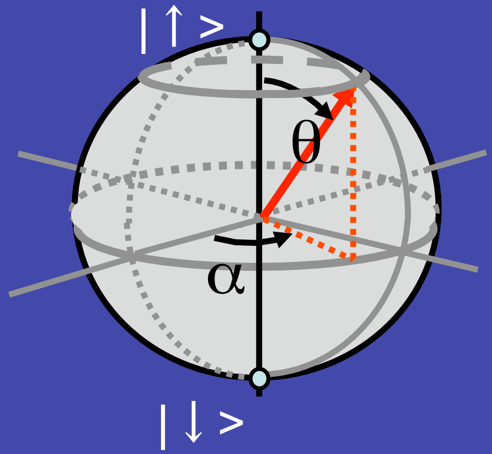
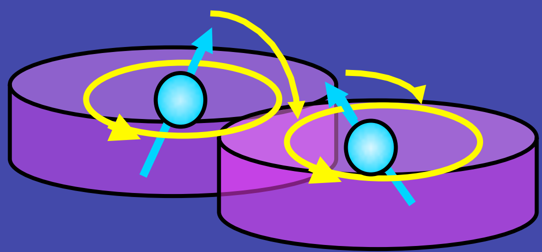


Summer school of FIRST/Q-cybanetics
Chinen, Okinawa, Aug. 21, 2010

Qubits by electron spins in quantum dot system – Basic theory



Yasuhiro Tokura

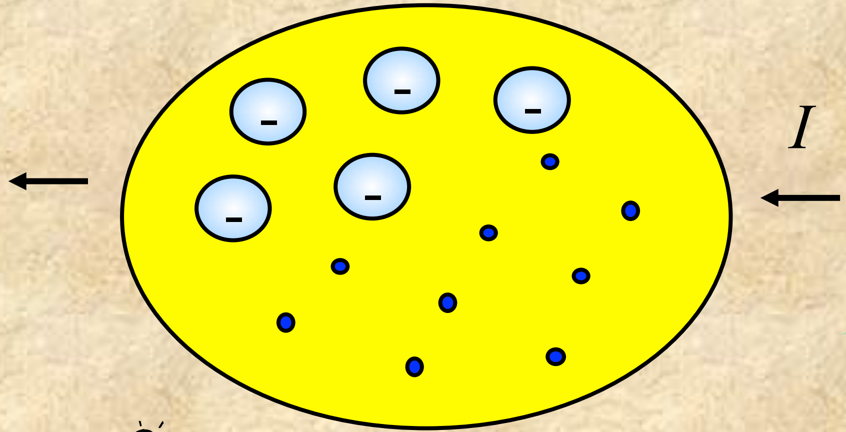
NTT Basic Research Laboratories
FIRST project, theory subgroup
Quantum Cybanetics, semiconductor qubits

Self-introduction

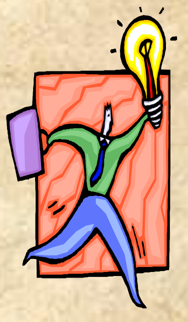
- 相関理化学という分野で修士課程修了。
- 1985年　NTT入社　基礎研究所所属
- 半導体物性、メゾスコピック、ナノサイエンスに従事
- 1998年　オランダ・デルフト工科大　客員研究員
- 2004年　東京理科大　客員教授
- 2005年　量子光物性研究部　部長

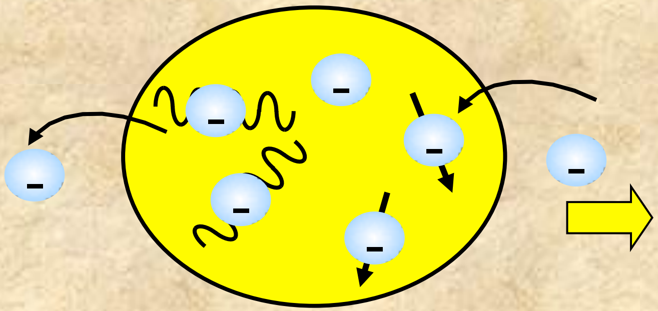
- 興味のある研究分野
  - 量子輸送現象、非平衡現象、量子情報処理
- 趣味等
  - バドミントン、沖縄は家内の故郷で馴染み深い

Solid state qubits –microscopic coherence-



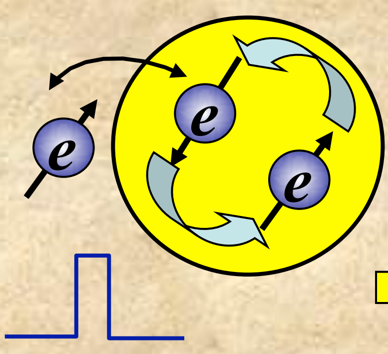
Macroscopic system
+
Ensemble measurement





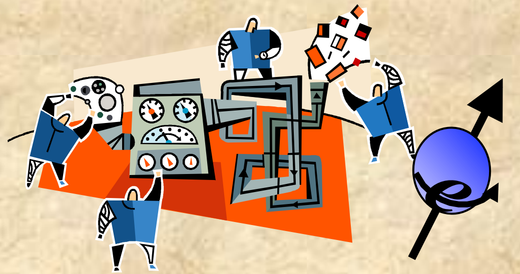
Nanostructures: Small ensemble
+
Single electron spectroscopy

Mesoscopic physics: quantum interference, low-dimensionality,….



System of just one or two electrons
+
Dynamics in single shot

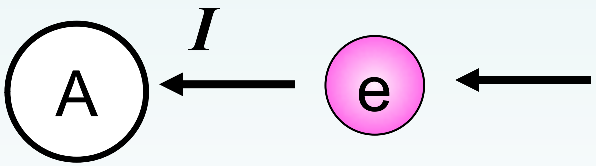
Control over microscopic nature of energy quanta, correlation



Also, challenge to quantum information

Charge and spin: Tiny quantities to detect

Charge "*e*"=1.6X10^{-19} C



Best resolution measurement with low-T and low-noise "fA"…10^{4} electrons/sec

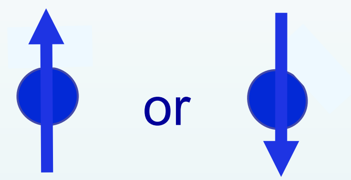
⇨ Ensemble measurement : 10$^{4\text{-}13}$ times/sec

Spin Magnetic moment "μ"

$$\mu_B = g\mu_B S_z = \frac{eh}{2mc} = 9.27\text{x}10^{-24}\ \text{J/T}$$ for electron spin

Standard measurement using a Hall device and a SQUID device 10^{-10} J/T

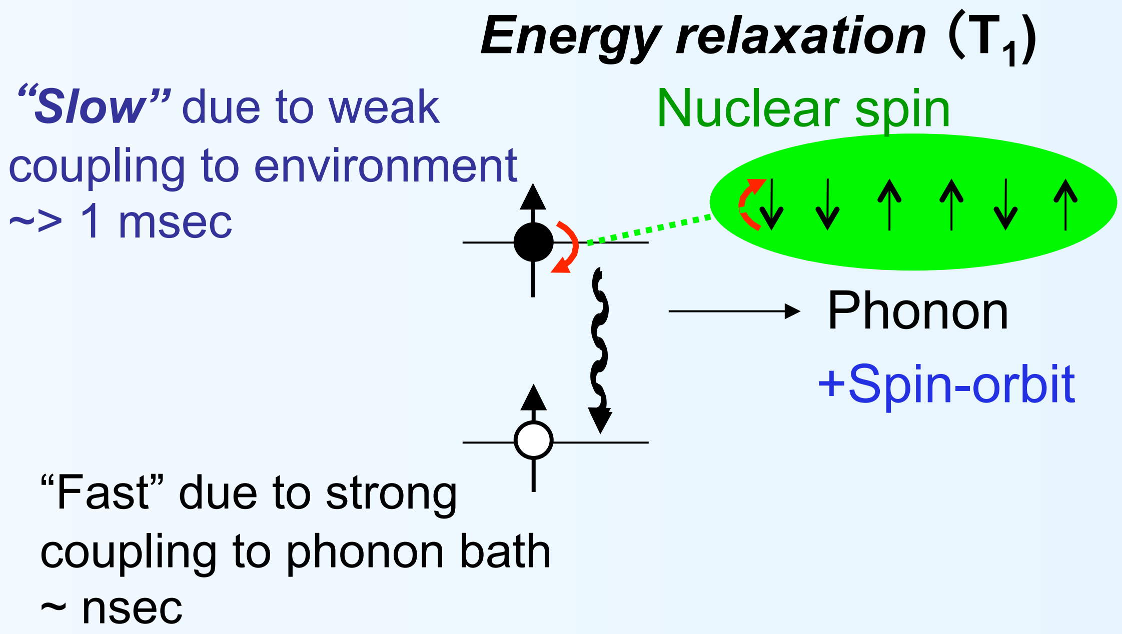
⇨ Ensemble measurement: 10^{14} spins



How to identify single "e" and "μ"? …

Manipulate/Readout of quantum information

Orbital and spin degrees of freedom



Charge = Orbital
T_1~T_2 ~ nsec
due to strong phonon coupling

Spin…robust quantum number

→ ***Spin qubits and quantum computing***

Use of Quantum Dots (QDs)

Part I, Aug. 21

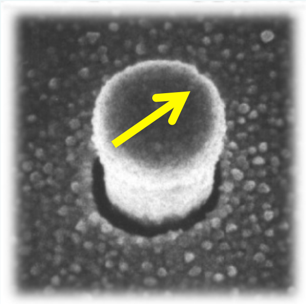
Basic theory of spin qubits in QDs
(Y. Tokura, NTT)

Part II, Aug. 26

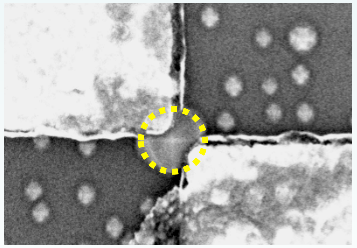
Experiments and future problems
(Prof. S. Tarucha, Univ. Tokyo)

Single and double QDs holding a few electrons

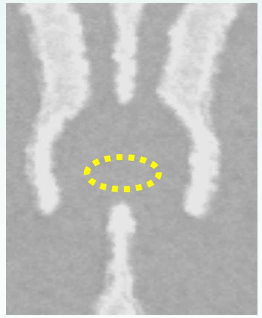
Advent of one-electron single QDs



Tarucha et al. *PRL* 96



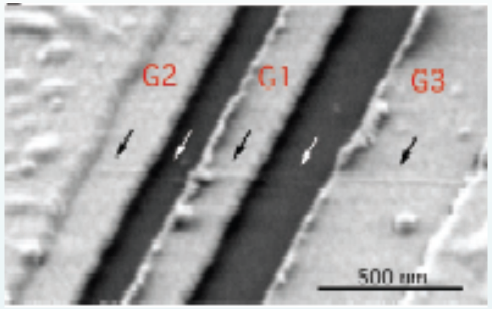
Jung et al. *APL*05



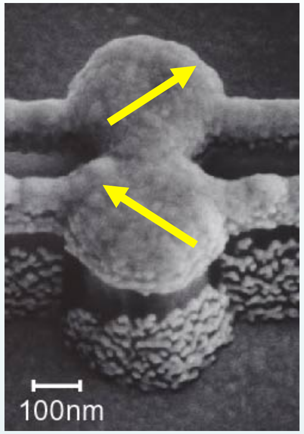
Ciorga et al. *PRB* 02

Advent of two-electron double QDs

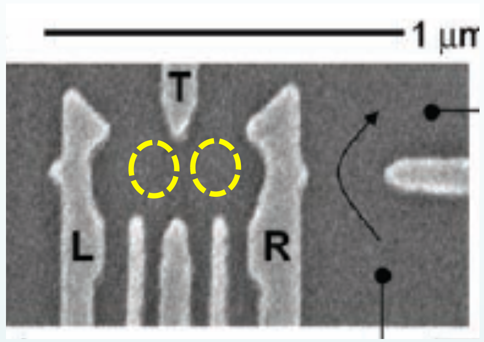
nanotube



Mason et al. *Science* 04



Hatano et al. *Science* 05



Petta et al. *Science* 04

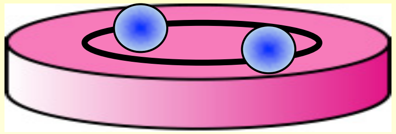
Energy spectrum of a quantum dot

-Hamiltonian: Quantum mechanical effect
and interaction effect

-Tunneling spectroscopy : Conductance

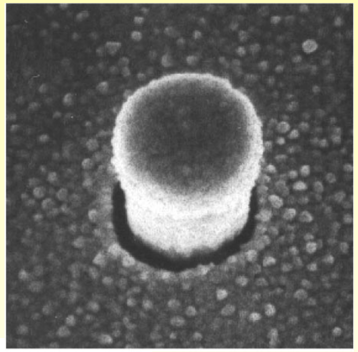
- Isolation of single electron

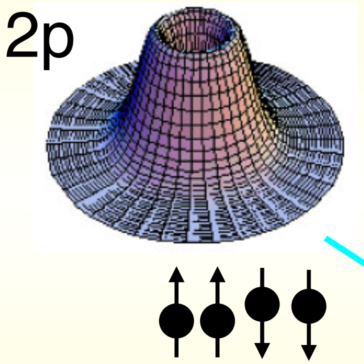
Eigen energy of two-dimensional harmonic QD

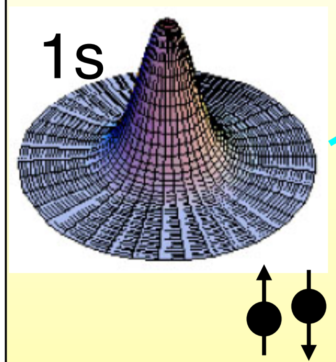


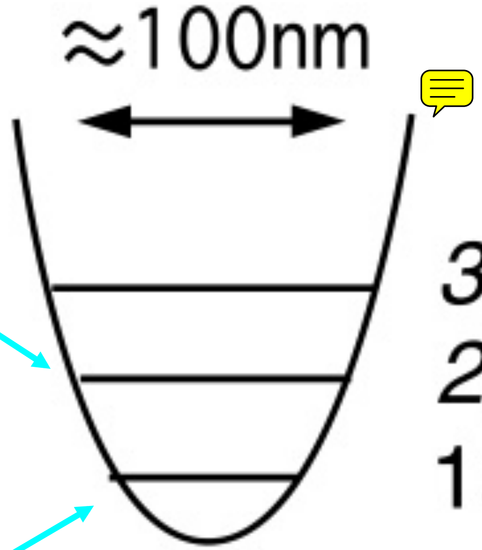
$H = \hbar^2 \boldsymbol{k}^2/2m^* + V(\boldsymbol{r})$

$V(\boldsymbol{r}) = (1/2) m \omega_0^2 r^2$









3s3d **(n, l)= (1, 0)(0, ±2)**

2p **(n, l)=(0, ±1)**

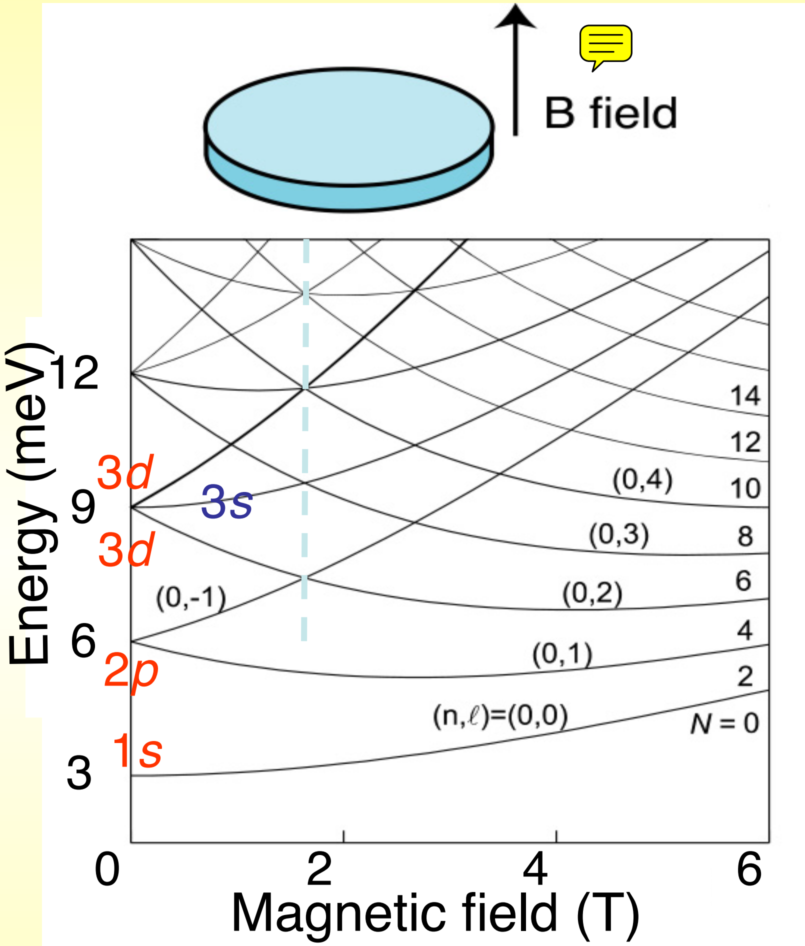
1*s* **(n, l)=(0, 0)**

$$E_{n\ell} = (2n + |\ell| + 1)\hbar\omega_0$$

n *radial quantum number*

l *angular momentum quantum number*

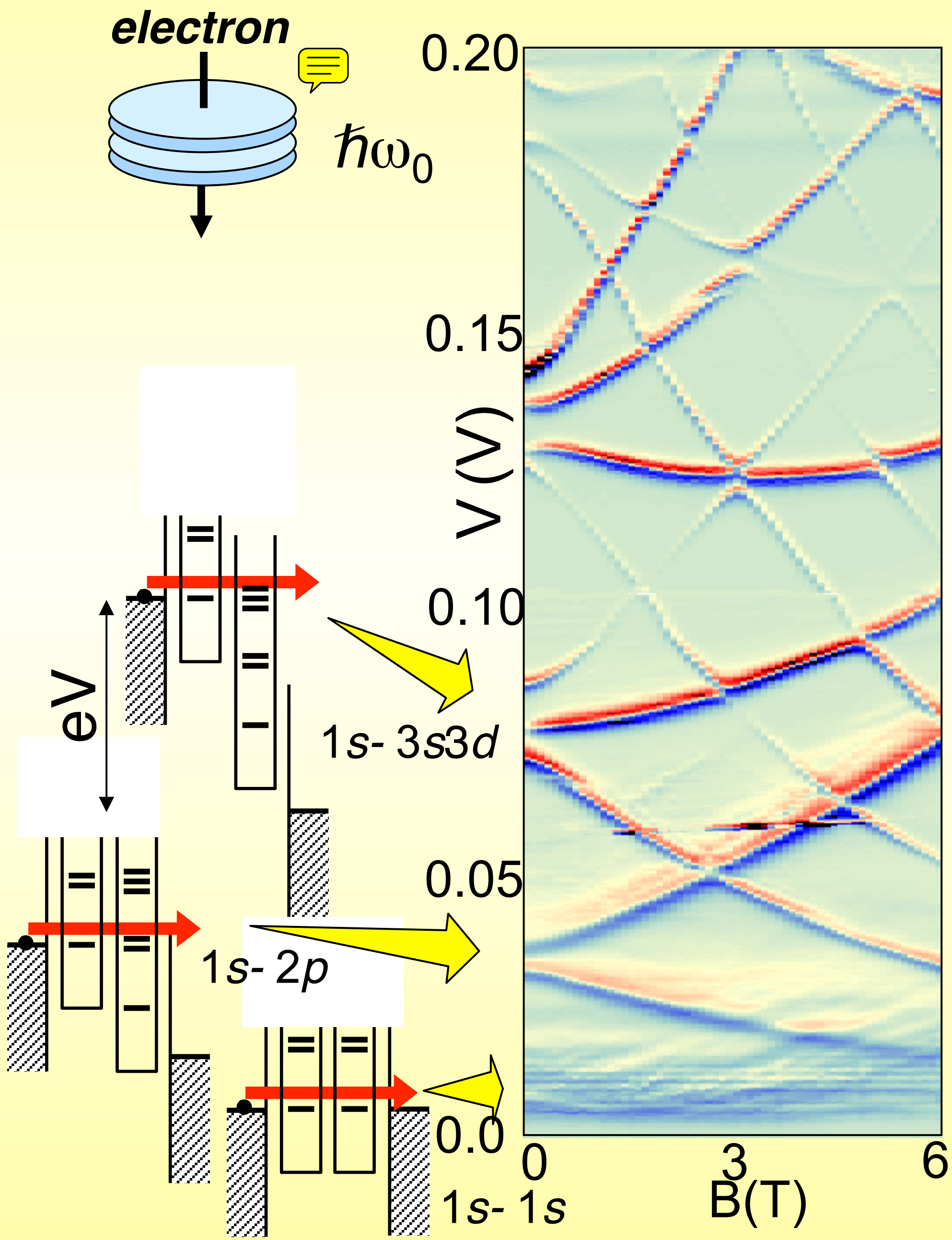
Single-particle states in a 2D harmonic QD



Fock-Darwin states

$E_{n\ell} = -(\ell/2)\hbar\omega_c$ .... $\propto$ B

$+(n+|\ell|/2+1/2)\hbar\sqrt{4\omega_0^2+\omega_c^2}$



Model Hamiltonian for an isolated QD

Constant Interaction model:

$$\mathcal{H}_{CI} = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{2C}(e\mathcal{N} - Q_0)^2$$

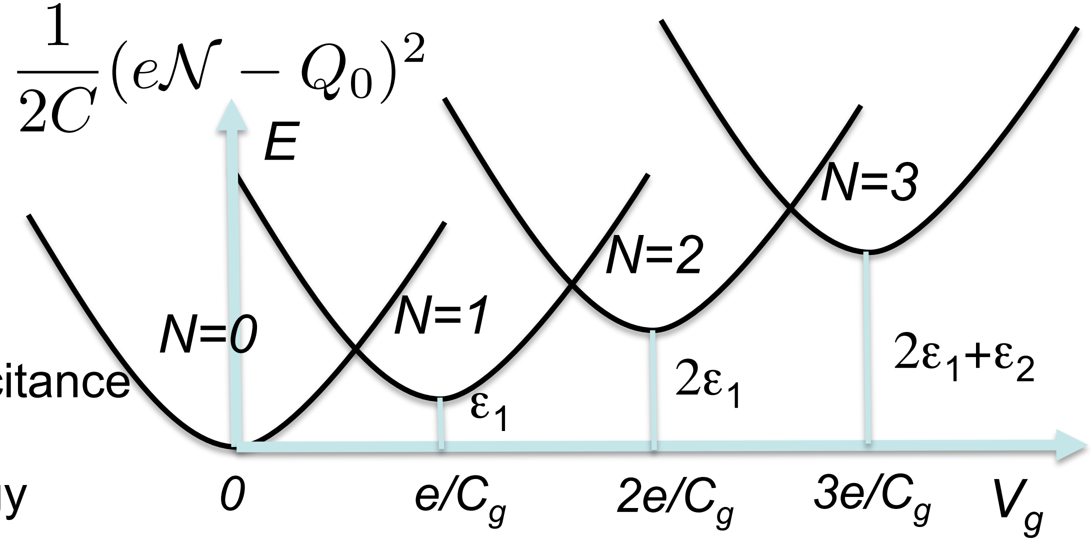
$N = \Sigma_{k\sigma} c^{+}_{k\sigma} c_{k\sigma}$

$Q_0 = V_g C_g$: offset charge

V_g, C_g: Gate voltage, capacitance

$U = e^2/2C$: Charging energy

C: Total capacitance



S= 0 for even N
S=1/2 for odd N

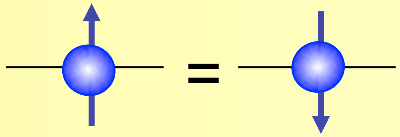
Even-odd filling, Spin pairing

Assumption: $\Delta\varepsilon_k \sim 2\varepsilon_F/N$ for large N and =0 for spin degeneracy
$U \gg k_B T$ (low temperature)

One and two electron states in QD

N=1

$\phi(\boldsymbol{r})$ ε_1

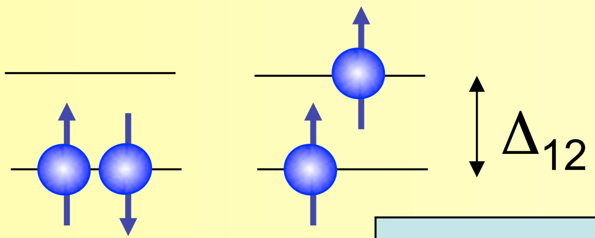


$U(1)=\varepsilon_1$

N=2

ϕ_2 ε_2

ϕ_1 ε_1



$\Delta_{12} > V_{ex}$

GS $U(2)=2\varepsilon_1 + V_{intra}$ ***Singlet***

ES $U^*(2)=\varepsilon_1 + \varepsilon_2 + V_{intra} - V_{ex}$

$|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ ***Triplet***

Hartree

$$V_H(r) = \frac{e^2}{\kappa}\int d^2r' \frac{n_s(r')}{|\mathbf{r}-\mathbf{r}'|}$$

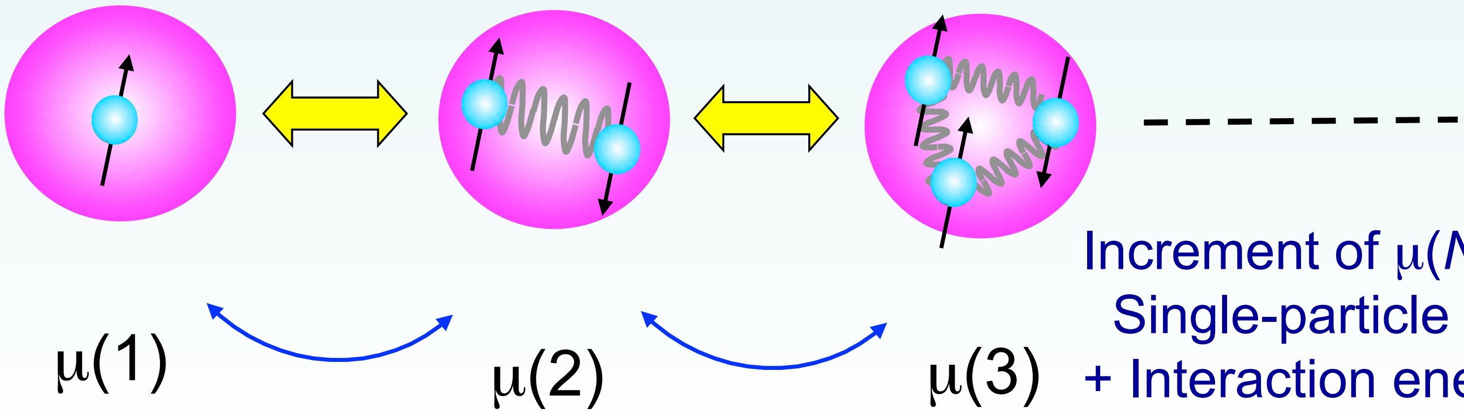
$$V_{intra} \equiv \langle V_H(r) \rangle$$

Fock (exchange energy)

$$\Delta(\mathbf{r},\mathbf{r}') = \frac{e^2}{\kappa}\sum_\beta f(\epsilon_\beta - \mu)\frac{\psi^*_\beta(\mathbf{r}')\psi_\beta(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}$$

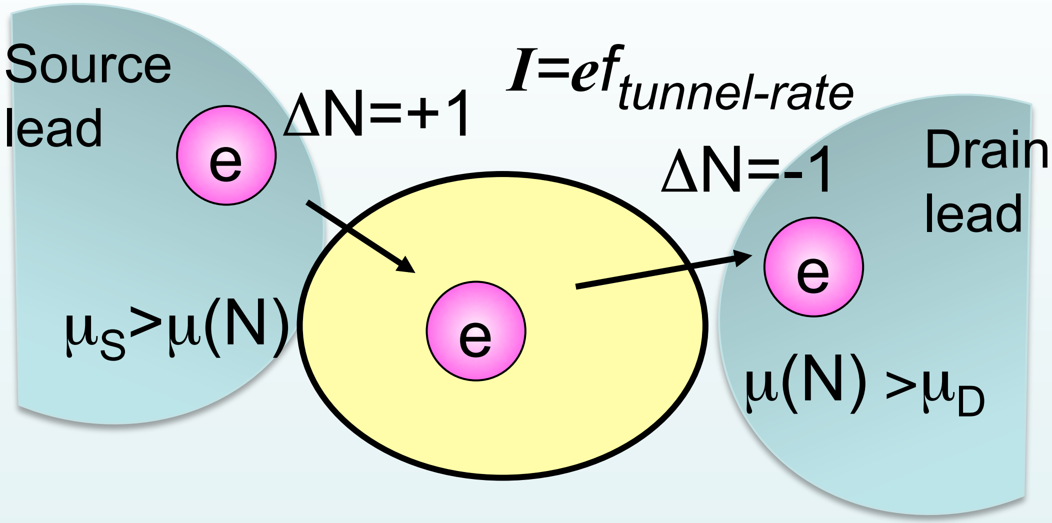
$$V_{ex} \equiv \langle \Delta(r,r') \rangle$$

How do we detect the energy spectra ?

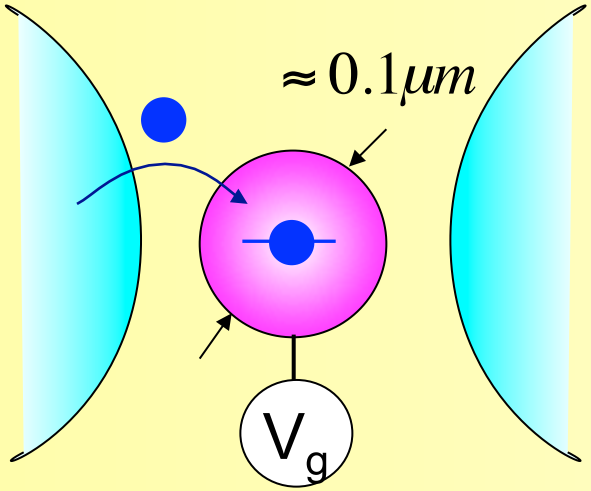


Increment of $\mu(N)$
Single-particle level spacing
+ Interaction energy

Current-sensitive measurement

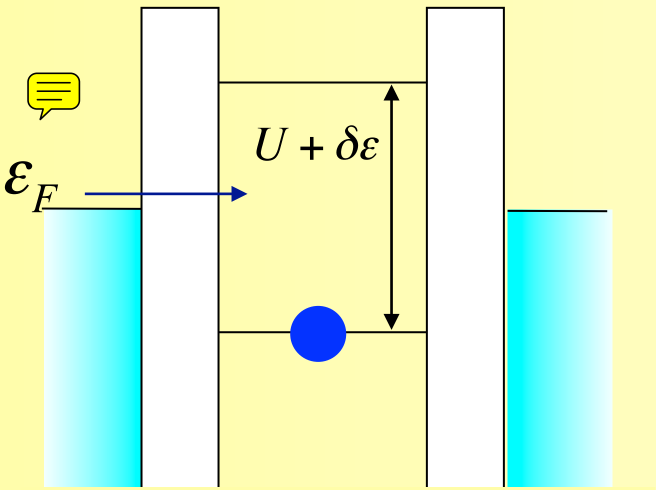


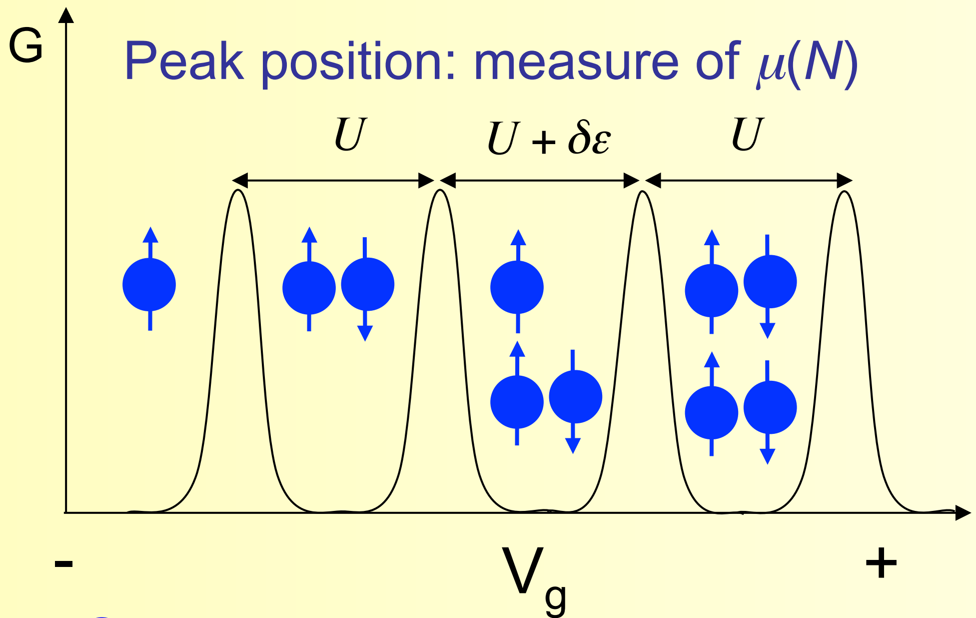
Probing electronic states in quantum Dot



Conductance peaks appear every time when the cost of $U+\delta\varepsilon$ is paid: "Coulomb oscillations"

U : On-site repulsion
$\delta\varepsilon$: Level spacing
(=0 for spin pairs)

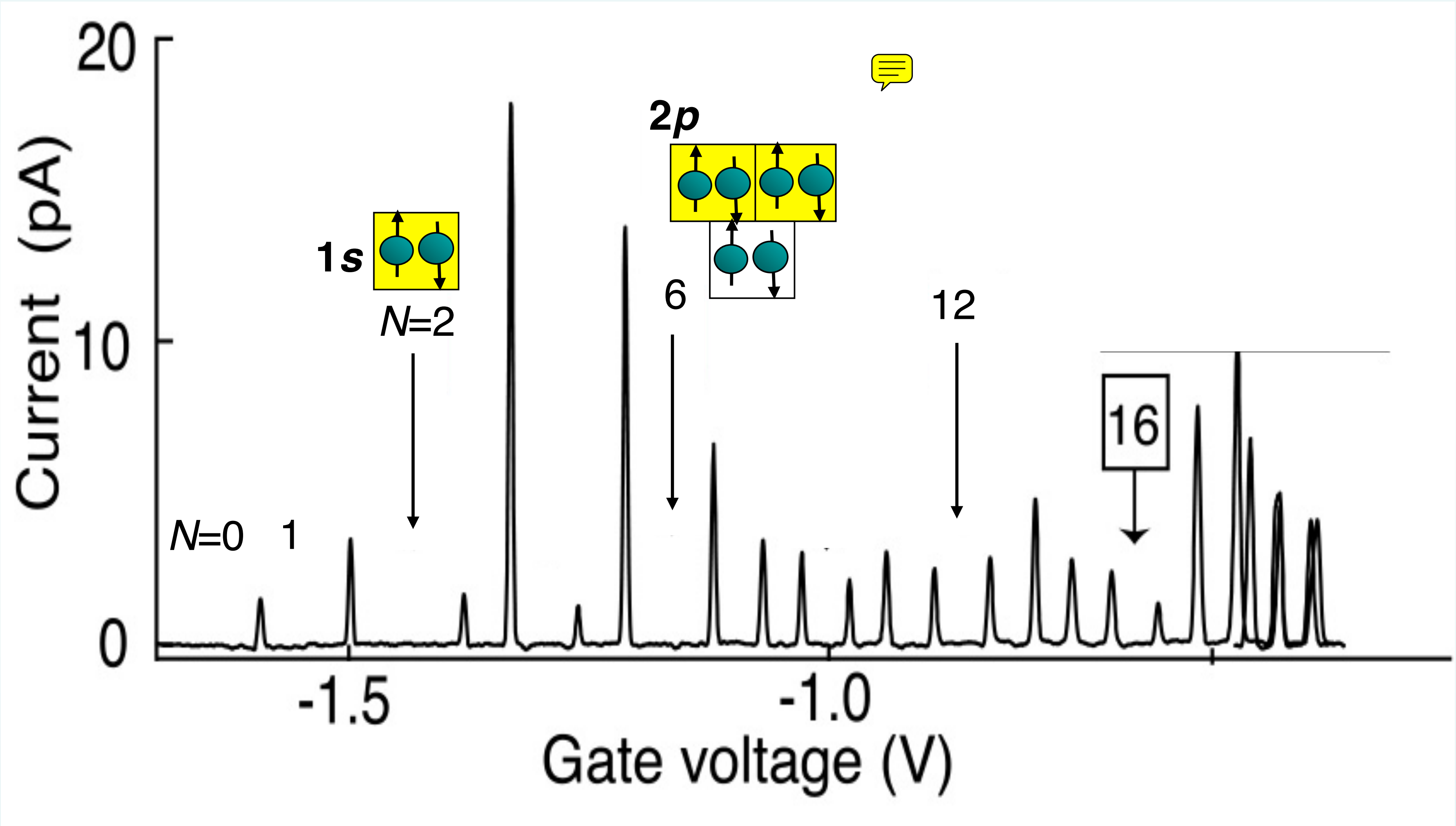




…. Spin paired states

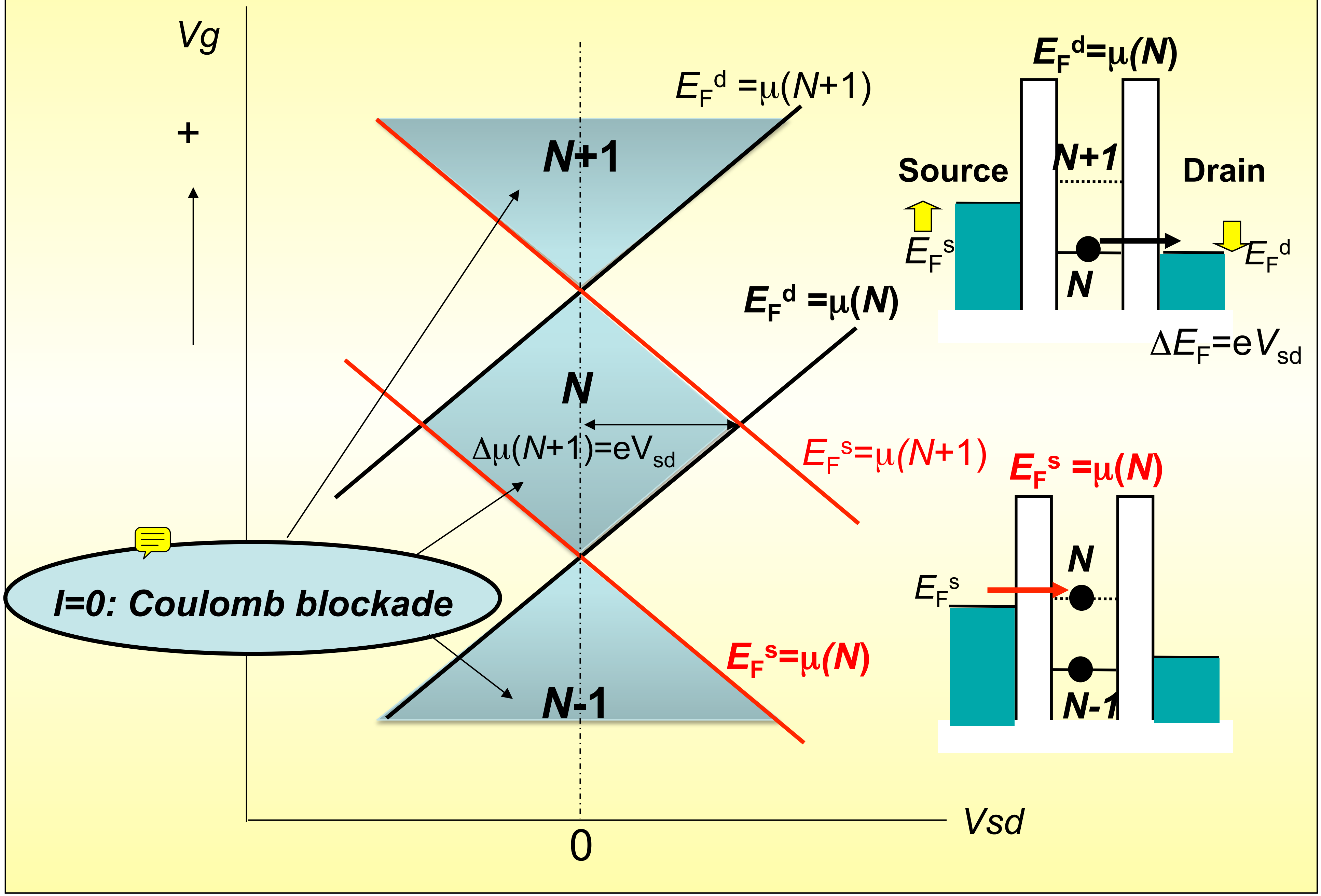
Single electron tunneling (SET) transistor

"Atom-like shell filling"

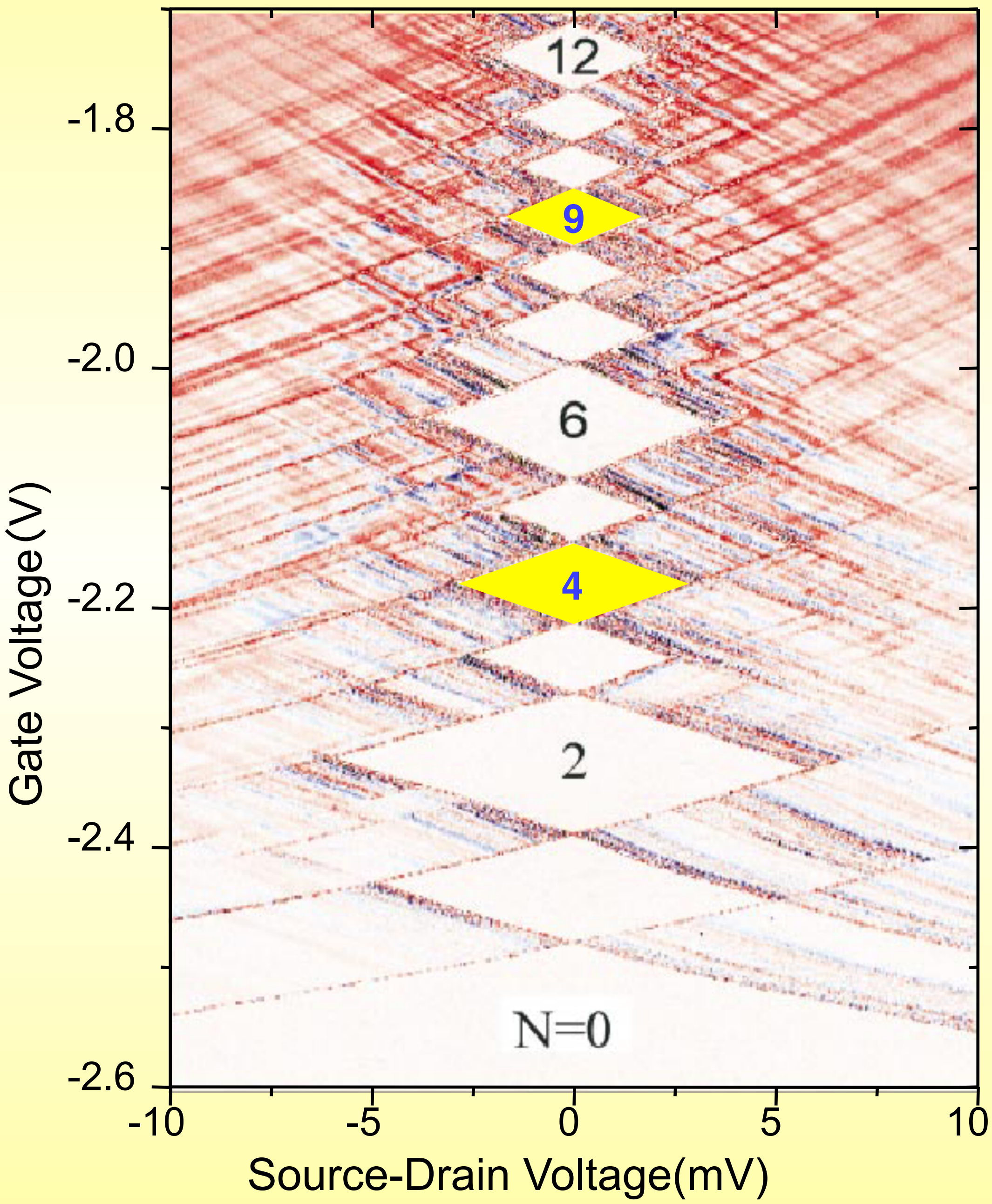


Single electron tunnel=ensemble measurement : *I=fxe*

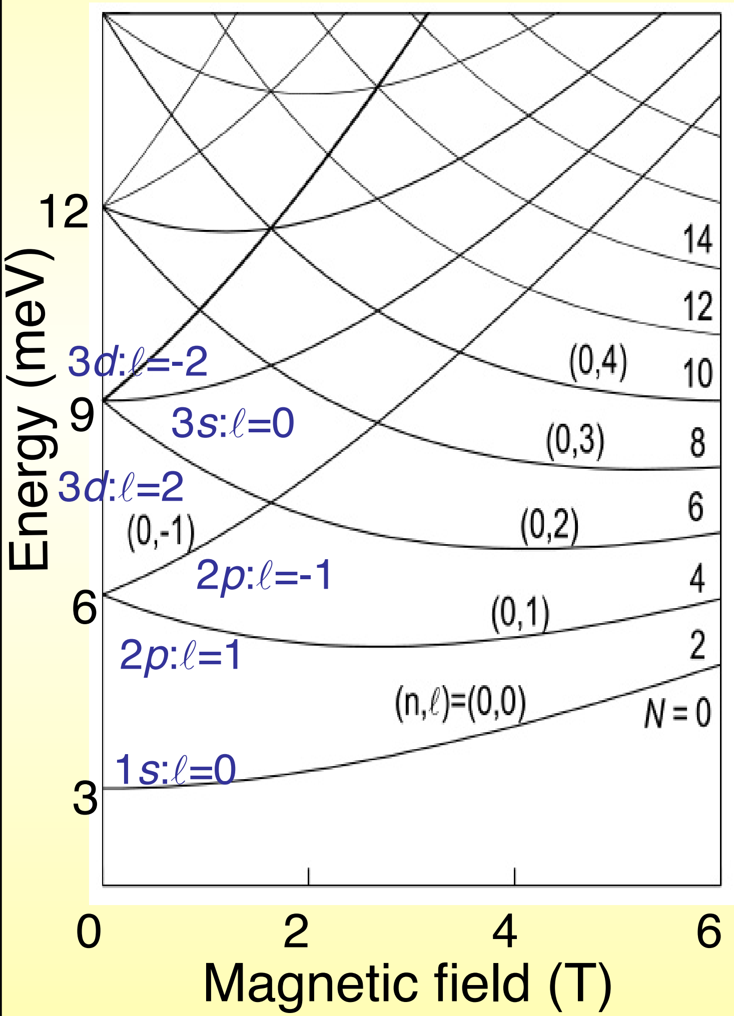
Coulomb diamond: dI/dV_{sd}-V_{sd} and V_g

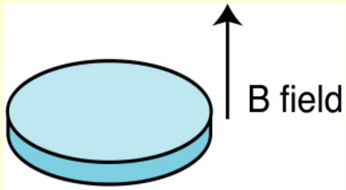


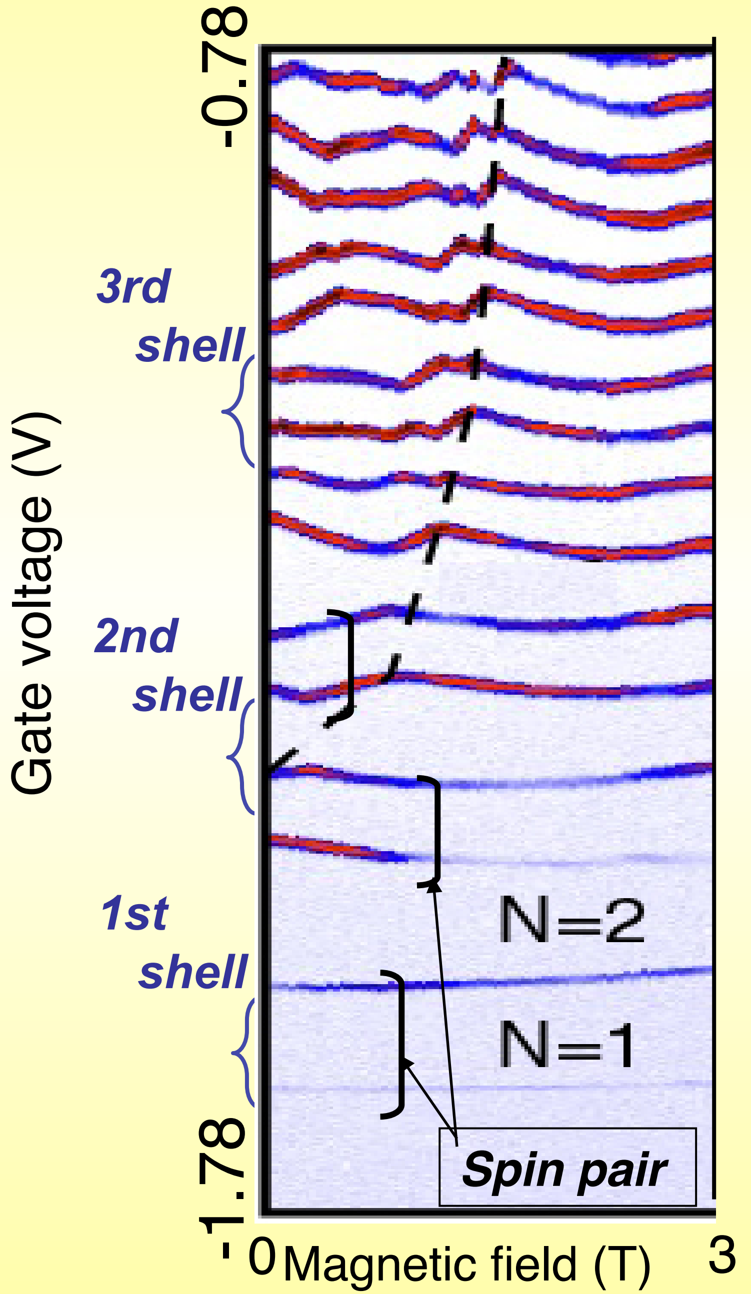
Coulomb diamond



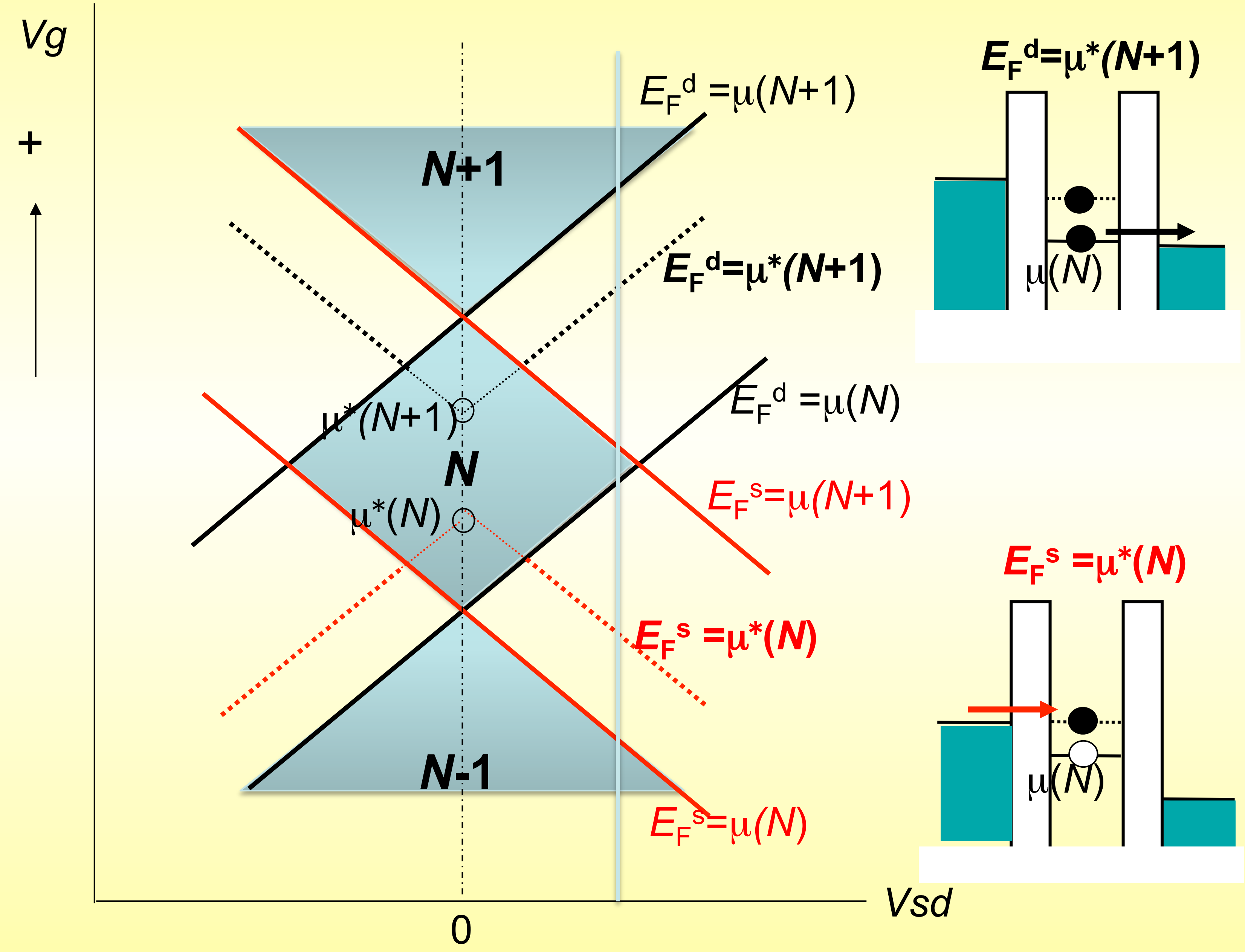
Evolution of Coulomb peaks (μ(N)) with B





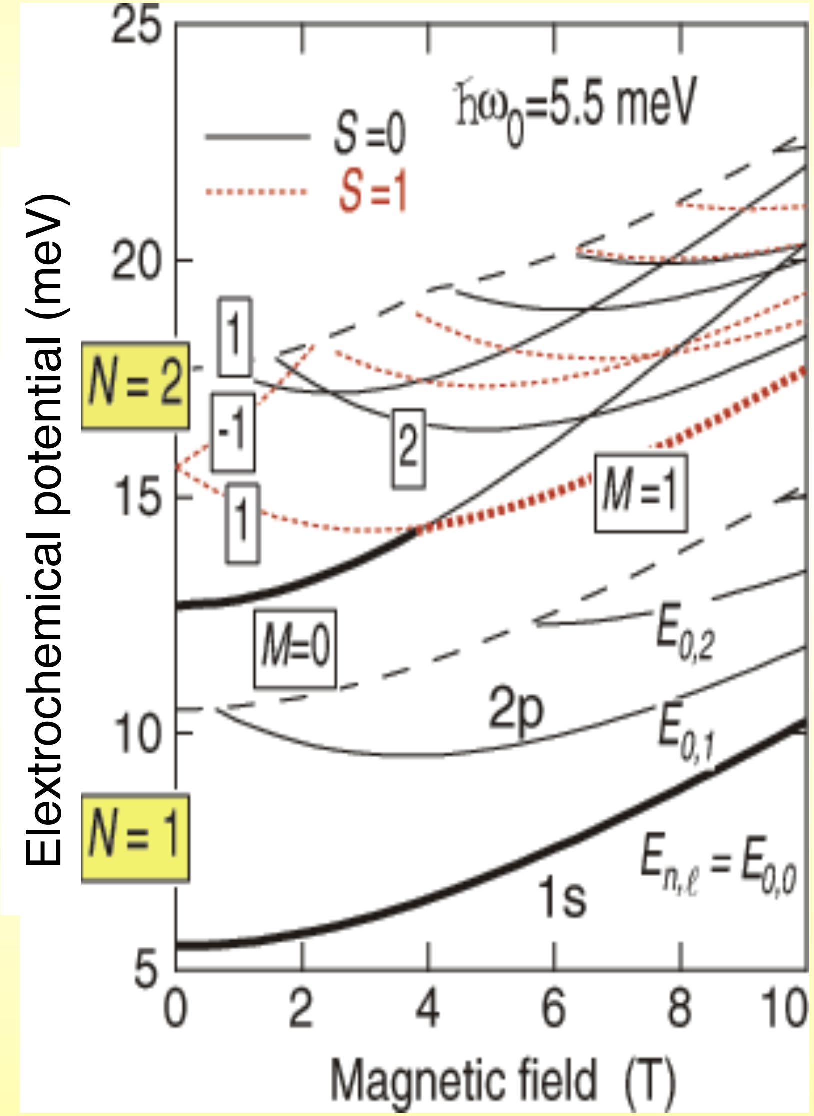


Excited states

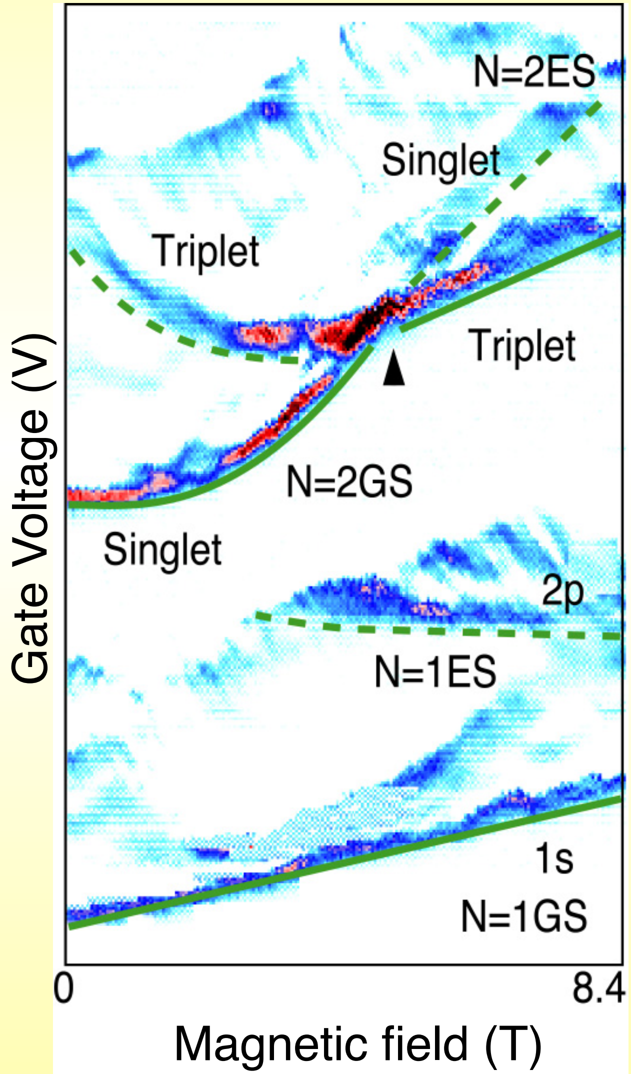


Spin singlet-triplet transition

Exact diagonalization

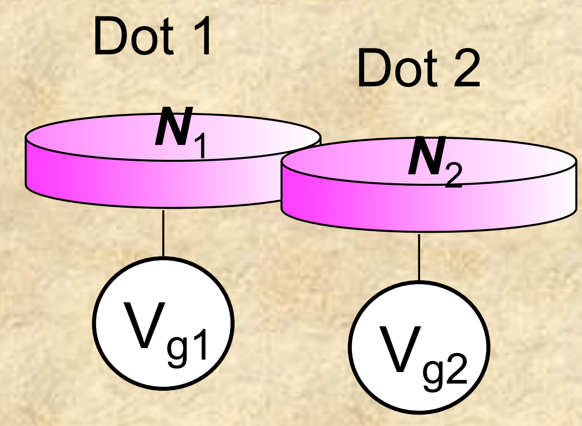


Experiment:
Excitation spectrum

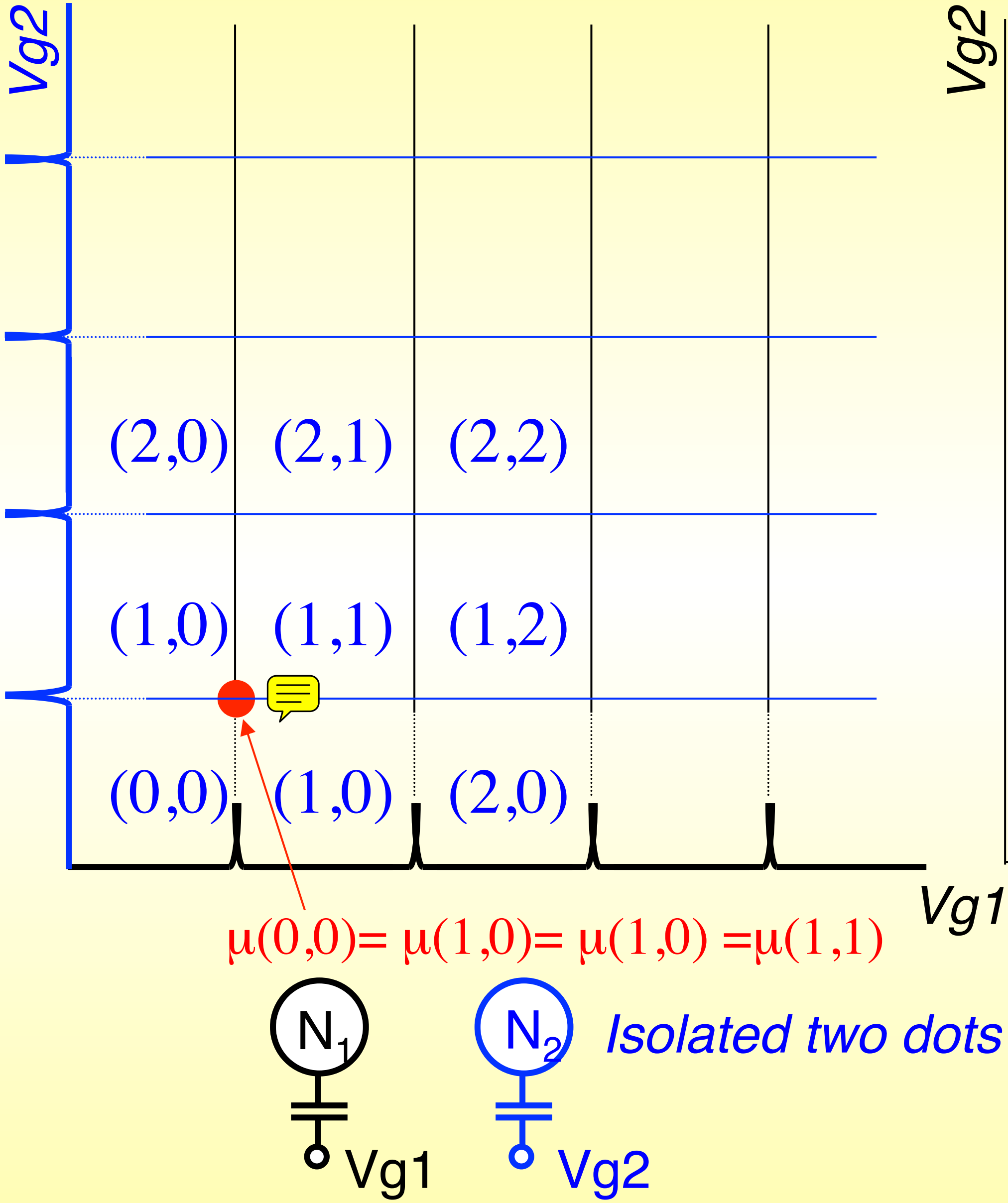


Two-electrons in two quantum dots

- Heitler-London state
- Exchange coupling



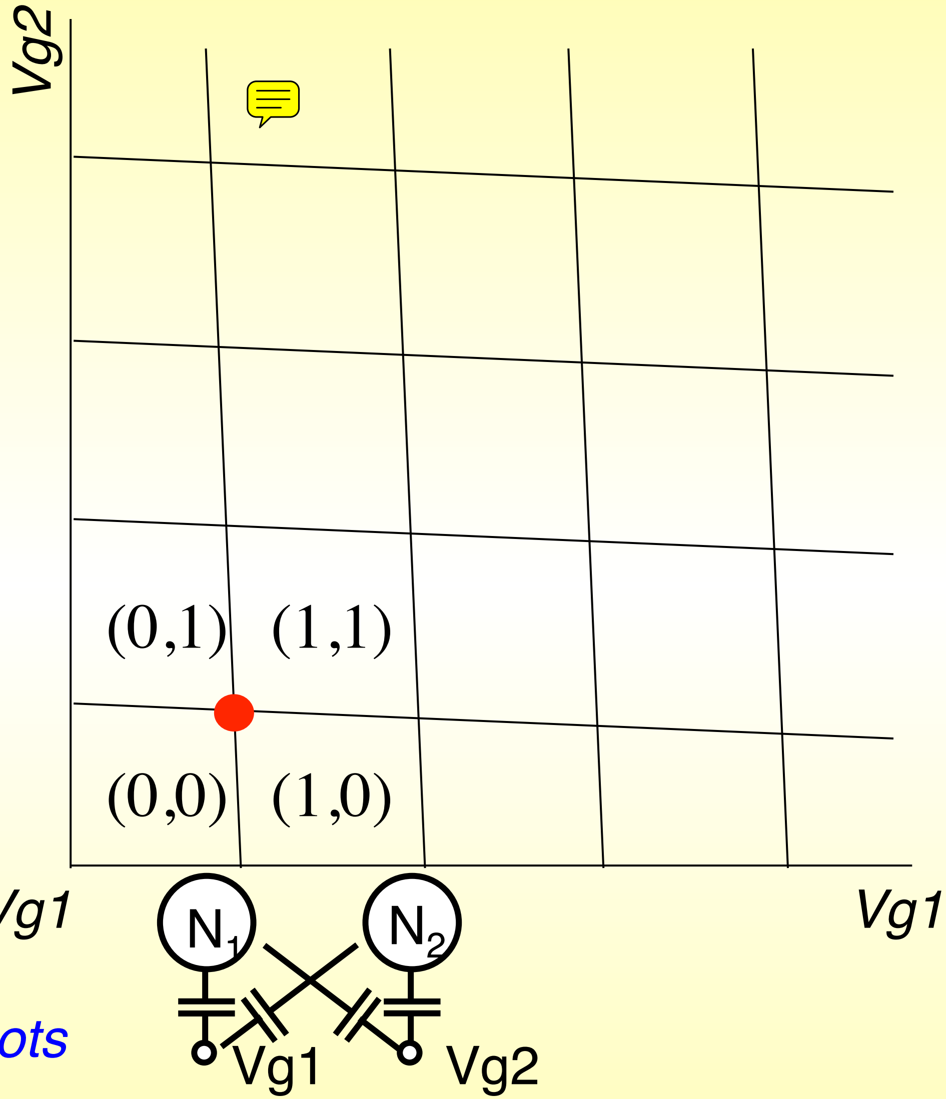
Stability diagram of double dot system



μ(0,0)= μ(1,0)= μ(1,0) =μ(1,1)

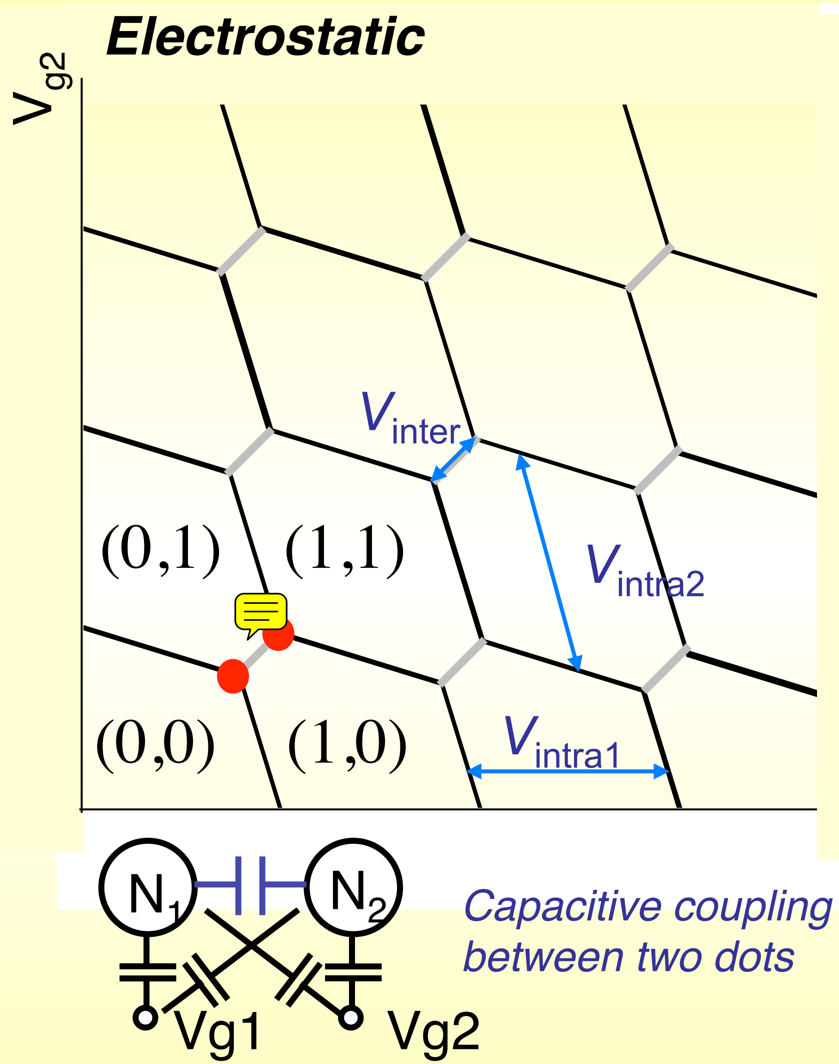
Isolated two dots

$\mu(N_1)$ and $\mu(N_2)$ independent

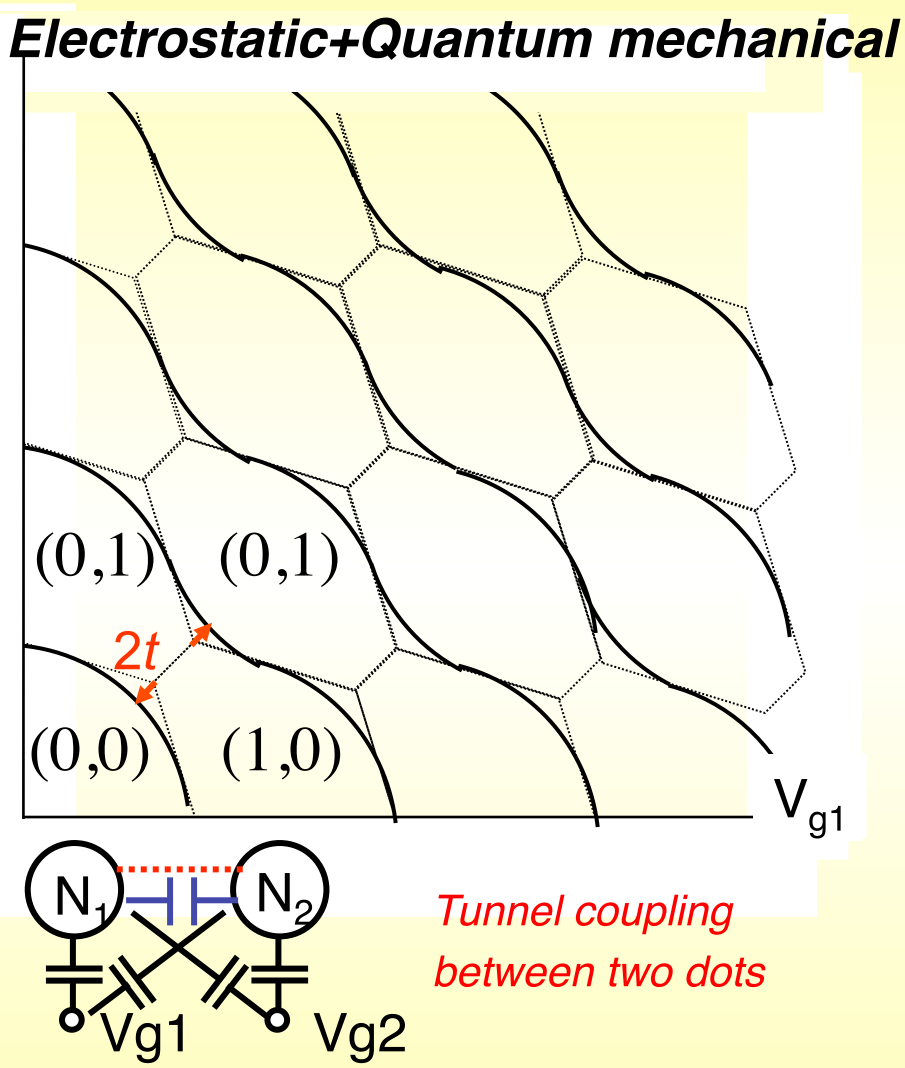


Two dots are only coupled through the gates.

Stability diagram of double dot system



One-electron charging in one dot raises the electrostatic potential of the other dot by $E_c=e^2/C_{inter}$,

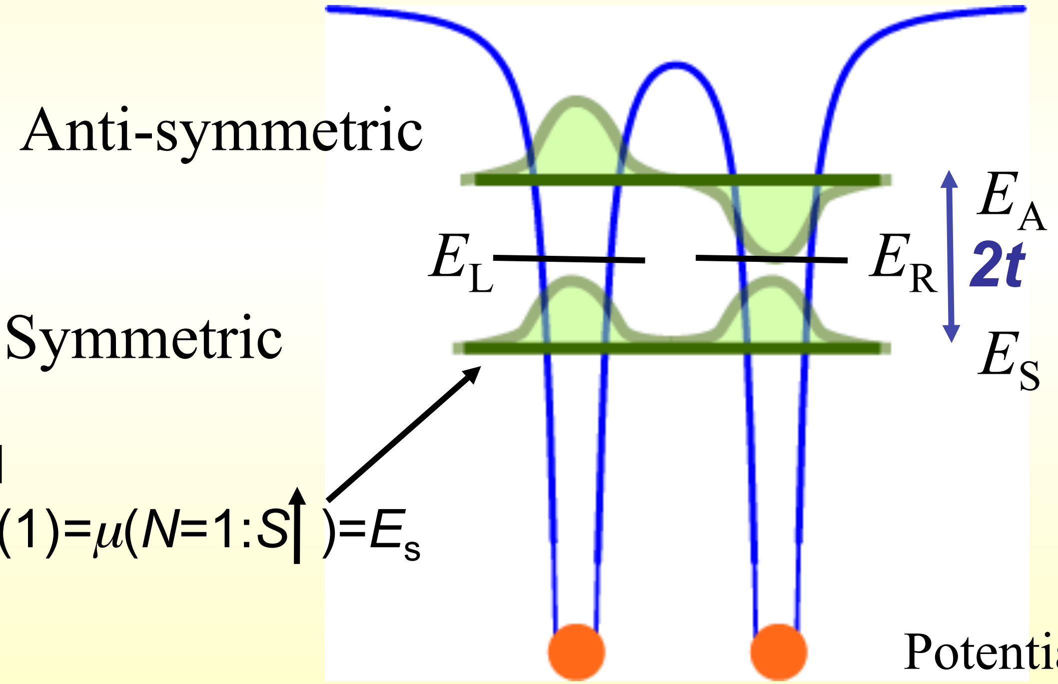


Degeneracies between different charge states are lifted by the tunnel coupling. The total electron number is only well defined.

Single electron states in coupled dots

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V_{ext}(\mathbf{r})\right]\phi_p(\mathbf{r}) = E_p\phi_p(\mathbf{r})$$

$\phi_{\mathrm{L}}(\boldsymbol{r})$ $\phi_{\mathrm{R}}(\boldsymbol{r})$



$$\phi_{\mathrm{A}}(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[\phi_L(\mathbf{r}) - \phi_R(\mathbf{r})\right]$$

$$\phi_S(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[\phi_L(\mathbf{r}) + \phi_R(\mathbf{r})\right]$$

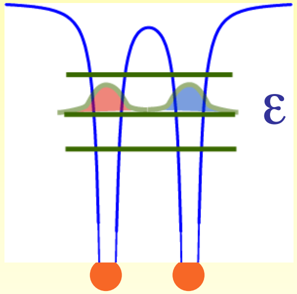
N=1

$U(1)=\mu(N=1:S\uparrow)=E_s$

Potential offset between two dots: δ

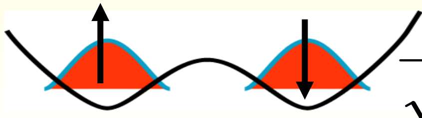
$$E_{\mathrm{S}}\text{-}E_{\mathrm{A}} = \sqrt{4t^2 + \delta}$$

Two electron states: Hund-Mulliken approach



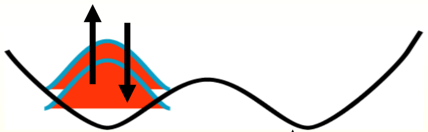
$$\mu_{HM\text{-}S}(2) = E_{HM\text{-}S}(2) - E_S(1)$$

Three singlet states:



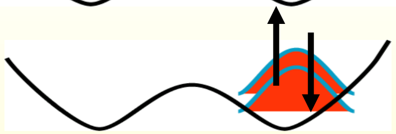
$$\frac{1}{\sqrt{2}}\left(|LR\rangle + |RL\rangle\right)$$

$$E_{HM-S} = 2\varepsilon + V_{inter} + V_{ex} - 4\frac{t^2}{V_{intra} - V_{inter} - V_{ex}}$$



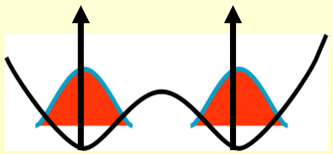
$$|LL\rangle$$

$$E_{HM-T} = 2\varepsilon + V_{inter} - V_{ex}$$



$$|RR\rangle$$

Exchange energy:

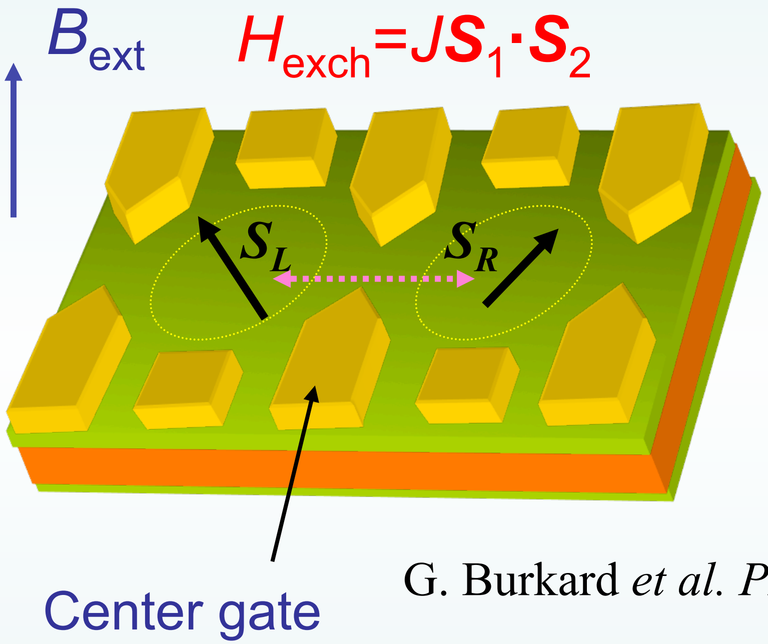


$$\frac{1}{\sqrt{2}}\left(|LR\rangle - |RL\rangle\right)$$

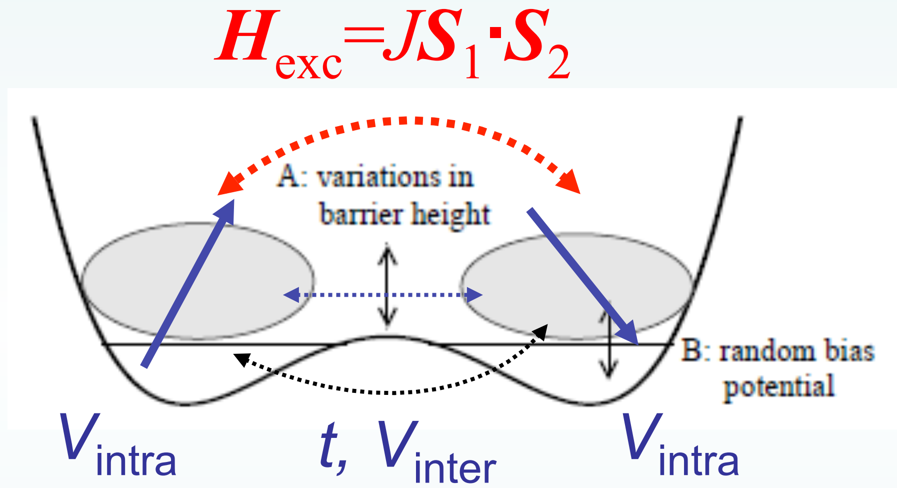
$$E_{HM\text{-}T} - E_{HM\text{-}S} = 4t^2/(V_{intra} - V_{inter} - V_{ex}) - 2V_{ex}$$
$$\equiv J$$

Exchange coupling in DQD

$J \sim 4t^2/(V_{intra}-V_{inter})$



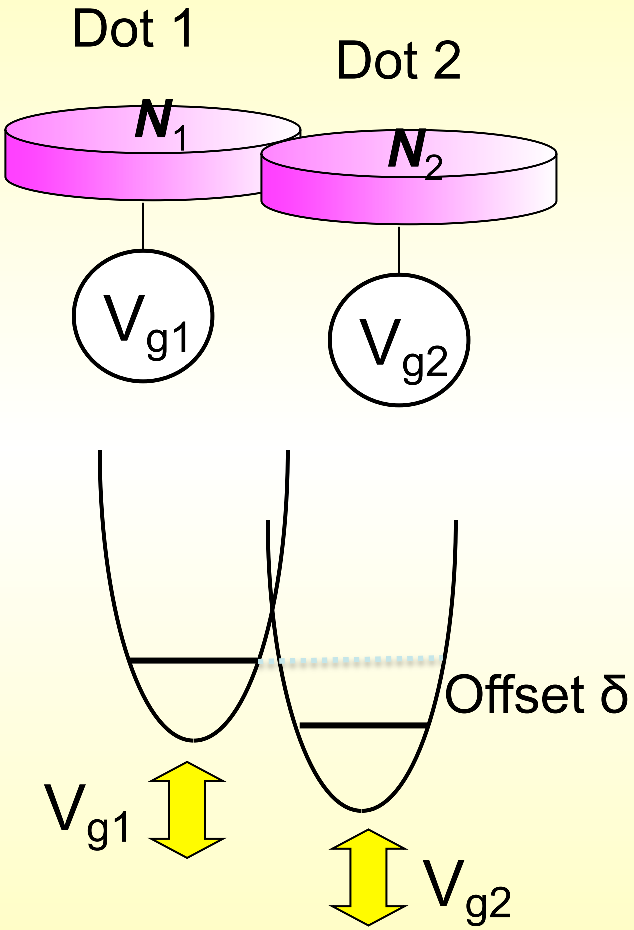
G. Burkard *et al.* *PRB* 00



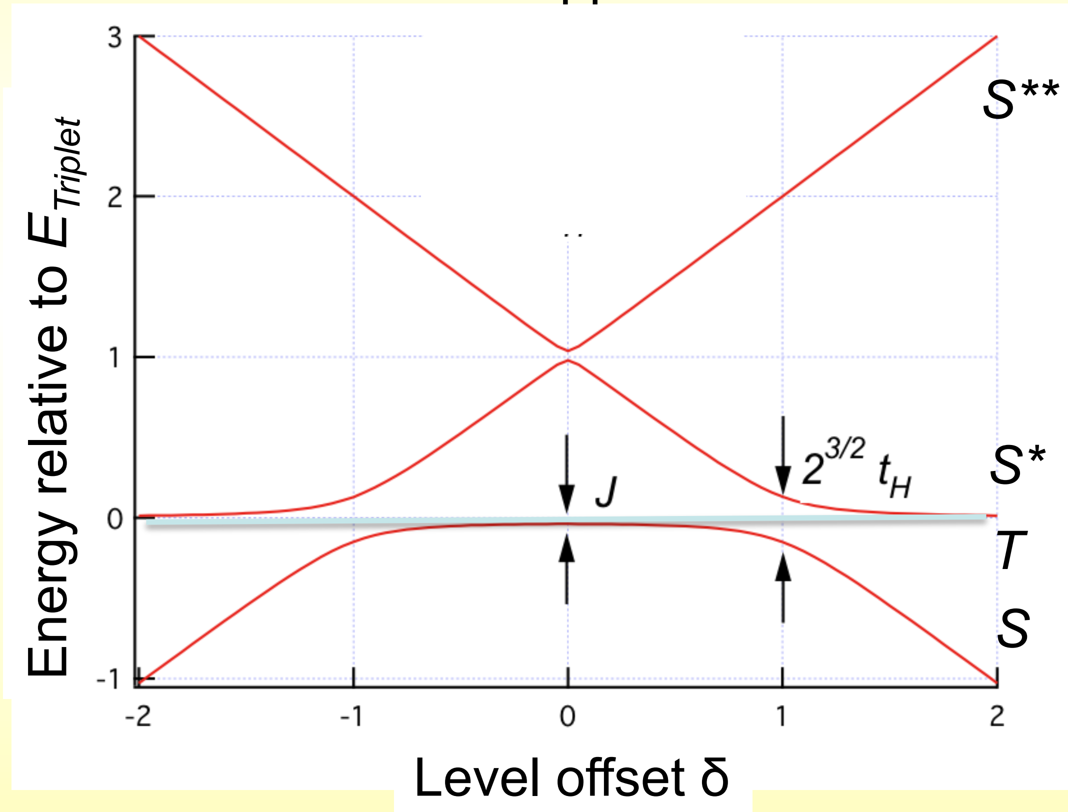
t is controlled by center gate voltage.

$\Rightarrow J=J(t : V_g, B_{ext})$

Offset dependence of exchange J



Two-electron spectra by
Hund-Mulliken approach



Spin qubit using quantum dots

- Concept
- Initialization
- two-qubit operations

Use electron spin for making qubit…Why!?

Natural two level system

Qubit = a|↑> + b|↓>

Correlation of spin exchange

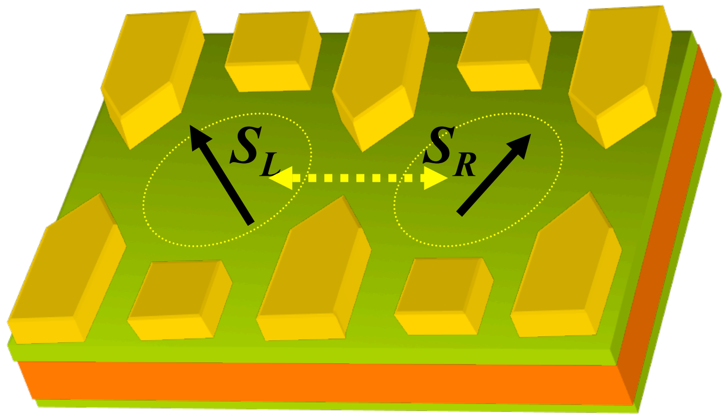
$H_{\mathrm{int}} = J\mathbf{S}_1 \cdot \mathbf{S}_2$

Robust quantum number

Long T_1 and T_2

Scalable in solid state system

Toward > 10^4 -10^5



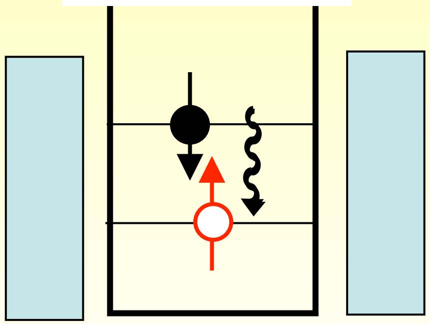
Loss and DiVincenzo PRA (98)

Possible information transfer

Charge….useful for measurement
Atom….useful for storage
Photon….useful for communication

Initialization

Zeeman splitting $E_{Zeeman} = g_{dot}\,\mu B$ ($|g_{dot}| < |g_{bulk}|=0.44$ GaAs)



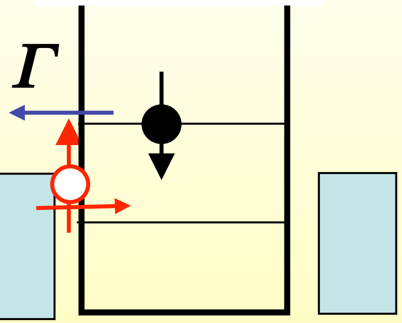
Polarization $=1 - \exp[-E_{Zeeman}/k_BT]$

>99% pure state : $|\uparrow\rangle$ at 300mK

for E_{Zeeman} (B=8 T) $>k_BT$

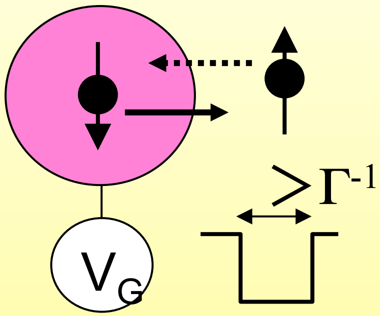
…Easy Initialization by waiting for a time longer than T_1 (ms)

For fast Initialization



Spin exchange by tunneling between the QD and contact leads

Initialization time $< \Gamma^{-1} \sim$ nsec



Universal logic gates

Classical calculation

completeness : {AND, OR, NOT}
{NAND}
{XOR}
.....

input		output	
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0
Bit A	**Bit C**		

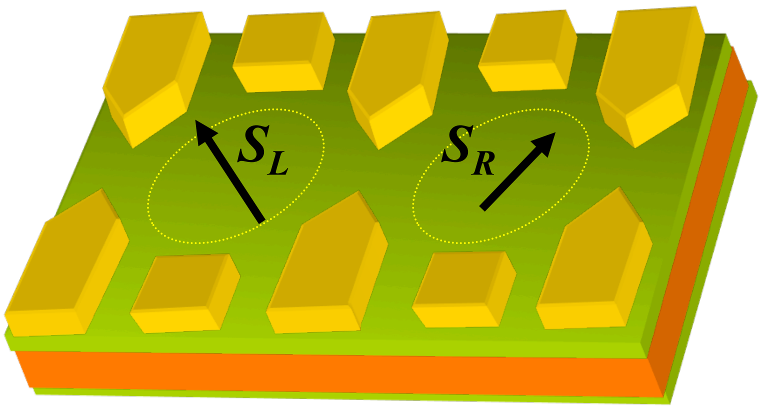
If Bit A input "1", then Invert Bit C

Quantum calculation

{Rotation, CNOT}

||

{Rotation, SWAP$^{1/2}$}



CNOT = XOR
can be prepared using SWAP$^{1/2}$

CNOT(Nonentangled state)=Entangled state
....SWAP$^{1/2}$ "***Entangler***"

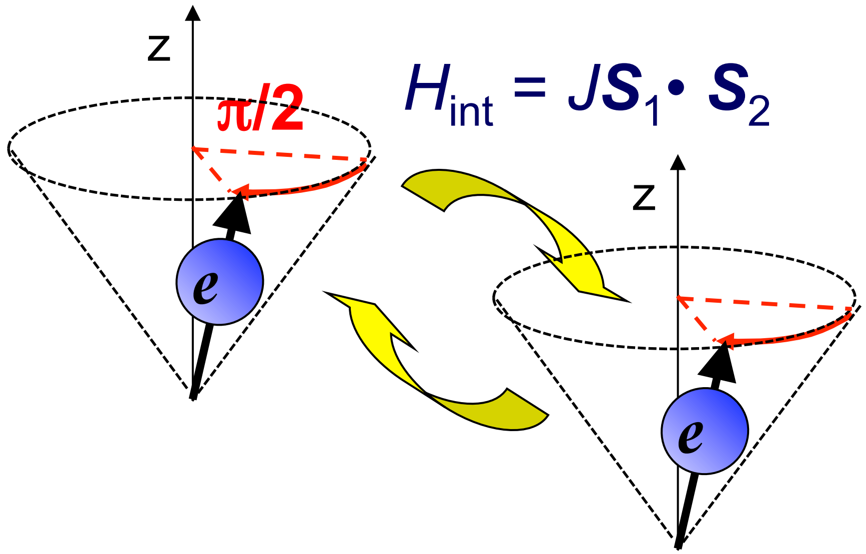
CNOT using exchange coupling

D. Loss and D. DiVincenzo, PRA98

$$U_{CNOT} = U_{sw}^{1/2} \exp[i(\pi/2) S_1^z]\, U_{sw}^{1/2} \exp[i(\pi/2) S_1^z] \exp[-i(\pi/2) S_2^z]$$

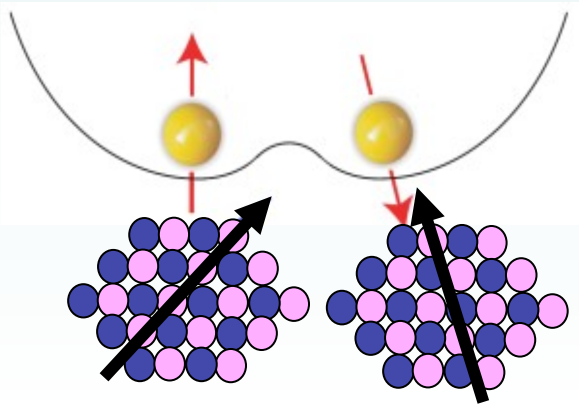
$\pi/2$ rotation of spin 1, 2 about z-axis

Square root SWAP of U_{SW} between spin 1 and 2



Effect of fluctuating nuclear field

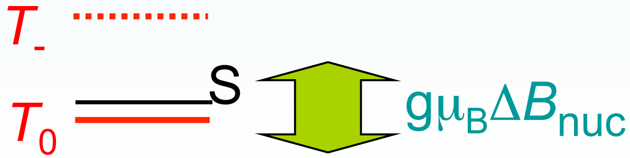
Mixing of singlet and triplet states in a double QD when $J < g\mu_B \Delta B_{nuc}$



$\Delta \mathbf{B}_{nuc} = \mathbf{B}_{1nuc} - \mathbf{B}_{2nuc}$

If $J < g\mu_B \Delta B_{nuc} < E_{Zeeman}$,

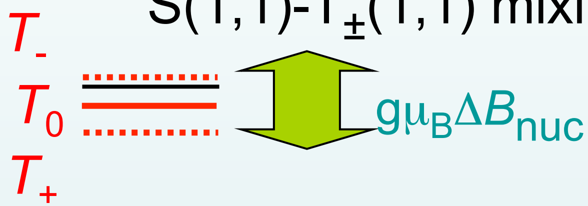
S(1,1)-T_0(1,1) mixing due to ΔB_{nuc}^{z}



E(↑,↓) <> E(↓,↑)

If J, $E_{Zeeman} < g\mu_B \Delta B_{nuc}$,

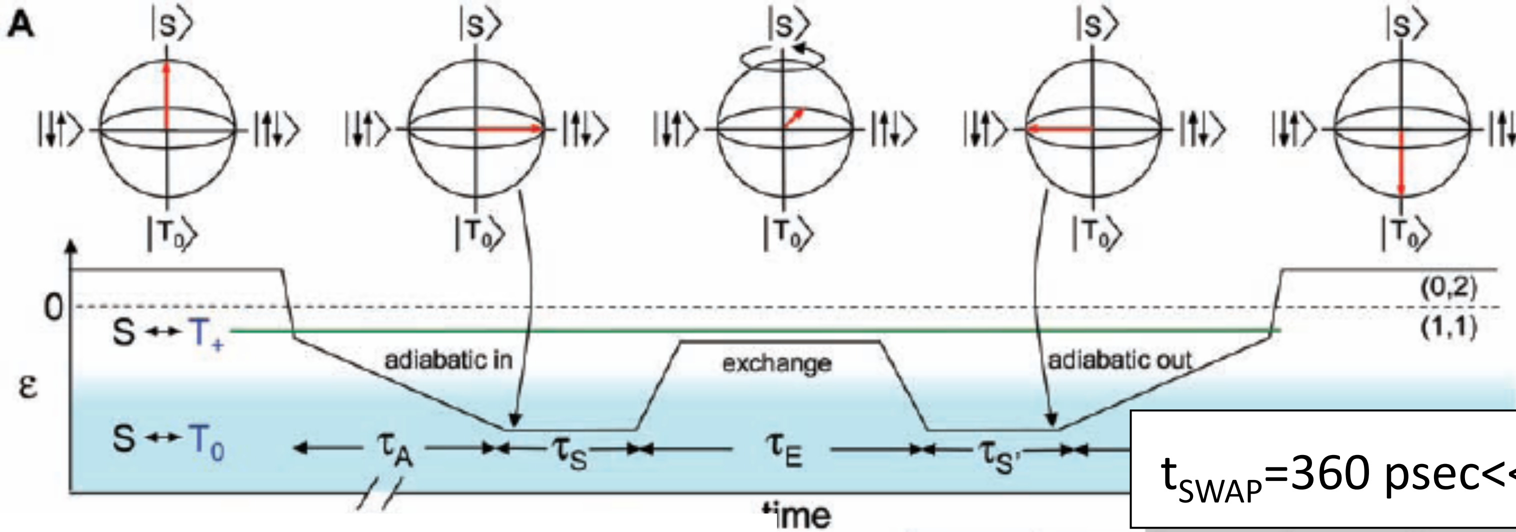
S(1,1)-T_0 (1,1) mixing due to ΔB_{nuc}^{z}
S(1,1)-$T_\pm$(1,1) mixing due to $\Delta B_{nuc}^{x,y}$



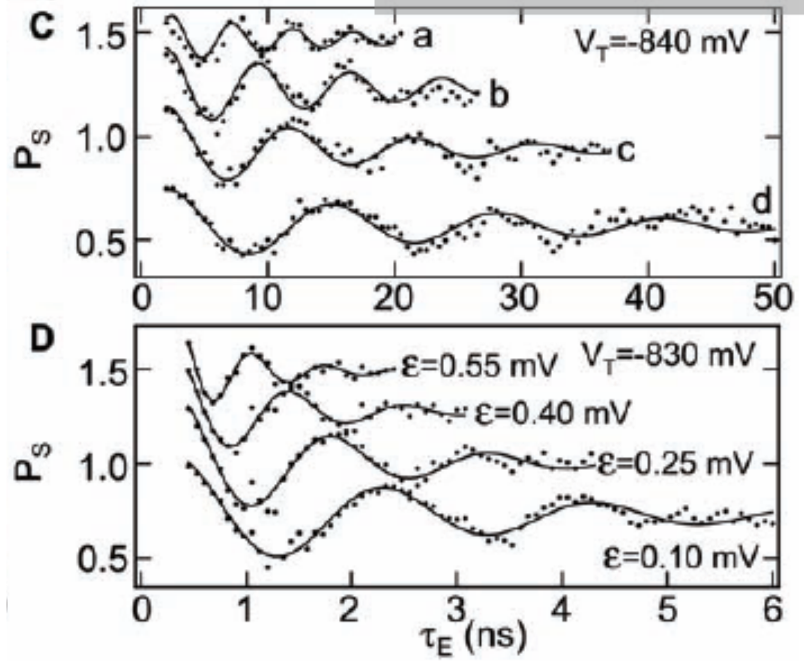
Johnson *et al. Nature* 05
Koppens *et al. Sciece* 05
Kodera, *et al. Physica E* 08

J manipulation: SWAP between |↑↓> and |↓↑>

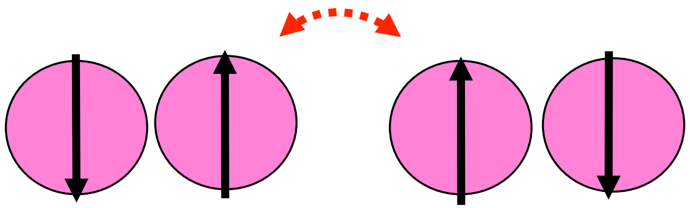
S(0,↑↓) → (↑,↓) —SWAP→ (↓,↑) → T_0 =(|↑↓>+|↓↑>)/√2



t_{SWAP}=360 psec<< t_{Rabi}



control tunnel coupling
to control SWAP



|0> |1>

Petta *et al. Science* 05

Single spin rotation using

Global dc B field and local ac B field

....On-chip coil

....Electrically driven spin resonance

Coherent manipulation of single electron spins

Local ac magnetic field has been generated by various ways

electrically by injecting an ac current to an on-chip coil

Koppens *et al.* Science 2006

and by applying an ac electric field

Nowack *et al.* Science 2007

Pioro-Ladriere *et al.* Nature Physics 2008

optically by applying an off-resonance laser pulse to induce optical Stark effect

F. Jelezko et al. PRL2004

R. Hanson et al. PRL 2006; Science 2008

D. Press et al. Nature 2008

Note: First electrical control of spin qubit made out of |↑↓> and |↓↑> using SWAP in double QD

Petta *et al.* Science 2005

Optical control of single electron spins

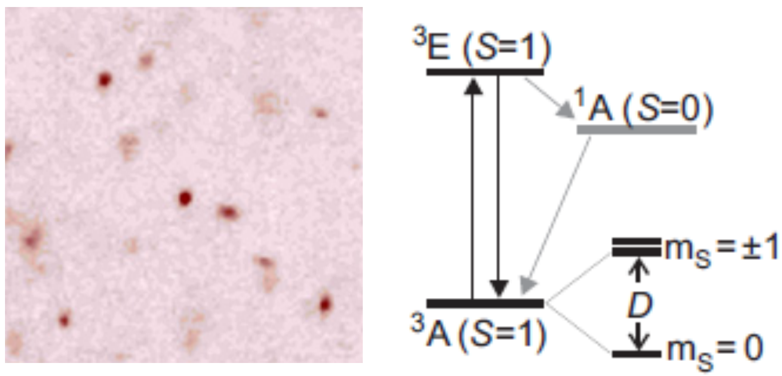
N-V center in diamond

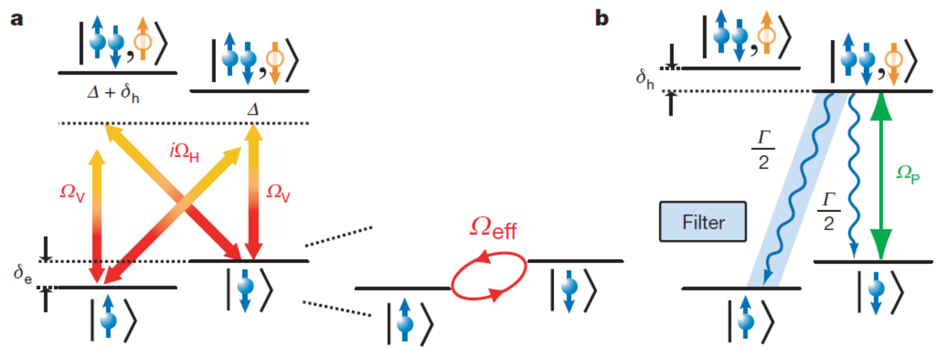
F. Jelezko *et al.* PRL2004

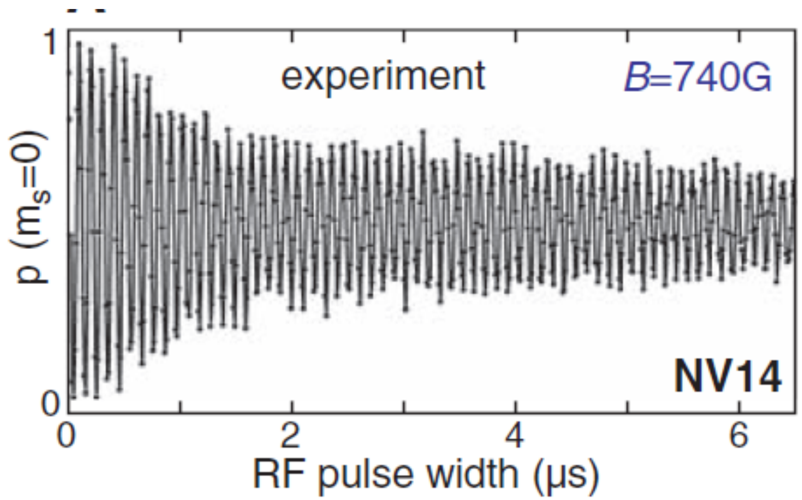
R. Hanson *et al.* PRL 2006; Science 2008

Self-assembled InGaAs quantum dots

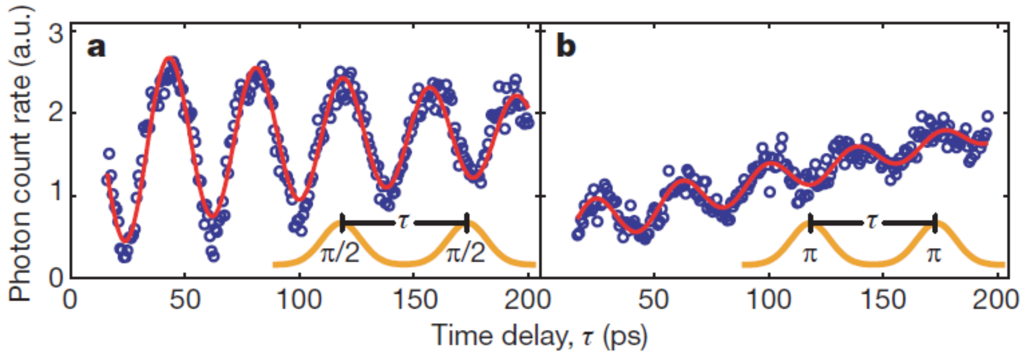
D. Press *et al.* *Nature* 2008







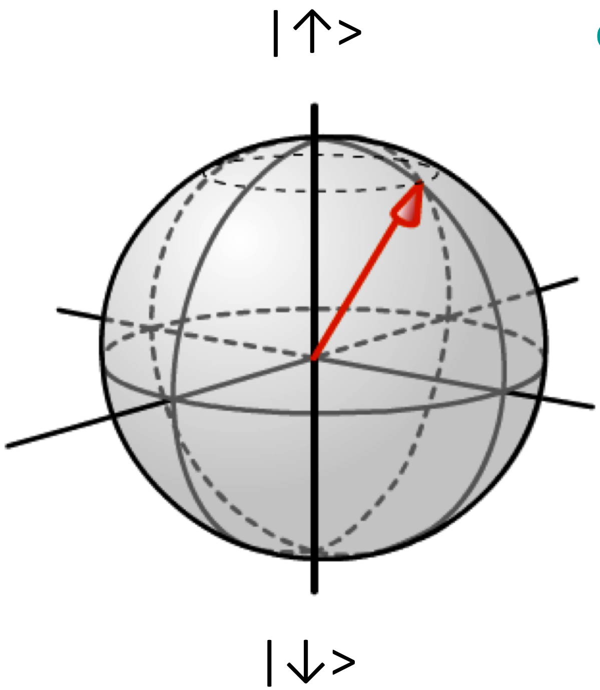
Ramsey

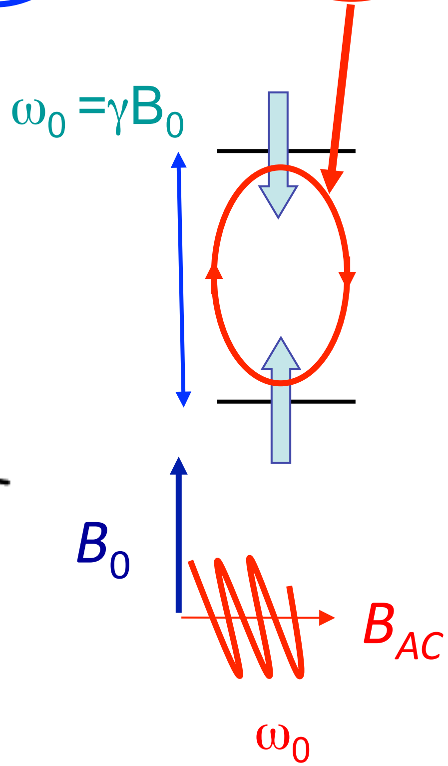


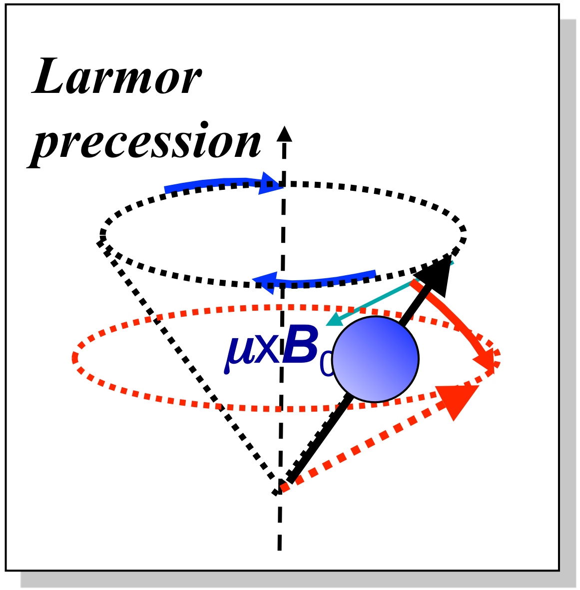
Concept of spin rotation=ESR

ESR Hamiltonian

$$H_{2level} = \frac{\varepsilon_z}{2}\sigma_z + \frac{\varepsilon_x(t)}{2}\sigma_x$$







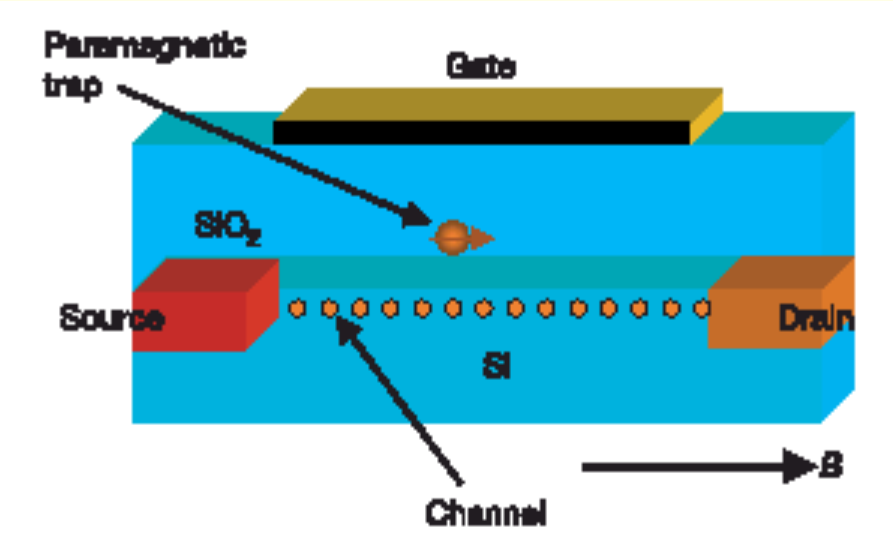
Scalable qubits

To manipulate a single spin in a QD:	To manipulate more spins in a multiple QD:
Global DC B_0 + **Local** AC B_{AC}	**Local** DC B_0 + **Local** AC B_{AC}

Paramagnetic defect located near Si/SiO2

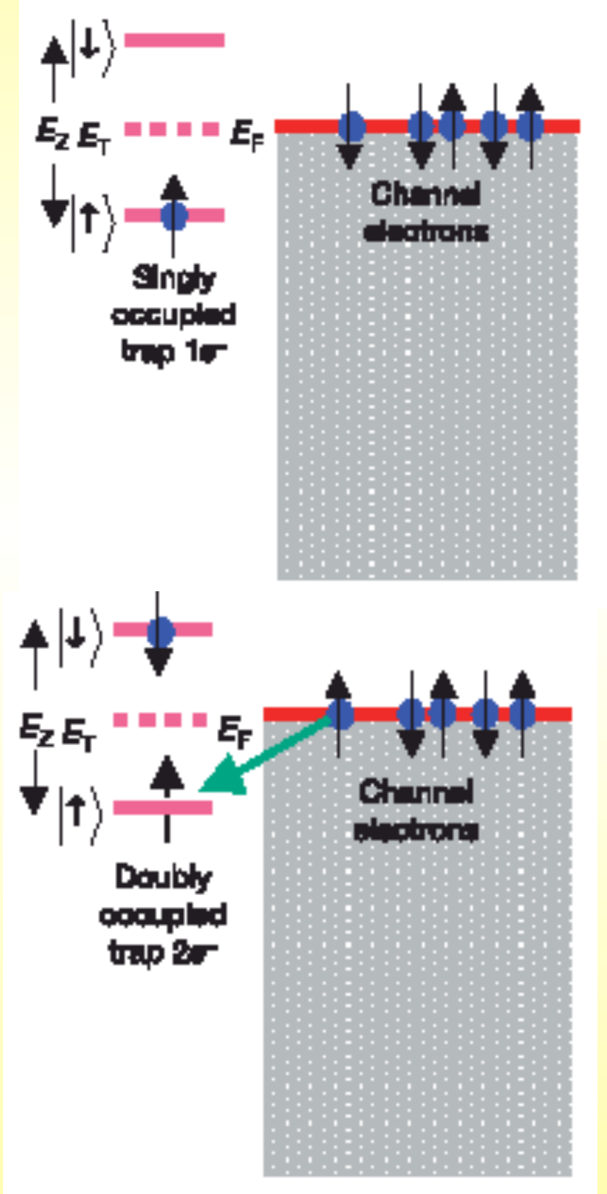
Electrical detection of the spin resonance of a single electron in a silicon field-effect transistor

M. Xiao[1], I. Martin[2], E. Yablonovitch[3] & H. W. Jiang[1]

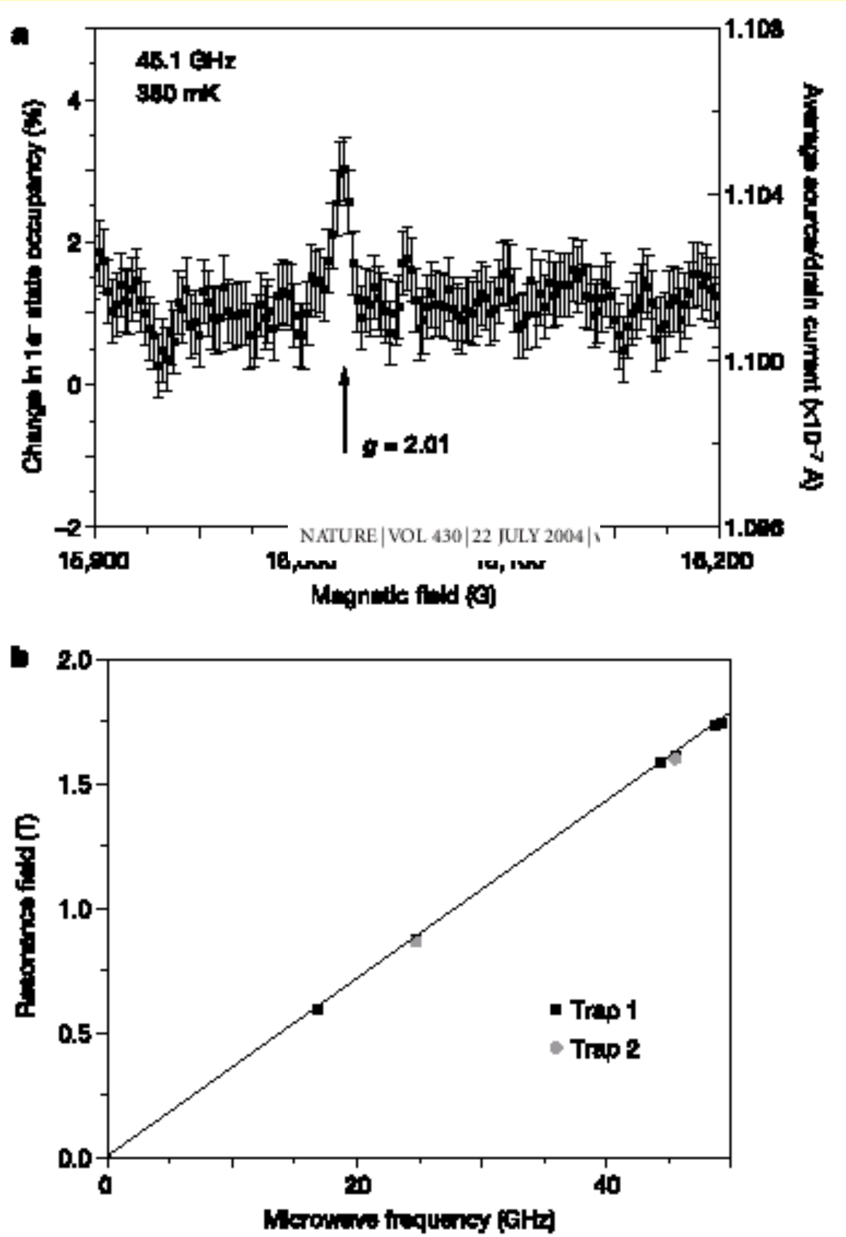


Cavity ESR:
An ESR peak in the shot noise current signal
⟶ T2*= 100 nsec, due to spin-spin relaxation

Note:T2 ~ 100 μsec for isolated paramagnetic defect
T2* ~ 100 ns for Si/SiGe 2DEG



ESR of a single defect

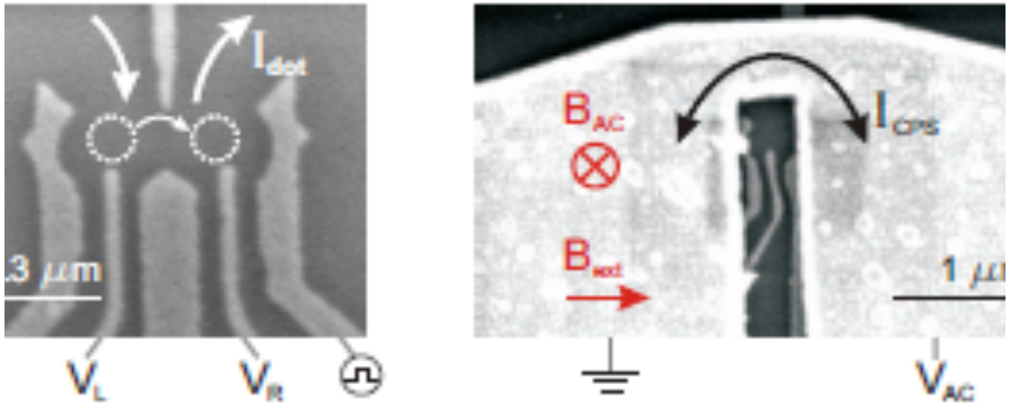


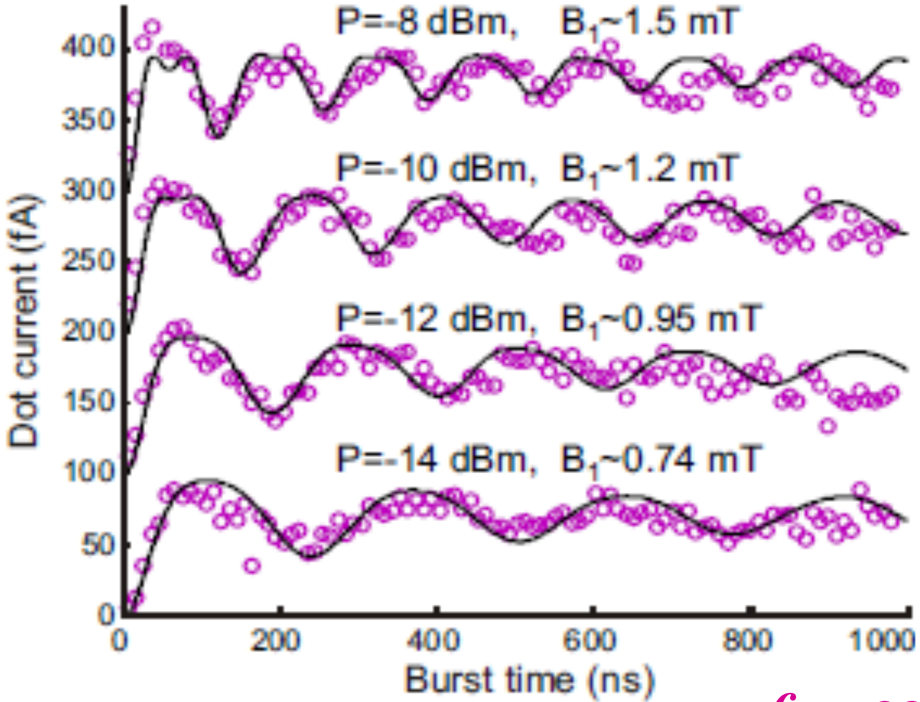
T_2 derived from FWHM = 0.1 μsec, due to spin-spin relaxation
Note: T_2 ~ 100 μsec for isolated paramagnetic defect

Linear relationship of resonant magnetic field versus microwave frequency: g-factor ~2.0

Straightforward technique: on-chip coil

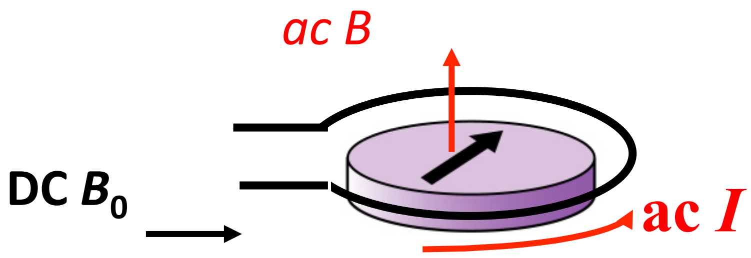
Koppens *et al Science* 06





$f_{\mathrm{Rabi}} \propto I_{\mathrm{ac}}$

B_{ac} induced by I_{ac} flowing a coil

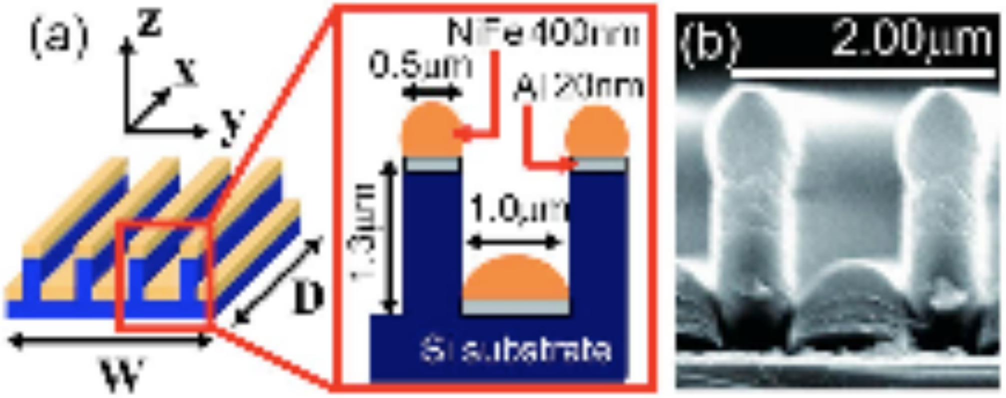


I_{AC} = 1 mA B_{ac}~ 1 mT
π rotation: ~ 80 ns

accompanied heating and difficulty in localizing the field
….Problem in qubit scalabilty

Voltage driven ESR

Spin address with magnetic field gradient



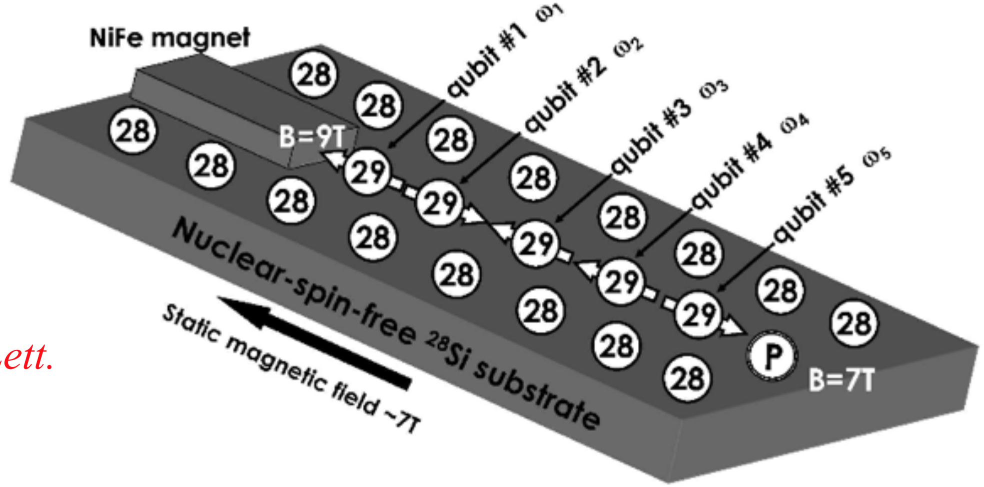
b_{SL} ~ -0.38 T/μm

Direct NMR observation of local magnetic field generated by micromagnet

S. Watanabe et al. Appl. Phys. Lett. 92, 253116 (2008)

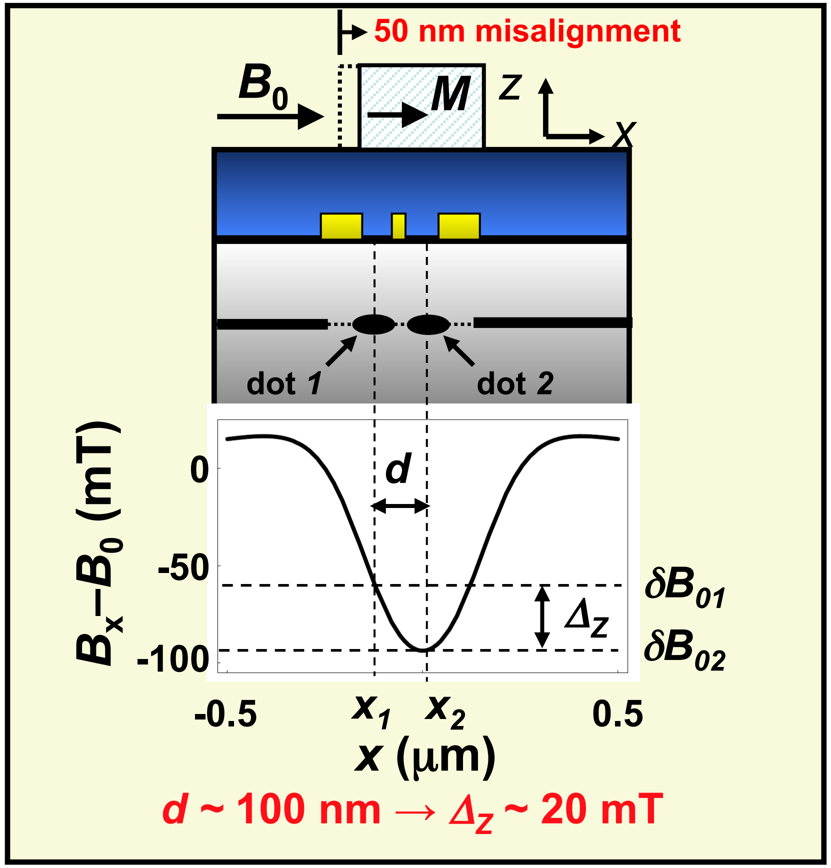
Proposal of an all Silicon quantum computer

T. D. Ladd et al. Phys. Rev. Lett. 89, 017901 (2002)



Selectivity: nano-MRI

Stray field parallel to external field



E. A. Laird et al., Phys. Rev. Lett. (2007)

M. Piore-Ladriere et al. Nat. Phys. 4, 776 (2008)

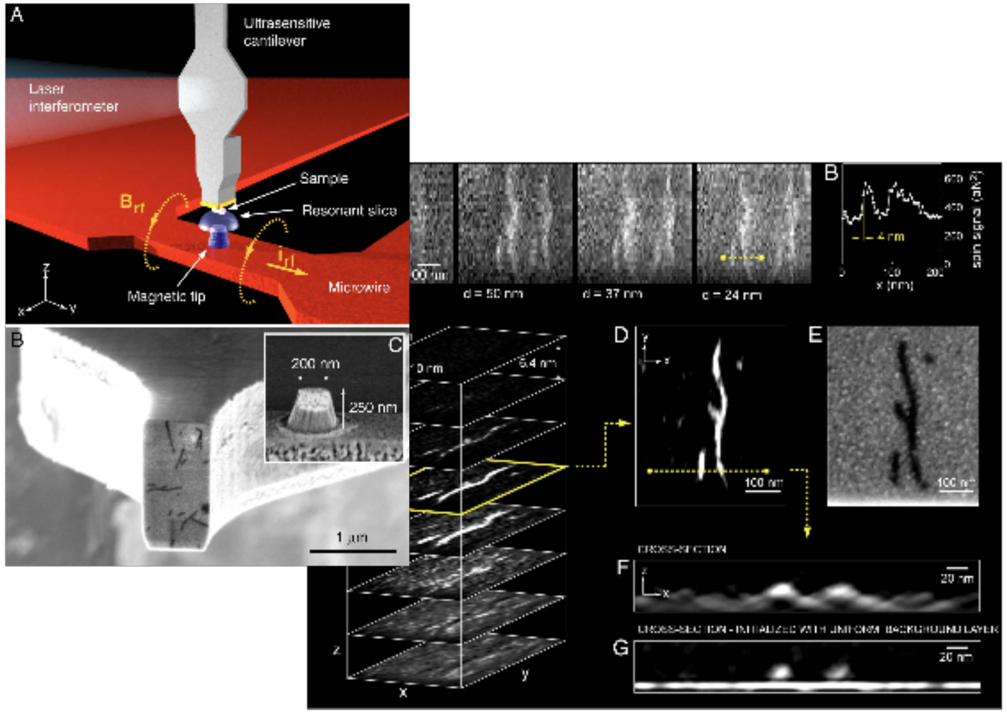
Nanoscale magnetic resonance imaging

C. L. Degen[a], M. Poggio[a,b], H. J. Mamin[a], C. T. Rettner[a], and D. Rugar[a,1]

[a]IBM Research Division, Almaden Research Center, 650 Harry Road, San Jose, CA 95120; and [b]Center for Probing the Nanoscale, Stanford Uni… Lomita Mall, Stanford, CA 94305

PNAS ***106***, *1313 (2009).*

Tobacco mosaic virus

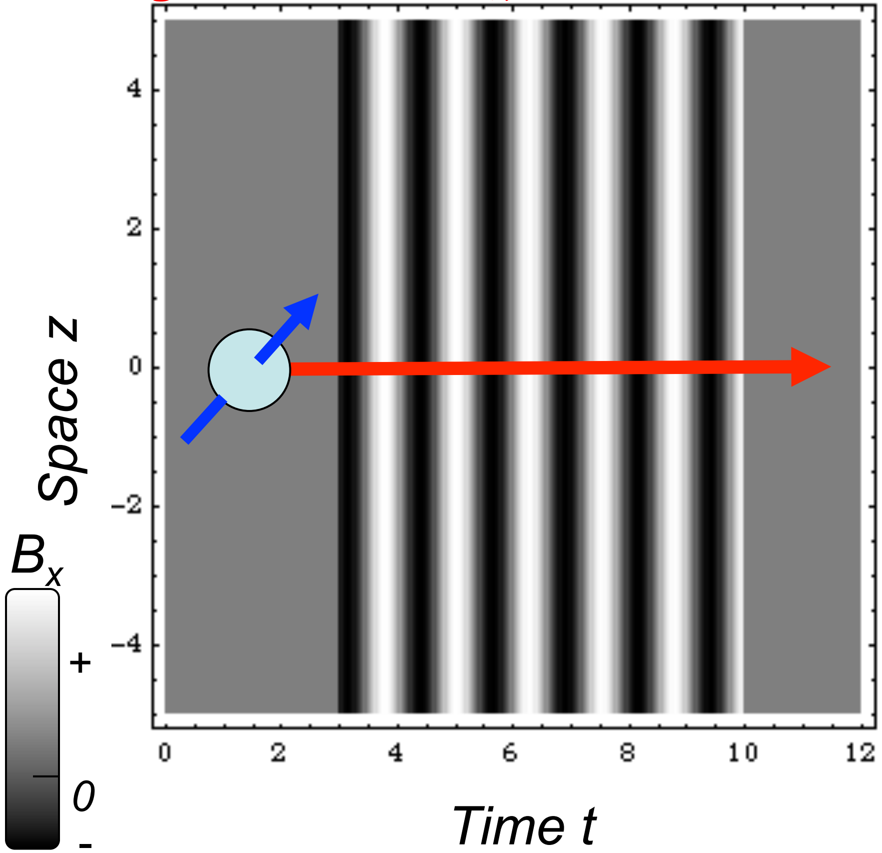


Magnetic field gradient enables selective ESR addressing for each spins like MRI.

Electric dipole Single ESR

Standard spin resonance

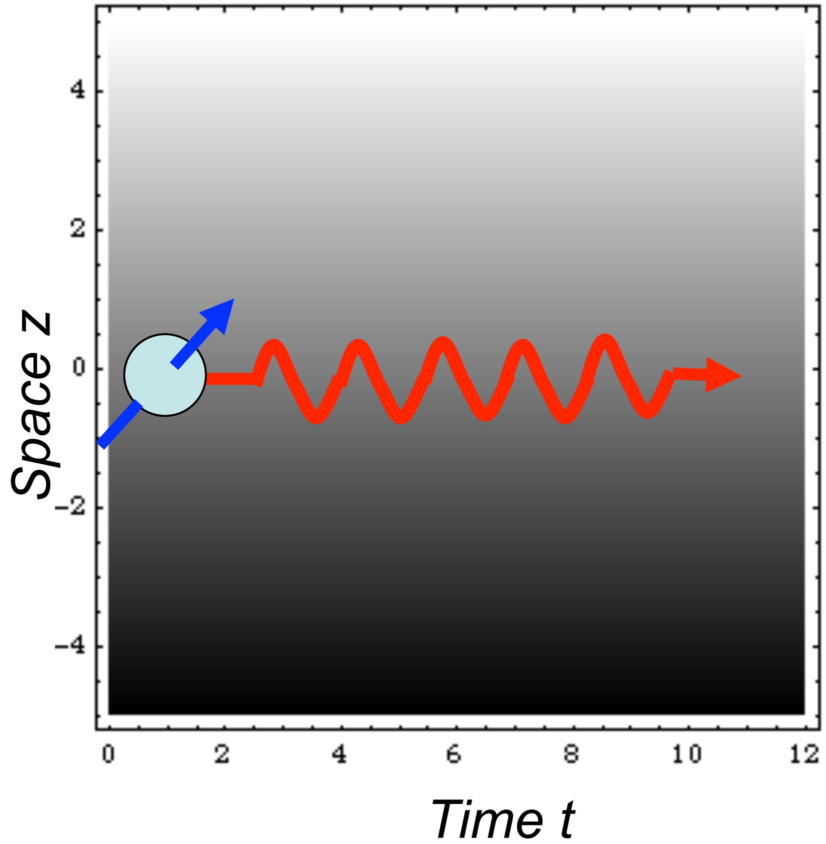
Time-dependent uniform magnetic field $B_x(t)$



~20 GHz local B field: $B_x(t)$

Electric driven resonance

Non-uniform static magnetic field $B_x(z)$

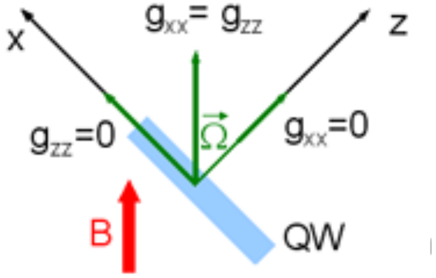


~20 GHz local E field: z(t)

No heating and better designed for making multiple qubits

Physical systems of non-uniform field

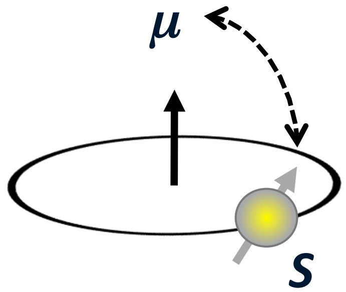
g-tensor modulation



Y. Kato
Science 2003

g-tensor engineering ...Optical control

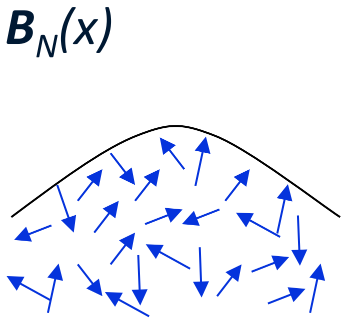
Spin-orbit



K. C. Nowack
Science 2007

$\mathbf{B}_{loc}=(\nabla V \times \mathbf{p})\sigma$ ...realistic but not general and small in GaAs

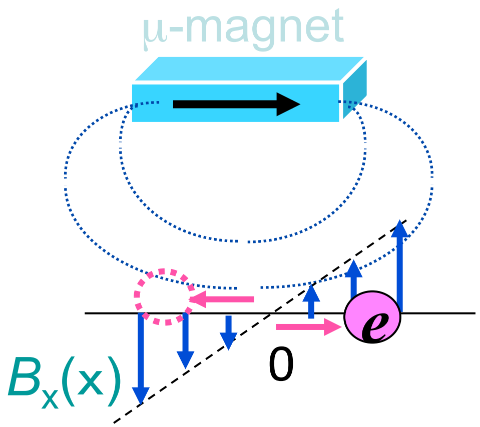
Hyperfine int.



E. A. Laird
PRL (2007)

not-coherent ... Local, static B field

Slanting Zeeman field



Y. Tokura
PRL 2006

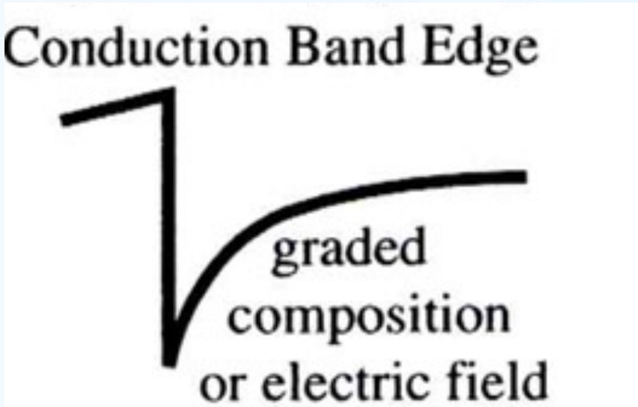
Micro-magnet induced field gradient…slanting Zeeman field

Spin-orbit Interaction (SOI)

$$\boldsymbol{H}_{SOI} = -\frac{e\hbar}{4m^2c^2}\,[\,E \times \boldsymbol{v}\,]\cdot \boldsymbol{s}$$

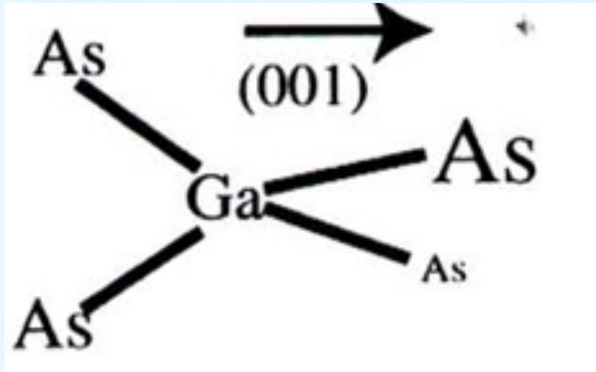
Rashba SOI

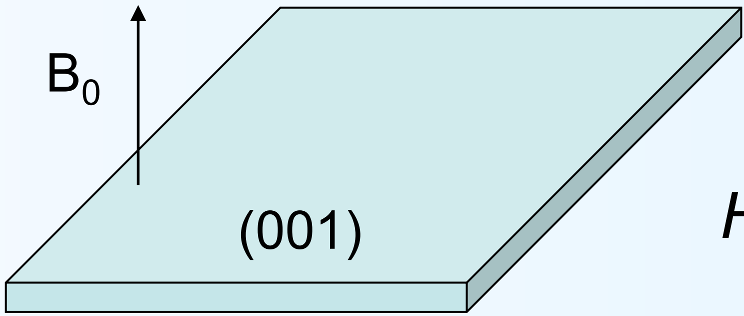
Structural inversion symmetry



Dresselhaus SOI

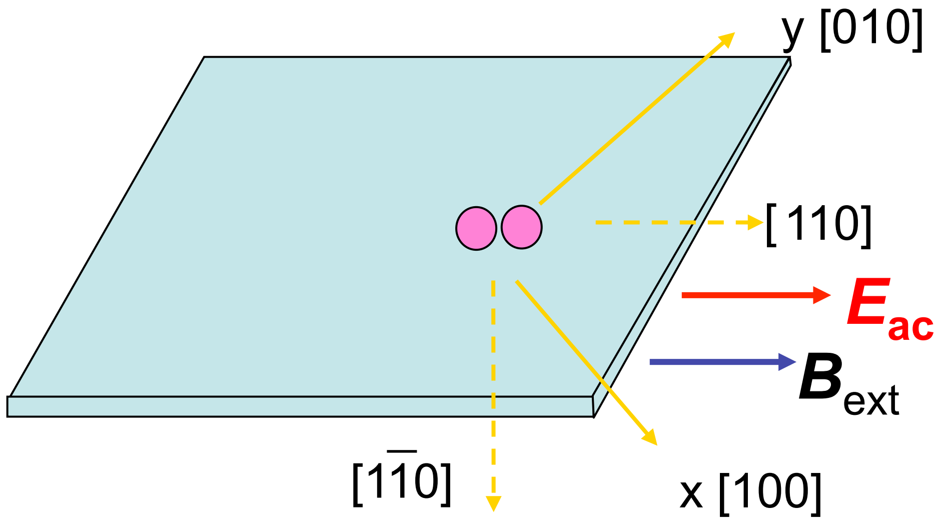
Bulk inversion asymmetry





$$H_{SOI} = \alpha(-p_y\sigma_x + p_x\sigma_y) + \beta(-p_x\sigma_x + p_y\sigma_y)$$

Local B field generation by SOI



$H_{spin} = H_{Zeeman} + H_{SOI}$

$H_{Zeeman} = -(1/2) g_B \mu B_{ext} \boldsymbol{\sigma}$

$H_{SOI} = \alpha(-p_y\sigma_x + p_x\sigma_y) + \beta(-p_x\sigma_x + p_y\sigma_y)$

Rashba *Dresselhaus*

External $\boldsymbol{B}_{ext}$ -> position-dependent $\boldsymbol{B}_{loc}$

$$\mathcal{U} = \exp[-i\frac{m}{\hbar}\{(\alpha x + \beta y)\sigma_y - (\beta x + \alpha y)\sigma_x\}],$$

$$\tilde{\mathcal{H}}_{spin} \equiv \mathcal{U}^\dagger \mathcal{H}_{spin} \mathcal{U}$$

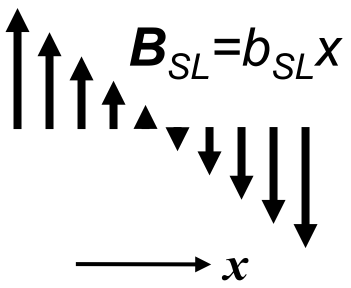
$$= -\frac{1}{2} g\mu_B [B_{ext}\sigma_x + B_{SOI}(x)\sigma_z],$$

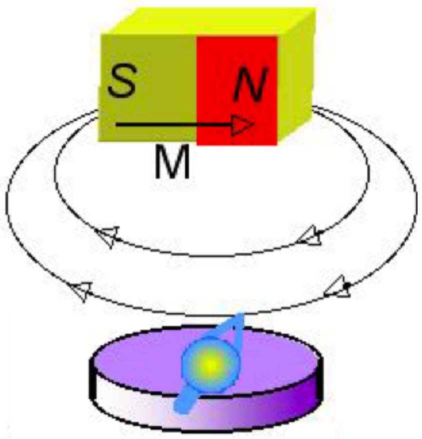
$$B_{SOI}(x) = B_{ext}\frac{2m}{\hbar}(\alpha \pm \beta)x$$

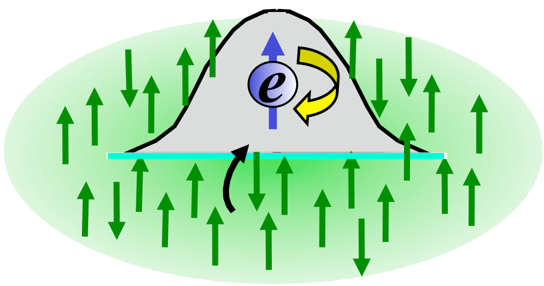
\+ Bext // [110]
\- Bext // [1-10]

Golovach et al., PRB 06, 都倉,固体物理 44, 17 (2009).

Generic slanting Zeeman fields







Effective slanting field by SOI

$$B_{SOI}(x) \quad = \quad B_{ext}\frac{2m}{\hbar}(\alpha \pm \beta)x$$

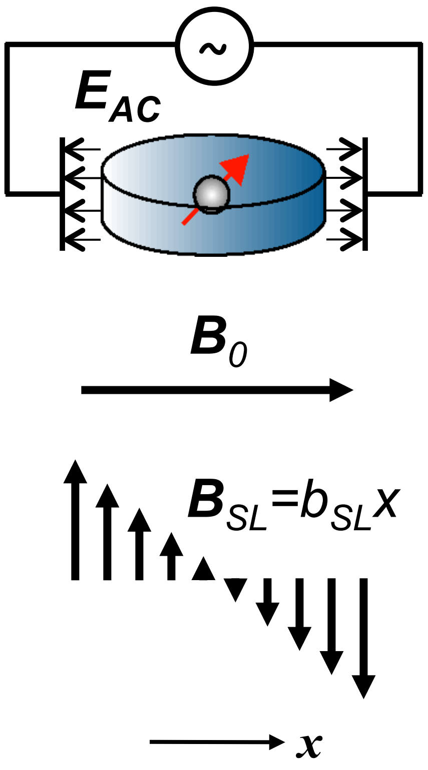
Slanting field by on-chip micro-magnet

$$\mathbf{B}_{magnet} \quad = \quad [\delta B + b_{SL}z]\hat{\mathbf{x}} + b_{SL}x\hat{\mathbf{z}}$$

Effective slanting field by nuclear spin

$$\begin{aligned}
\mathcal{H}_{HF} &= \frac{A}{2}\sum_j \Psi_0^2(\mathbf{r}_j)\mathbf{I}_j \cdot \sigma \\
&= \mathcal{H}_{HF}^0 + \frac{A}{4}\sum_j (\mathbf{r}\cdot\partial_{\mathbf{r}_j})\Psi_0^2(\mathbf{r}_j)\{I_j^+\sigma^- + I_j^-\sigma^+\} \\
&\equiv \mathcal{H}_{HF}^0 - \frac{1}{2}g\mu_B\{\mathbf{b}_{HF}^+\sigma^- + \mathbf{b}_{HF}^-\sigma^+\}\cdot\mathbf{r}
\end{aligned}$$

AC electric field with a slanting Zeeman field



AC electric field E_{AC} **affects the electron charge:**

$$\mathcal{H}_{el} = e\mathbf{E}_{AC}(t)\cdot\mathbf{r}$$

Introduction of canonical transformation

$$\Psi_0(\mathbf{r},t) = e^{-i\mathbf{k}\cdot\mathbf{R}(t)}\Psi_0^{osc}(\mathbf{r},t),$$

$$\mathbf{R}(t) \equiv -\frac{e\mathbf{E}_{AC}(t)}{m\omega_0^2}$$

which transforms position operator with a time-dependent displacement

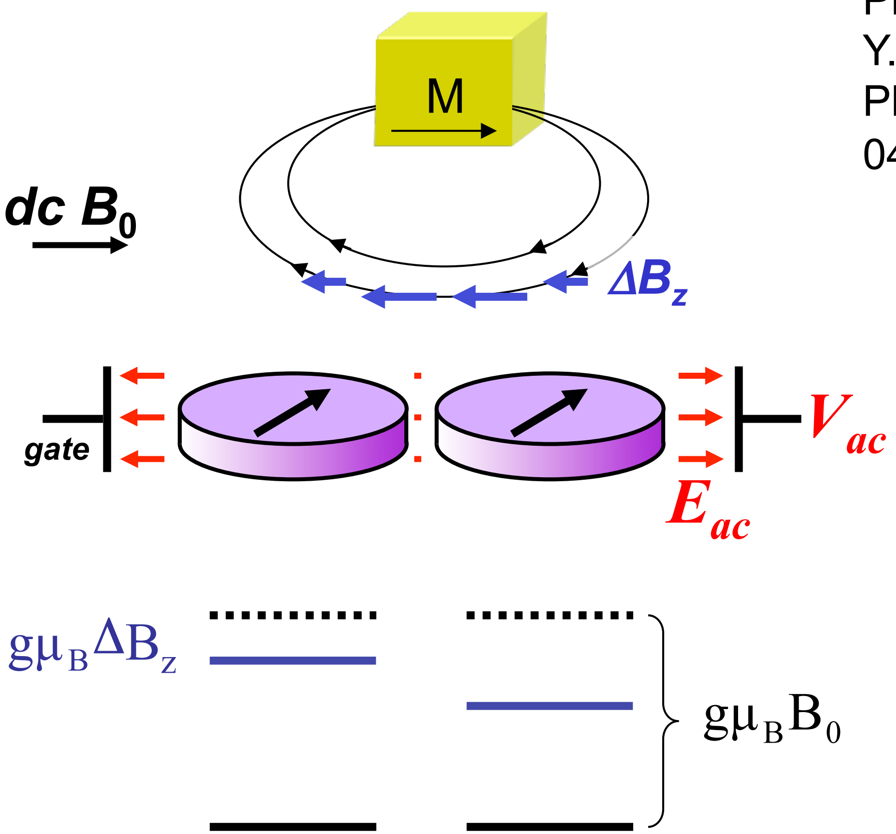
$$\tilde{\mathbf{r}} \equiv e^{i\mathbf{k}\cdot\mathbf{R}(t)}\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{R}(t)} = \mathbf{r}+\mathbf{R}(t)$$

Hence, the slanting field become equivalent to time-dependent transverse field:

$$\tilde{B}_{SL}(t) \equiv b_{SL}R_x(t)$$

ESR Hamiltonian

EDSR with Slanting Zeeman Field by Micro-magnet

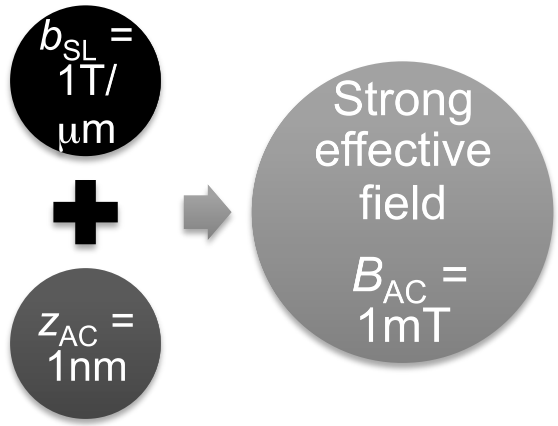


ΔB_z different in two dots.
Address two spins independently.

Proposal
Y. Tokura *et al.*,
Phys. Rev. Lett. **96**, 047202 (2006)

Experiment
M. Pioro-Ladrière *et al.*,
Nat. Phys. **4**, 776 (2008).

Large gradient



Small displacement
(V_{AC} ~ 1 mV)

No need for spin-orbit coupling, hyperfine interaction or g-factor engineering.

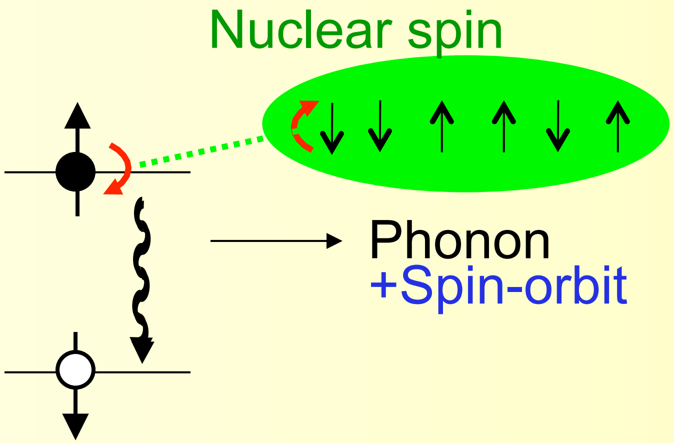
Decoherence problem

- T_1, T_2, and T_2^*
- Spin-orbit and nuclear spin coupling

Spin relaxation

Spin scattering in nonmagnetic semiconductor

Energy+Spin relaxation **(T_1)**



SO coupling + coupling to phonons

$T_1 > 100\ \mu sec$

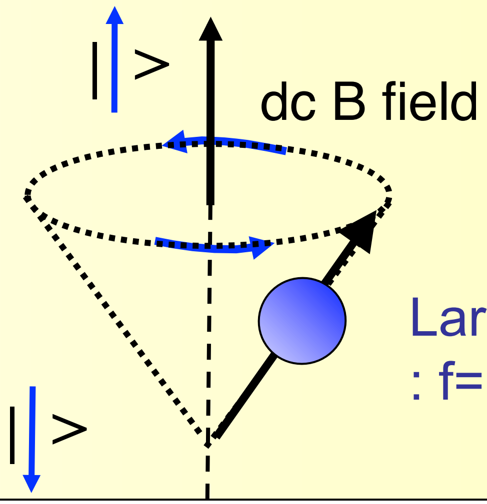
Khaetskii and Nazarov, *PRB* (00) for GaAs; Golovach *et al*. *PRL* (04) for GaAs QD

"Slow" due to weak coupling to environment

Hyperfine coupling to nuclear spins

$T_1 >> \mu sec$ due to DOS discreteness

Erlingsson *et al*. PRB (01); Khaetskii *et al*. *PRL* (02)



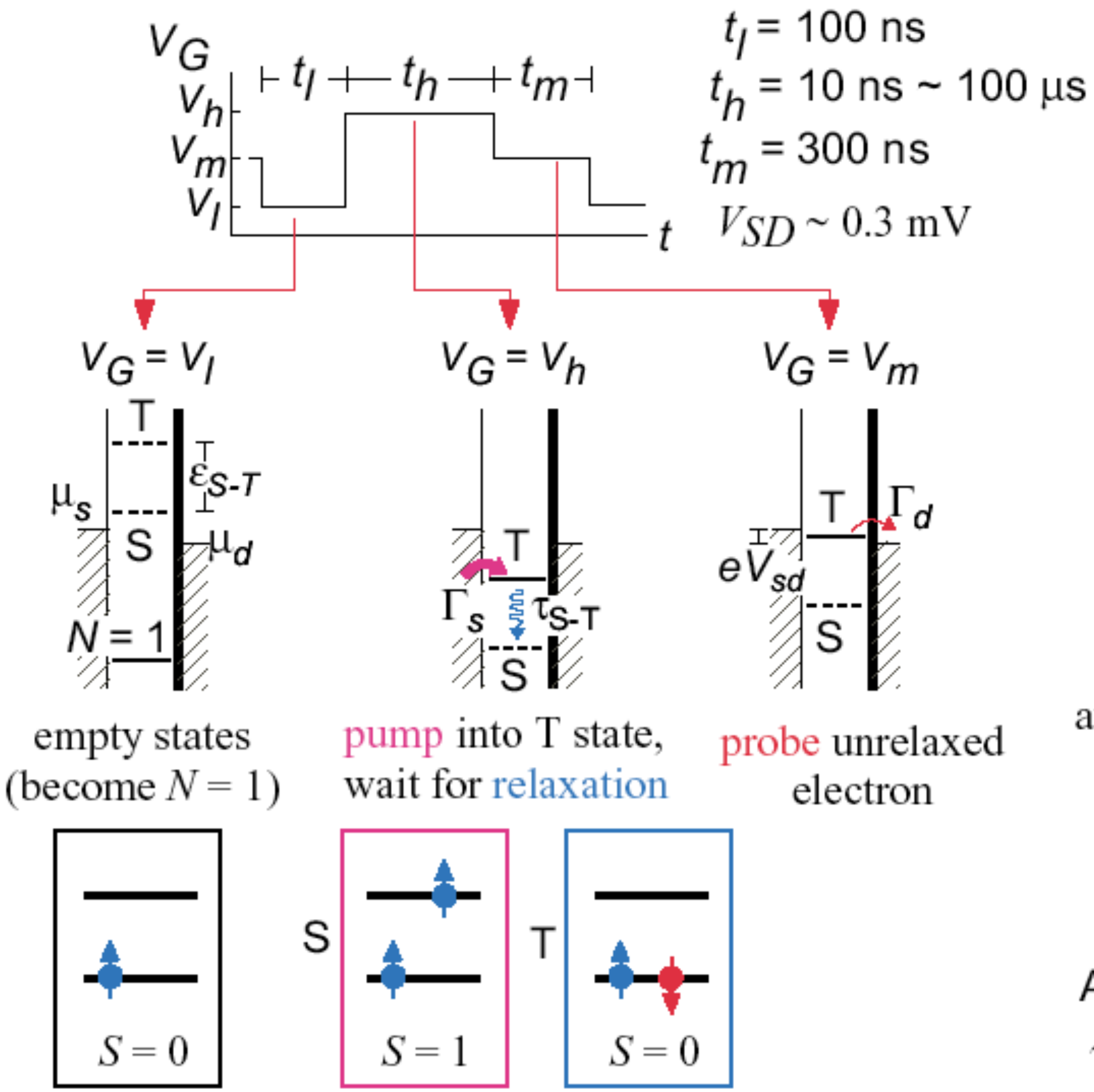
Phonon only influences $\delta B(t)$ along xy.

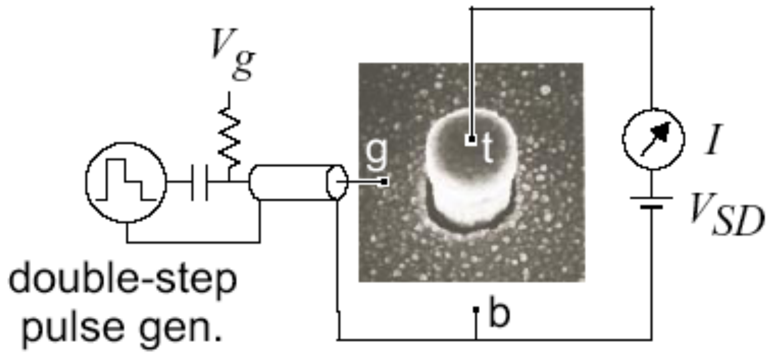
Larmor precession : $f = g\mu B./h$

$T_2 = 2T_1$ > msec

Electrical Pump & Probe Measurement

Fujisawa *et al.* *Nature* **419**, 278 (02)





rise time at sample: ~ 0.7 ns

($< \Gamma_d^{-1}, \Gamma_s^{-1}, \tau_{S\text{-}T}$)

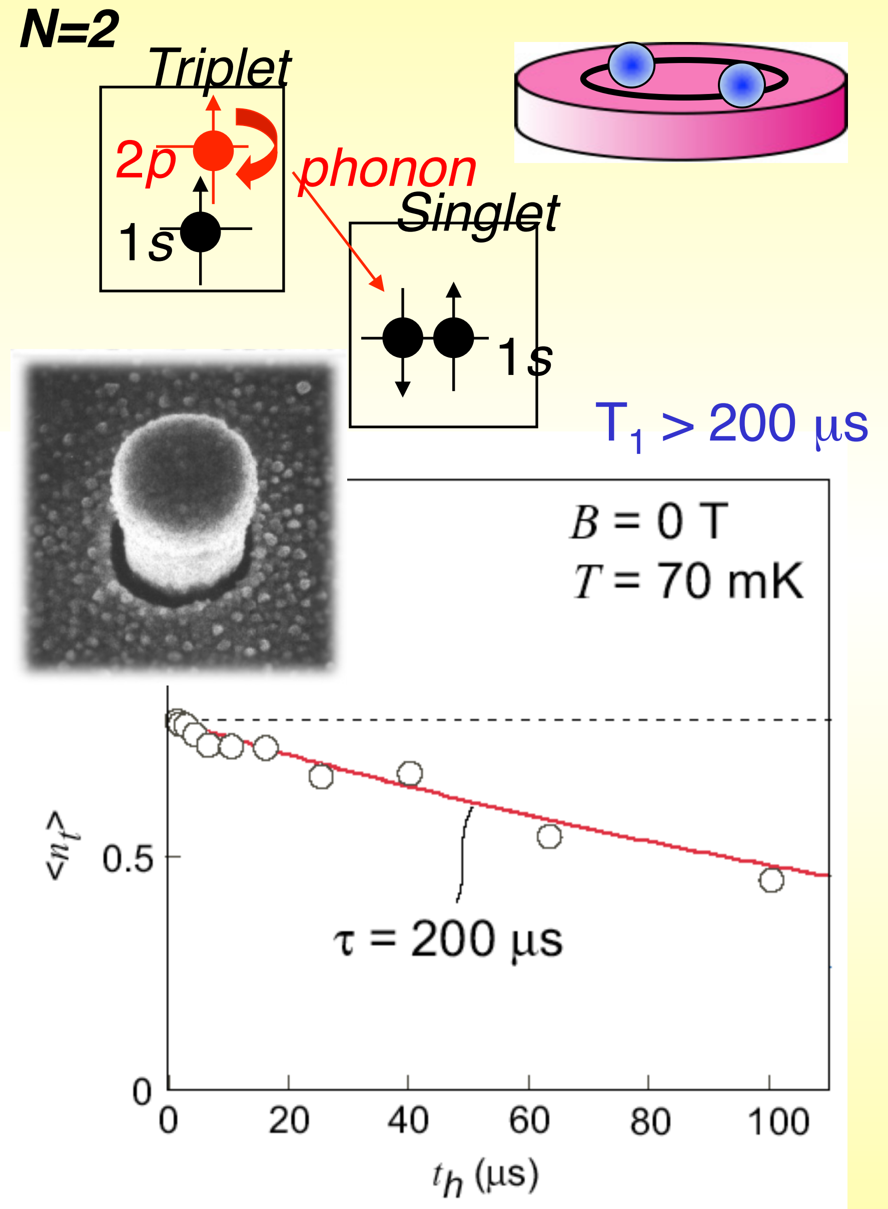
average number of tunneling electrons during one pulse

$$\langle n_t \rangle = A\exp(-t_h/\tau_{S\text{-}T})$$

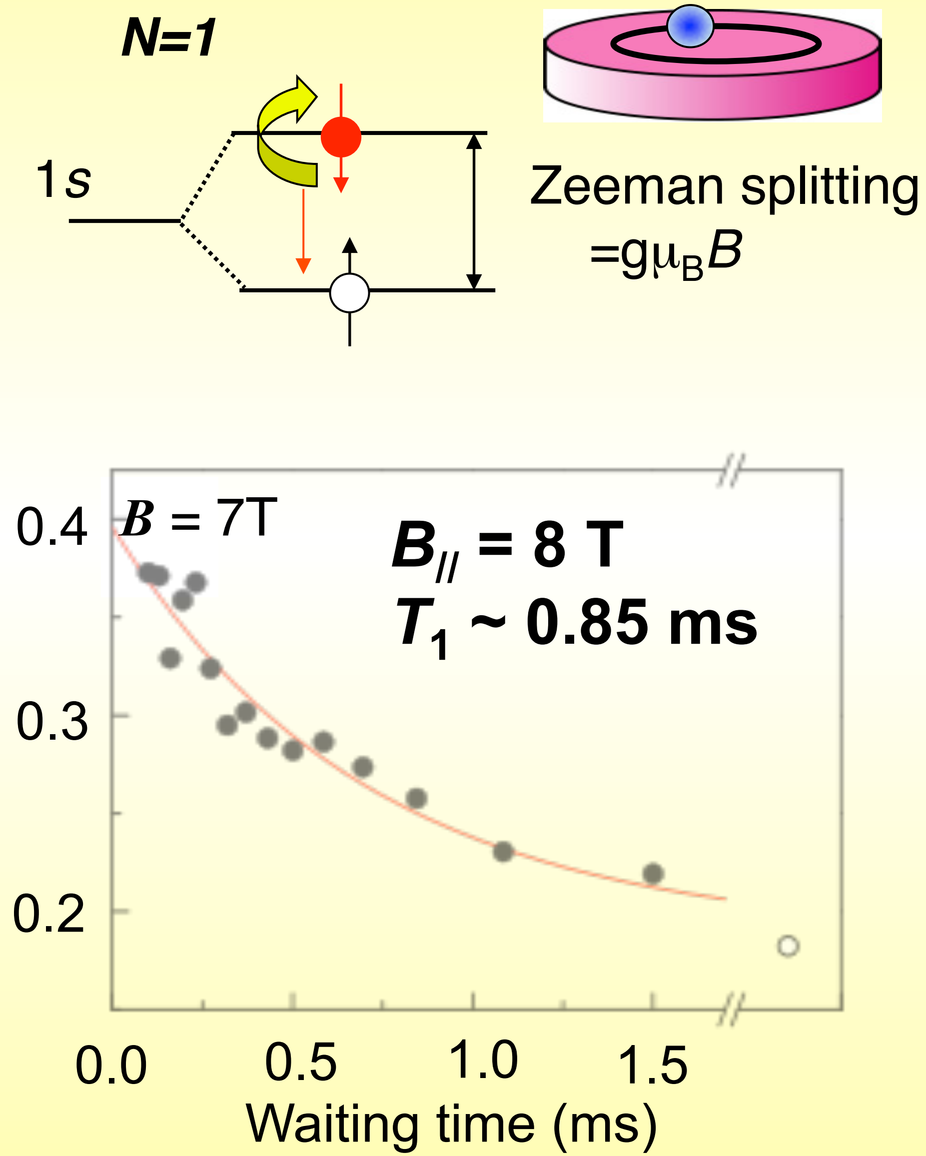
A~1: related to the injection efficiency

$\tau_{S\text{-}T}$: spin-flip energy relaxation time

Measurement of Spin LifetimeT_1: SO effect



Fujisawa *et al.* *Nature* (2002)

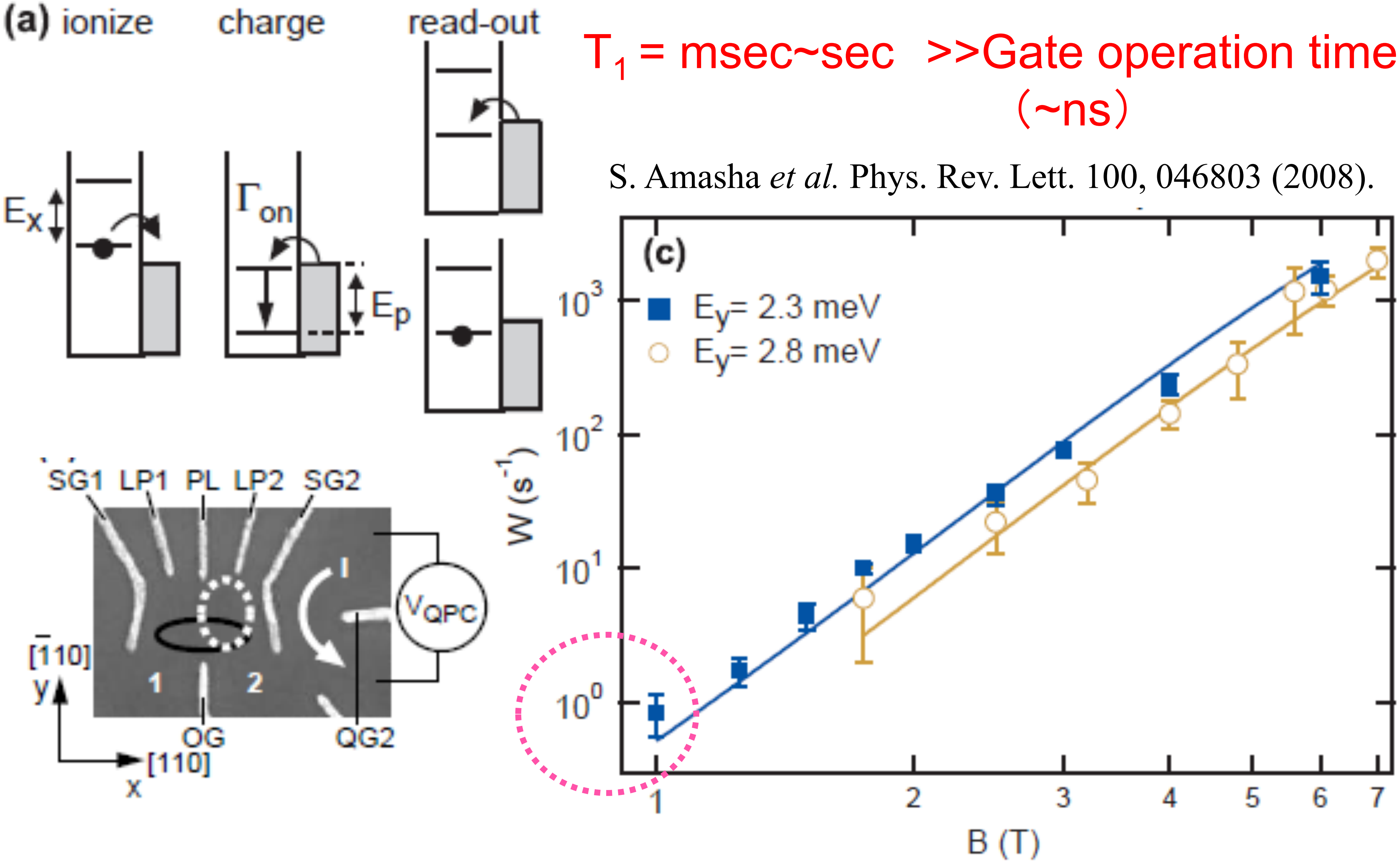


Elzerman *et al.* *Nature* (2004)

Energy relaxation time: field dependence

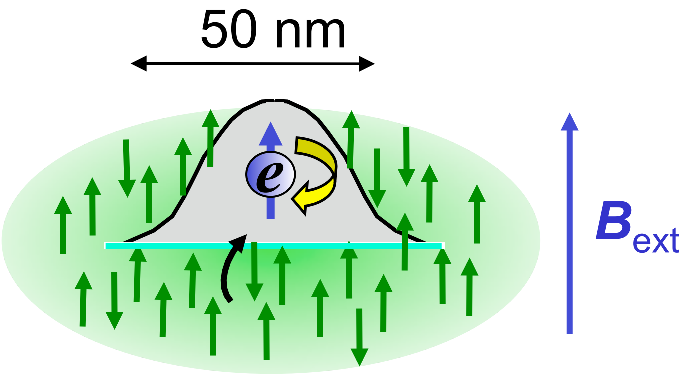
T_1 = msec~sec >>Gate operation time (~ns)

S. Amasha *et al.* Phys. Rev. Lett. 100, 046803 (2008).



Nuclear Spin Bath Problem

Contact interaction to nuclei: ^{69}Ga, ^{71}Ga, ^{75}As (I=3/2) in GaAs QD

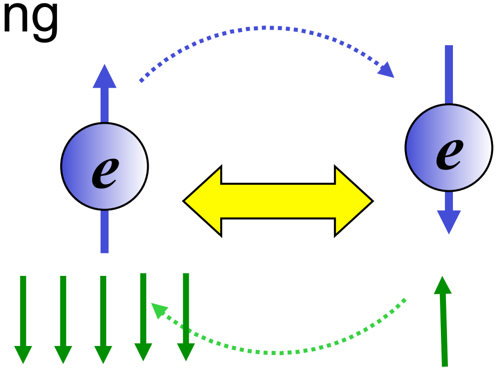


N=10^5 to 10^6 nuclei in GaAs QD

Flip-flop

$$H_{HF} = A|\psi(x)|^2 \left(\frac{I_+S_- + I_-S_+}{2} + I_Z S_Z \right)$$

Nuclear spins are dynamically polarized because of the long lifetime (~min.).



Overhauser shift (ΔE_{Zeeman})

…Shift of ESR condition

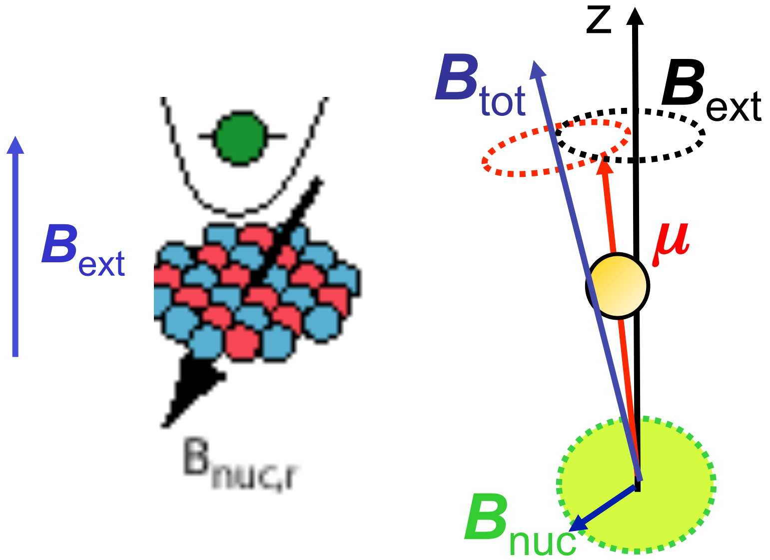
Usually very weak for ESR because of the large difference in the Zeeman energy

Fal'ko *et al.* J. Phys: Condense Matter 91; Khaetskii *et al.* *PRL* 02; Erlingsson *et al.* PRB 01

But can influence the ensemble measurement of ESR and Rabi

Decoherence by a nuclear spin bath

Electron Zeeman states in the statistically fluctuating nuclear spin bath



$\boldsymbol{B}_{ext}$=a few T >> $\boldsymbol{B}_{nuc}=A/\sqrt{N}$ ~ a few mT (~30 MHz)

B^{z}_{nuc} // B_{ext}

Fluctuate Larmor frequency by B^{z}_{nuc}/B_{ext} (~0.1 %) but very slowly (~sec)

…. Influence of B^{xy}_{nuc} is small, because

$\Delta B^{xy} \sim (B_{ext}^{2} + B^{xy}_{nuc}{}^{2})^{1/2} - B_{ext} \sim 0.0001\ \%$

⇨ *Inhomogeneous broadening of ESR condition*

Phase fluctuation (or dephasing) in the ensemble measurement → T_2^*=10 to 30 ns

Bracker *et al. PRL* 04; Petta *et al. Science* 05; Koppens *et al. Nature* 05 Pioro-Ladriere, *et al..Nature Physics* 08, Tokura, Nature Physics 09.

⇨ *Decoherence mechanism by electron spin mediated spin diffusion*

T_2~> 1μs

L. Cywinski, *et al.*, Phys. Rev. B 79, 245314 (2009).
W. A. Coish, *et al.*, Phys. Rev. B 81, 165316 (2010).

Summary

Spin qubits with quantum dots

Isolation of single electron spin in each quantum dots

Exchange control

Electrical modulation of exchange energy available for SWAP

Exchange control

EDSR with Spin-orbit+micromagnet useful for multiple qubits

Decoherence problem

Spin-orbit interaction
Hyperfine coupling with nuclear spin bath