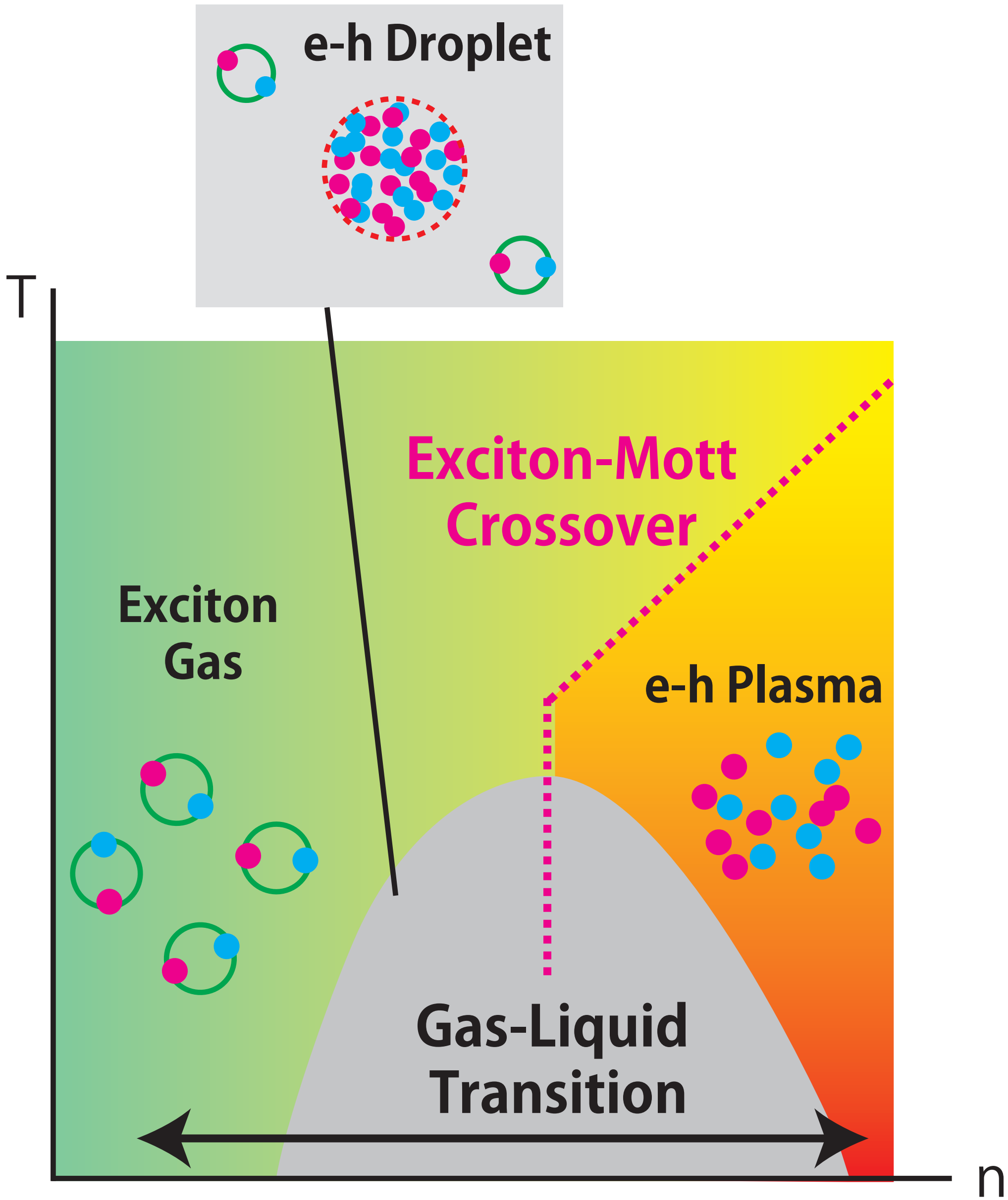
Sign change
at $\mu = \mu_e + \mu_h$.
(KMS Relation)

Bosonic
Exciton Peak
(Insulator)

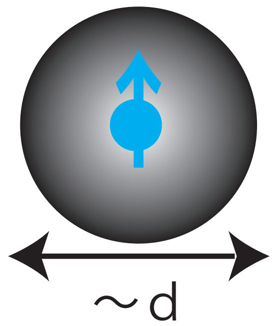
⇔
Exciton-Mott

Fermionic
Gain
(Metal)

Gas-Liquid Transition



Exchange Hole (HF Approximation)

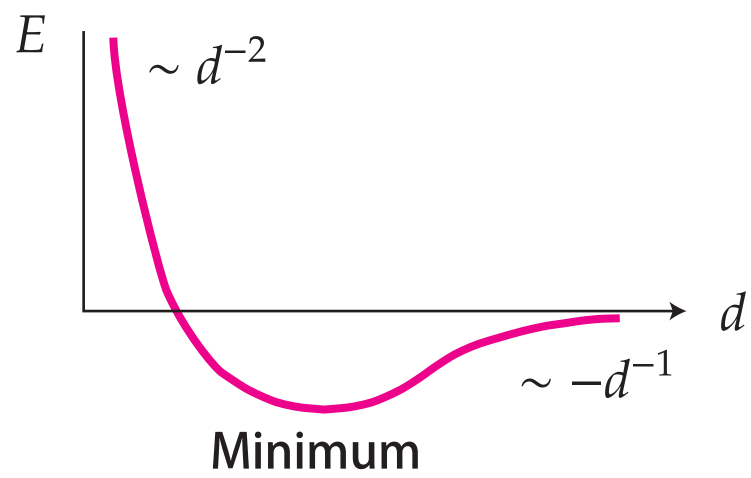


Energy decrease

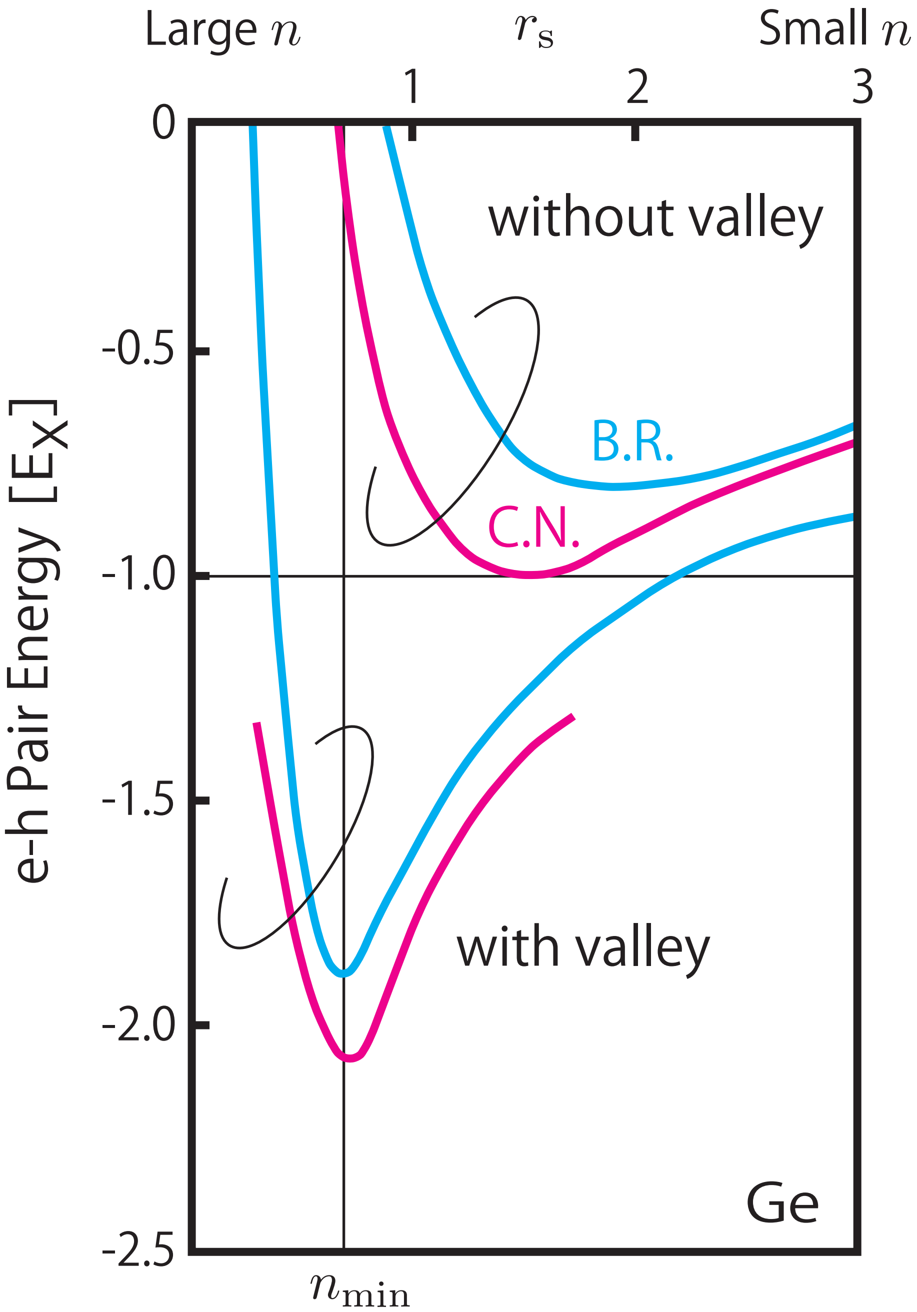
$\sim e^2/\epsilon d$

Energy per an e-h pair

$$K \sim \frac{(\hbar/d)^2}{2m_\mathrm{r}} \propto d^{-2} \qquad U \sim -\frac{e^2}{\epsilon d}$$



Electron Hole Droplet

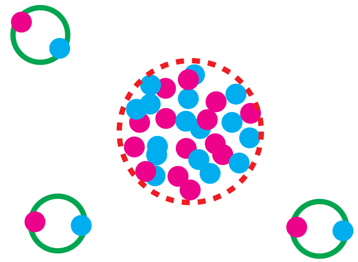


RPA Calculation

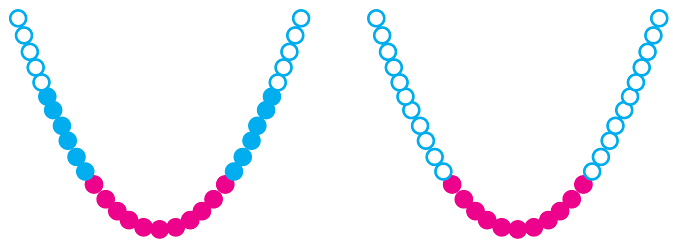
Brinkman and Rice, PRB **7**,1508 (1973)
Combescot and Nozieres, J. Phys **C5**, 2369 (1972)

$E_{min} < -E_X$

→ Formation of e-h droplet



Valley Degeneracy

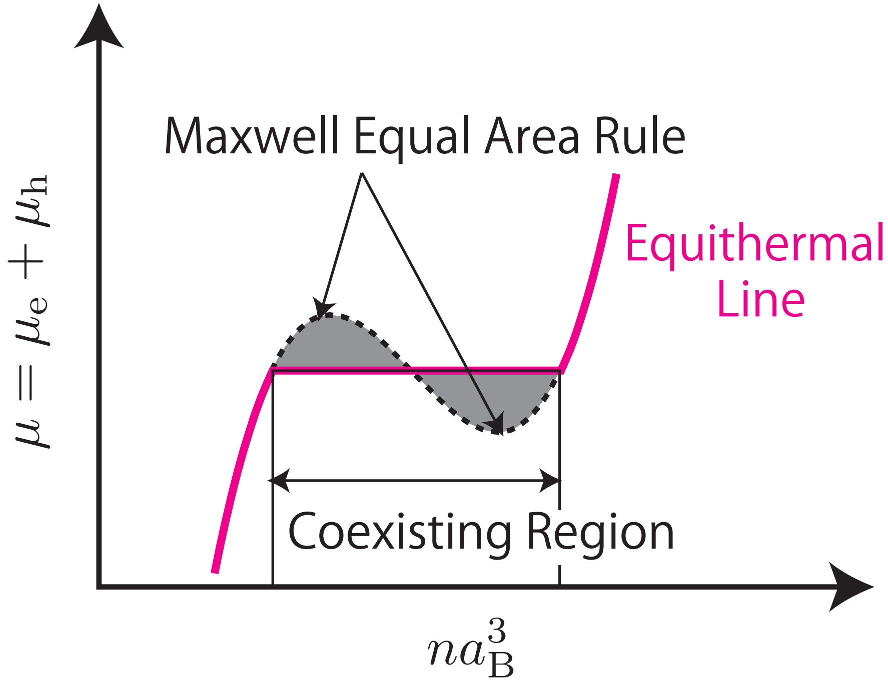


Decrease in kinetic energy

“Pure” Mott Transition

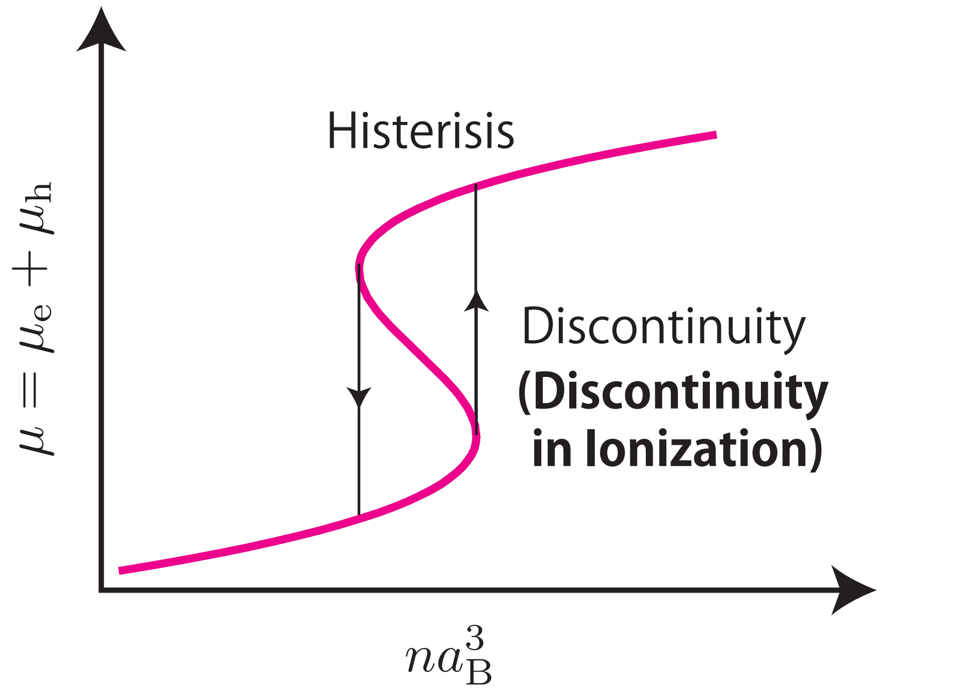
Two Possibilities of First Order Insulator-Metal Transition

① Gas-Liquid Transition

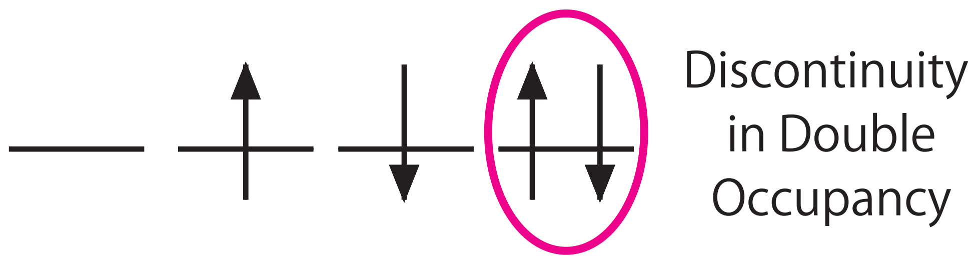


Analogue of van der Waals Gas-Liquid Transition

② “Pure” Mott Transition



Analogue of Mott-Hubbard Transition



BCS-BEC Crossover (Quantum Condensation)

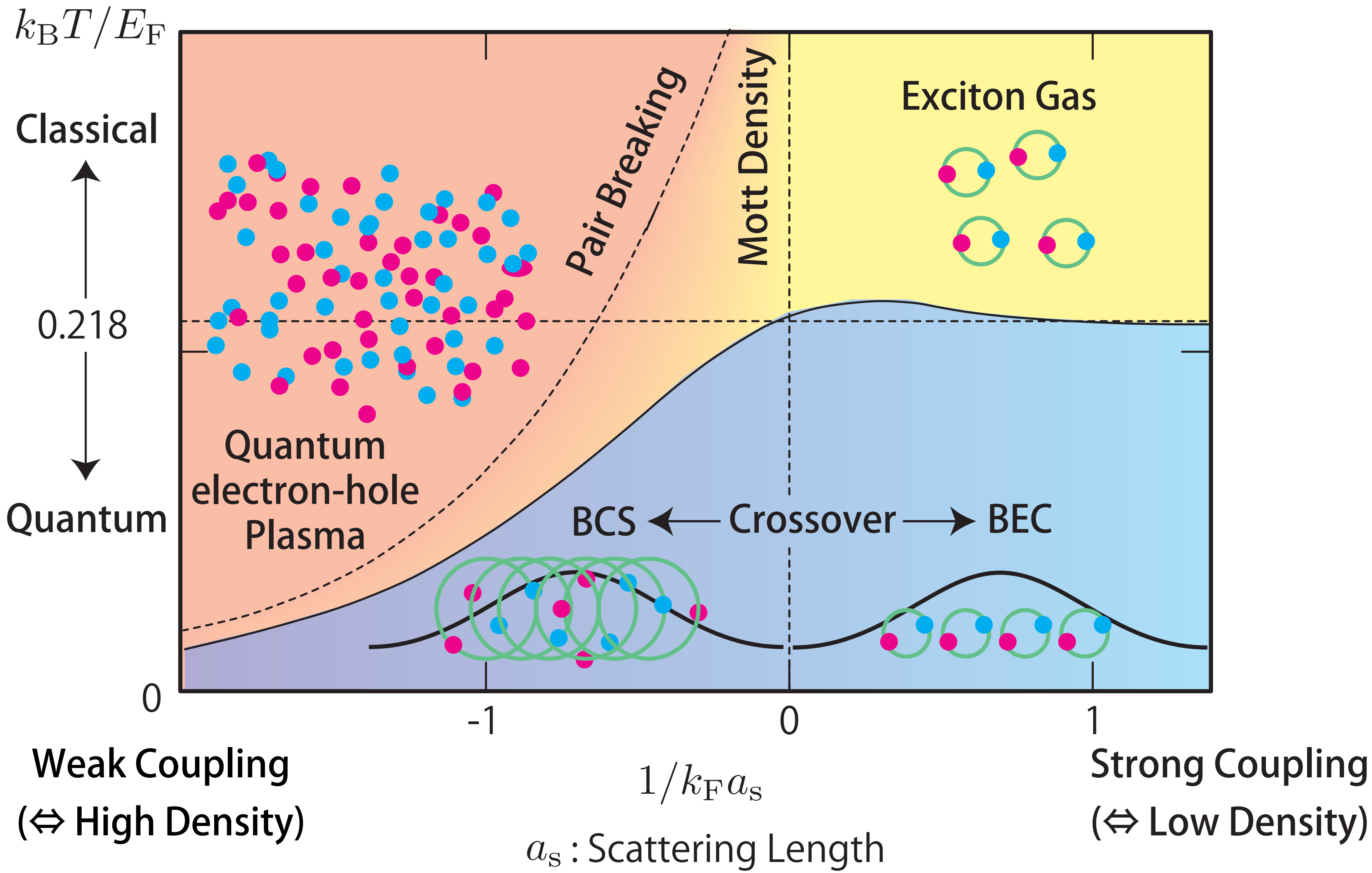
Thouless Criterion : Divergence of Pair Susceptibility

Nozieres and Schmitt-Rink, J. Low. Temp. Phys. **59**, 195 (1985).

Short-Range Attractive Interaction

Pair Susceptibility (Ladder)

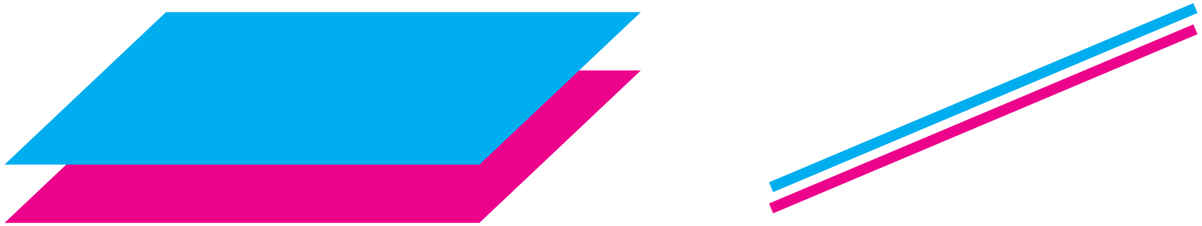
Thermodynamic Potential



Our Research Interest

① Low Dimensional e-h Systems

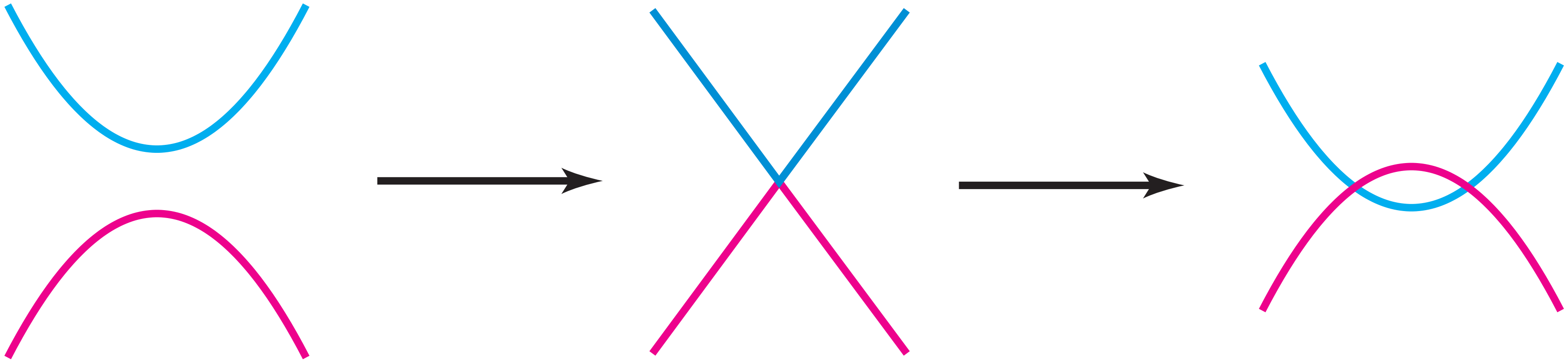
2D (Quantum Well) | 1D (Quantum Wire) | 0D (Quantum Dot)



Magnetic field → quantum Hall systems

② Band Gap Control : Go back to the original Mott’s idea !

Semiconductor | Dirac | Semimetal



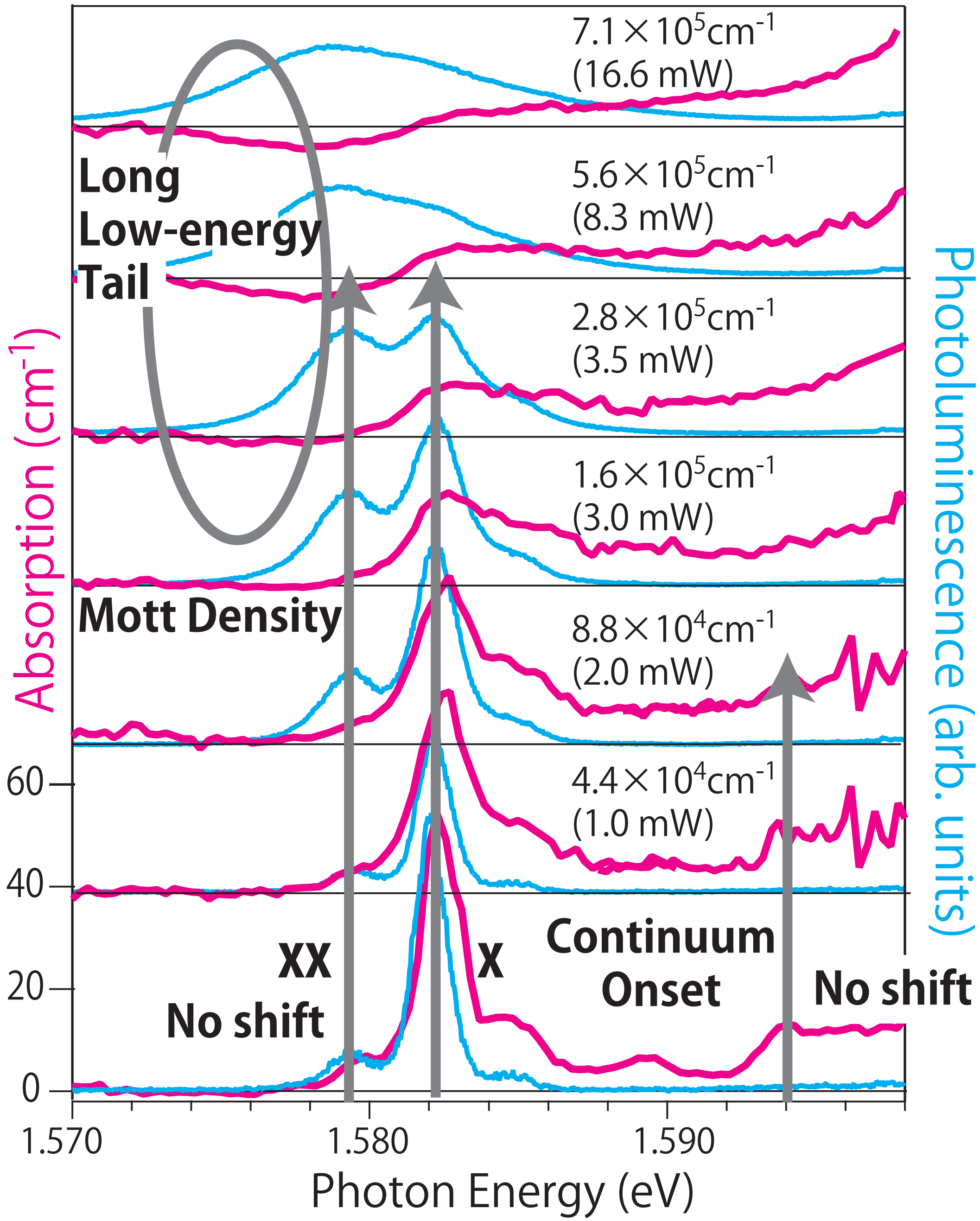
Band Gap > 0 | Band Gap = 0 | Band Gap < 0

Topic 1

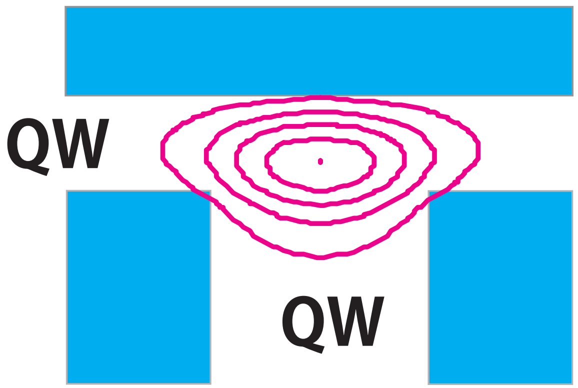
One-Dimensional Electron-Hole Systems

T. Yoshioka and K. Asano

Experiments on T-shaped Quantum Wire



Hayamizu et al., PRL **99**, 167403 (2007).

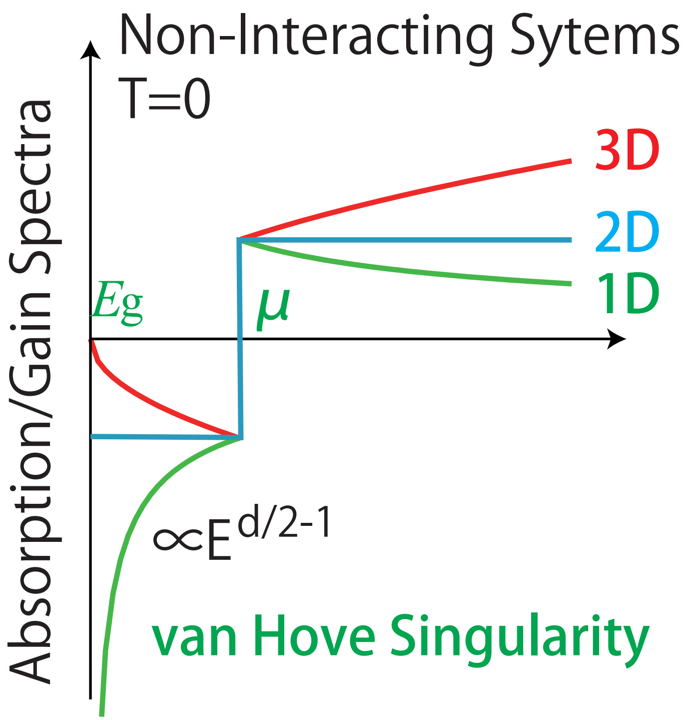


① Highly Clean
② Long-Range Coulomb
(No gate structure)

Optical Gain (Laser Application) & Dimensionality

Advantage

1. Large DOS at Band-Edge



2. Strong Phase Space Filling Effect

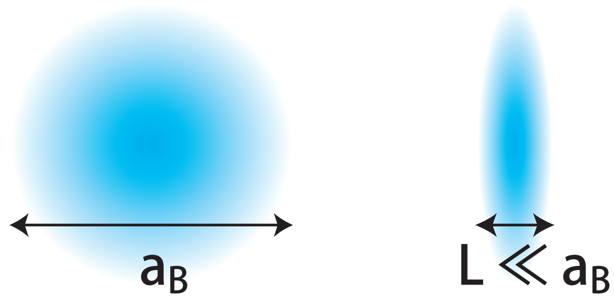
3. Strong Excitonic Enhancement near $E \sim \mu$

v.s.

Disdvantage

(Enemies of Exciton-Mott Transition)

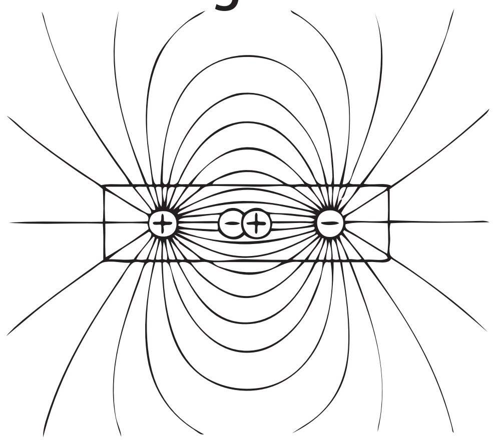
1. Huge Exciton Binding Energy



① $E_X \sim$ 100K (Ideal 1D → +∞)

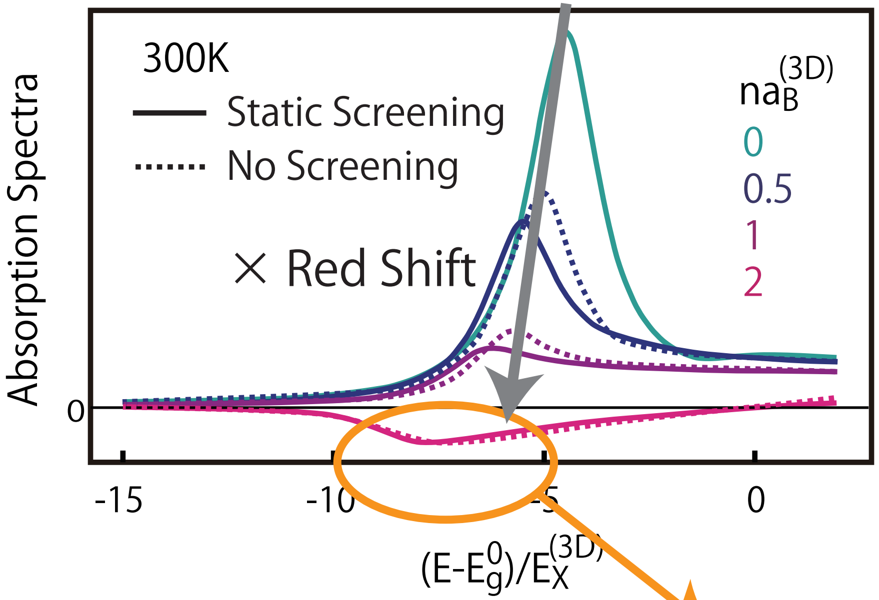
② Infinitesimal attraction ⇒ Bound

2. Small Screening

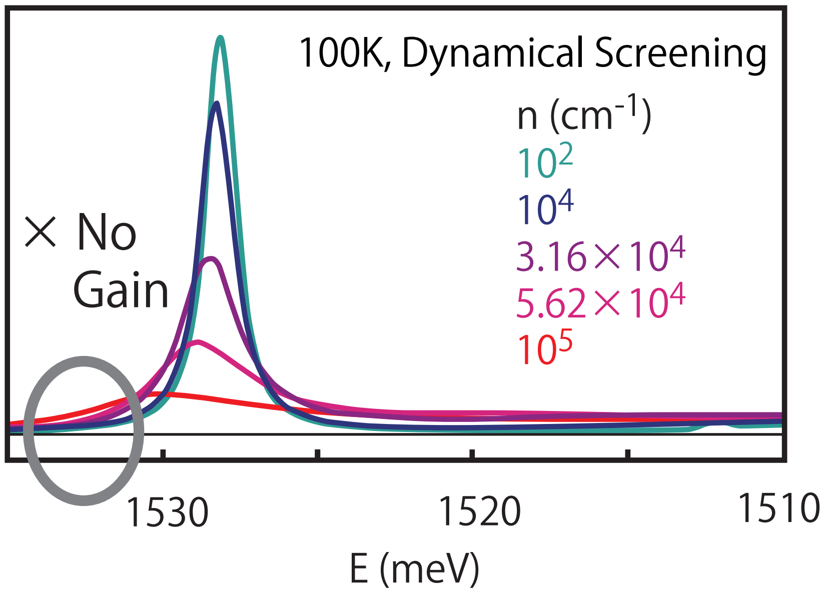


Theories by Traditional Approach

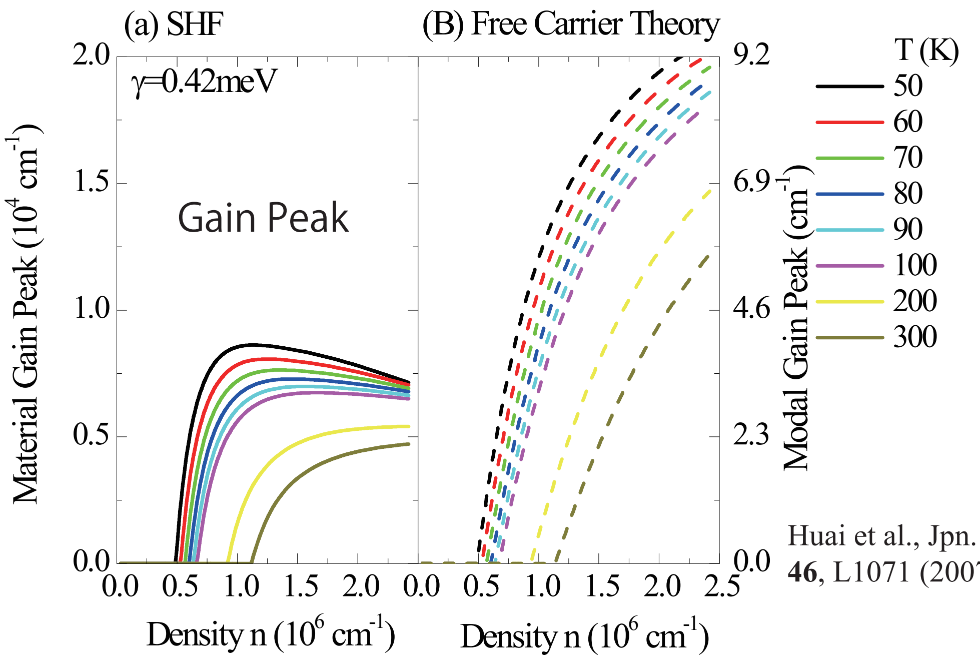
Benner and Haug, EuroPhys. Lett. **16**, 579 (1991).



Wang and Das Sarma, PRB **64**, 195313 (2001).

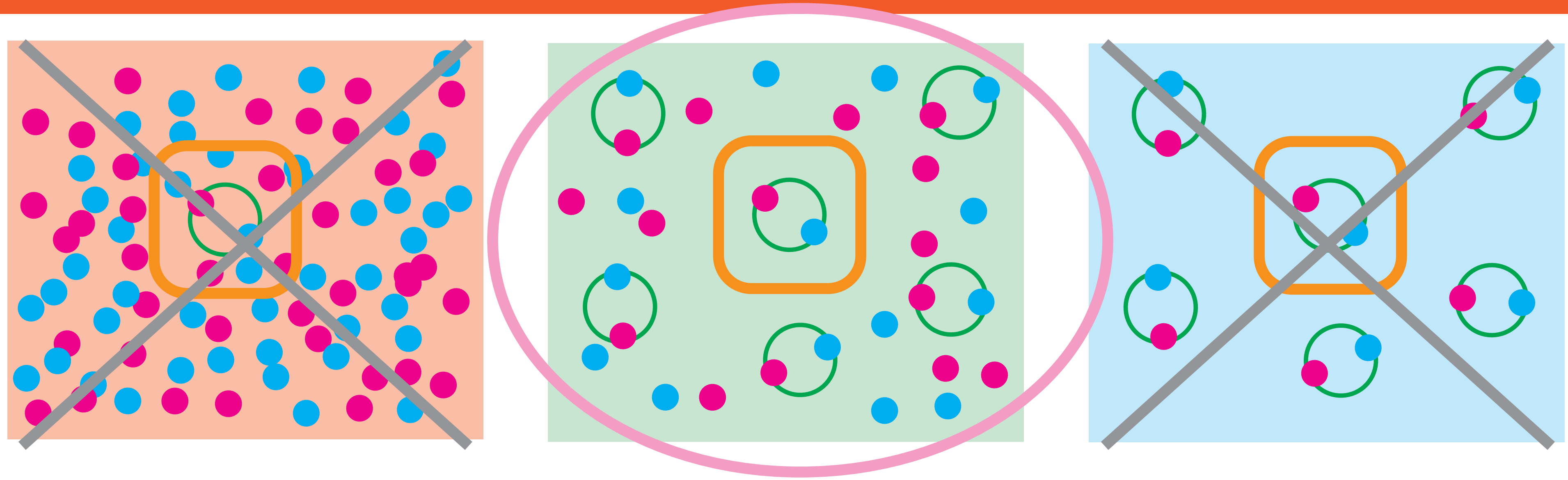


Traditional Approach
Screened Hartree Fock
+ Ladder Approx.



Huai et al., Jpn. J. Appl. Phys. **46**, L1071 (2007).

Our Approach to Exciton-Mott Crossover/Transition



X in e-h Plasma | **X in X Gas + e-h Plasma** | **X in X Gas**

Benner and Haug, EuroPhys. Lett. **16**, 579 (1991).
Wang and Das Sarma, PRB **64**, 195313 (2001).
Huai *et al*., Jpn. J. Appl. Phys. **46,** L1071 (2007).

Yoshioka and Asano 投稿中

Hanamiya, Asano and Ogawa: physica E **40**, 1401 (2008).

(1) T-Matrix Contribution in Self Energy
⇒ Excitonic Effect in DOS

↑

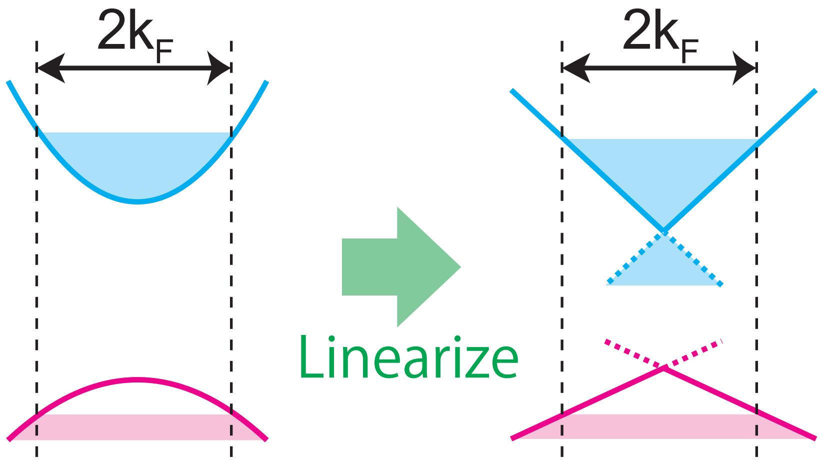
Ionization Ratio Self-Consistent

↓

(2) Excitonic Suppression of Screening

Theory for High n & Low T Regime

Bosonization Approach



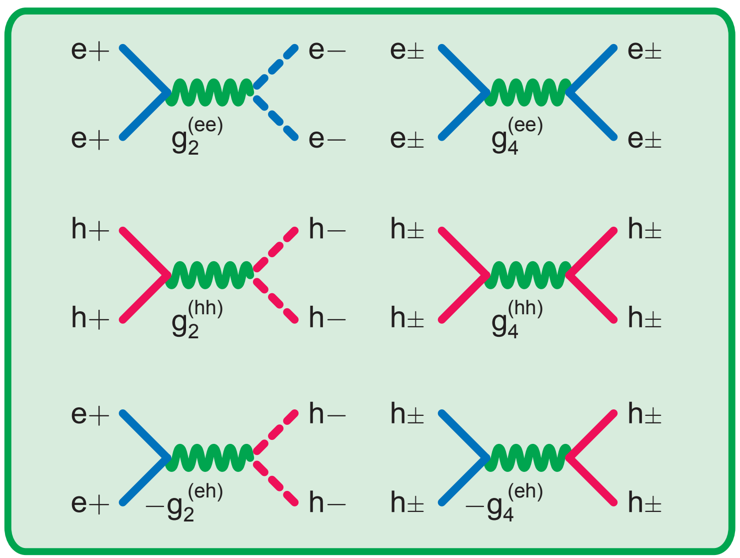
~~Fermi liquid~~ ~~quasi-electrons quasi-holes quasi-excitons~~

Importance of collective modes

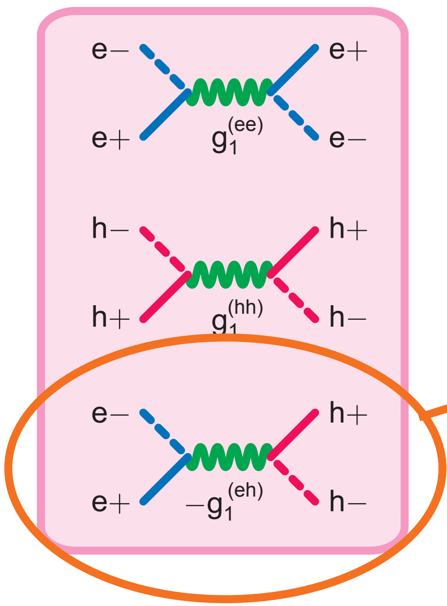
Charge	**Massive**
Mass	**Massless**
e-Spin	**Massive**
h-Spin	**Massive**

C1S0 Phase

Forward ⇒ Solvable



Backward ⇒ RG



e-h Backward Scattering is relevant!

Always insulating at T=0.

Algebraic Order of Ground State

Low energy physics is dominated by the mass density mode.

$$\mathcal{H}_\rho^{(\mathrm{m})} = \frac{v^{(\mathrm{m})}}{2\pi} \int dx \left[K^{(\mathrm{m})} \left(\partial_x \Theta_\rho^{(\mathrm{m})} \right)^2 + \frac{1}{K^{(\mathrm{m})}} \left(\partial_x \Phi_\rho^{(\mathrm{m})} \right)^2 \right]$$

$2k_F$ Mass Density Wave (Biexcitonic Crystal) $\sim x^{-K_m/2}$

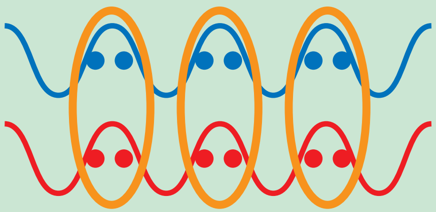
Biexcitonic Condensation $\sim x^{-2/K_m}$

$\Rightarrow$ Biexcitonic Supersolid

c.f. Andreev and Lifshitz (1969).

$K^{(\mathrm{m})}$: 1 (non-int.) — 2 — ∞

$2k_F$ Mass Density Wave
(Biexcitonic Crystal)



"Acoustic Mode" $\Leftrightarrow$ Mass
"Optical Mode" $\Leftrightarrow$ Charge

Biexcitonic Condensation

Phase Seperation ?

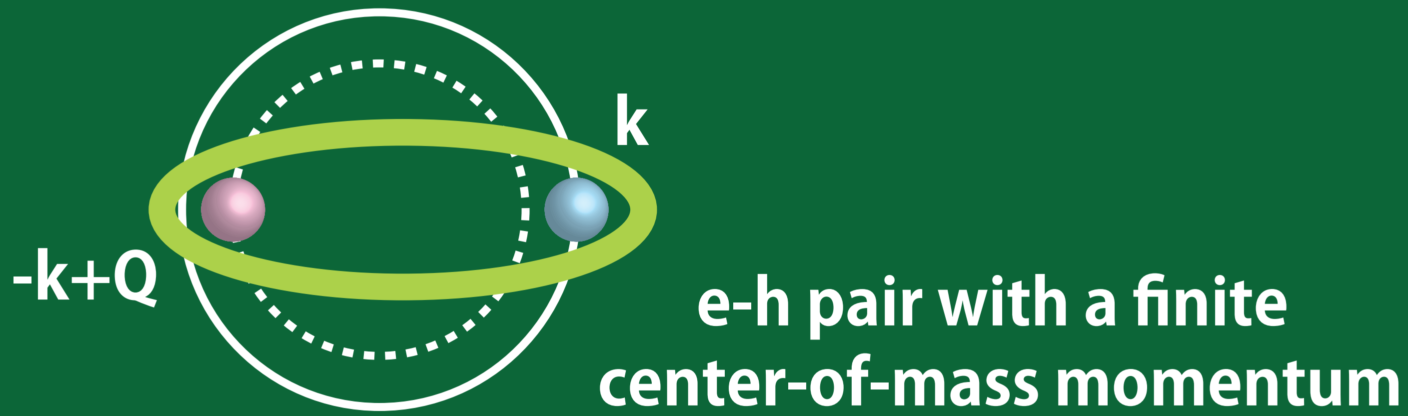
Asymmtery of e and h mass
$\Rightarrow$ Stabilize the biexcitonic crystal order

K. Asano and T. Ogawa, J. Lumin. **112**, 200 (2005).

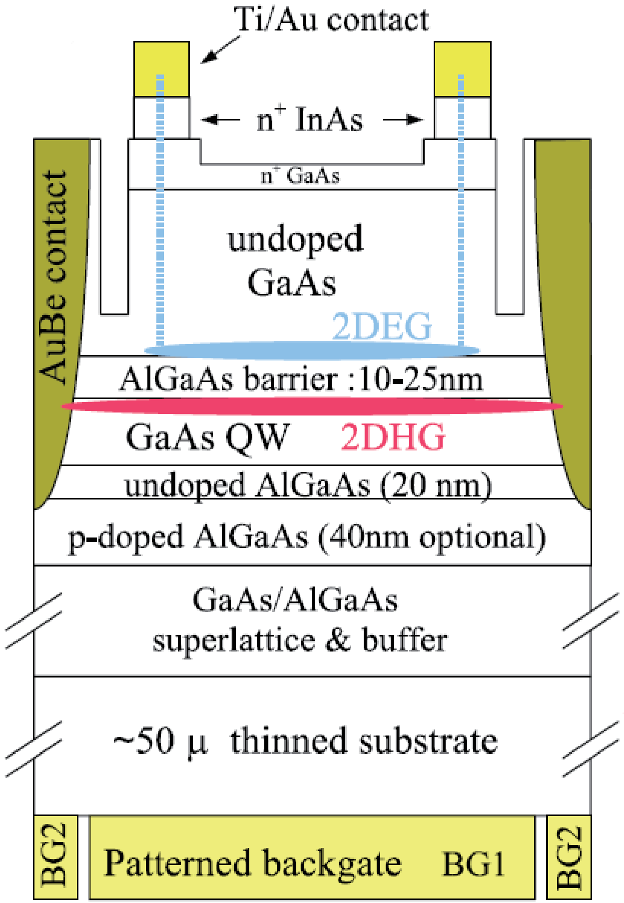
Topic 2

Fulde-Ferrell Phase in Electron-Hole Systems with Density Imbalance

K. Yamashita, K. Asano and T. Ohashi



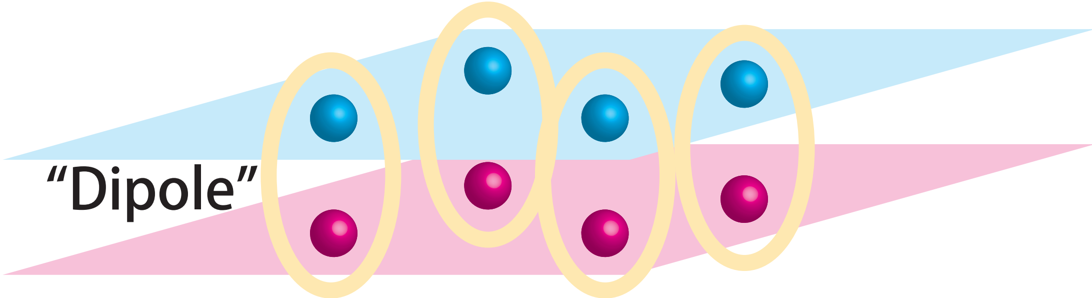
Electron-Hole Bilayer Systems



e & h densities
→ Independently controlled.

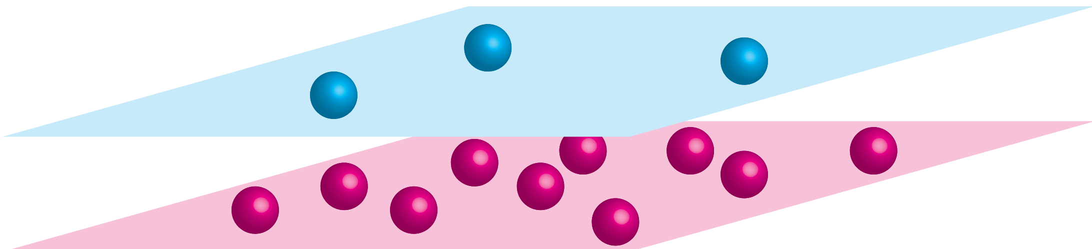
Optical spectra
Transport (Coulomb drag)

Density Balanced Case



Exciton Mott Transition
BCS-BEC Crossover

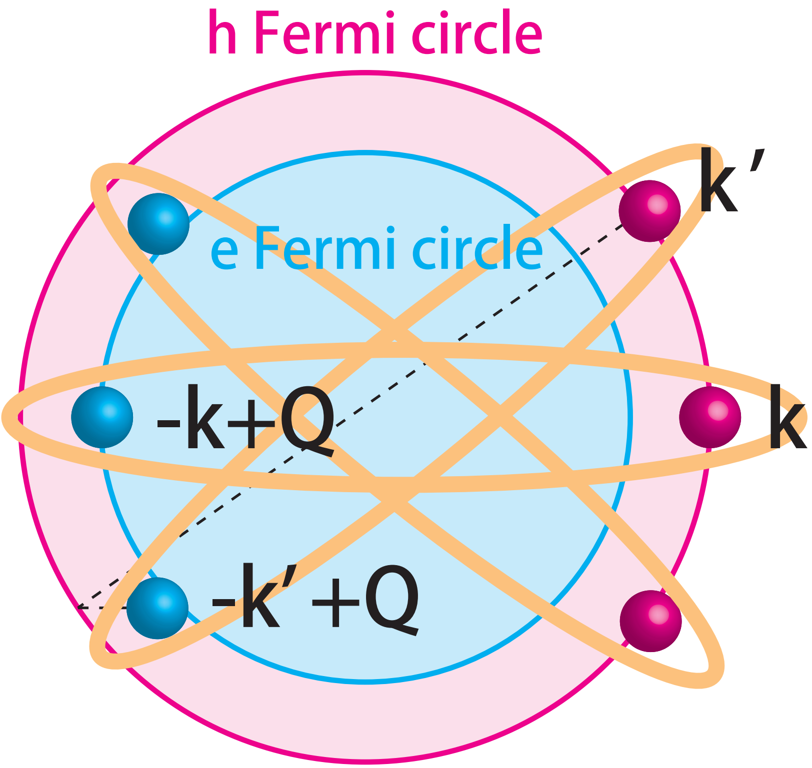
Density Imbalanced Case



Trions ?
Deformation of Fermi circles ?
Phase Seperations ?
Exotic Quantum Condensations ?

Quantum Condensations in Imbalanced e-h Systems

Fulde-Ferrell Phase



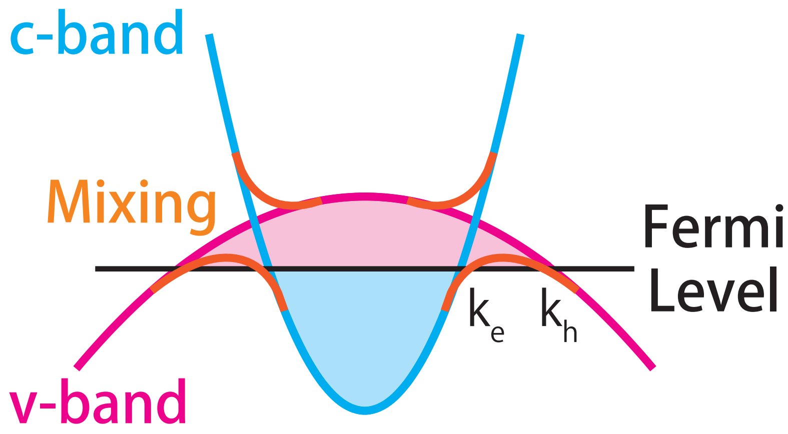
e-h pair with CM momentum Q

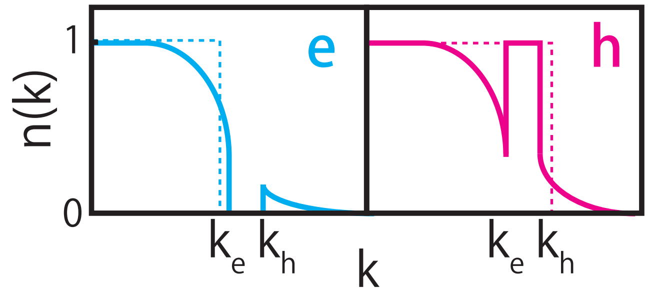
Fulde and Ferrell: PR **135**, 705(1964).

c.f. Inhomogeneous solution:
A. I. Larkin and Y. N. Ovchinnikov,
Sov. Phys. JETP **20**, 762 (1965).

Sarma Phase

(Breached pair phase)





Condensation of e-h pair with Q=0
+ Normal hole liquid

Sarma: J. Phys. Chem. Sol. **24**, 1029 (1963).
W. V. Liu and F. Wilczek: PRL **90**, 047002 (2003).

BCS Mean Field Approximation

Model Hamiltonian

$$H = \sum_{\boldsymbol{k}} \epsilon^{(\mathrm{e})}_{\boldsymbol{k}} e^{\dagger}_{\boldsymbol{k}} e_{\boldsymbol{k}} + \sum_{\boldsymbol{k}} \epsilon^{(\mathrm{h})}_{\boldsymbol{k}} h^{\dagger}_{\boldsymbol{k}} h_{\boldsymbol{k}} - \sum_{\boldsymbol{k}\neq\boldsymbol{k}',\boldsymbol{q}} V_{\boldsymbol{k}\boldsymbol{k}'} e^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}/2} h^{\dagger}_{-\boldsymbol{k}+\boldsymbol{q}/2} h_{-\boldsymbol{k}'+\boldsymbol{q}/2} e_{\boldsymbol{k}'+\boldsymbol{q}/2}$$

$$V_{\boldsymbol{k}\boldsymbol{k}'} = \frac{1}{S}\int v(\boldsymbol{r}) e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} d\boldsymbol{r} = \frac{1}{S}\cdot\frac{e^2}{2\epsilon|\boldsymbol{k}-\boldsymbol{k}'|} e^{-|\boldsymbol{k}-\boldsymbol{k}'|d}$$

Long-range Coulomb

~~Spin~~
~~e-e & h-h interactions~~
~~Interlayer charging energy~~

BCS Mean Field Approximation

$$\Omega = \sum_{\boldsymbol{k}} (\eta^{+}_{\boldsymbol{k}} - E_{\boldsymbol{k}}) + \sum_{\boldsymbol{k}\boldsymbol{k}'} \Delta_{\boldsymbol{q}}(\boldsymbol{k}) [V_{\boldsymbol{k},\boldsymbol{k}'}]^{-1} \Delta_{\boldsymbol{q}}(\boldsymbol{k}') + \sum_{\boldsymbol{k}} E^{+}_{\boldsymbol{k}} f(E^{+}_{\boldsymbol{k}}) + \sum_{\boldsymbol{k}} E^{-}_{\boldsymbol{k}} f(E^{-}_{\boldsymbol{k}})$$

$$E^{\pm}_{\boldsymbol{k}} = E_{\boldsymbol{k}} \pm \eta^{-}_{\boldsymbol{k}}\ ,\ E_{\boldsymbol{k}} = \sqrt{(\eta^{+}_{\boldsymbol{k}})^2 + \Delta_{\boldsymbol{q}}(\boldsymbol{k})}\ ,\ \eta^{\pm}_{\boldsymbol{k}} = \frac{1}{2}(\epsilon^{(\mathrm{e})}_{\boldsymbol{k}+\boldsymbol{q}/2} - \mu^{(\mathrm{e})}) \pm \frac{1}{2}(\epsilon^{(\mathrm{h})}_{-\boldsymbol{k}+\boldsymbol{q}/2} - \mu^{(\mathrm{h})})$$

Numerical optimization:

CM momentum of e-h pair $\boldsymbol{q}$
Order parameter $\Delta_{\boldsymbol{q}}(\boldsymbol{k})$
→ Minimize thermodynamic potential Ω

FF and Sarma phases are considered on an equal footing !
Thermodynamical stability is automatically considered.

Phase Diagram at Zero Temperature

Parameters

Mass ratio

$$\frac{m^{(h)}}{m^{(e)}} = 4.3$$

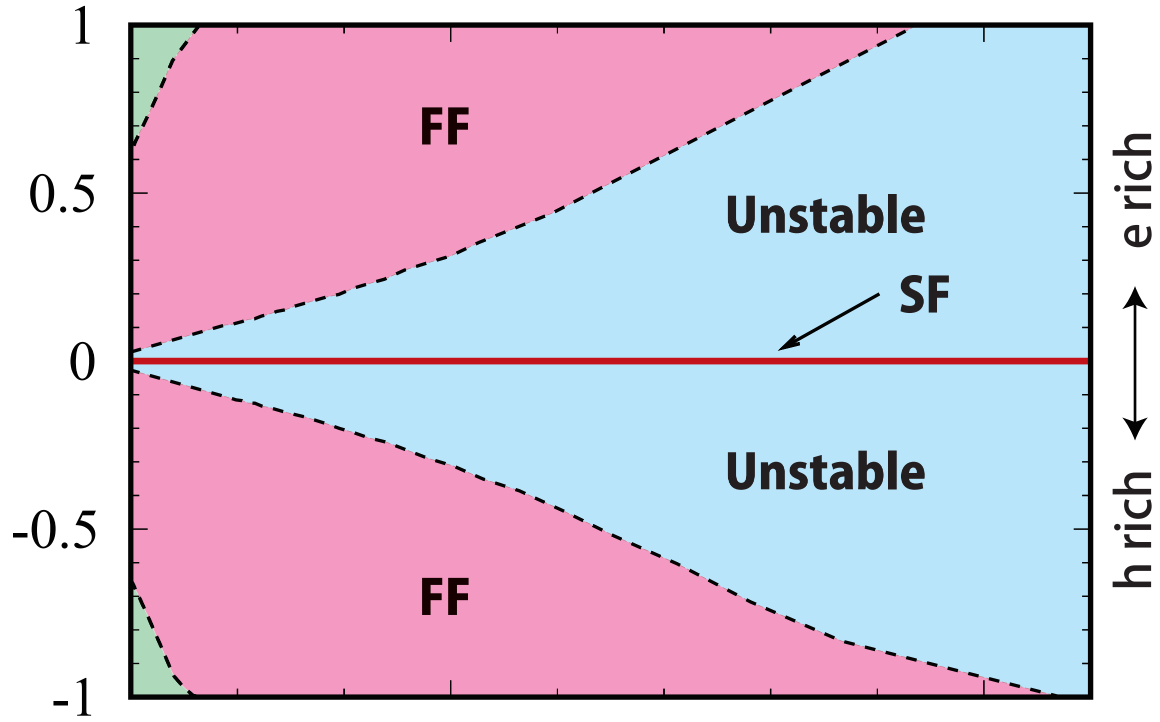
Interlayer distance

$$d = 2a_B^{(2D)} = \frac{\epsilon}{e^2 m_r}$$

SF: superfluid phase (excitonic insulator)

FF: Fulde-Ferrell phase

Unstable: no uniform solution

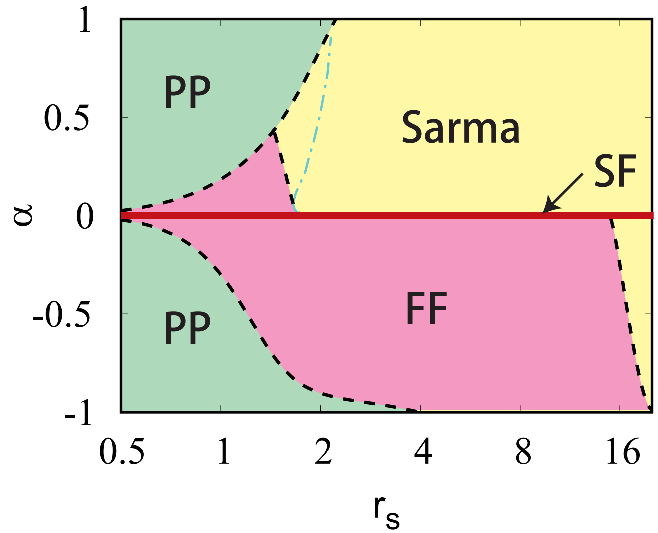


$$\alpha = \frac{N^{(e)} - N^{(h)}}{N^{(e)} + N^{(h)}}$$

Interaction Strength $r_s = \frac{1}{a_B^{(2D)}} \sqrt{\frac{2S}{\pi(N^{(e)} + N^{(h)})}}$

c.f. Previous calculation

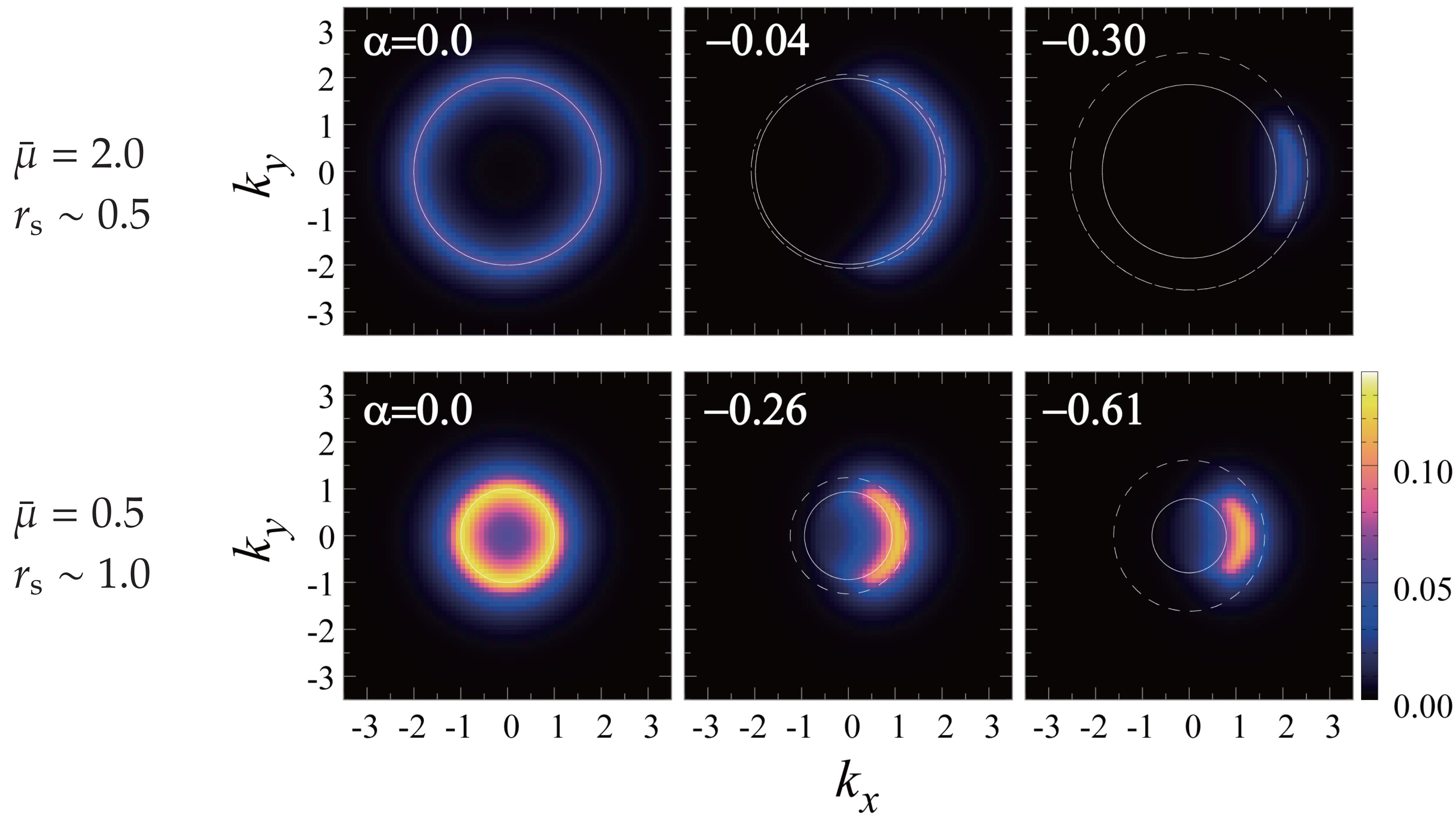
Instability of Sarma phase toward FF phase



Pieri et al. PRB75,113301 (2007).

Order Parameters

Order parameter mixing effects stabilize the FF phase.



Epitome of Condensed Matter Physics !