

イオントラップを用いた 量子ゲート実験

大阪大学大学院基礎工学研究科

占部伸二

H22. 8. 27 量子情報処理サマースクール

The contents of this lecture

1. Introduction
2. Ion trap and ion qubit
3. Initialization and state detection
4. Coherence time
5. Quantum gate
6. Spin dependent force and its application

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Graduate students

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Master course

K.Uchida, K.Tamura, K.Masuda, N.Wada,

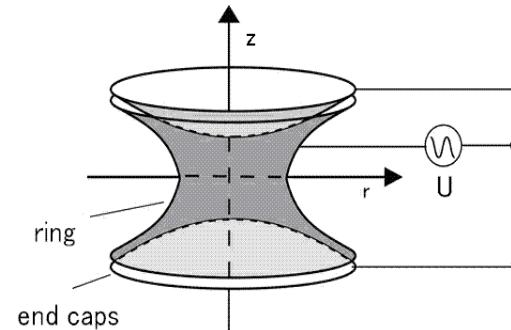
T.Watanabe,

Y.Ibaragi, T.Ohno, T.Kanda, K.Kohda, Y.Tateishi,

T.Fujiwara

1. Introduction

Ion trap: Trapping of charged particles with electromagnetic fields
Mass spectrometry (1950's),
Spectroscopy ← Laser cooling (1975)
Wineland & Dehmelt,
Hänsch & Schawlow

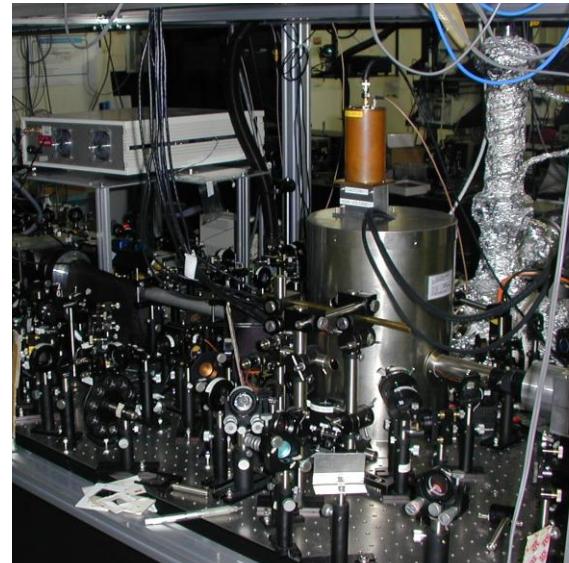
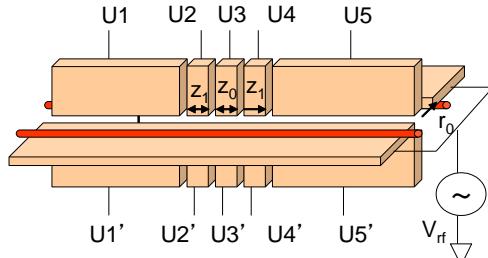
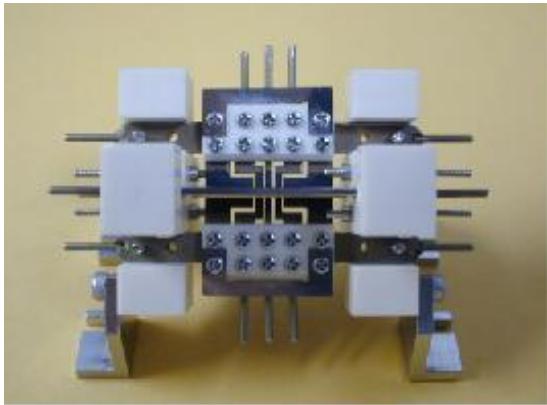


High resolution spectroscopy:
Isolation from the environment
→ Optical frequency standard with single ions($Q \sim 10^{15}$)
 $Mg^+ - Al^+$: 8×10^{-18} : quantum logic spectroscopy(NIST, 2010)

Quantum information processing
qubit: internal states of an ion in an ion string
Cirac & Zoller (1995)

2. Ion traps and ion qubit

Linear rf traps



Typical trap operating parameters:

ultra high vacuum: less than 10^{-8} Pa

trap dimension (r_0) : 0.6 mm

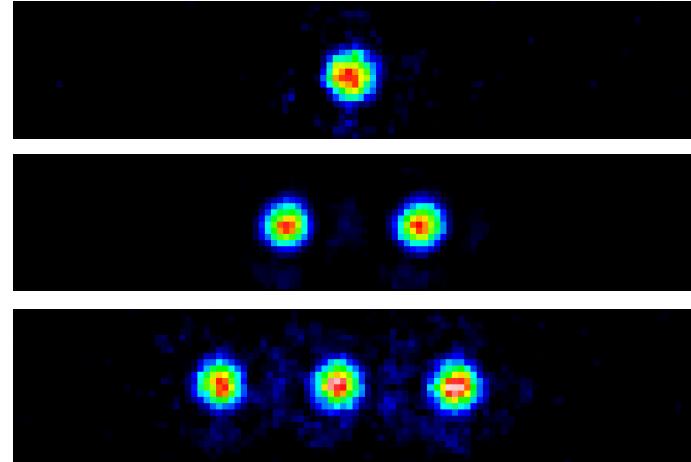
rf frequency (Ω) : 24 MHz

effective potential depth (V_{eff}) : ~ 10 V

collective motional frequency

z : 0.7 MHz, x : 2.1 MHz, y : 2.3 MHz

distance between ions in strings: about 7μ



Images of 1,2,3 ions

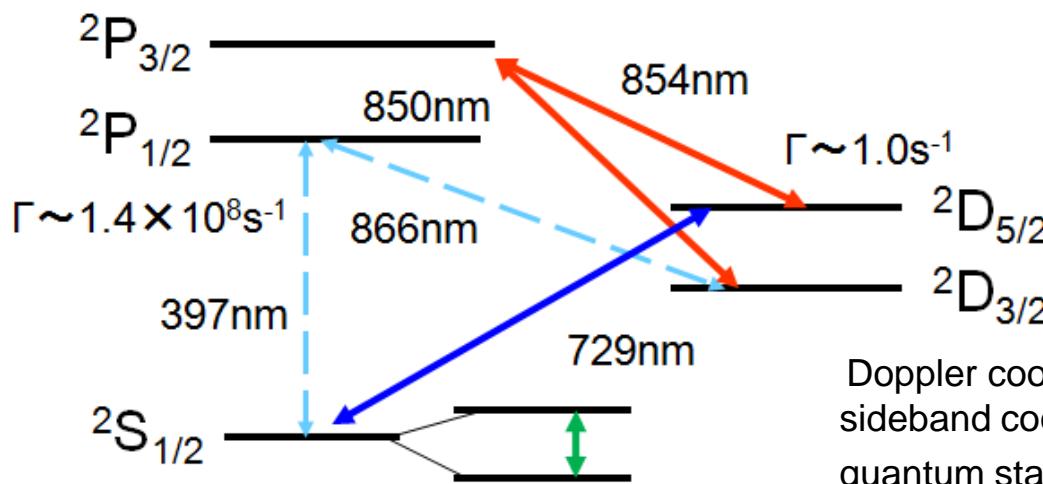
Ion qubit

Hyperfine levels, ground state - metastable state

Ions: Be⁺, Mg⁺, Zn⁺, Cd⁺ (hyperfine qubits)

Ca⁺, Sr⁺, Ba⁺, Yb⁺, Hg⁺ (hyperfine or metastable qubits)

$^{40}\text{Ca}^+$ ion



Doppler cooling : 397nm, 866nm
sideband cooling : 729nm, 854nm
quantum state control : 729nm etc.

Zeeman qubits: $^2\text{S}_{1/2}, m=-1/2$ - $^2\text{S}_{1/2}, m=1/2$ ($\sim 10\text{MHz}$)

Optical-transition qubits: $^2\text{S}_{1/2}$ - $^2\text{D}_{5/2}$ (729nm)

Terahertz-separated qubits : $^2\text{D}_{3/2}$ - $^2\text{D}_{5/2}$ (1.82THz)

Terahertz-separated qubit:

Raman transitions driven by phase locked lasers bridged by a frequency comb

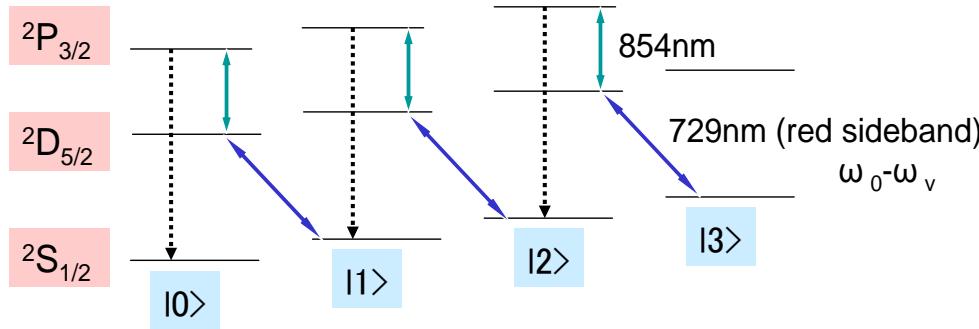
3. Initialization and state detection

Initialization

internal state: optical pumping $\sim 100\%$

external state: Doppler cooling : from 10000 K to mK

sideband cooling : to the motional ground state

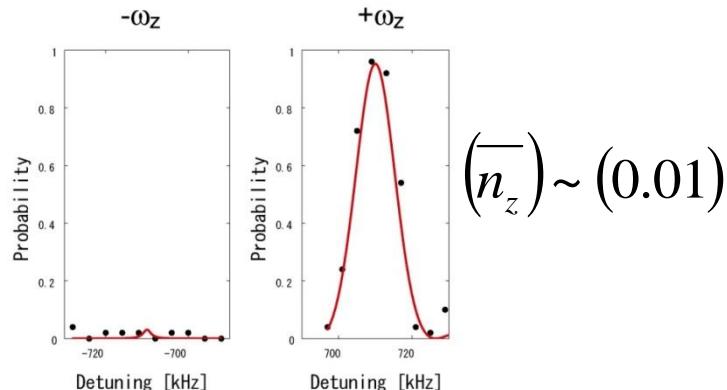


$$S_- / S_+ = \langle n_z \rangle / (\langle n_z \rangle + 1)$$

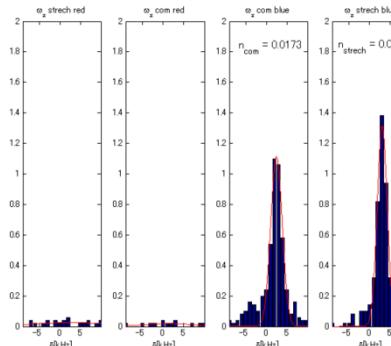
S_r: height of red sideband spectrum

S_+ : height of blue sideband spectrum

Red and blue sideband spectra of axial modes in S-D transitions after sideband cooling



Single ions



Two ions

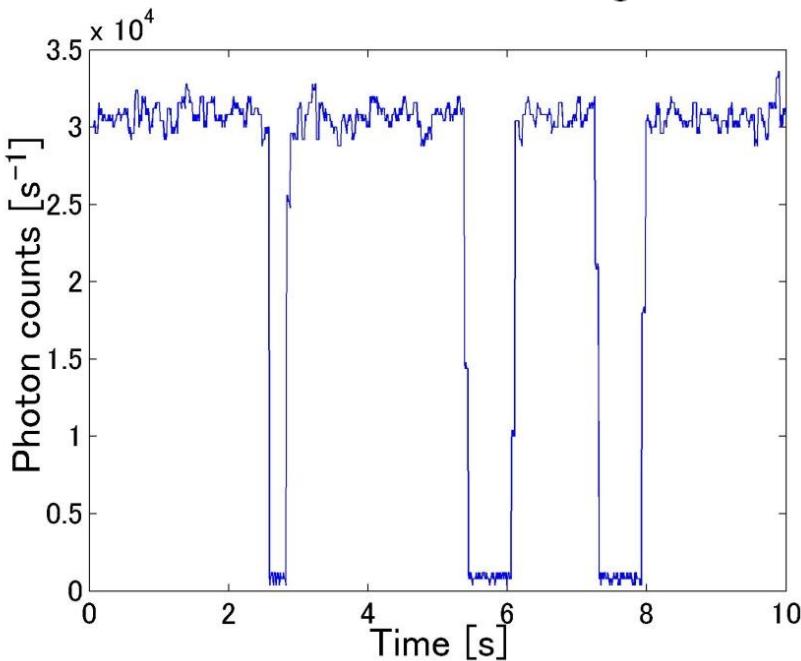
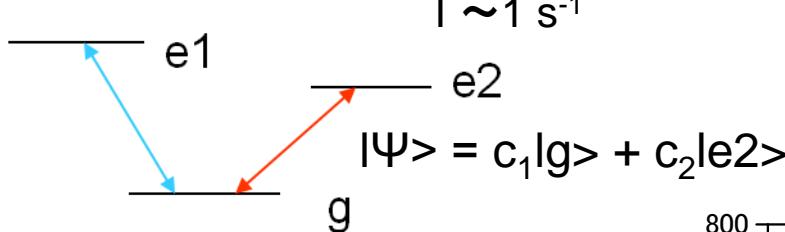
$$\left(\overline{n_{COM}}, \overline{n_{st}}\right) \\ \sim (0.017, 0.020)$$

State detection

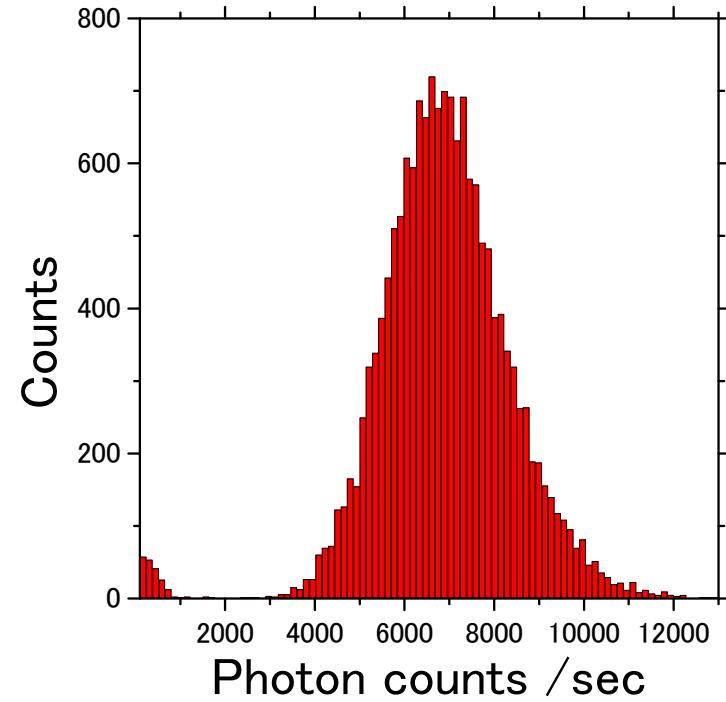
Electron shelving method: nearly 100% efficiency

Cycling transitions \cdots fluorescence signal $\sim 3 \times 10^4$ photons/s/ion

$$\Gamma \sim 1.4 \times 10^{-8} \text{ s}^{-1}$$



Quantum jump signal of single ions



Histogram of photon counts

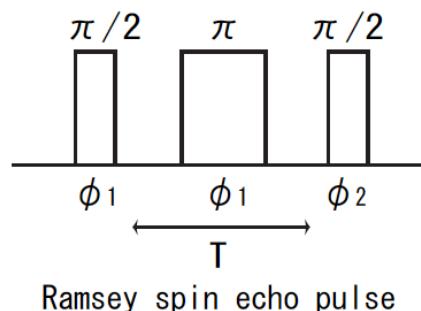
4. Coherence time

Coherence time: hyperfine ground state ••• several minutes
metastable state ••• about 1.0 s ($^{40}\text{Ca}^+$)

External disturbance: magnetic field fluctuation, laser linewidth etc.

Coherence time of terahertz-separated ($D_{3/2}$ - $D_{5/2}$) qubits

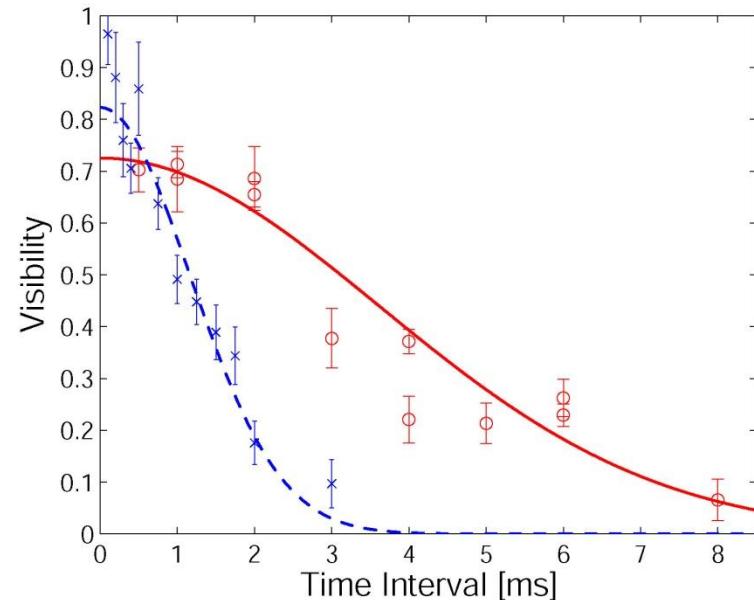
- Spin echo sequence



Coherence Revival by Spin echo π pulse
(Effective against slow magnetic field fluctuation)

Coherence time of normal and spin echo sequence

	Spin echo sequence	Normal sequence
Coherence time (e^{-1} decay)	5.1 [ms]	1.7 [ms]



Fringe visibility of Ramsey signal

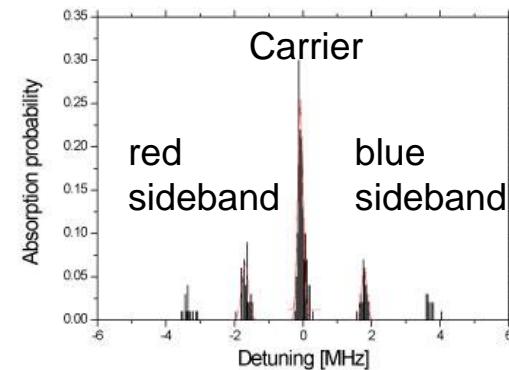
K.Toyoda, H.Shiibara, S.Haze, R.Yamazaki,
S.Urabe, Phys. Rev. A 79, 023419, 2009

5. Quantum gate

Entanglement between qubits ... mediated by the collective motional states.

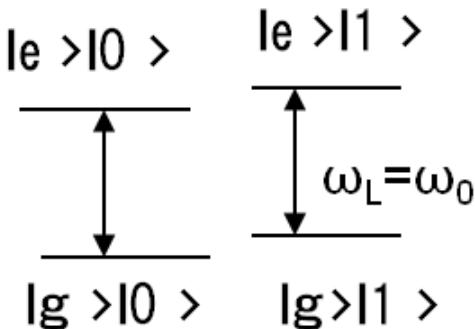
manipulation of qubits ... laser pulses
 carrier pulse,
 red sideband pulse,
 blue sideband pulse

hyperfine qubit: Raman transitions,
 metastable qubit: E2 transitions

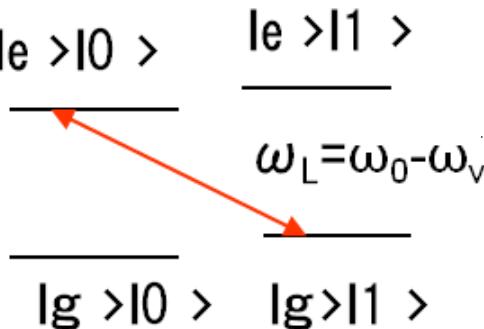


Optical spectrum of single ions
 (Electric quadrupole transitions)

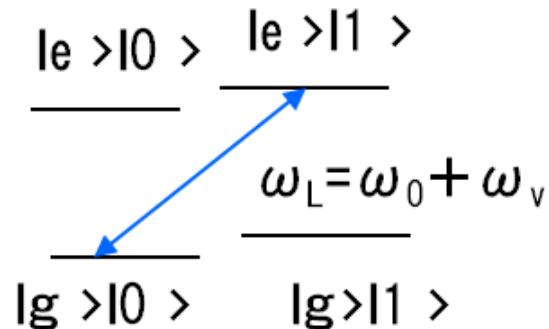
carrier transitions



red sideband transitions



blue sideband transitions



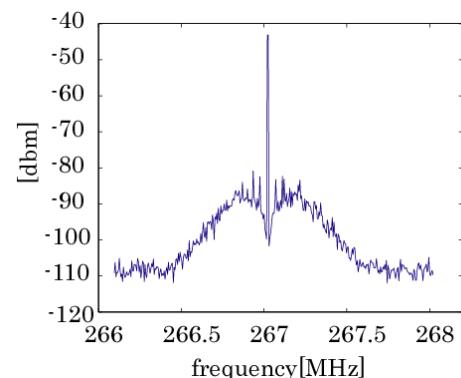
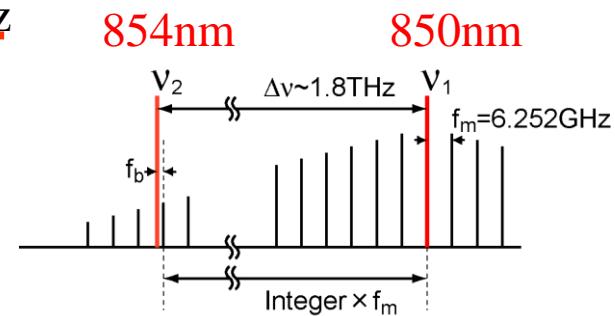
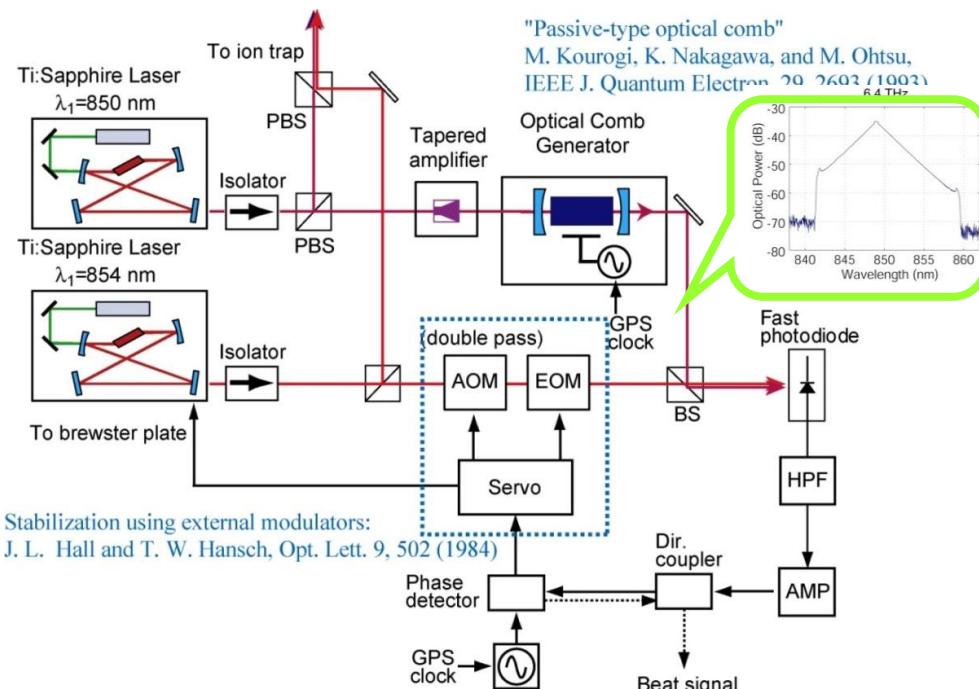
Qubit : internal states, $|g\rangle$, $|e\rangle$, Bus bit : motional states, $|I0\rangle$, $|I1\rangle$

729nm Ti : sapphire laser for S-D qubits

The frequency is locked to a ULE cavity by the Pound-Drever-Hall method.
line width : less than 1KHz,
frequency drift : less than 3kHz/h

Phase-locked laser system with a frequency comb for D-D qubits

$$1.8194 \cdots \text{THz} \sim 6.252 \text{ GHz} \times 291 \text{ lines} + 267. \cdots \text{MHz}$$



R.Yamazaki, T.Iwai, K.Toyoda, and S.Urabe, Opt. Lett. Vol.32,No.5,(2007)2085
S.Haze, Y.Senokuchi, R.Yamazaki, K.Toyoda, S.Urabe, Appl. Phys. B to be published

$$\Delta\Phi_{\text{rms}} = 50 \text{ mrad}$$

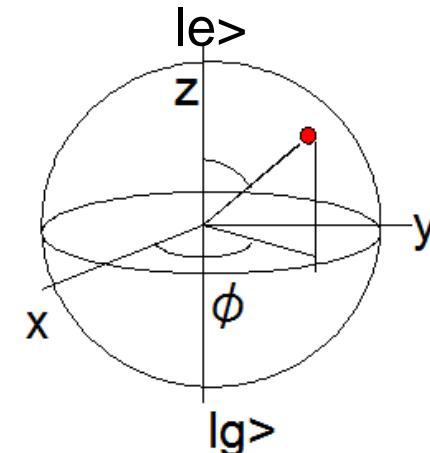
5.1 Single qubit rotation

carrier pulse, pulse area : $\Omega_0 t = \theta$, phase: φ

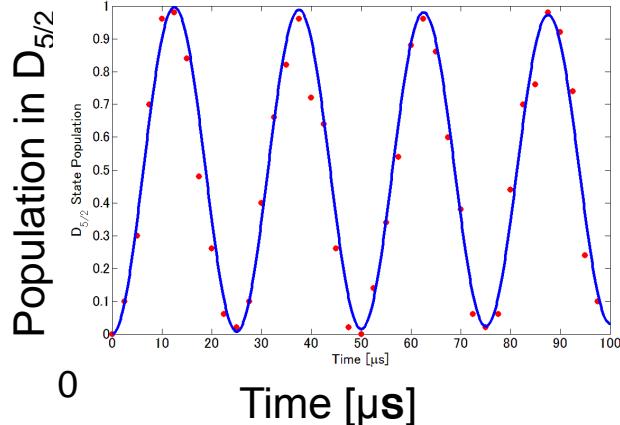
$$|\Psi(t)\rangle = \hat{R}(\theta, \varphi) |\Psi(0)\rangle$$

$$\hat{R}(\theta, \varphi) = \exp\left[i \frac{\theta}{2} (\hat{\sigma}_x \cos \varphi - \hat{\sigma}_y \sin \varphi)\right]$$

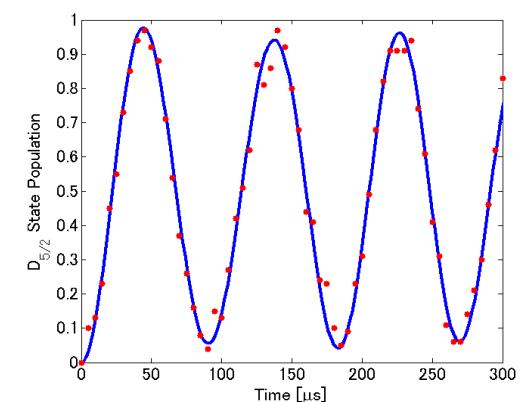
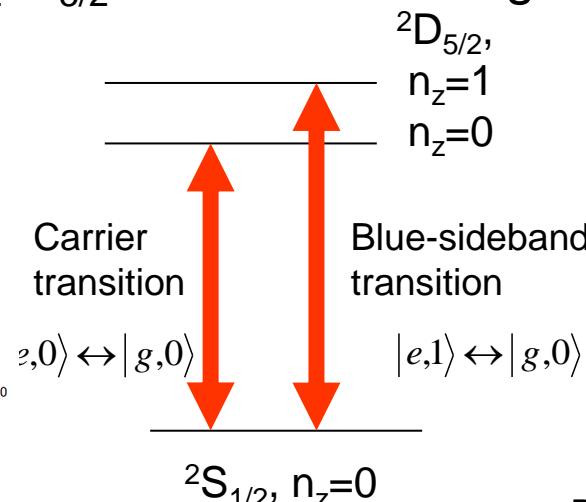
$$\Rightarrow \begin{pmatrix} \cos(\theta/2) & ie^{i\varphi} \sin(\theta/2) \\ ie^{-i\varphi} \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$



Rabi oscillations of $S_{1/2}$ - $D_{5/2}$ transitions in single $^{40}\text{Ca}^+$ ions



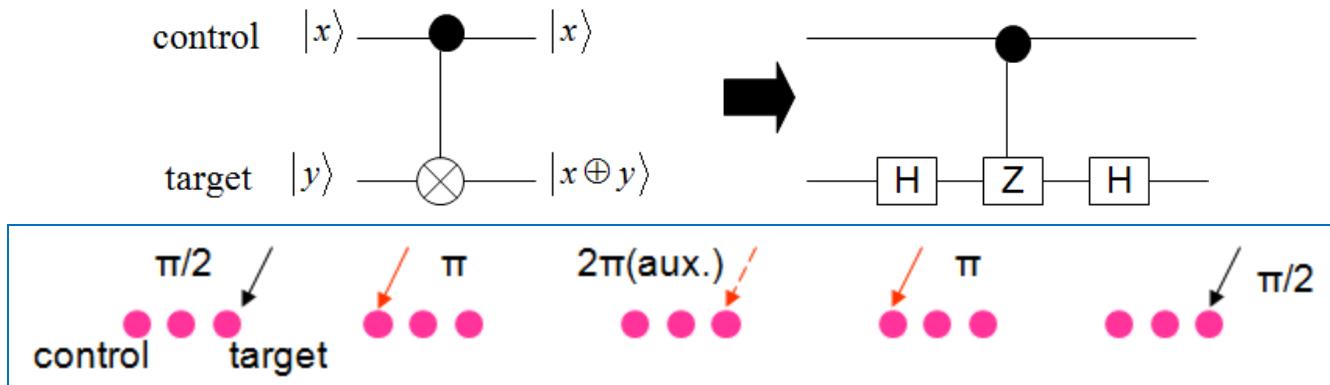
Carrier Rabi oscillation



Blue-sideband Rabi oscillation

5.2 Two-qubit gate

Cirac-Zoller C-Not gate



carrier $\pi/2$ pulse: Hadamard gate $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$,

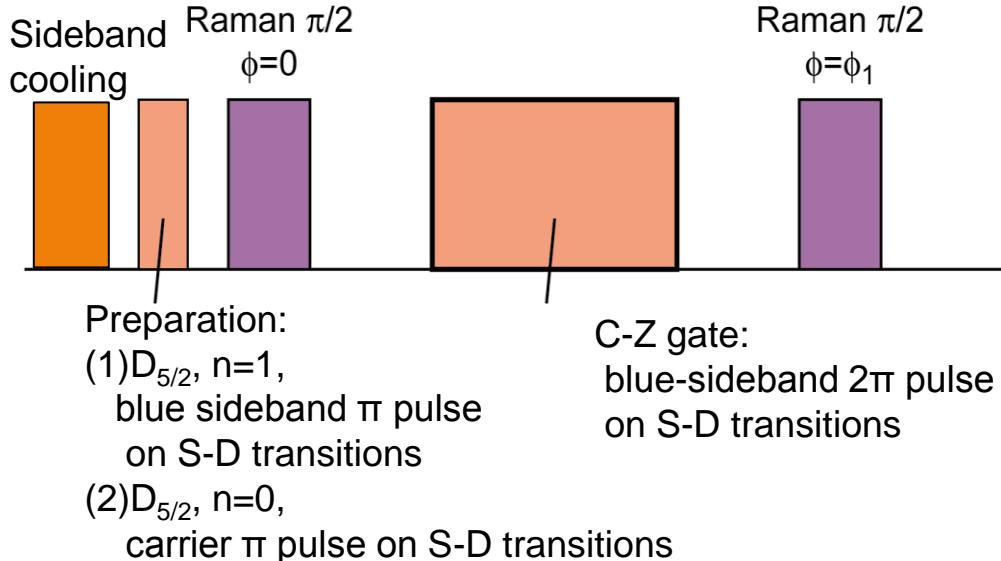
red sideband π pulse: swap operation

$$|0, g\rangle \rightarrow |0, g\rangle, \quad |0, e\rangle \rightarrow |1, g\rangle, \quad |0\rangle (\alpha|g\rangle + \beta|e\rangle) \rightarrow (\alpha|0\rangle + \beta|1\rangle)|g\rangle$$

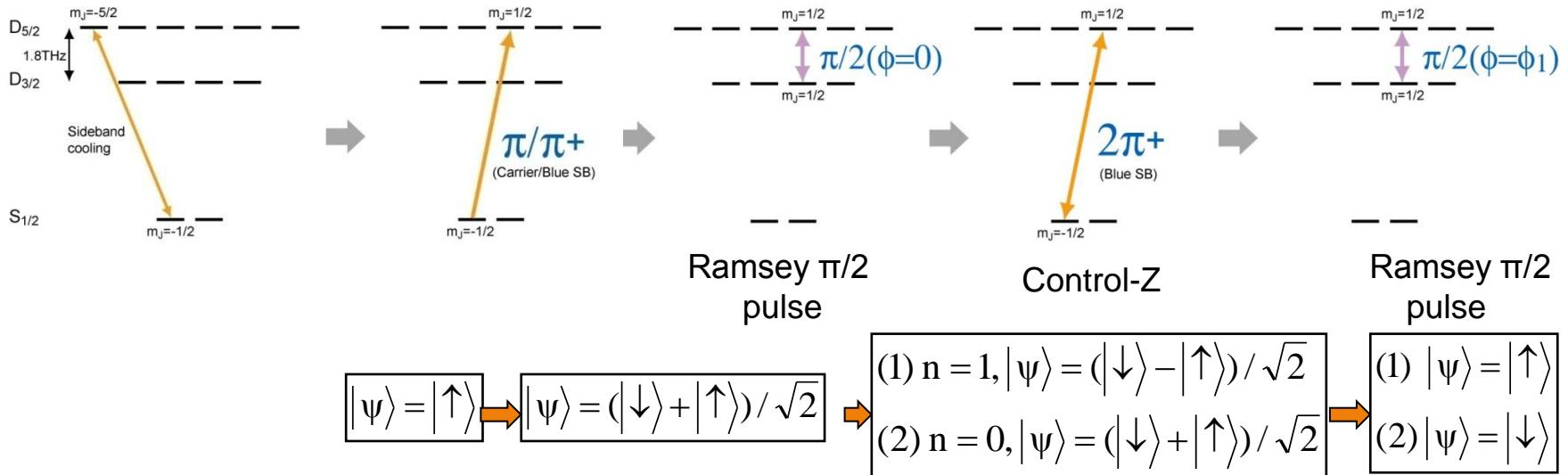
red sideband 2π pulse(aux) : control- σ_z : $C - Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \hat{I} & \hat{0} \\ \hat{0} & Z \end{pmatrix}$

Cirac-Zoller Gate experiment using terahertz-separated qubits

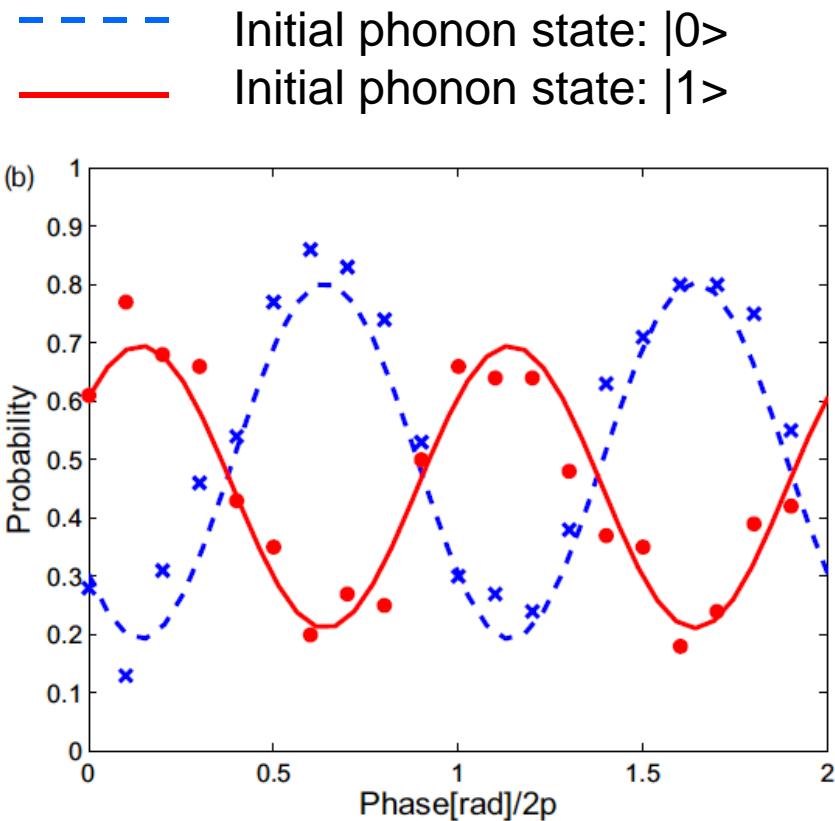
Single $^{40}\text{Ca}^+$,
 Control bit : phonon states
 $|0\rangle$: $n=0$
 $|1\rangle$: $n=1$
 Target bit: internal states
 $| \uparrow \rangle$: $D_{5/2}(m=1/2)$
 $| \downarrow \rangle$: $D_{3/2}(m=1/2)$



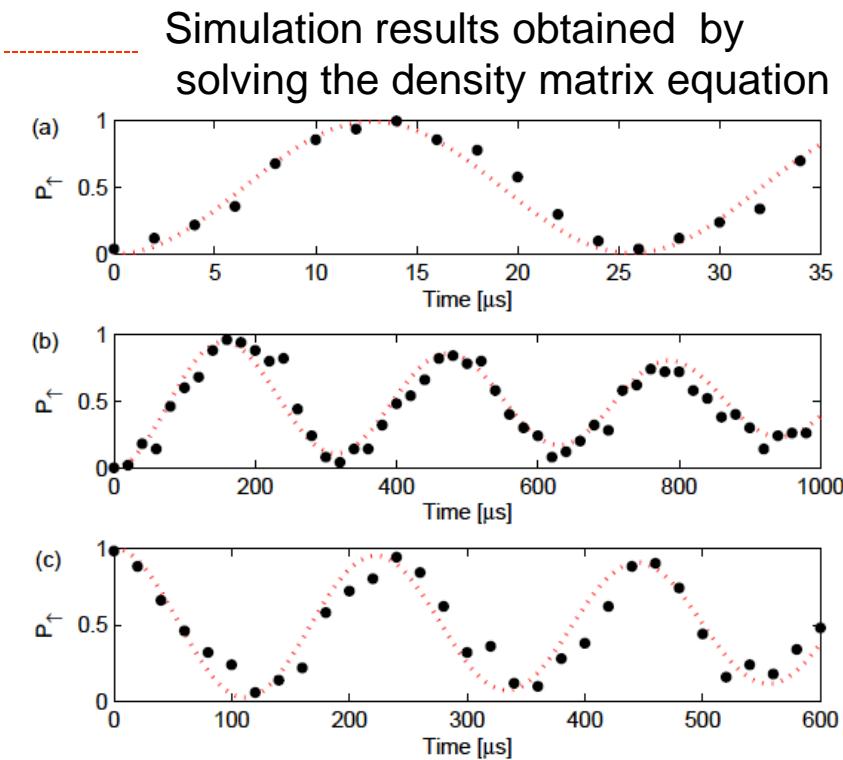
CZ gate excitation scheme



CZ gate result



Fidelity ~ 0.74



5.3 Creation of two-particle entanglement

2 two-level ions are irradiated equally with a laser pulse, whose frequency is tuned near the first red sideband of the COM mode.

The total number of excitation of quanta is conserved and the basis of one-quantum Hamiltonian is composed of : $\{|g,g\rangle|1\rangle, |e,g\rangle|0\rangle, |g,e\rangle|0\rangle\}$

Interaction Hamiltonian (rotating frame)

Basis : $\{|g,g\rangle|1\rangle, |e,g\rangle|0\rangle, |g,e\rangle|0\rangle\}$

$$\hat{H}_I = \begin{bmatrix} \hbar\Delta_1 & \hbar\Omega/2 & \hbar\Omega/2 \\ \hbar\Omega/2 & \hbar\Delta_0 & 0 \\ \hbar\Omega/2 & 0 & \Delta_0 \end{bmatrix}$$

$$\Delta_0 = -\Delta_1 = \delta/2$$

$$\delta = \omega_0 - \nu - \omega_L, \quad \Omega = \eta\Omega'/\sqrt{N},$$

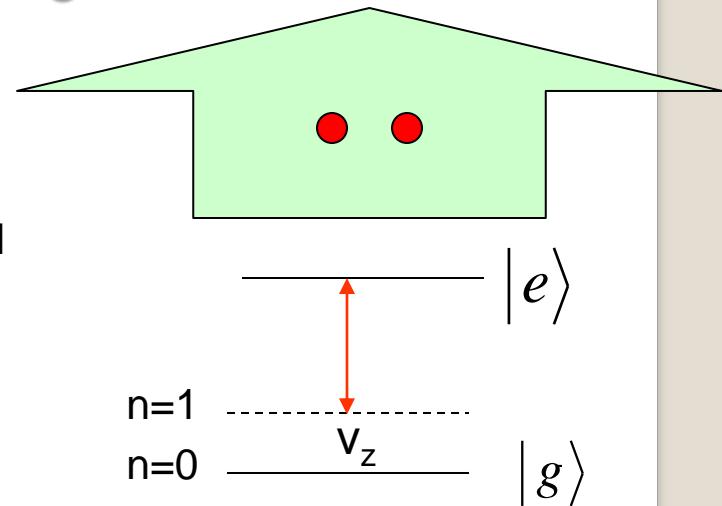
η : Lamb - Dicke parameter

Basis : $\{|g,g\rangle|1\rangle, |\Psi_+\rangle|0\rangle, |\Psi_-\rangle|0\rangle\}$

$$\hat{H}_I = \begin{bmatrix} \hbar\Delta_1 & \hbar\Omega/2 & 0 \\ \hbar\Omega/2 & \hbar\Delta_0 & 0 \\ 0 & 0 & \hbar\Delta_0 \end{bmatrix}$$

$$|\Psi_+\rangle = (|e,g\rangle + |g,e\rangle)/\sqrt{2},$$

$$|\Psi_-\rangle = (|e,g\rangle - |g,e\rangle)/\sqrt{2}$$



I.E.Linington & N.V. Vitanov:
Phys. Rev. A77,010302 (2008)

symmetric two-state subspace: $\{|g, g\rangle|1\rangle, |\Psi_+\rangle|0\rangle\}$

(1) Rapid adiabatic passage method

$$|g, g\rangle|1\rangle \Rightarrow \frac{1}{\sqrt{2}}(|g, e\rangle + |e, g\rangle)|0\rangle$$

Adiabatic condition

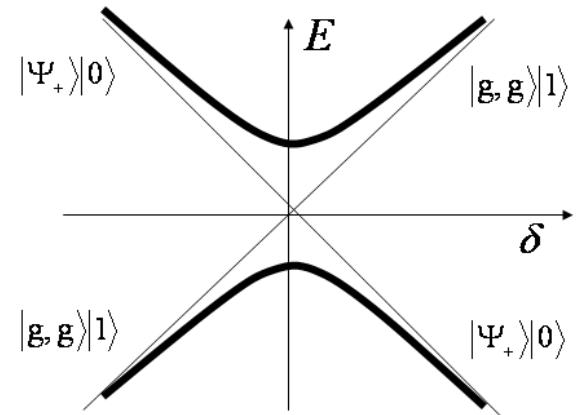
$$\frac{1}{2}|\dot{\Omega}\delta - \Omega\dot{\delta}| \ll (\Omega^2 + \delta^2)^{3/2}$$

N ions

$$|g, g, \dots, g\rangle|m\rangle \Rightarrow |W_m^N\rangle|0\rangle,$$

RSB RAP pulse

Energy of dressed states



$$\text{Dicke state: } |W_m^N\rangle = \frac{1}{\sqrt{C_m^N}} \sum_k P_k |e, e, \dots, e, g, \dots, g\rangle$$

I.E.Linington & N.V. Vitanov: Phys. Rev. A77,010302 (2008)

(2) Red sideband π pulse method (two-state system)

$$\Delta_0 = -\Delta_1 = 0 \quad |\Psi\rangle = \cos(\Omega t/2)|g, g\rangle|1\rangle - i \sin(\Omega t/2)|\Psi_+\rangle|0\rangle$$

$$|g, g\rangle|1\rangle, (t=0) \Rightarrow \frac{1}{\sqrt{2}}(|g, e\rangle + |e, g\rangle)|0\rangle, (t=\pi/\Omega)$$

D.B.Hume et al. PRA, 80, 052302, 2009

Important process:
initialization to $|g_1, g_2\rangle|1\rangle$

Individual addressing to
one ion is necessary.

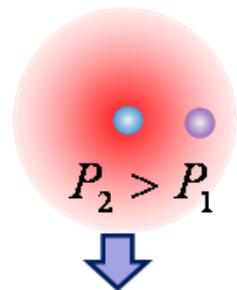
$$|g_1, g_2\rangle|0\rangle \xrightarrow{\text{blue sideband } \pi} |e_1, g_2\rangle|1\rangle \xrightarrow{\text{carrier } \pi} |g_1, g_2\rangle|1\rangle$$

blue sideband π

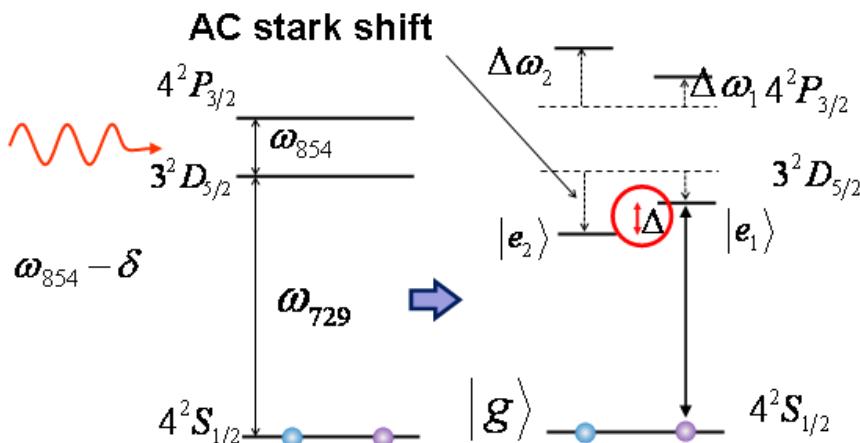
carrier π

differential AC Stark shift methods

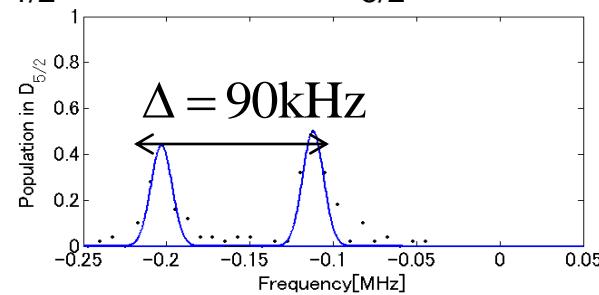
Off resonant 854nm laser
(100GHz)



$$\Delta\omega_2 > \Delta\omega_1$$

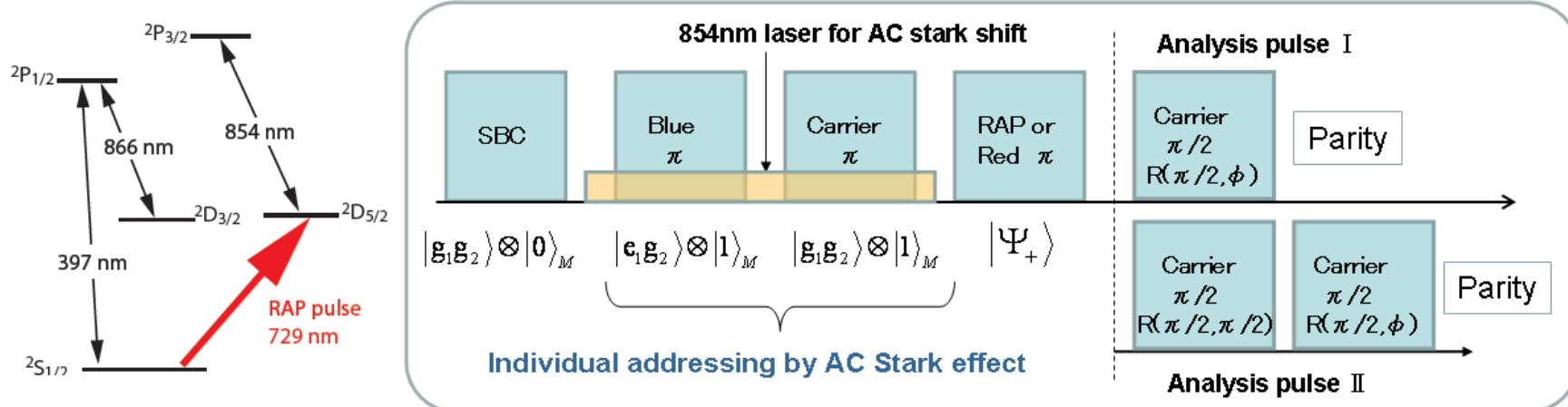


Qubit: $S_{1/2}, m=-1/2$ and $D_{5/2}, m=-5/2$



Carrier spectrum of two ions:
AC Stark shift, on

Pulse sequence of generation of two-particle entangled states



RAP: rapid adiabatic passage pulse

Fidelity of the generated states

$$F \equiv \langle \Psi_+ | \rho | \Psi_+ \rangle = \frac{1}{2} (\underbrace{\rho_{ge,ge} + \rho_{eg,eg}}_{\text{Diagonal terms}} + \underbrace{\rho_{eg,ge} + \rho_{ge,eg}}_{\text{Off diagonal terms}})$$

Diagonal terms: probability of single-ion fluorescing events after the state creation

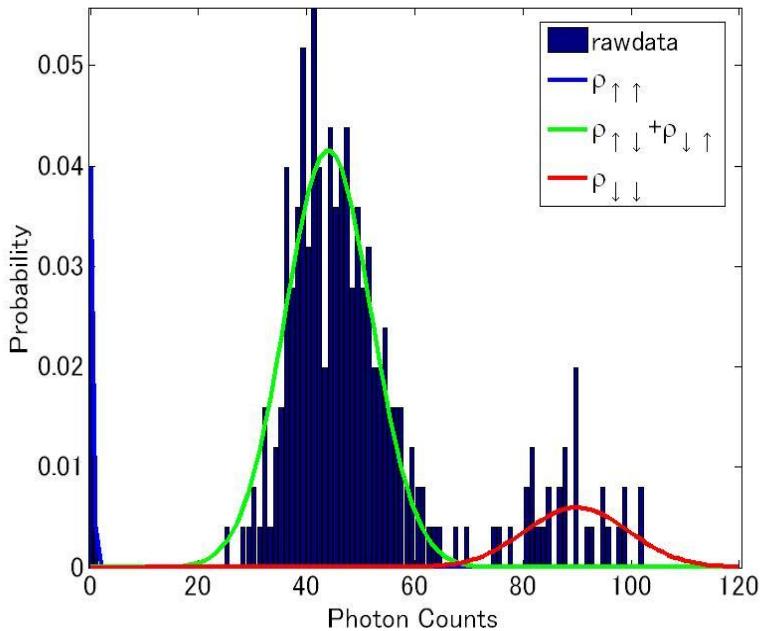
Off diagonal terms: parity signal after analysis pulse I or II

$$\text{Analysis Pulse I : Parity } P(\phi) \equiv \langle \sigma_{z1} \sigma_{z2} \rangle = 2 |\rho_{gg,ee}| \overset{0 \text{ for Dicke states}}{\cos(2\phi)} + (\rho_{ge,eg} + \rho_{eg,ge})$$

$$\text{Analysis Pulse II : Parity } P(\phi) = \langle \sigma_{1z} \sigma_{2z} \rangle = \cos 2\phi, \text{ for } \rho = |\Psi_+\rangle \langle \Psi_+|$$

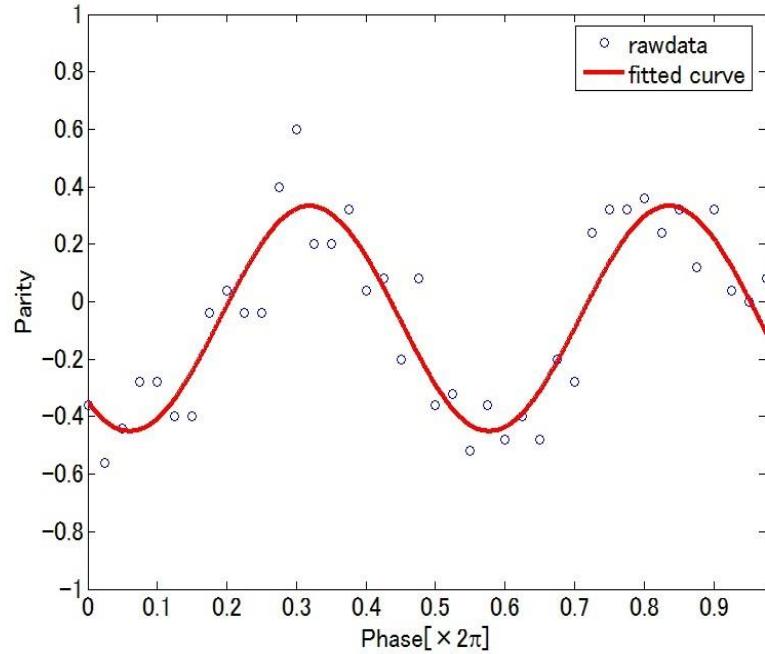
Result of the RAP method

Histogram of photon counts



$$\rho_{\uparrow\downarrow\downarrow} + \rho_{\downarrow\uparrow\uparrow} = 0.83 \pm 0.06 (\pm \sigma)$$

Parity signal after analysis pulse II

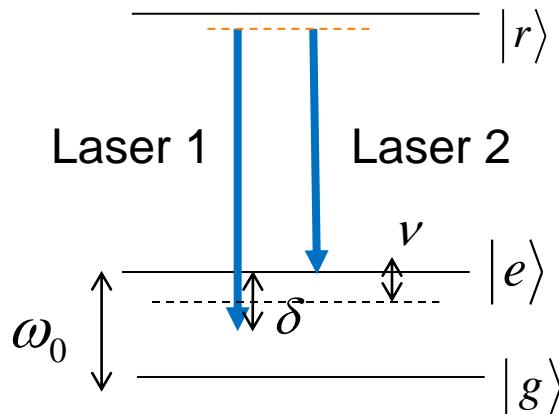


$$\rho_{\uparrow\downarrow\downarrow} + \rho_{\downarrow\uparrow\uparrow} \geq 0.40 \pm 0.03 (\pm \sigma)$$

$$F \geq 0.62 \pm 0.06$$

6. Spin dependent force and its applications

σ_z dependent force



D.Leibfried et al., Nature,422,412,2003
G.J.Milburn et al, Fortschr. Phys.,48,801,2000

$$k_z = (\vec{k}_{L1} - \vec{k}_{L2}) \cdot \vec{z}$$

$$z_0 = n(2\pi / k_z)$$

Optical dipole force (state dependent)

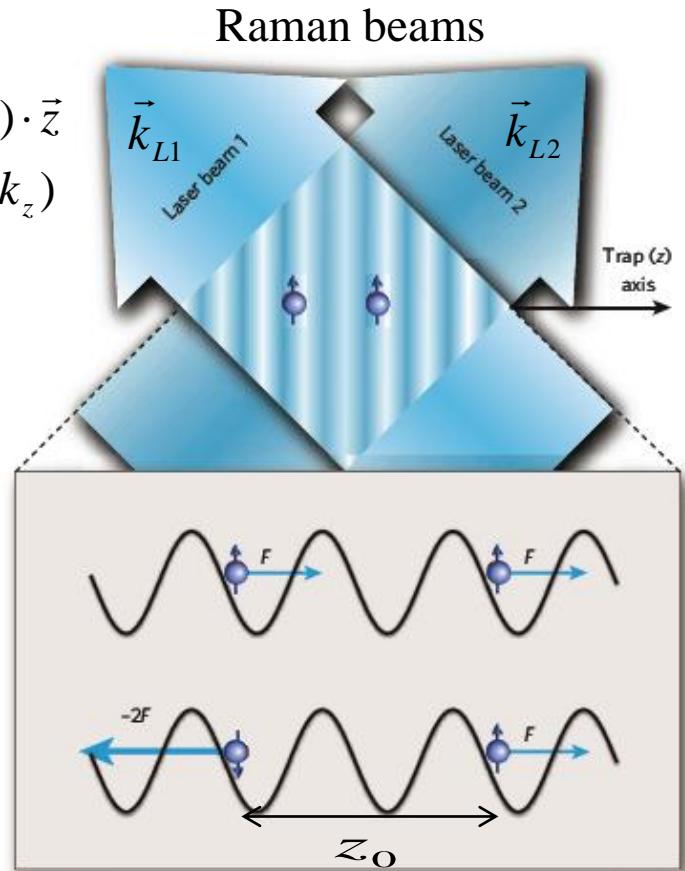
Spin dependent force Hamiltonian, $\Omega_j = \Omega$

$$\hat{H}_{\text{int}} = \hbar\Omega \sum_j k_z \hat{z} \sigma_{jz} \cos \delta t$$

COM mode: $k_z \hat{z}_j \rightarrow \eta(\hat{a} + \hat{a}^\dagger)$,

Interaction picture and rotating wave approximation

$$\hat{H}_{\text{int}} = (\hbar\Omega\eta) \left(\sum_j \hat{\sigma}_{j,z} / 2 \right) (\hat{a} e^{-i(\nu-\delta)t} + \hat{a}^\dagger e^{i(\nu-\delta)t})$$

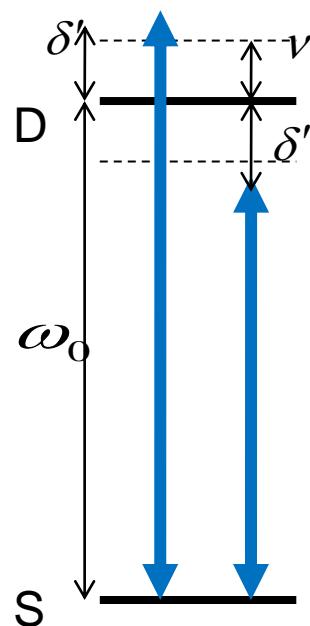


σ_ϕ dependent force (Molmer & Sørensen gate)

A.Sørensen & K.Molmer, PRA,62,022311, 2000
P.C.Haljan et al, PRA, 72,062316, 2005
C.F Roos, New J. Phys., 10, 013002, 2008

Two-color beams(Ω_1, Ω_2), 2 ions

E2 transition



Dressed states

$$|+\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle)$$

Amplitude-modulated beams

$$\begin{aligned} & \cos[(\omega_0 + \delta')t + k_1 z] + \cos[(\omega_0 - \delta')t + k_2 z] \\ &= 2 \cos[\delta' t + (k_1 - k_2)z/2] \cos[\omega_0 t + (k_1 + k_2)z/2] \end{aligned}$$

Modulation signal Resonant carrier wave

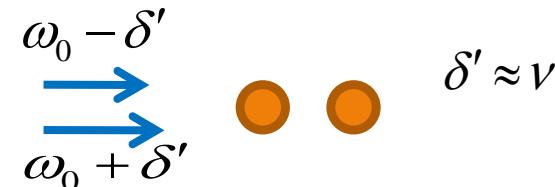
$$\hat{H}_{\text{int}} = \hbar \Omega_0 e^{-i\varphi} \sum_{i=1,2} \sigma_{+j} (e^{-i\delta't + ik_1 z} + e^{i\delta't + ik_2 z}) + h.c.$$

COM mode: $k_z \hat{z}_j \rightarrow \eta_c (\hat{a}_c + \hat{a}_c^\dagger)$, $\eta_c = k_z \sqrt{\hbar/2M\nu N}$

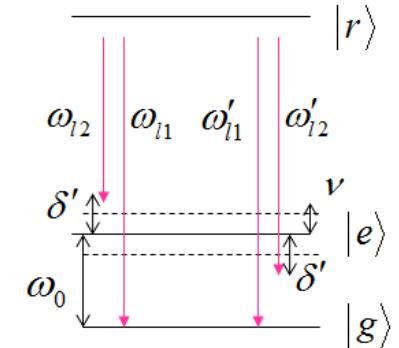
stretch mode: $k_z \hat{z}_1 \rightarrow -\eta_s (\hat{a}_s + \hat{a}_s^\dagger)$, $k_z \hat{z}_2 \rightarrow \eta_s (\hat{a}_s + \hat{a}_s^\dagger)$, $\eta_s = k_z \sqrt{\hbar/2\mu\nu N}$

collective addressing: $\Omega_1^j = \Omega_2^j = \Omega_0$

detuning: $\omega_1 = \omega_0 + \delta'$, $\omega_2 = \omega_0 - \delta'$



Two pairs of Raman beams



(1) General field coupling

close to the sidebands : $\nu - \delta' \ll \delta'$, and $\Omega_0 \ll \delta'$

Interaction picture and rotating wave approximation

$$\hat{H}_{\text{int}} = -\hbar \eta \Omega_0 [\hat{J}_x \cos \varphi' + \hat{J}_y \sin \varphi'] (\hat{a}^\dagger e^{i(\nu-\delta')t} + \hat{a} e^{-i(\nu-\delta')t})$$

$\underbrace{\sigma_\Phi \text{ dependence}}$

$$\hat{J}_x = \frac{1}{2} \sum_j \hat{\sigma}_{xj}, \quad \hat{J}_y = \frac{1}{2} \sum_j \hat{\sigma}_{yj}$$

Time evolution: $|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$

$$\hat{U}(t) = \underbrace{\exp[-\hat{J}_x \{\alpha(t)\hat{a}^\dagger - \alpha^*(t)\hat{a}\}]} \cdot \exp(-i\Phi_g \hat{J}_x^2)$$

displacement operator

$$\alpha(t) = \alpha_0 (1 - e^{i(\nu-\delta')t}),$$

$$\Phi(t) = \alpha_0^2 \{(\nu - \delta')t - \sin(\nu - \delta')t\}$$

Spin dependent circular motion in the phase space

$$|\uparrow_x \downarrow_x\rangle \Rightarrow \text{radius: } J_x \alpha_0, \text{ center: } (-J_x \alpha_0, 0), \quad |\downarrow_x \uparrow_x\rangle \Rightarrow \text{radius: } J_x \alpha_0, \text{ center: } (J_x \alpha_0, 0)$$

Disentangling condition

\Rightarrow closed trajectory

$$(\nu - \delta')t = 2\pi n, \quad n: \text{integer}$$

$$\text{gained phase: } \Phi_g = 2\pi n \left(\frac{\Omega_0 \eta}{\nu - \delta'} \right)^2$$

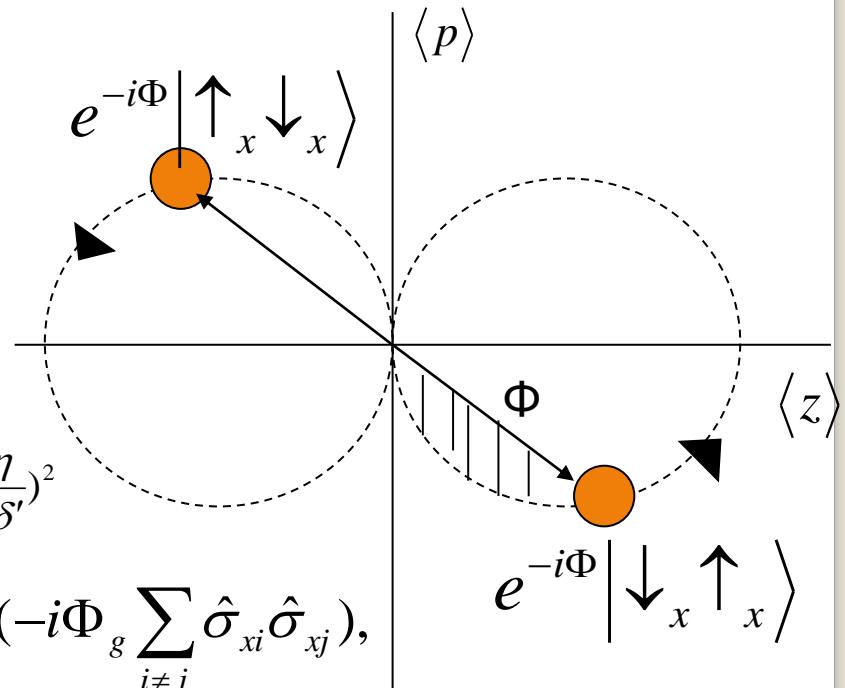
$$\text{ex: } n=1 \rightarrow \tau_g = 2\pi / (\nu - \delta'), \quad \Phi_g = 2\pi \left(\frac{\Omega_0 \eta}{\nu - \delta'} \right)^2$$

$$\hat{U} = \exp[-i\Phi_g J_x^2] = \exp(-i\Phi_g N/4) \exp(-i\Phi_g \sum_{i \neq j} \hat{\sigma}_{xi} \hat{\sigma}_{xj}),$$

effective Hamiltonian : $\hat{H}_{eff} = \hbar \Phi_g \sum_{i \neq j} \hat{\sigma}_{xi} \hat{\sigma}_{xj}$ Spin-spin interaction

Φ_g : unconventional geometric phase

$\Phi_g = (\text{geometric phase}) + (\text{dynamic phase}),$
 $(\text{dynamic phase}) = -2x(\text{geometric phase})$



Generation of entangled states

$$\Phi_g = \frac{\pi}{2}, \text{ (or } \frac{\Omega_0 \eta}{\nu - \delta'} = \frac{1}{2} \text{)} \rightarrow \hat{U}_{\max} = \exp[-i\pi J_x^2 / 2]$$

Entanglement of two ions: (stretch mode) P.C.Haljan et al, PRA, 72,062316, 2005	$ ee\rangle \Rightarrow \Psi_1\rangle = (1/\sqrt{2})(ee\rangle + i gg\rangle)$ $ gg\rangle \Rightarrow \Psi_2\rangle = (1/\sqrt{2})(ee\rangle - i gg\rangle)$ $ eg\rangle \Rightarrow \Psi_3\rangle = (1/\sqrt{2})(eg\rangle + i ge\rangle)$ $ ge\rangle \Rightarrow \Psi_4\rangle = (1/\sqrt{2})(eg\rangle - i ge\rangle)$
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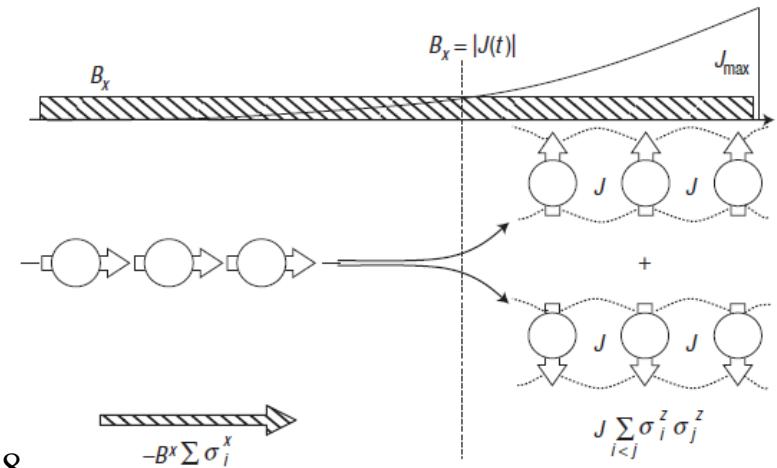
High fidelity Bell state generation: F=0.993,
 J.Benheim et al, Nature phys. Vol.4,463,2008

Quantum simulation: Ising model

$$\hat{H}_{\text{Ising}} = \frac{1}{2} \sum_{i,j} J_{i,j}^x \sigma_{z,i} \sigma_{z,j} + \sum_i B_x \sigma_{x,i}$$

Phase transition: paramagnetic order
 \Leftrightarrow (anti-) ferromagnetic order

Two ions:A.Friedenauer et.al, Nature phys. 4,757,2008



(2) Weak-field coupling

A.Sorensen & K.Molmer, PRL,82,1971(1999)

$$\eta\Omega_0 \ll \nu - \delta' \rightarrow \alpha \ll 1$$

$$\hat{U}(t) = D(-\hat{J}_x \alpha) \exp(-i\Phi_g \hat{J}_x^2) \Rightarrow \hat{U}(t) = \exp(-i\tilde{\Omega} \hat{J}_x^2 t)$$

$$H_{eff} = \hbar\tilde{\Omega}\hat{J}_x^2 \approx (\hbar\tilde{\Omega}/4) \sum_{i \neq j} \hat{\sigma}_{xi} \hat{\sigma}_{xj}, \quad \tilde{\Omega} = (\eta\Omega_0)^2 / (\nu - \delta')$$

Quantum simulation : frustrated Ising spins

Three ions, transverse mode

$$H_{eff} = \sum_{i < j} J_{ij} \hat{\sigma}_x^i \hat{\sigma}_x^j + B_y \sum_i \hat{\sigma}_y^i$$

$$J_{ij} = \Omega_i \Omega_j \frac{\hbar k_x^2}{2M} \sum_m \frac{b_{i,m} b_{j,m}}{\mu^2 - \nu_m^2},$$

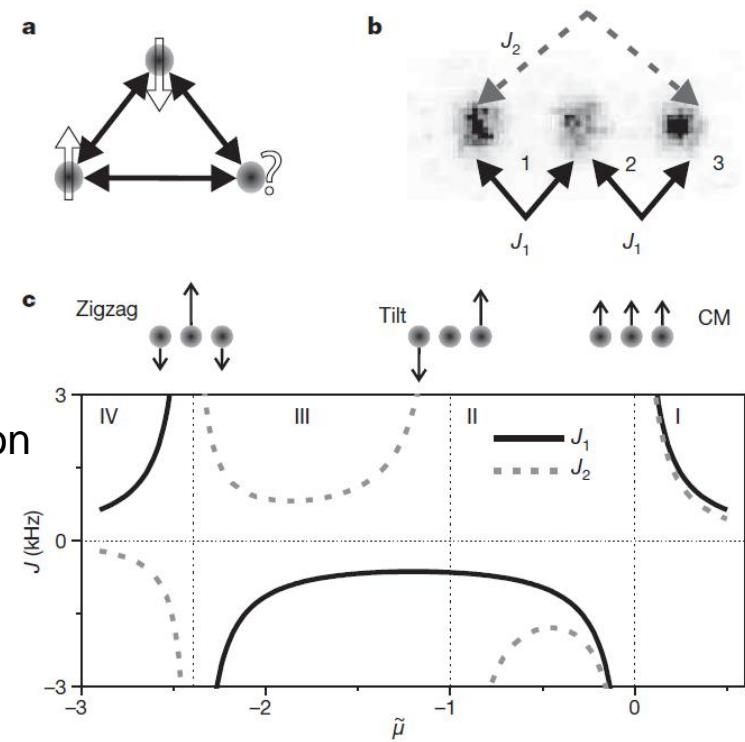
$b_{i,m}$: normal mode transformation matrix

$J_1 \equiv J_{12} = J_{23}$ Nearest-neighbor interaction

$J_2 \equiv J_{13}$ Next-nearest-neighbor interaction

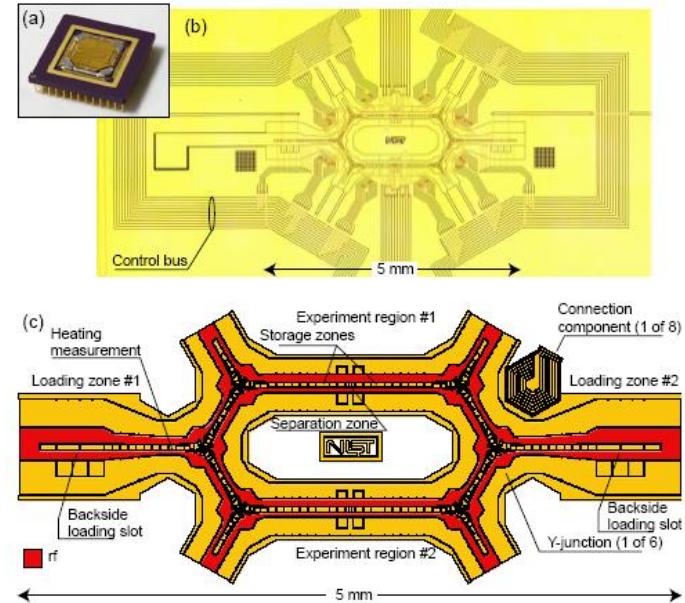
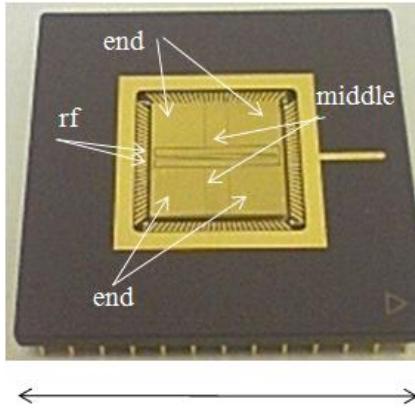
K.Kim et.al., PRL,103, 120502(2009)

K.Kim et .al., nature, 465,3(2010)



今後の方向

1. 大規模・集積化 プレーナートラップ



2. 他の量子糸との結合
3. 量子シミュレーション
4. 高速化