FIRST Quantum Information Processing Project
Summer School 2010

25 August 2010 Okinawa

Quantum Simulation of Hubbard Model Using Ultracold Atoms in an Optical Lattice

Kyoto University Y. Takahashi

Introduction(自己紹介)

Name(氏名)　:

Yoshiro Takahashi(高橋義朗)

Education(学歴):

Ohta High-School(群馬県太田高校）

Kyoto University,　Faculty of Science (京都大学理学部)

Kyoto University,　Graduate School of Science

(京都大学大学院理学研究科)

Degree(学位）:

Anomalous Behavior of Raman Heterodyne Signal in Pr^{3+}:LaF_3

Employment（職歴）:

Kyoto University,

Research Associate(助手):Atoms in Superfluid Helium

Lecturer(講師):Photo-excited triplet DNP

Associate Professor(助教授）:Laser Cooling

Professor（教授）

Introduction(自己紹介)

Research Topics:

Quantum Information Science Using Cold Atoms

Quantum Simulation (of Hubbard Model)

Spin Squeezing by QND Measurement

Fundamental Physics Using Cold Atoms:

(Searching for Permanent Electric Dipole Moment)

Test of Newton Gravity:

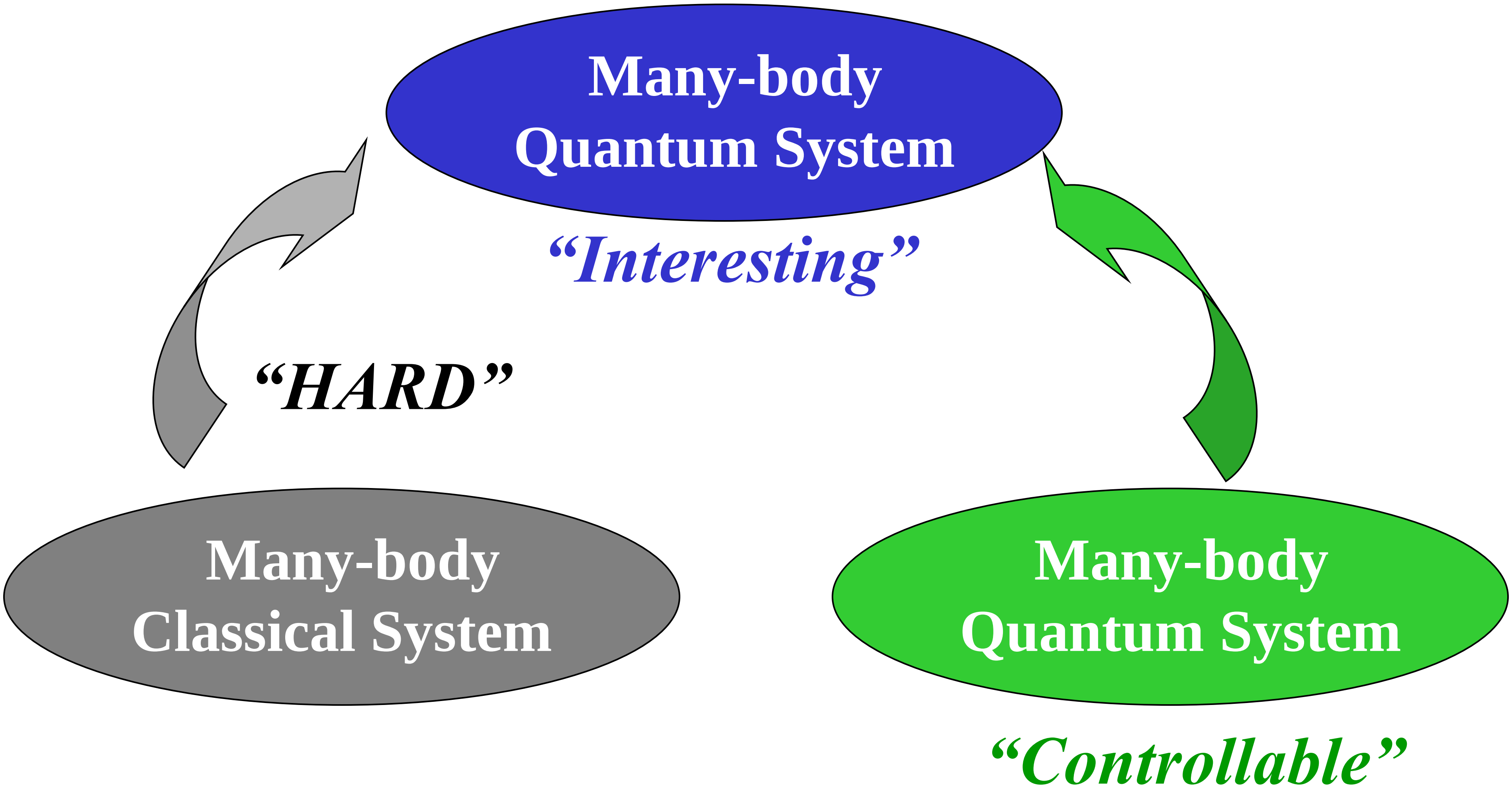
$$V = -G\frac{M_1 M_2}{r}(1+\alpha\exp(-\frac{r}{\lambda}))$$

Solid-State System

Quantum Computation ↓ ↑ **Quantum Simulation**

Atomic System

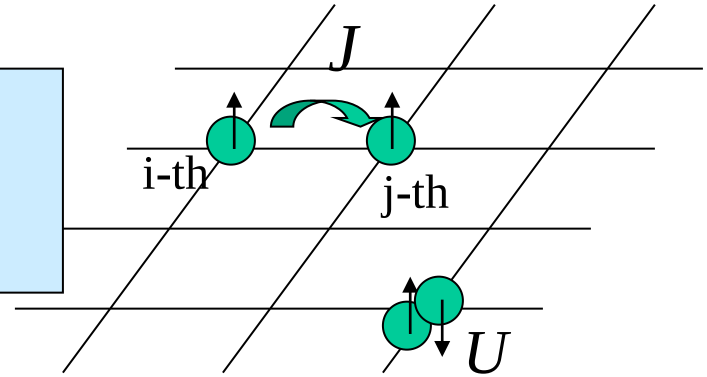
Quantum Simulation



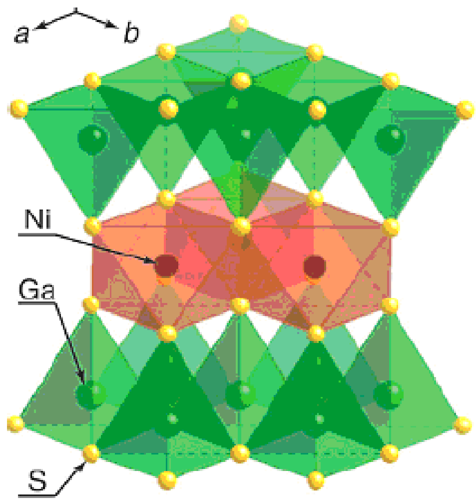
Quantum Simulation

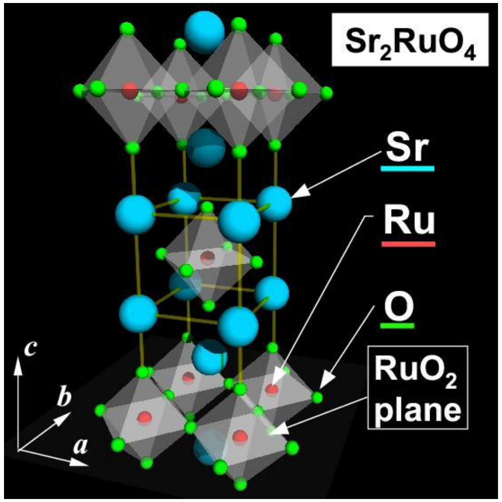
Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Magnetism, Superconductivity

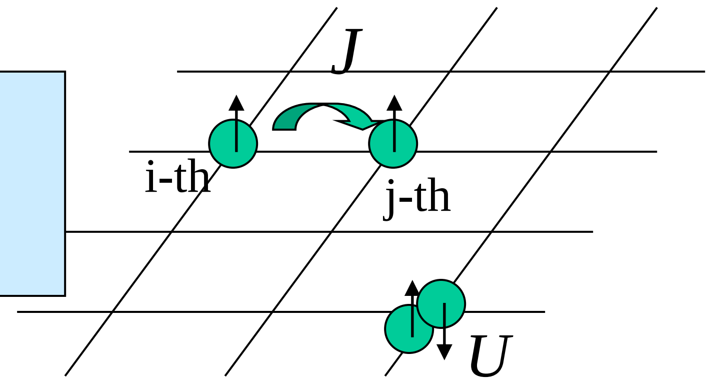




Quantum Simulation

Hubbard Model:

$$H = -J\sum_{<i,j>} c_i^+ c_j + U\sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Numerical Calculation

DMFT(動的平均場)
Gutzwiller
QMC(量子モンテカルロ)
DMRG(密度行列繰り込み群)
Exact Diagonalization (厳密対角化)

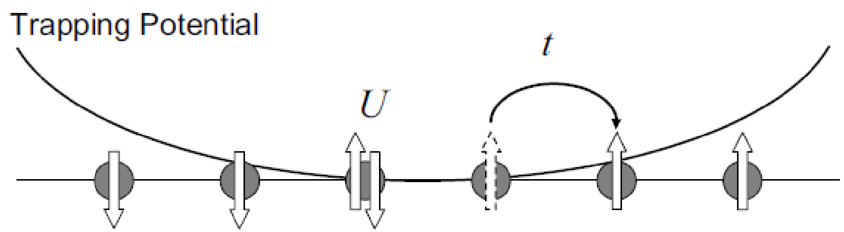
Quantum Simulation

Exact Diagonalization of Hubbard Model

S. Yamada, T. Imamura, M. Machida
Proceedings of the 2005 ACM/IEEE SC05 Conference(SC'05)

Earth Simulator:

1D Fermi Hubbard Model:
- Quarter Filling: 24 sits
- Half Filling: 20 sites



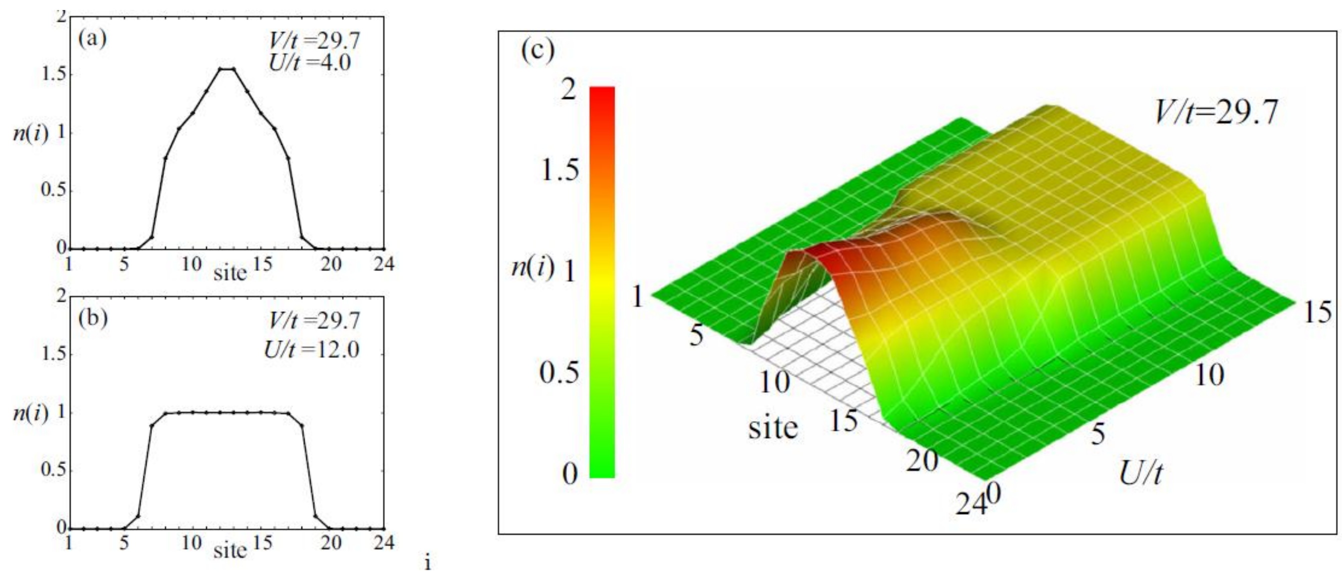


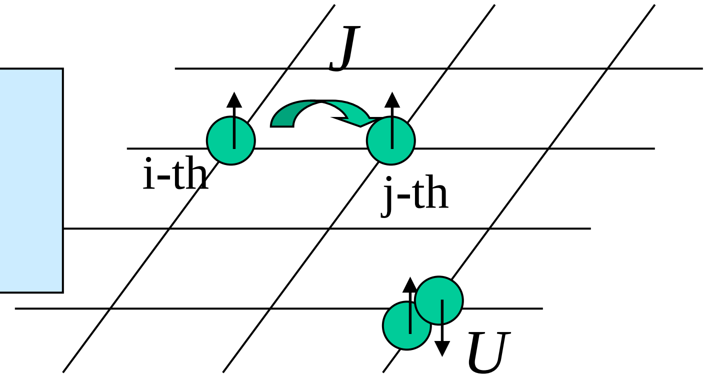
Figure 6: Particle density profile (a) $U/t = 4$, (b) $U/t = 12$, and (c) $0 \leq U/t \leq 15$ for 12 fermions$(6\uparrow, 6\downarrow)$ systems in 24-site Hubbard model with the trapped potential $(V/t = 29.7)$.

Next generation:
- Quarter Filling: 32 sites
- Half Filling: 26 sites

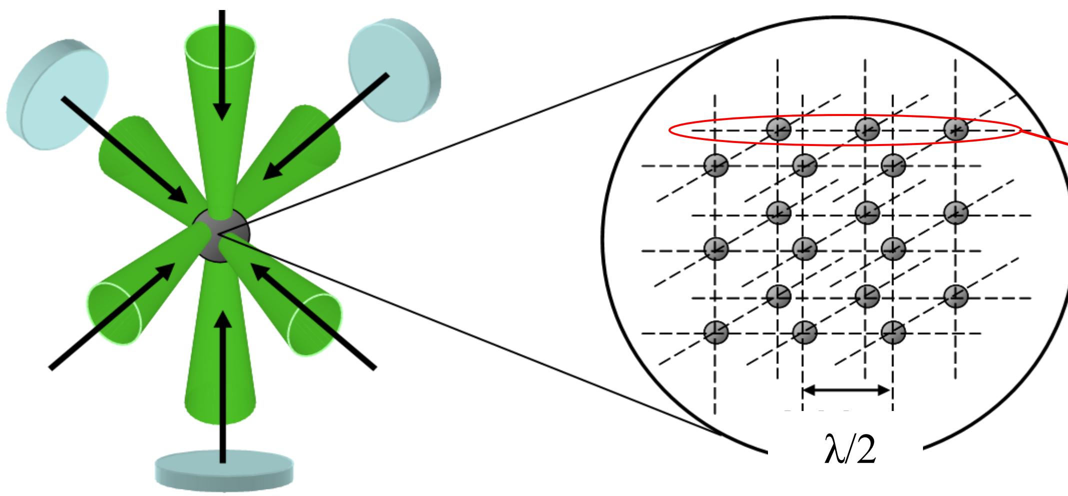
Quantum Simulation

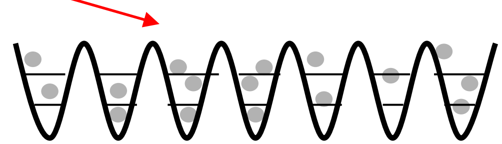
Hubbard Model:

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



→ Cold Atoms in Optical Lattice



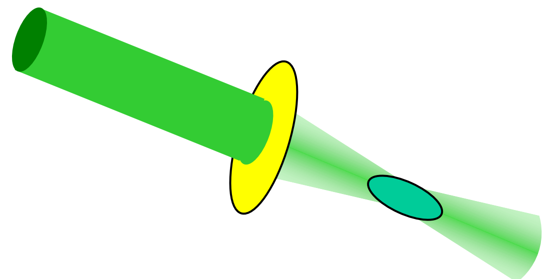


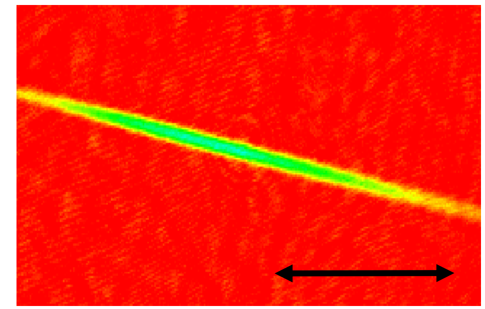
$$V = V_o \sin^2(kx)$$

Optical Trapping and Optical Lattice

"Optical Trap"

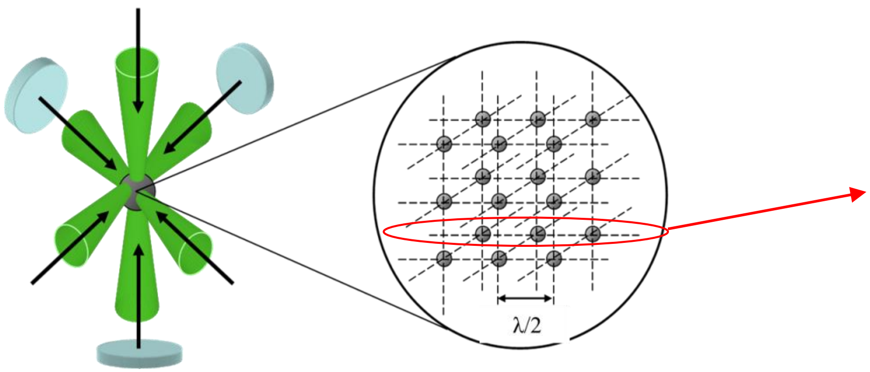
$$U = -\frac{1}{2}\alpha E^2 \approx -\hbar \frac{\Gamma^2}{8\Delta}\frac{I}{I_{sat}}$$



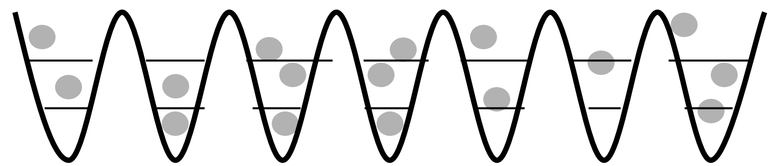


500 μm

"Optical Lattice"



$$V_o(x) = V_o \sin^2(k_L x)$$



$$V_o(\mathbf{x}) = \sum_{j=1}^{3} V_{oj} \sin^2(k_L x_j) = V_o \sum_{j=1}^{3} \sin^2(k_L x_j)$$

$$E_R = \frac{(\hbar k_L)^2}{2m}, s = \frac{V_0}{E_R}$$

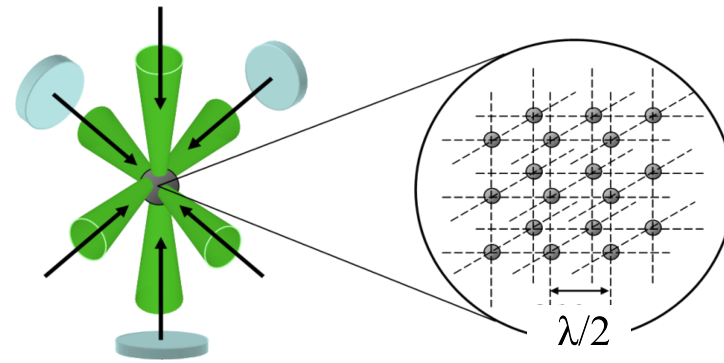
"to prevent mutual interference, frequency is shifted relative to each other by tens of MHz"

$$\omega_x \neq \omega_y \neq \omega_z \neq \omega_x$$

Quantum Simulation of Hubbard Model using "Cold Atoms in Optical Lattice"

[D. Jaksch *et al.*, PRL, **81** , 3108(1998)]

$$H = -J \sum_{<i,j>} c_i^{+} c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



$$J = E_R (2/\sqrt{\pi}) s^{3/4} \exp(-2\sqrt{s})$$

$$U = E_R a_s k_L \sqrt{8/\pi} s^{3/4}$$

$s \equiv V_o / E_R$, $E_R \equiv (\hbar k_L)^2 / 2m$, a_s : scattering length

Controllable Parameters

hopping between lattice sites	: J	lattice potential	: V_0
On-site interaction	: U	Feshbach Resonance	: a_s
filling factor (e- or h-doping)	: n	atom density	: n

Various geometry

Atomic Scattering Theory

$$\psi_{SC} = \exp(ikz) = \exp(ikr\cos\theta) = \sum_{l=0}^{\infty}(2l+1)i^l j_l(kr)P_l(\cos\theta)$$

$$R \to \infty \quad \approx \sum_{l=0}^{\infty}(2l+1)i^l P_l(\cos\theta)\frac{\sin(kr - l\pi/2)}{kr}$$

$$= \sum_{l=0}^{\infty}(2l+1)i^l \frac{P_l(\cos\theta)}{2ikr}\left[\exp(i(kr - \frac{l}{2}\pi)) - \exp(-i(kr - \frac{l}{2}\pi))\right]$$

"out-going" "in-coming"

With atom-atom interaction

$$\psi_{SC} \approx \exp(ikz) + \frac{f}{r}\exp(ikr)$$

f :scattering amplitude

$$\psi_{SC} \approx \sum_{l=0}^{\infty}(2l+1)i^l P_l(\cos\theta)\frac{\sin(kr - l\pi/2 + \delta_l)}{kr}$$

δ_l :phase shift

S_{00}

$$= \exp(-i\delta_l)\sum_{l=0}^{\infty}(2l+1)i^l \frac{P_l(\cos\theta)}{2ikr}\left[\exp(i(kr - \frac{l}{2}\pi))\exp(+2i\delta_l) - \exp(-i(kr - \frac{l}{2}\pi))\right]$$

Atomic Scattering Theory

$$f = \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) f_l \quad \text{with} \quad f_l = \frac{\exp(+2i\delta_l) - 1}{2ik} = \exp(+i\delta_l) \frac{\sin(\delta_l)}{k}$$

$$f_l = \frac{S_{00} - 1}{2ik}$$

$$\sigma_l = 4\pi(2l+1)|f_l|^2 = \frac{4\pi(2l+1)}{k^2} \sin^2(\delta_l)$$

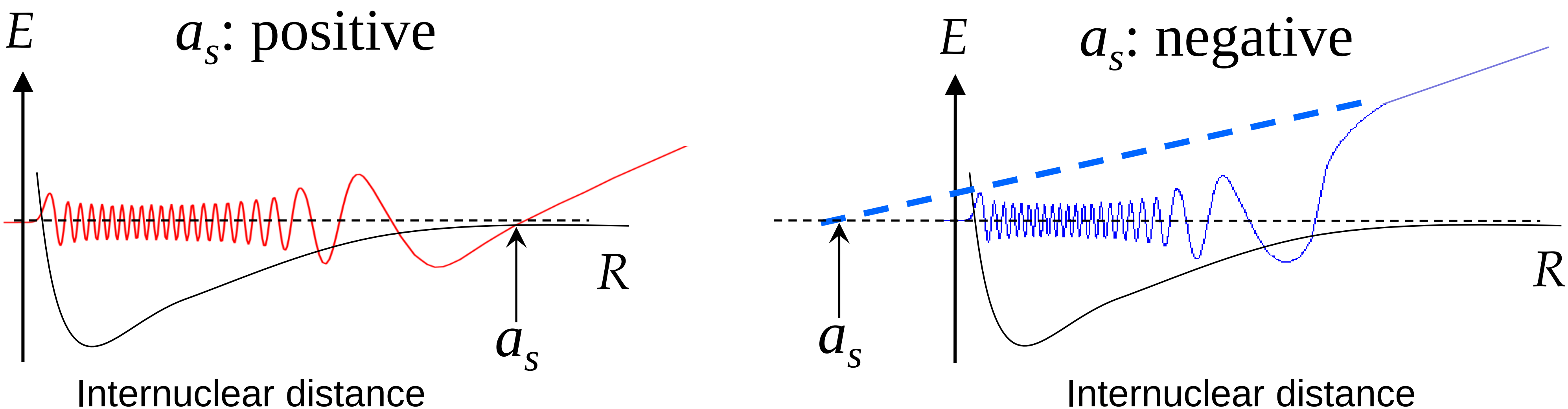
"At low temperature, only s-wave (l=0) scattering is important"

$$a_s \underset{k \to 0}{=} -\frac{\delta_l}{k}$$

$$f_0 = -a_s \qquad \sigma_0 = 4\pi |f_0|^2 = 4\pi |a_s|^2$$

Q. What is *Scattering Length* ?

$$\psi_{SC}(R) \underset{R\to\infty}{\propto} \frac{\sin(kR+\delta_0)}{kR} = \frac{\sin(k(R-a_s))}{kR}$$



$$V_{\text{int}} = \frac{4\pi\hbar^2 a_s}{m}\delta(\vec{r}_1 - \vec{r}_2)$$

$$U = \frac{4\pi a_s \hbar^2}{m}\int d^3x \left|w(x - x^{(i)})\right|^4$$

Analytical Expression of Scattering Length

$$V(R)\underset{R\to\infty}{\approx}-\frac{C_6}{R^6} \longrightarrow a_s=\bar{a}_s\times\left[1-\tan(\phi-\frac{\pi}{8})\right]$$

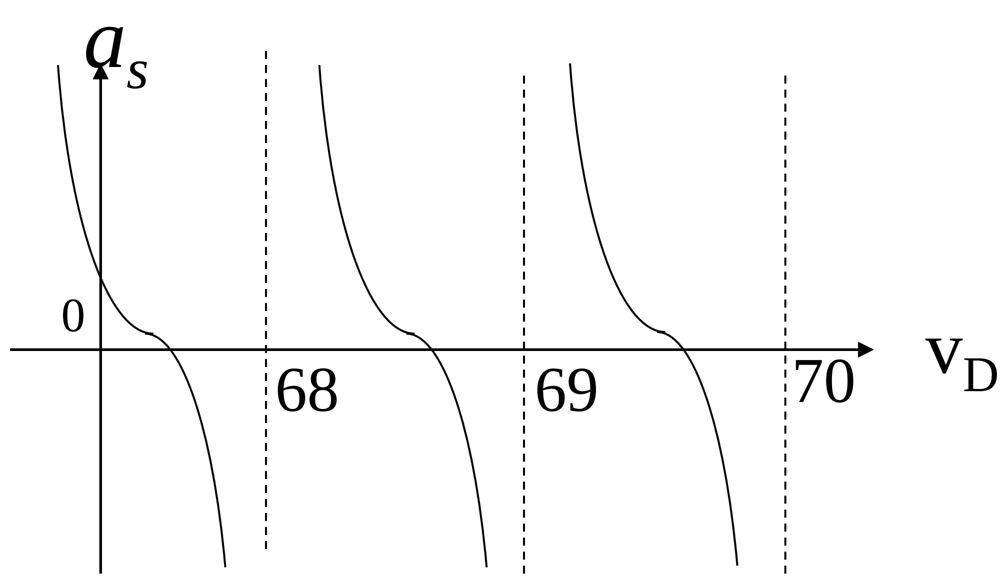
[Gribakin & Flambaum PRA, **48** 546(1993)]

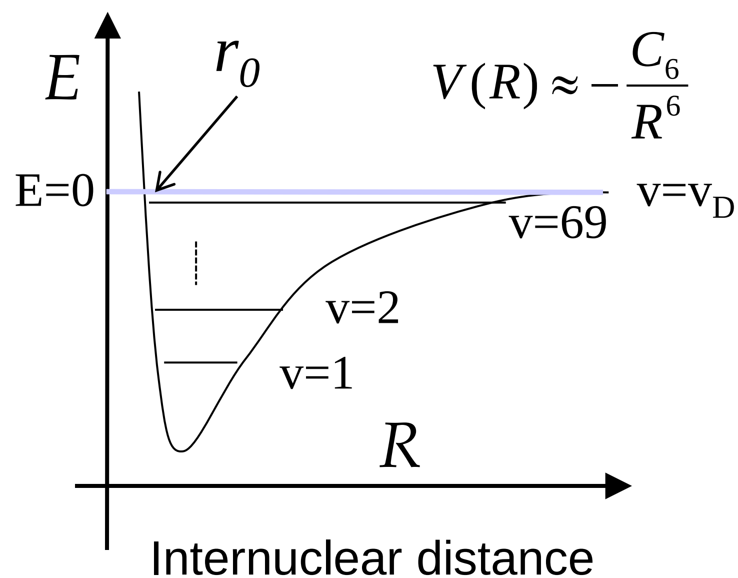
$$\bar{a}_s=\cos(\frac{\pi}{4})\left(\frac{\sqrt{2\mu C_6}}{4\hbar}\right)^{1/2}\left[\frac{\Gamma(3/4)}{\Gamma(5/4)}\right]$$

$$\phi=\frac{1}{\hbar}\int_{r_0}^{\infty}\sqrt{-2\mu V(R)}dR$$

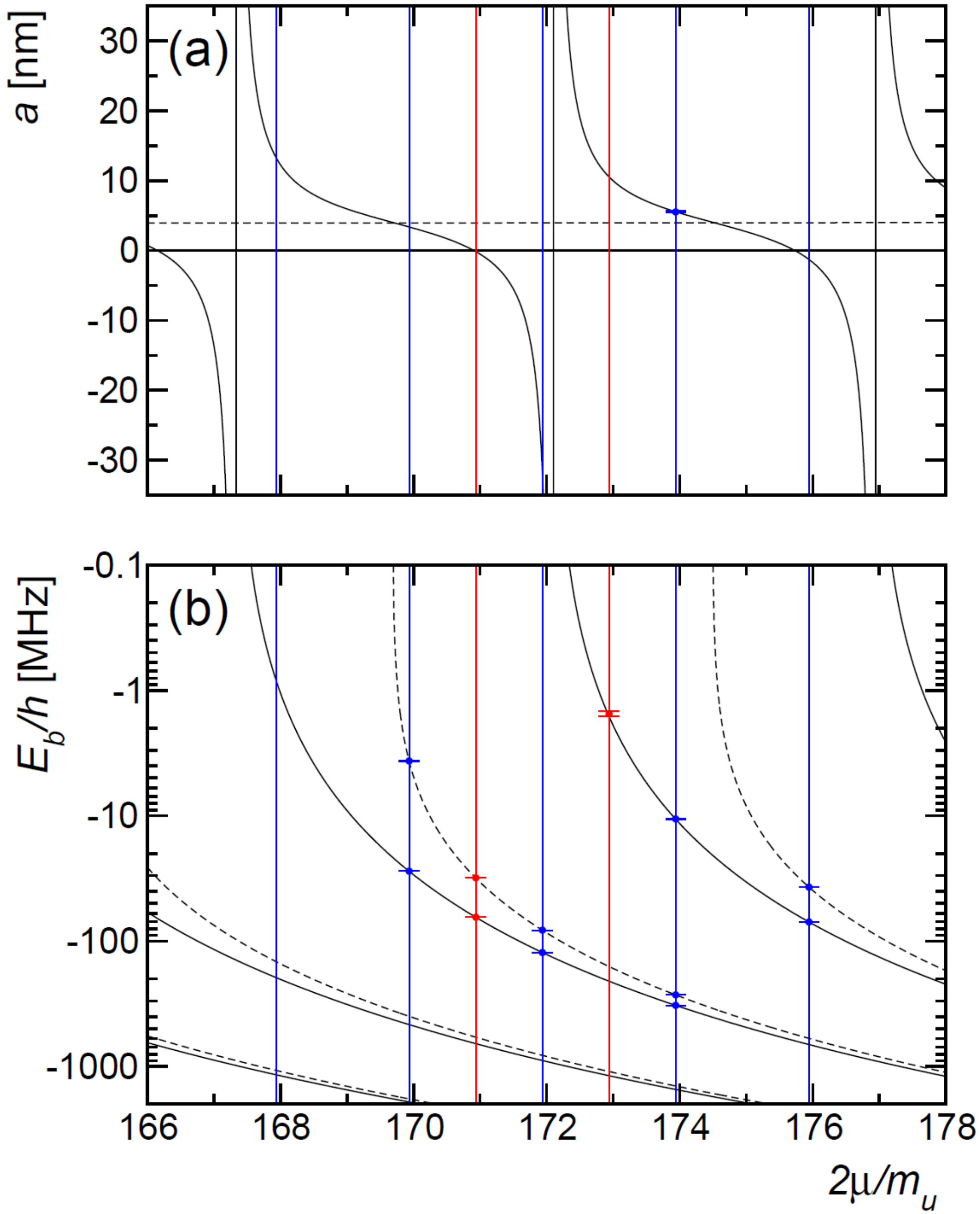
μ: Reduced mass

$$\phi-\frac{\pi}{8}=\pi(v_D+\frac{1}{2}) \longrightarrow a_s=\bar{a}_s\times\left[1-\tan(\pi(v_D+\frac{1}{2}))\right]$$





Binding Energy and Scattering Length : Case of Yb Atom



Lennard-Jones-like potential:

$$V(r)=\frac{C_{12}}{r^{12}}-\frac{C_8}{r^8}-\frac{C_6}{r^6}+\frac{\hbar^2 J(J+1)}{2\mu}\frac{1}{r^2}$$

$$\phi=\frac{1}{\hbar}\int_{r_0}^{\infty}\sqrt{-2\mu V(R)}dR$$

[M. Kitagawa, *et al*, PRA**77**, 012719 (2008)]

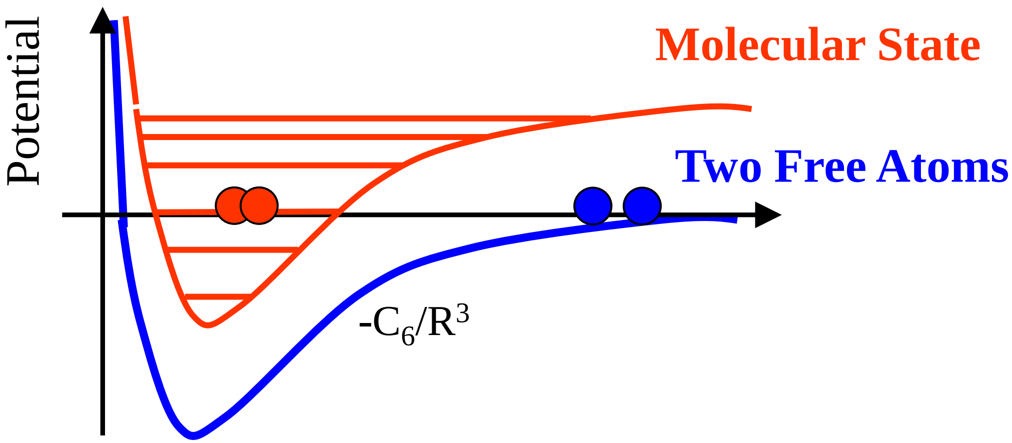
Q. What is *Feshbach Resonance* ?

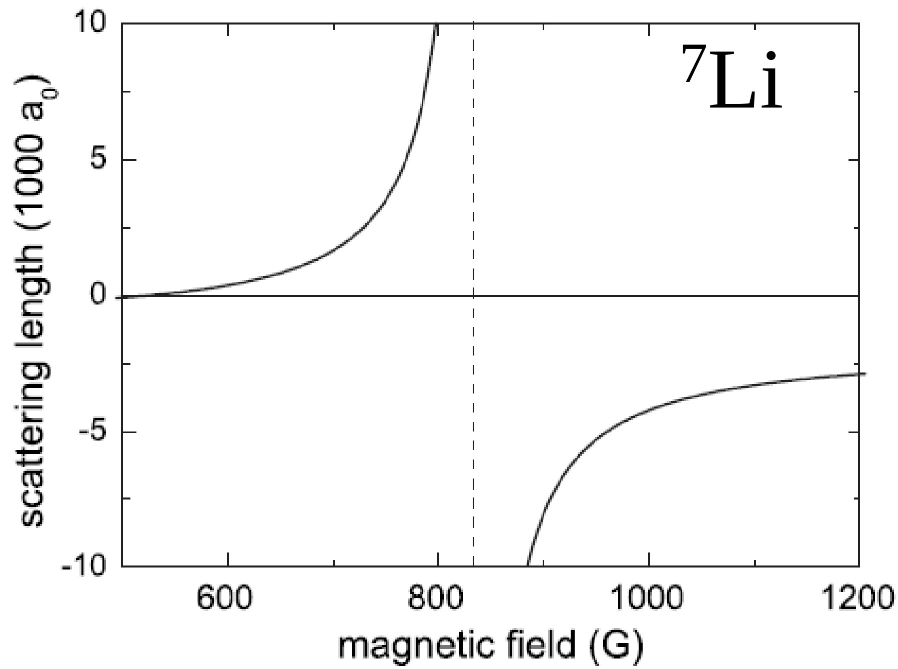
Coupling between "**Open Channel**" and "**Closed Channel**"

→ Control of a_s

$$a_s(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

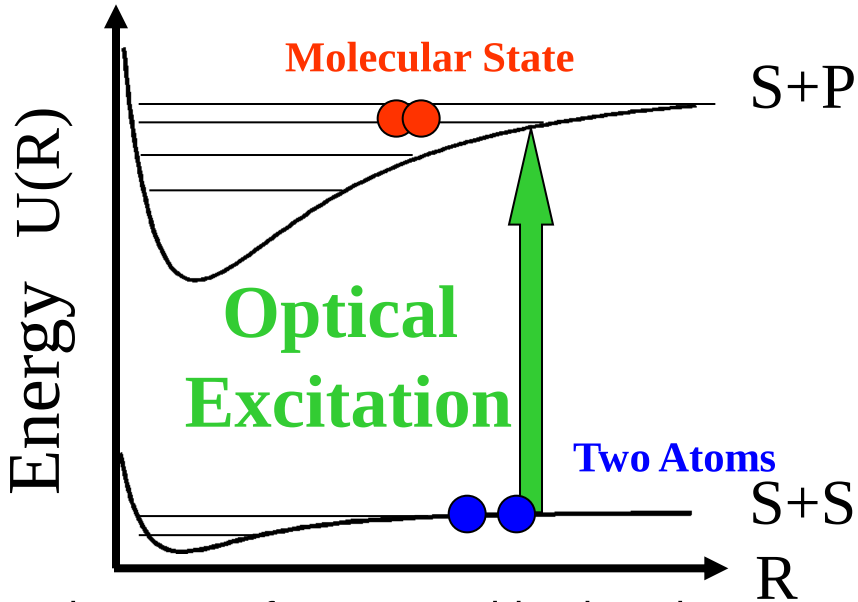
$$\Delta B = \frac{m(2\pi\hbar)^3}{4\pi\hbar^2 a_{bg}\mu_{res}} \left|\langle\phi_{res}|W|\phi_0^{(+)}\rangle\right|^2 \qquad B_0 = B_{res} - \langle\phi_{res}|W G_{bg}(0) W|\phi_{res}\rangle/\mu_{res}$$





[T. Kohler, K. Goral, P. S. Julienne, RMP **78**, 1311 (2006)]

Optical Feshbach Resonance



Advantages for Intercombination Lines

R. Ciurylo, *et al.* *Phys. Rev. A* **70**. 062710 (2004)

$$S_{00} = \frac{\Delta - i\Gamma_S/2 + i\gamma/2}{\Delta + i\Gamma_S/2 + i\gamma/2}$$

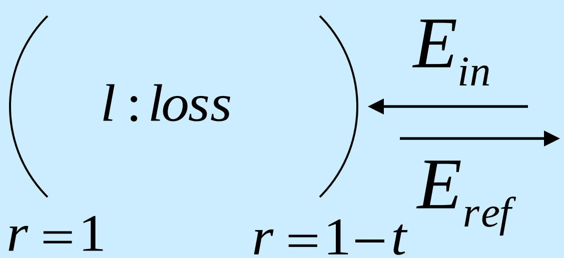
$\Gamma_S \propto \left|\langle b|V_{las}|f\rangle\right|^2$

γ :spontaneous decay rate

Δ :detuning from the PA resonance

[J. Bohn and P. Julienne PRA(1999)]

$$K_{PA} = \frac{\pi}{k}\left(1 - |S_{00}|^2\right) = \frac{\pi}{k}\frac{\Gamma_S\gamma}{\Delta^2 + (\Gamma_S/2 + \gamma/2)^2} \rightarrow 0 \ \text{for} \ \Gamma_S/\gamma >> 1$$



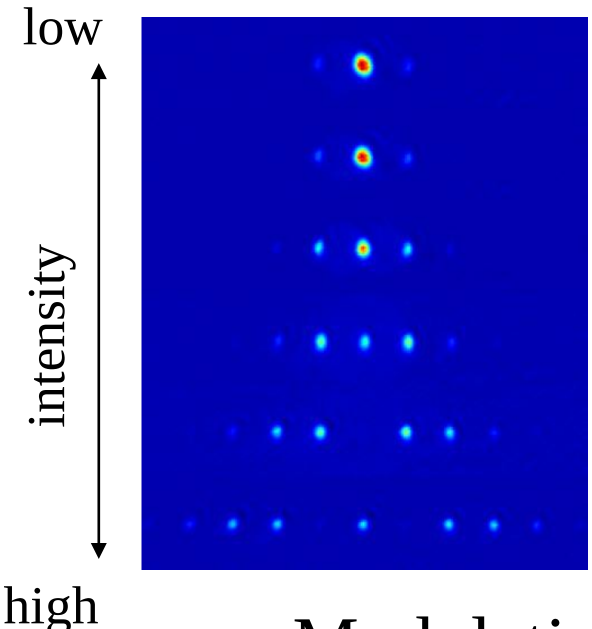
$$\frac{E_{ref}}{E_{in}} \equiv S = \frac{\Delta - i\Gamma + i\gamma}{\Delta + i\Gamma + i\gamma}$$

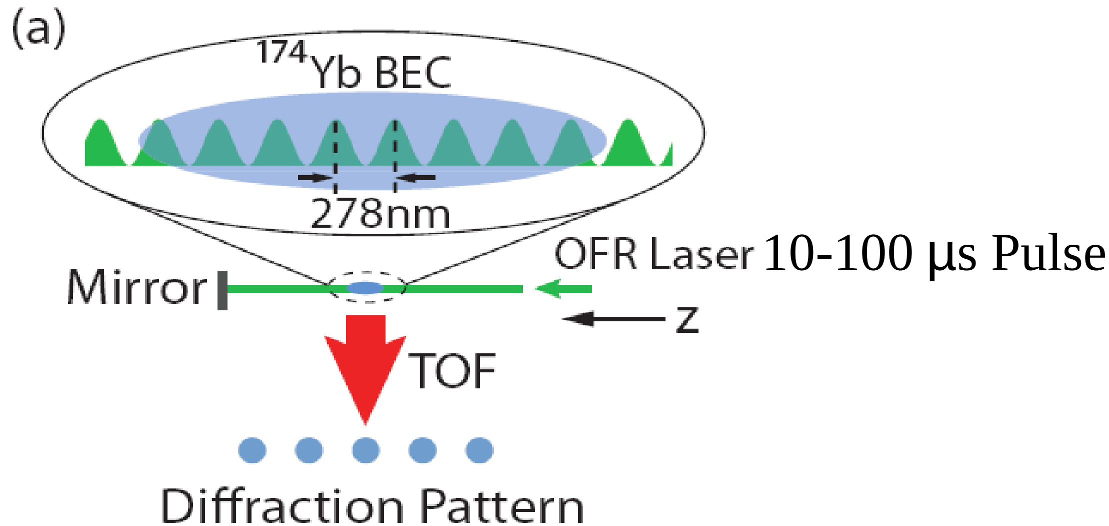
$$\Gamma = t \times c/(2L)$$

$$\gamma = l \times c/(2L)$$

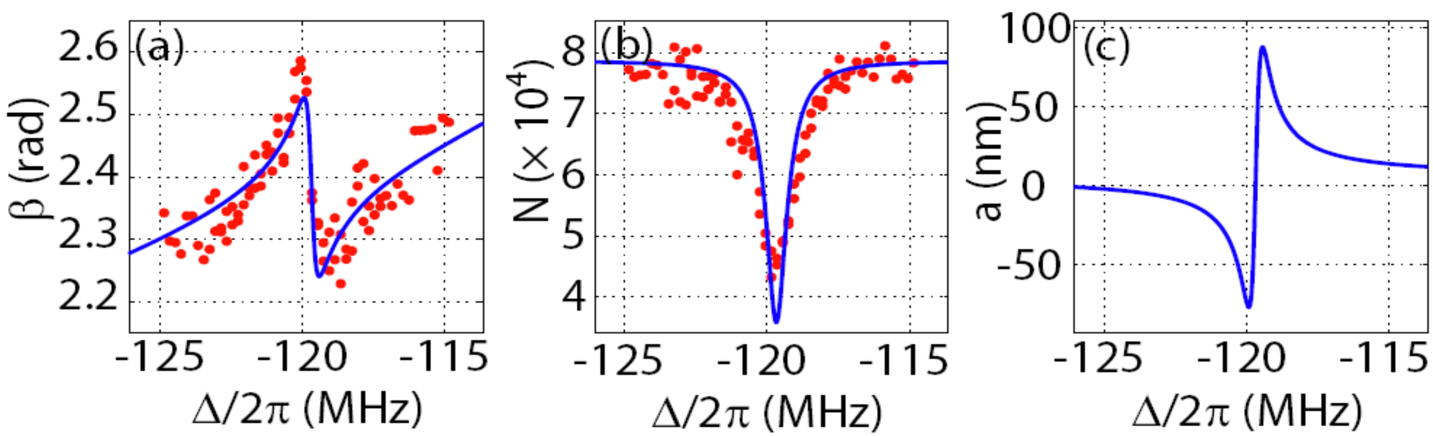
Nanometer-scale Spatial Modulation of an Inter-atomic Interaction

[R. Yamazaki *et al* ., PRL**105**, 050405 (2010)]



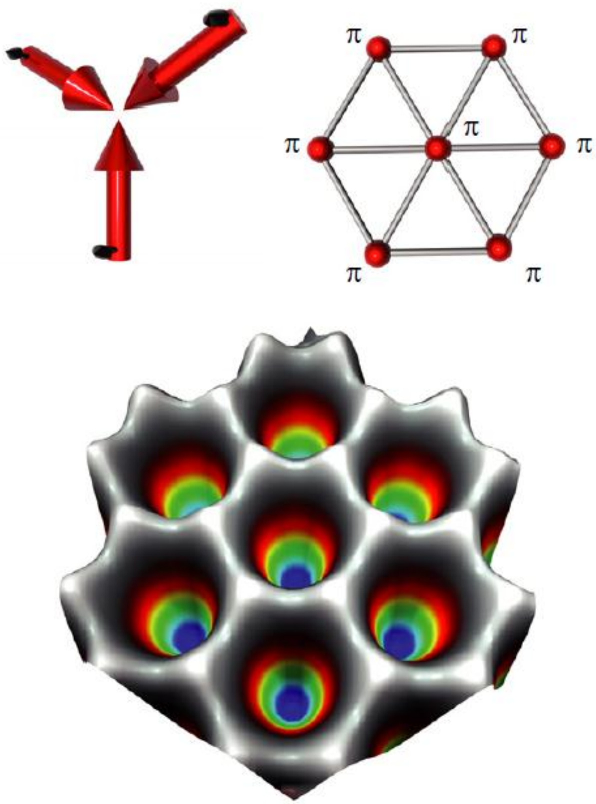


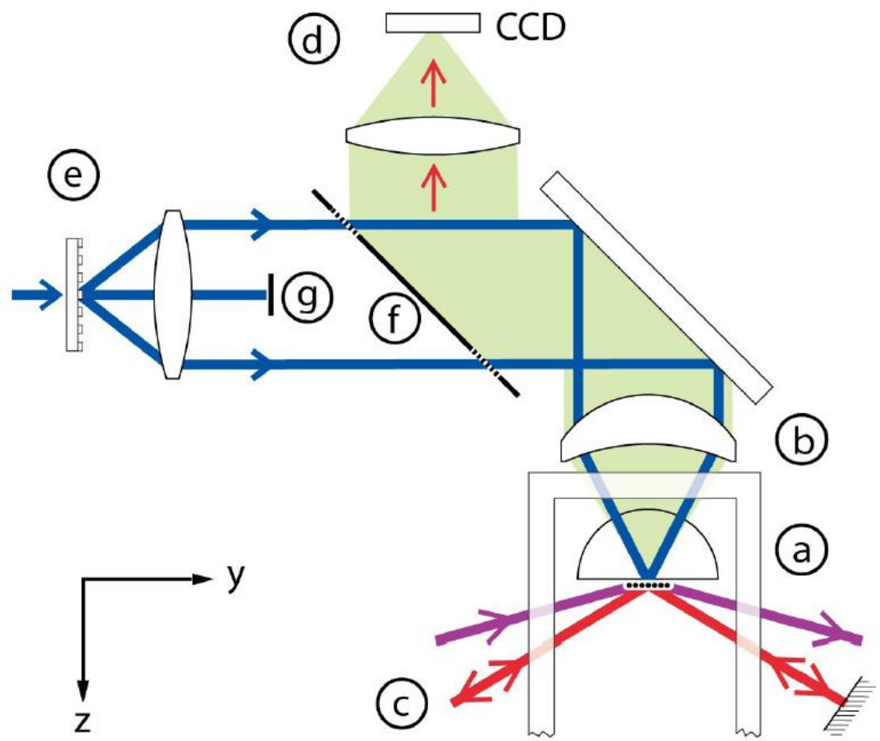
Modulation Index $\beta = \beta_{LS} + \beta_m$ $\quad \beta_{LS} = \frac{U_{LS}}{2}\tau \quad \beta_m = \frac{U_m}{2}\tau$



Q: How *Various Geometry* ?

An Example: Triangular Optical Lattice





[C. Becker *et al.*, New J. Phys. **12** 065025(2010)]

[M. Greiner Group)]

band structure

$$H_0 = -J \sum_{i,j,\sigma=\uparrow,\downarrow} c^{+}_{i,\sigma} c_{j,\sigma} \longrightarrow H_0 = \sum_{k,\sigma=\uparrow,\downarrow} c^{+}_{k,\sigma} c_{k,\sigma} \varepsilon(k)$$

,where

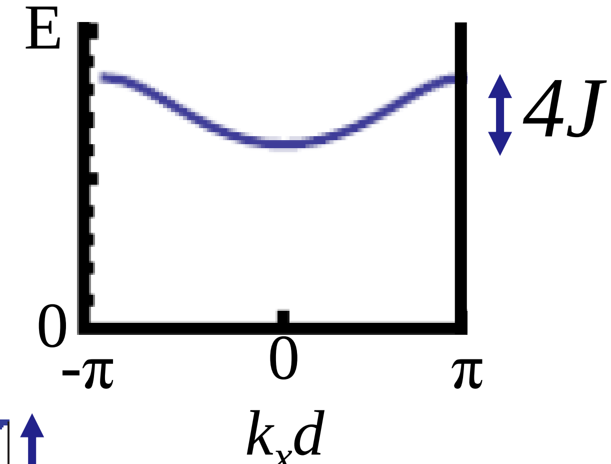
$$\varepsilon(k) = -J \sum_{<i,j>} \exp(-ik\cdot(x_i - x_j))$$

$c_{k,\sigma}$: annihilation operator of atom with spin σ for the wavevector k

1D case:

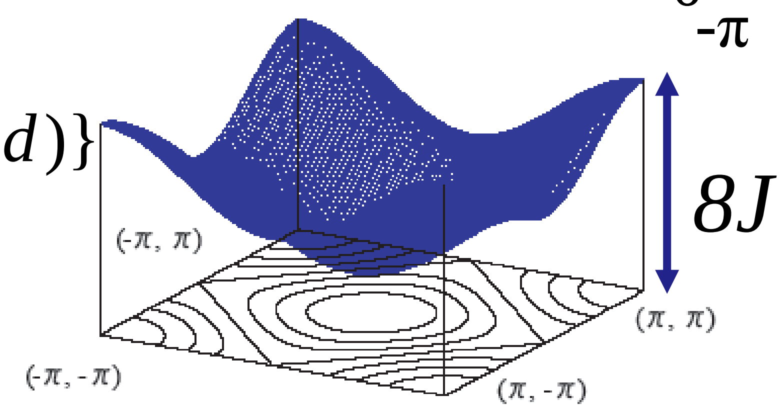
$$\varepsilon(k) = -J\{\exp(-ik_x d) + \exp(+ik_x d)\} = -2J\cos(k_x d)$$

(d :lattice constant)

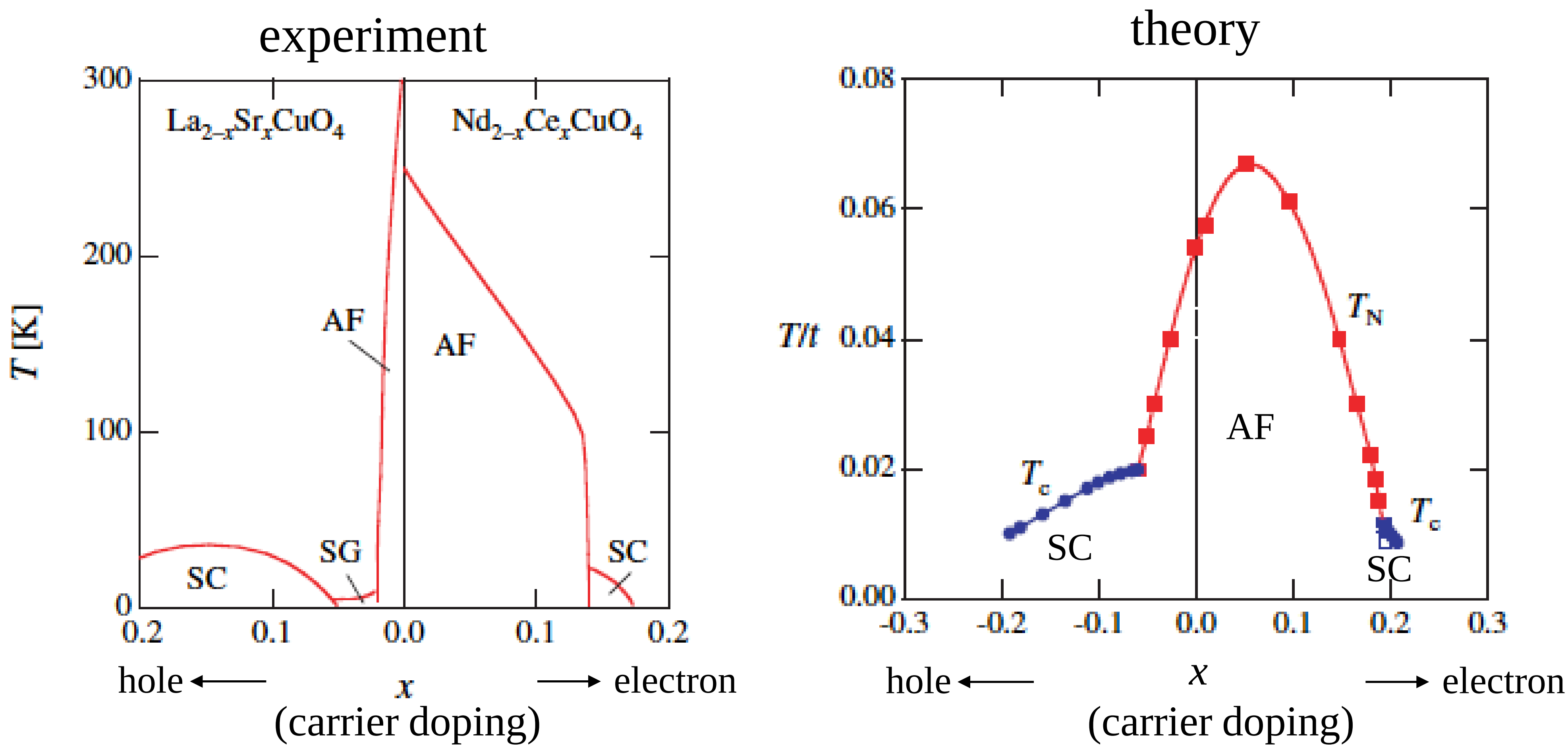


2D case:

$$\varepsilon(k) = -2J\{\cos(k_x d) + \cos(k_y d)\}$$



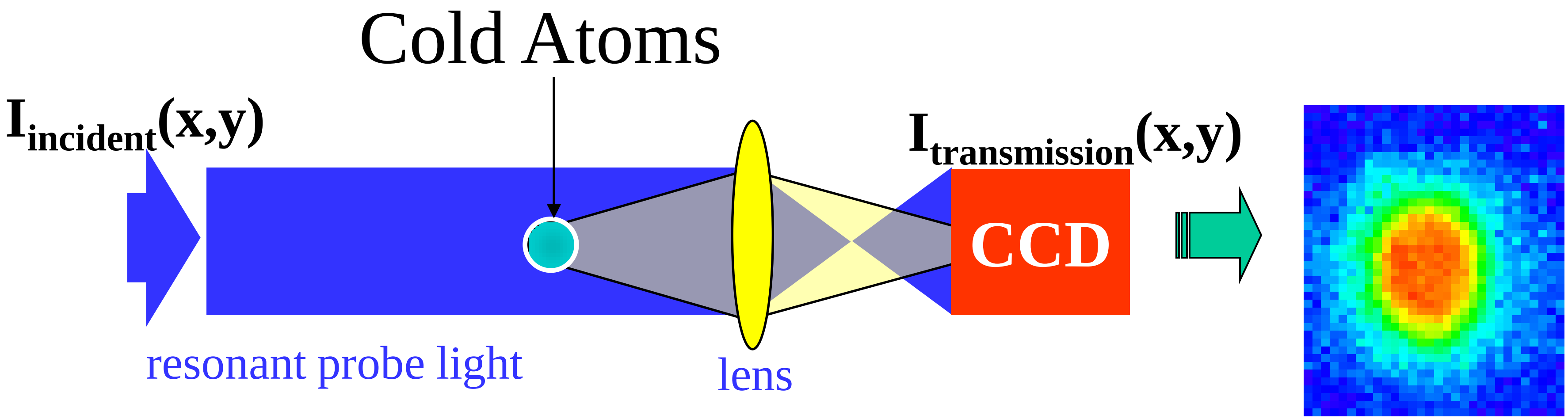
Phase Diagram of High-T_c Cuprate Superconductor



[in T. Moriya and K. Ueda, Rep. Prog.Phys.66(2003)1299]

There is controversy in the under-dope region

Optical Imaging



Time-of-Flight Image:

"The atom distribution after certain time from the sudden release of the atoms corresponds to **the momentum distribution**"

$$x = (P/M) \times t_{TOF}$$

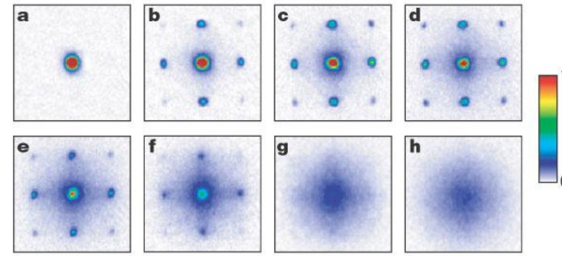
Quantum Simulators using Alkali Atoms

Bose-Hubbard Model:

"Superfluid - Mott-insulator Transition"

[M. Greiner, *et al*., Nature 415,39 (2002)]

…



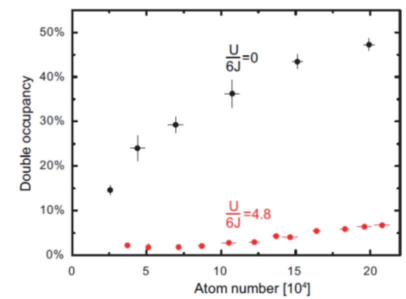
^{87}Rb

Fermi-Hubbard Model:

"Formation of Mott-insulator state"

[R. Jördens *et al.*, Nature **455**, 204 (2008)]

[U. Schneider, *et al.*, Science **322**,1520(2008)]



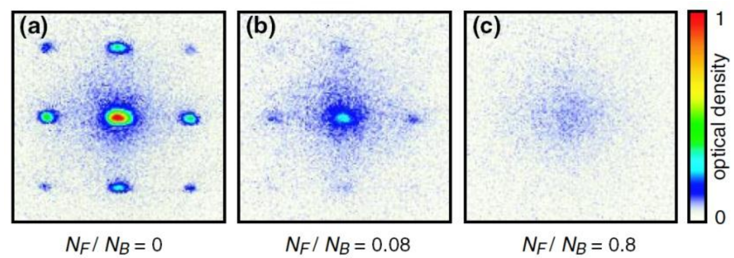
^{40}K

Bose-Fermi-Hubbard Model:

[K. Günter, et al, PRL**96**, 180402 (2006)]

[S. Ospelkaus, et al, PRL**96**, 180403 (2006)]

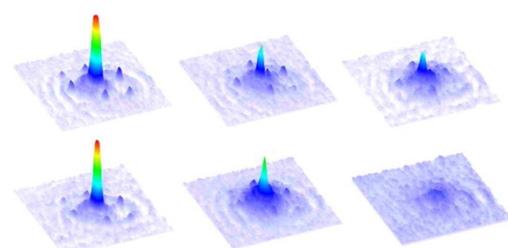
[Th. Best, *et al*, PRL**102**, 030408 (2008)]



^{87}Rb + ^{40}K

Bose-Bose-Hubbard Model:

[J. Catani, et al, PRA**77**, 011603(R) (2008)]

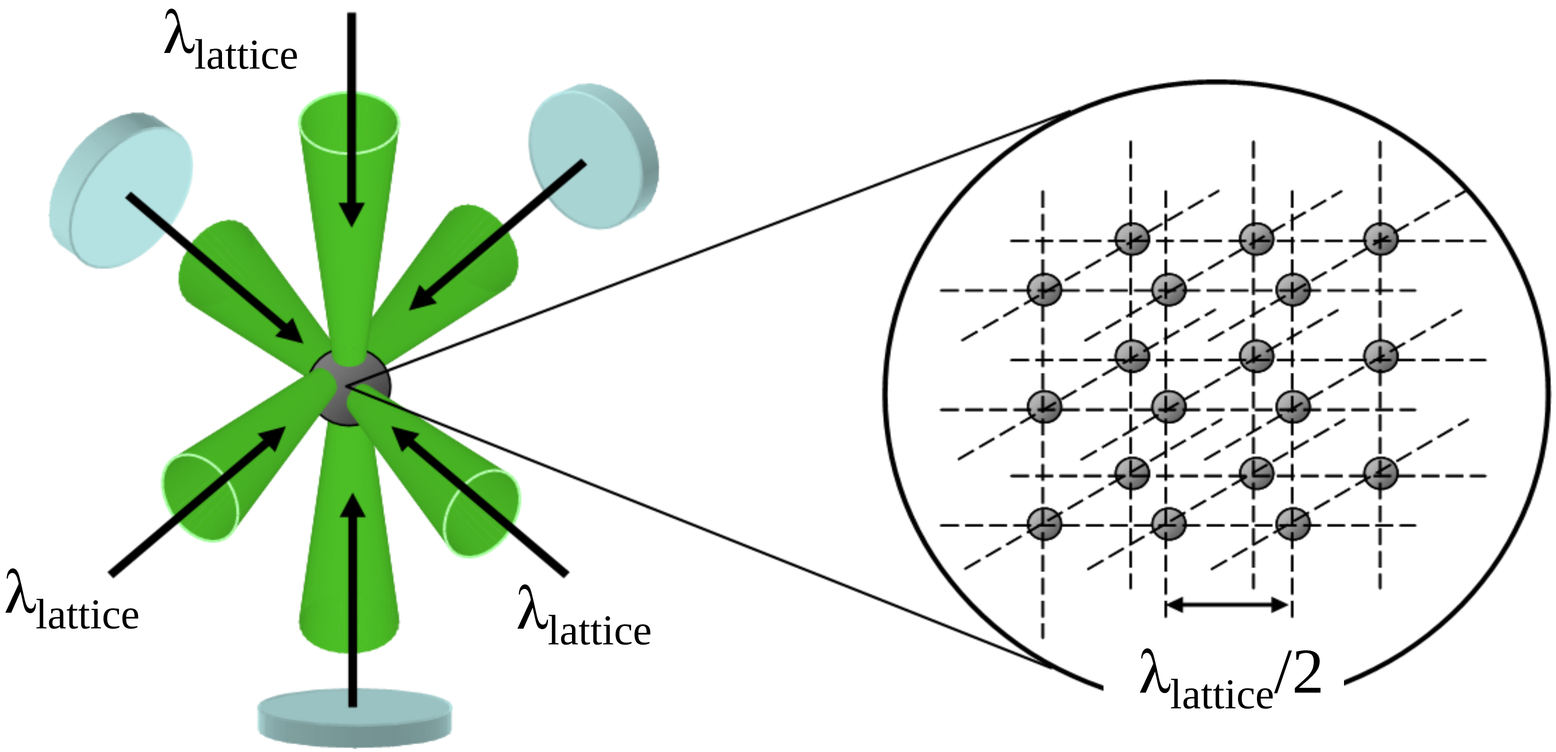


^{87}Rb + ^{41}K

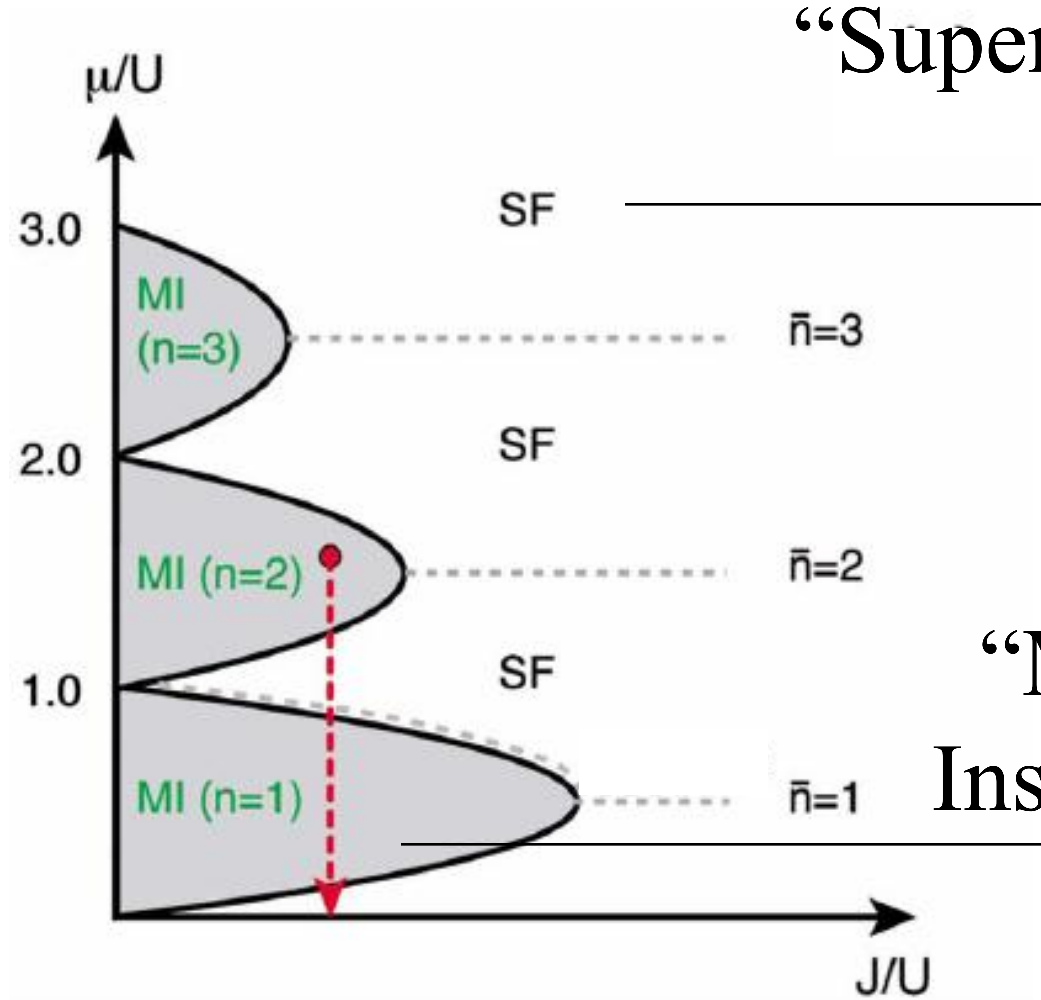
Bosons in a 3D optical lattice

$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$

"Bose-Hubbard Model"

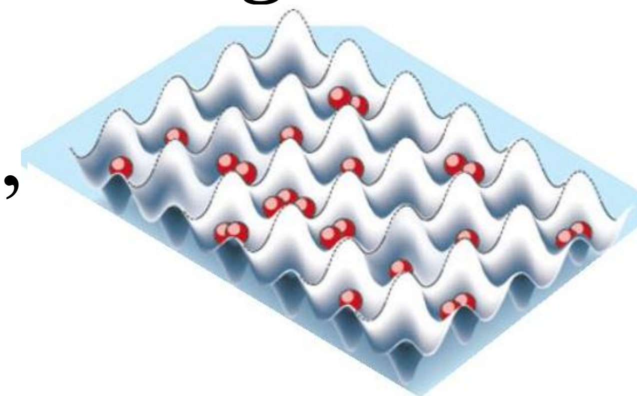


Phase Diagram of Repulsively Interacting Bosons



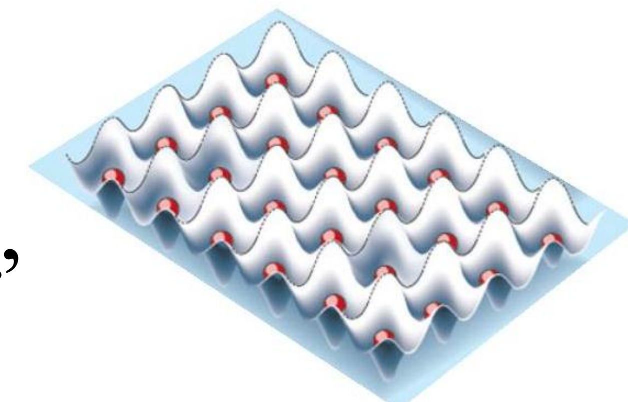
[RMP**80**,885(2008)]

“Superfluid”



$$\left|\Psi_{SF}\right\rangle \propto \left(\frac{1}{\sqrt{M}}\sum_{i=1}^{M} a_i^+\right)^N \left|0\right\rangle$$

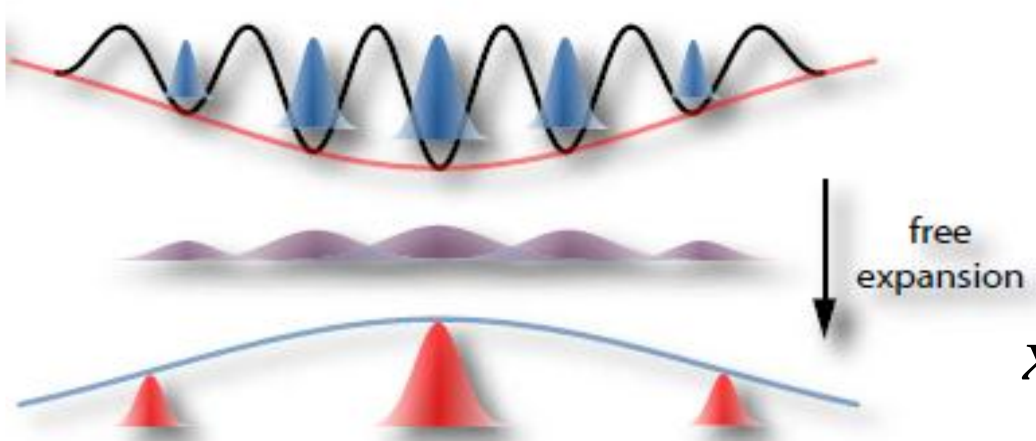
“Mott Insulator’



$$\left|\Psi_{MI}\right\rangle \propto \prod_{i=1}^{M}\left(a_i^+\right)^n \left|0\right\rangle$$

Interference Fringe : the direct signature of the phase coherence

"Sudden Release"



t_{TOF}

$x \leftrightarrow \hbar k$

$x = (\hbar k / M) t_{TOF}$

$$n(k) \propto \left|\tilde{w}(k)\right|^2 G(k)$$

Fourier Transform of the Wannier function

$$G(k) = \sum_{R,R'} \exp(ik \cdot (R - R')) \langle \hat{a}_R^+ \hat{a}_{R'} \rangle$$

no long-range order: $\langle \hat{a}_R^+ \hat{a}_{R'} \rangle = \delta_{R,R'} \rightarrow G(k) = N$

uniform long-range order: $\langle \hat{a}_R^+ \hat{a}_{R'} \rangle = 1 \rightarrow G(k) = \dfrac{\sin^2(kdN/2)}{\sin^2(kd/2)}$

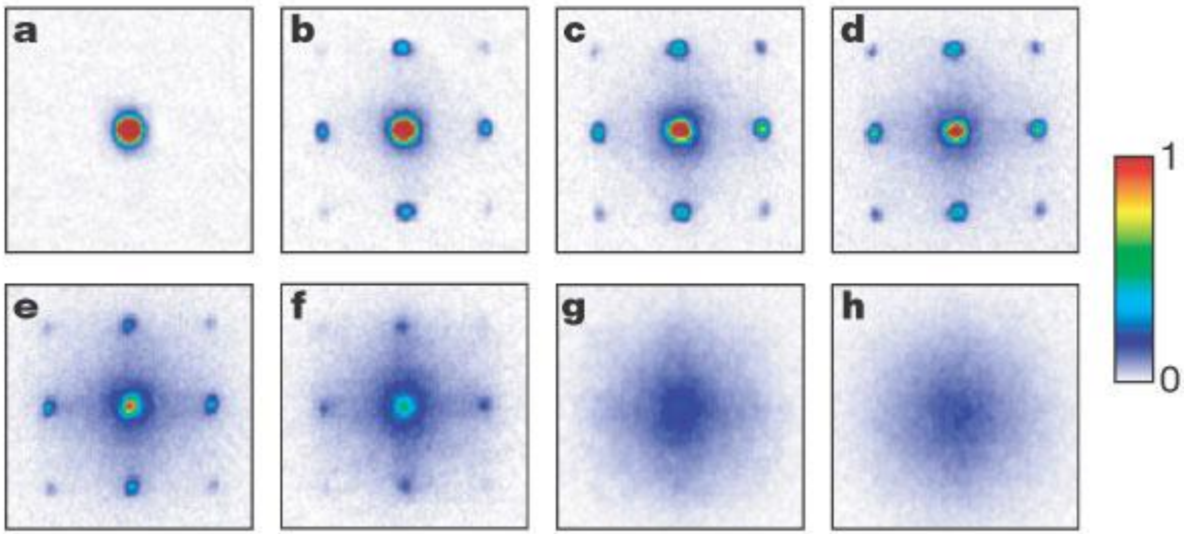
peaks at $\pm 2n\hbar k_L$ (n=0,1,2...)

Bose-Hubbard Model:
“Superfluid - Mott-insulator Transition”

[M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415,39 (2002)]

No lattice $V_0 / E_R = 3$ 7 10

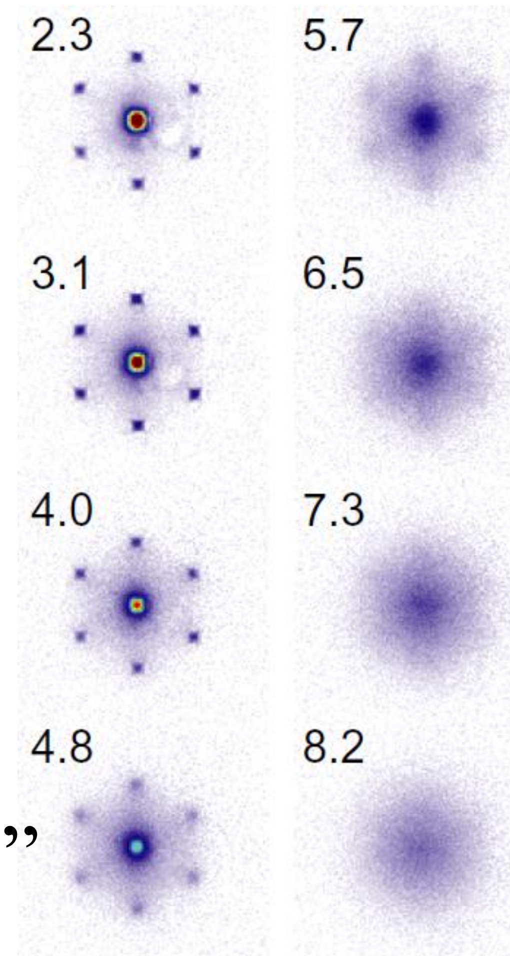
^{87}Rb



13 14 16 20

“cubic lattice”

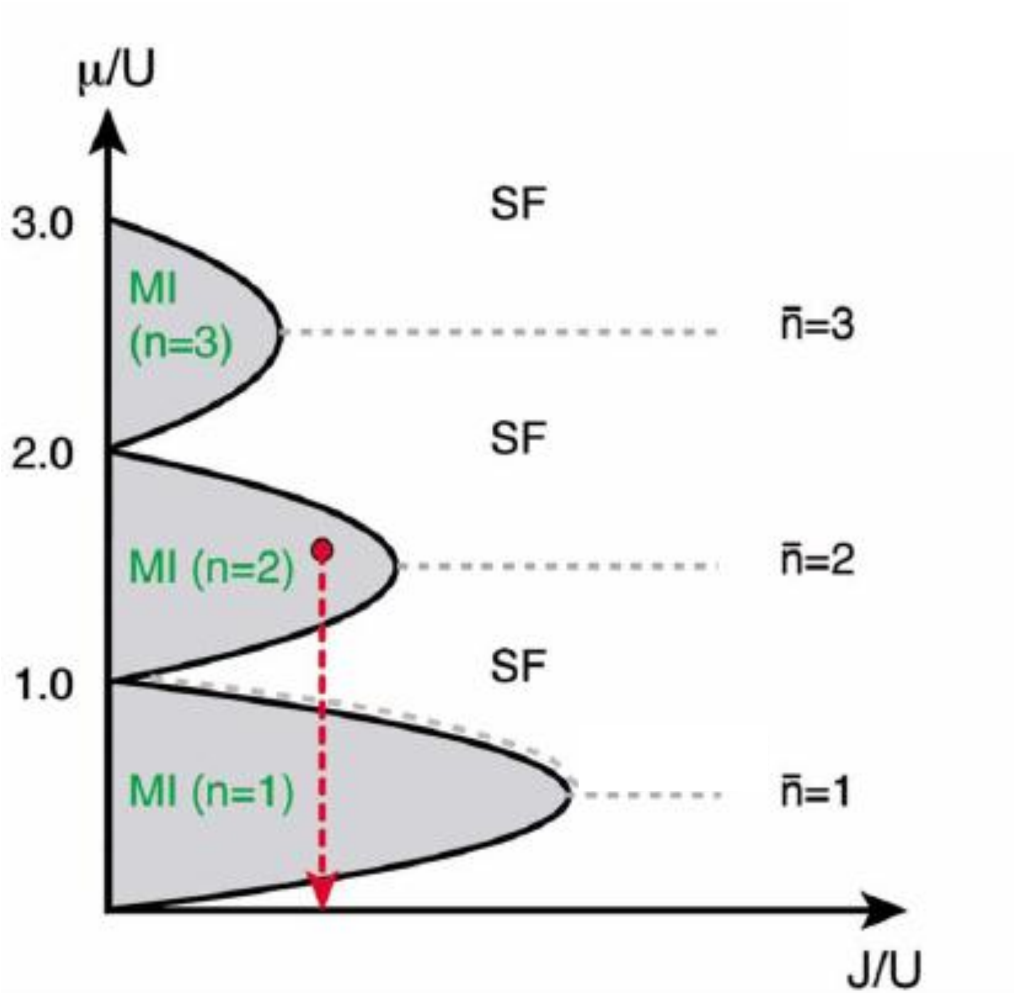
$s=$



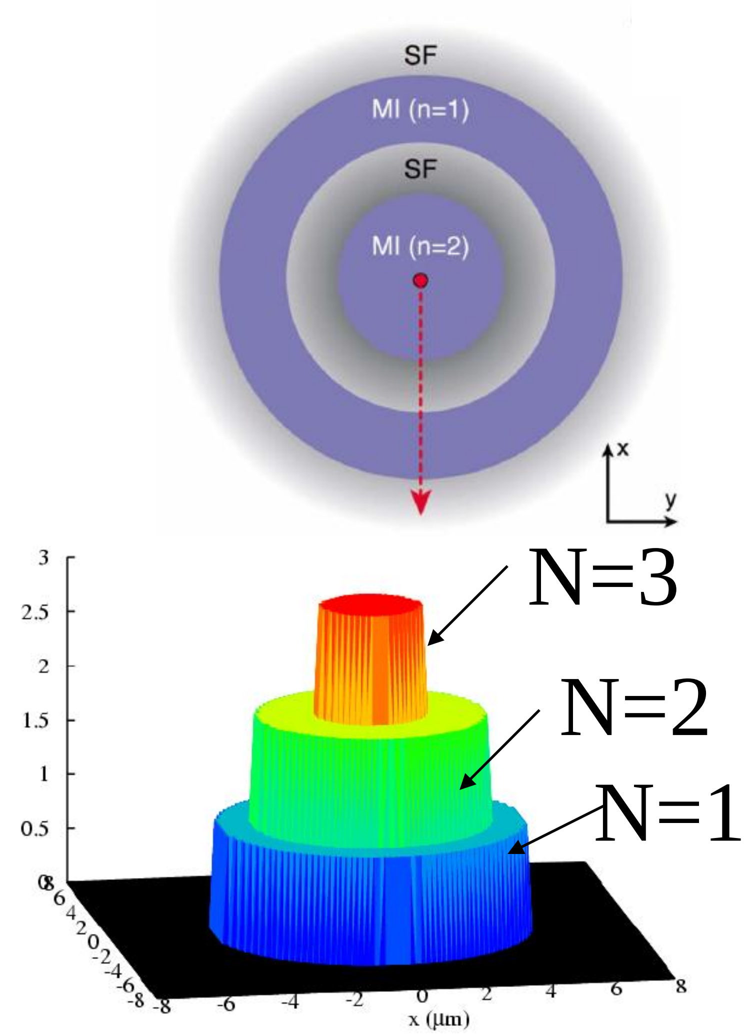
“triangular lattice”

[C. Becker *et al.*, New J. Phys. **12** 065025(2010)]

Phase Diagram of Repulsively Interacting Bosons



[RMP**80**,885(2008)]



Shell Structure of Mott States

High-Resolution RF Spectroscopy: Observation of Mott Shell Structure

[G. K. Campbell et al., Science 313, 649 (2006)]

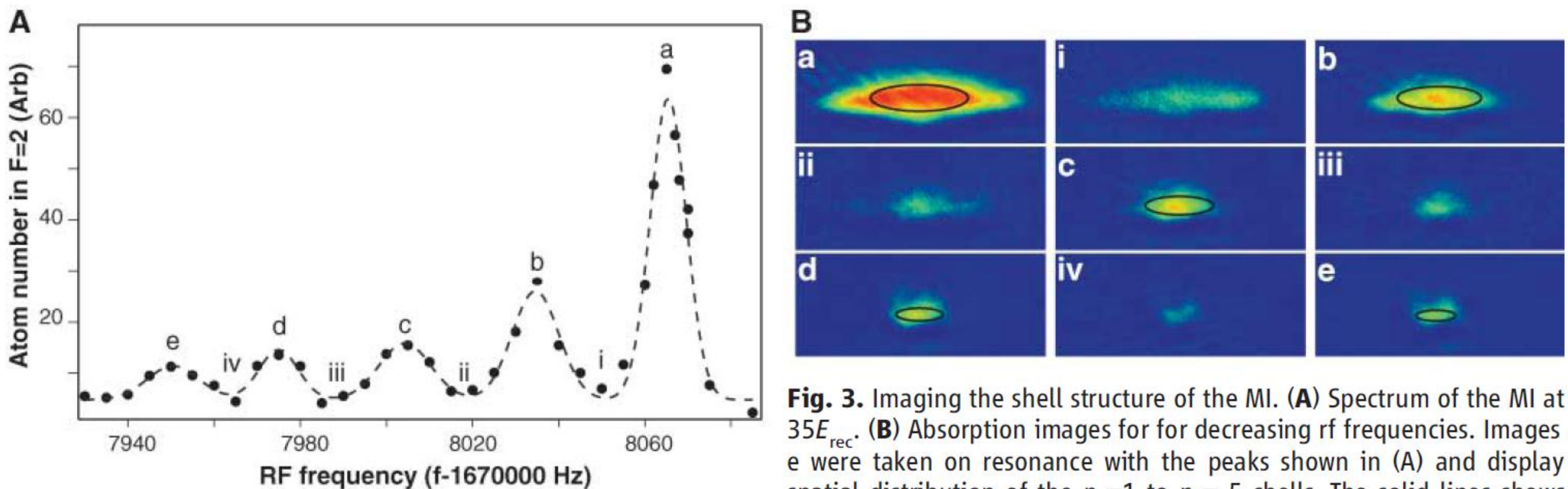


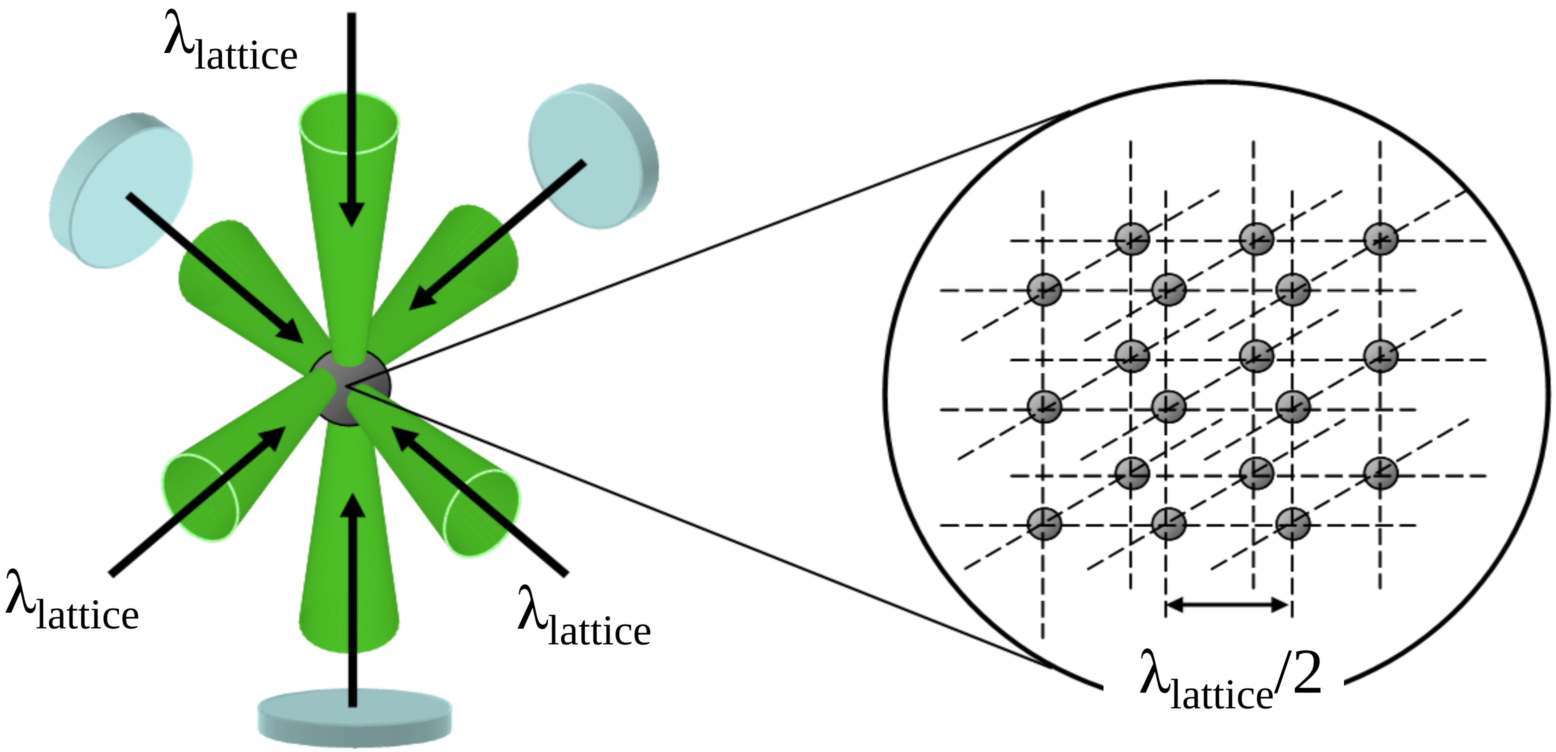
Fig. 3. Imaging the shell structure of the MI. (**A**) Spectrum of the MI at V = $35E_{rec}$. (**B**) Absorption images for for decreasing rf frequencies. Images a to e were taken on resonance with the peaks shown in (A) and display the spatial distribution of the n =1 to n = 5 shells. The solid lines shows the predicted contours of the shells. Absorption images taken for rf frequencies between the peaks (images i to iv) show a much smaller signal. The field of view was 185 μm by 80 μm.

$$h\nu_n = \frac{U}{a_{11}}(a_{12} - a_{11})(n-1)$$

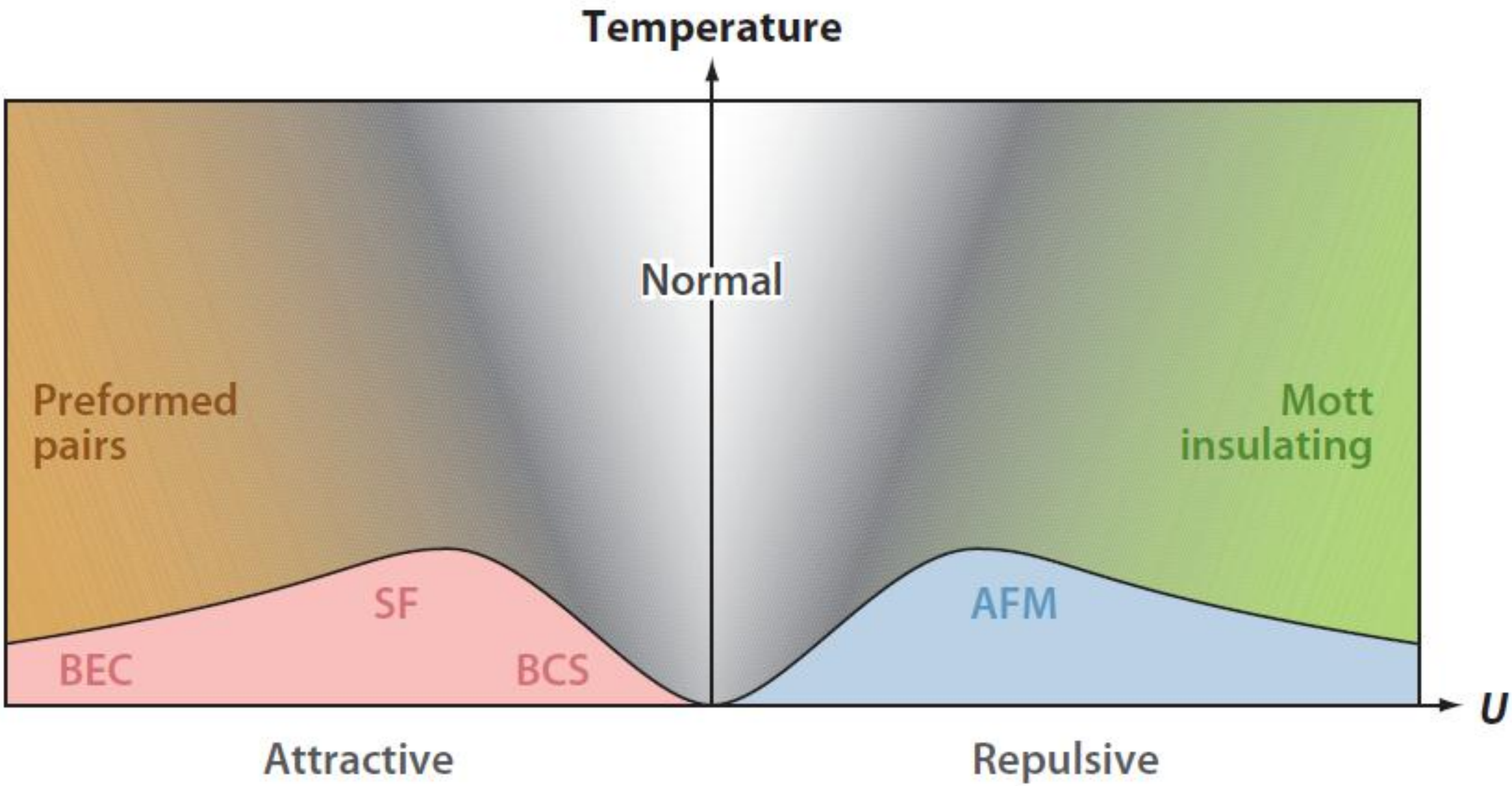
Fermions in a 3D optical lattice

$$H = -J\sum_{<i,j>} c_i^{+} c_j + U\sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_i \varepsilon_i n_i$$

"Fermi-Hubbard Model"



Phase Diagram of Repulsively and Attractively Interacting Fermions

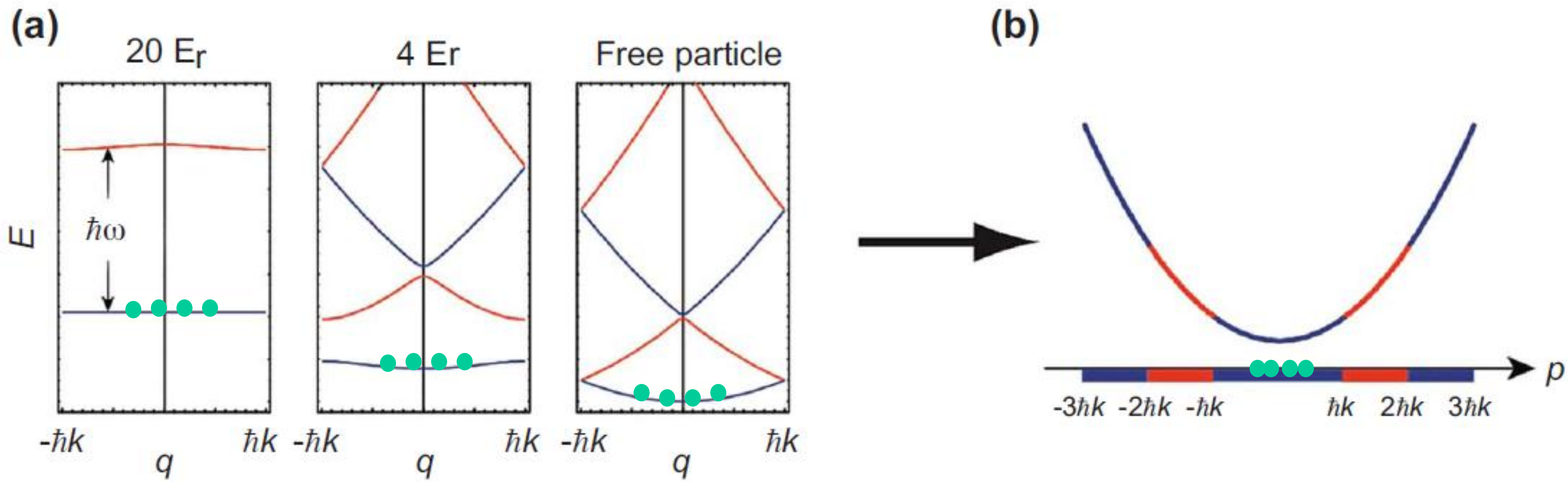


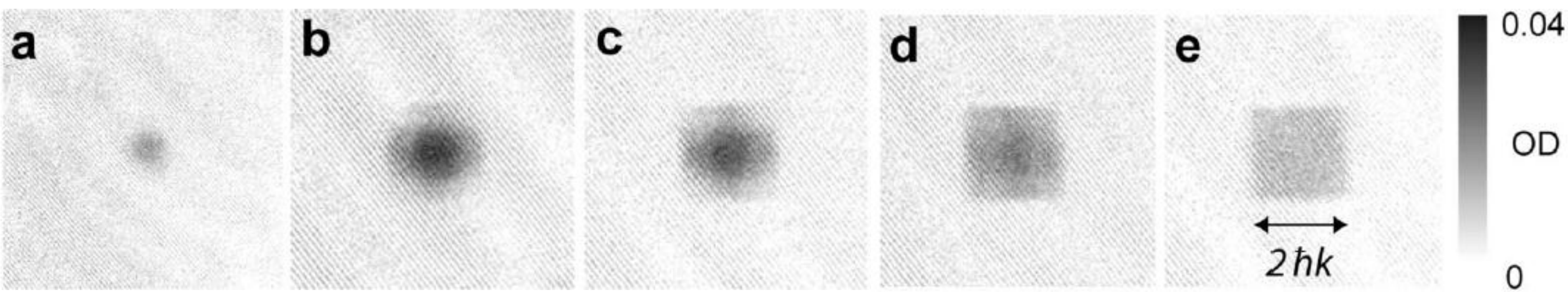
[T. Esslinger, Annu. Rev. Condens. Matter Phys. 2010. 1:129-152,
R. Micnas, J. Ranninger, S. Roaszkiewicw, Rev. Mod. Phys. 62, 113(1990)]

Observation of Fermi-Surface of ^{41}K

[M. Köhl, *et al.*, PRL **94**, 080403(2005)]

"band mapping"

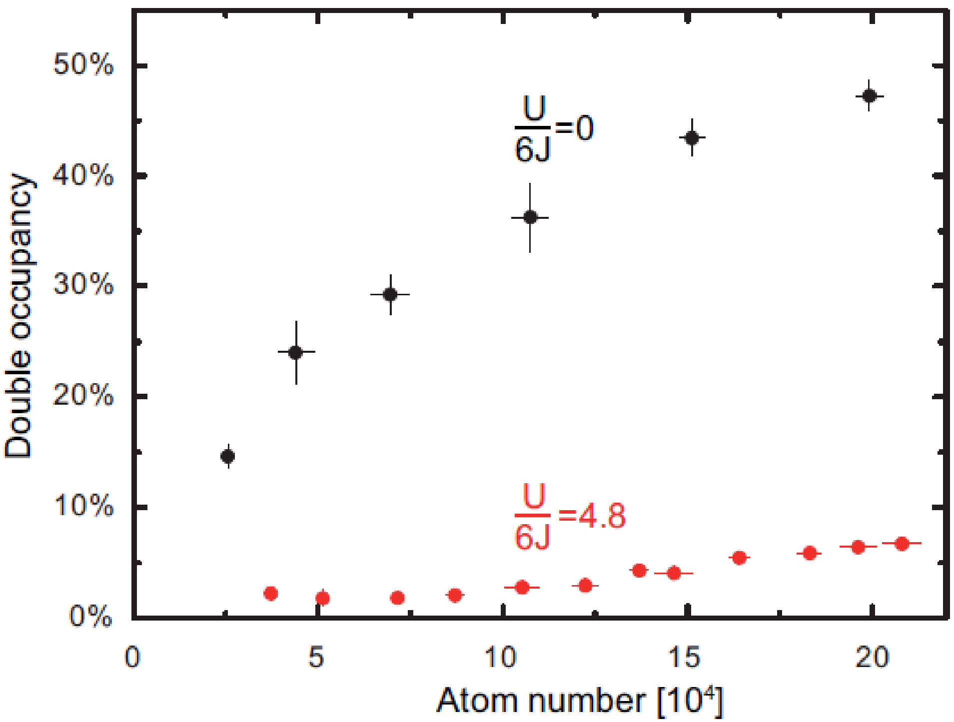


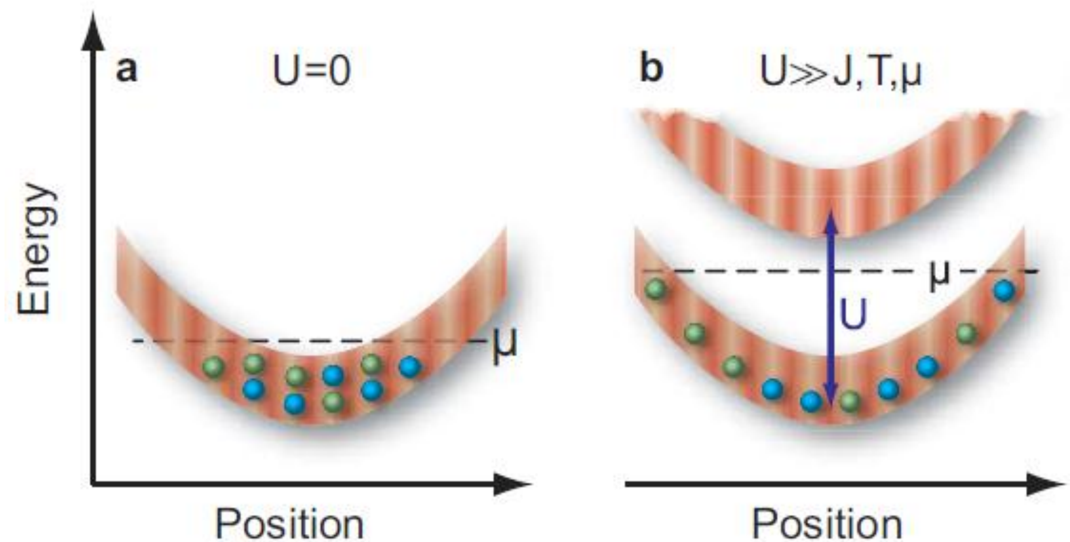


Fermi-Hubbard Model:

"A Mott insulator of ^{40}K atoms in an optical lattice"

[R. Jördens *et al.*, Nature **455**, 204 (2008)] [U. Schneider, *et al.*, Science **322**,1520(2008)]





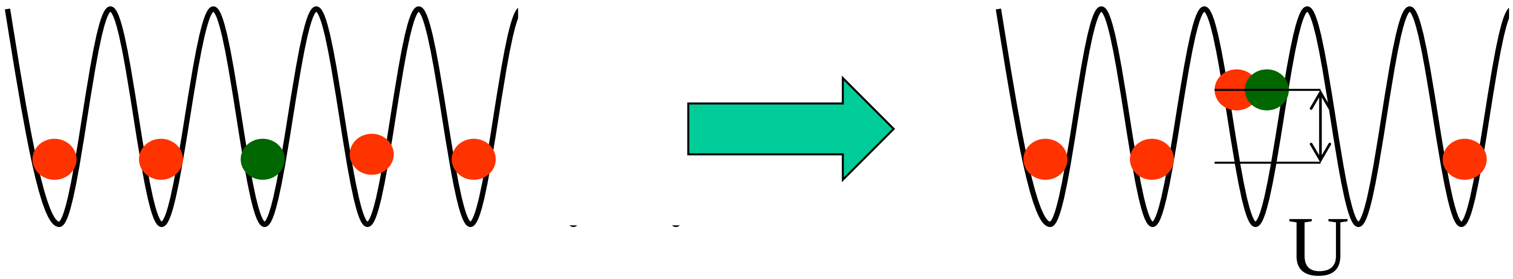
Fermi-Hubbard Model:

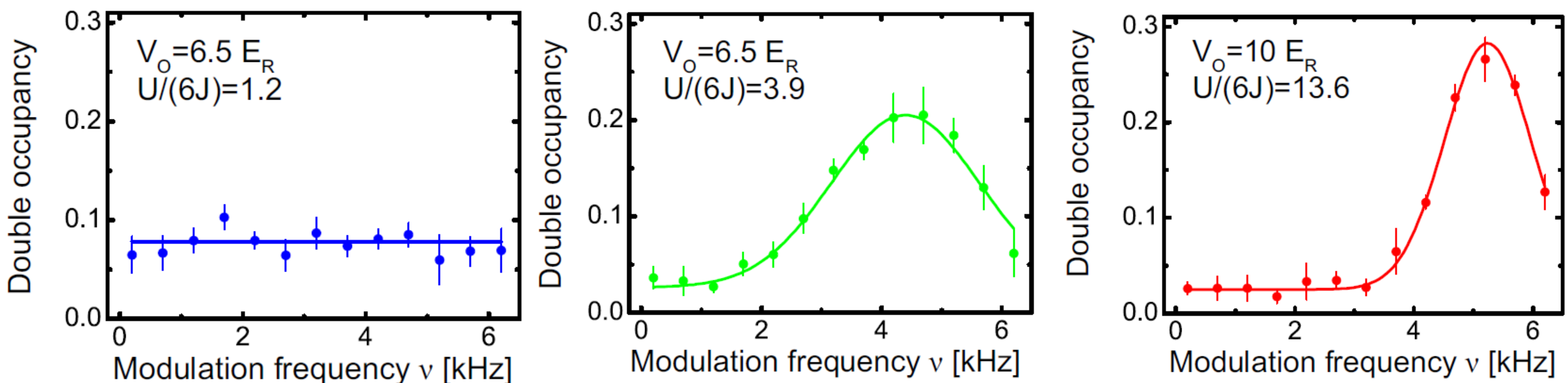
“A Mott insulator of ^{40}K atoms in an optical lattice”

[R. Jördens *et al.*, Nature **455**, 204 (2008)]

Modulation Spectroscopy of Mott Gap:

lattice intensity modulation results in creation of doublon



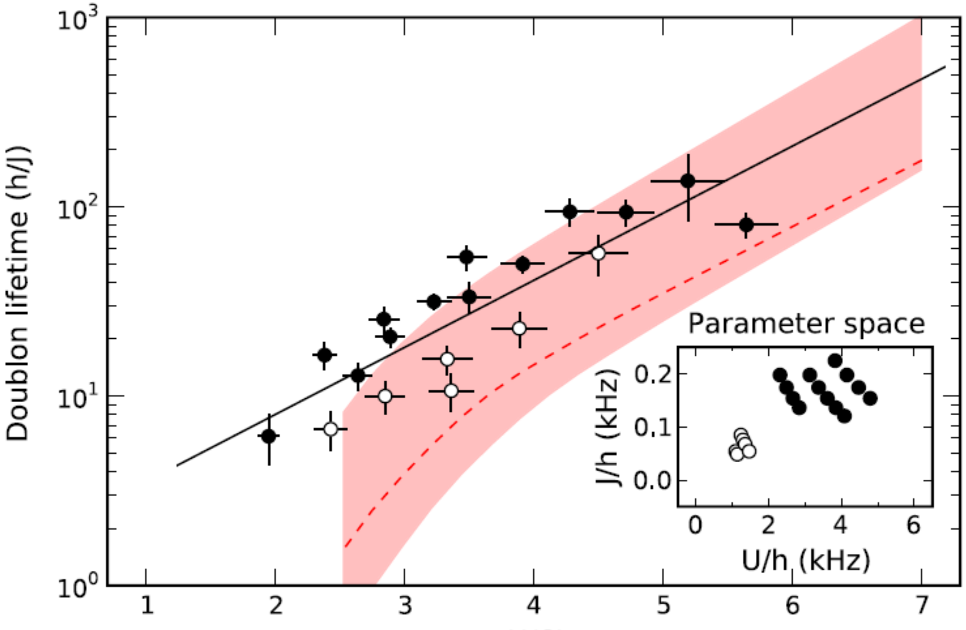


Fermi-Hubbard Model:

"A Mott insulator of ^{40}K atoms in an optical lattice"

[N. Strohmaier *et al.*, PRL **104**, 080401 (2010)]

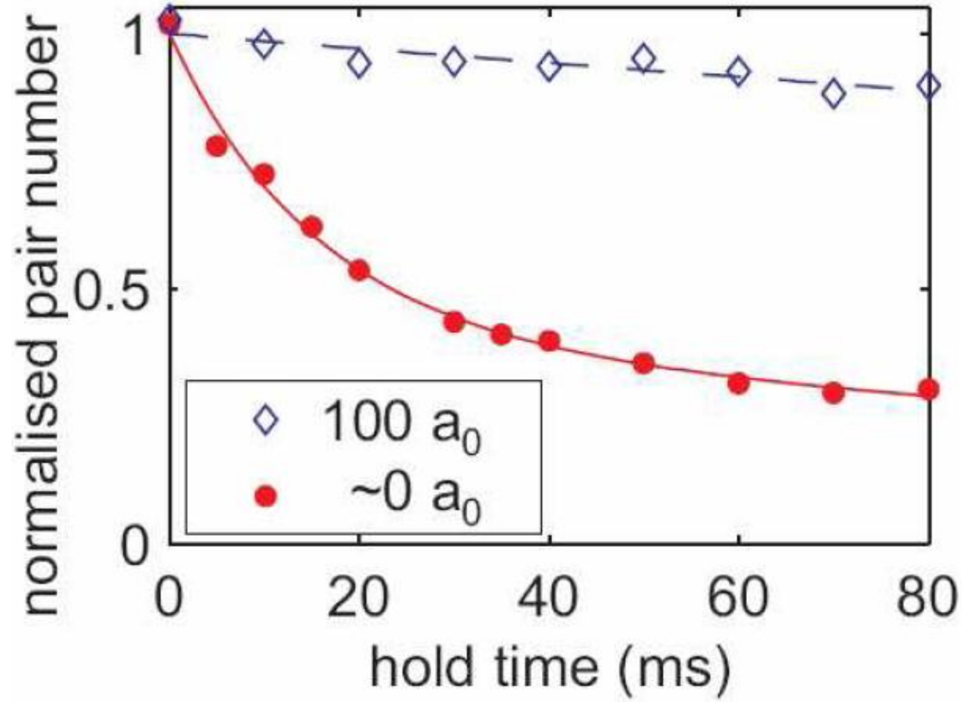
Doublon Decay



$$\frac{\tau_D}{h/J} = C \exp\left(\alpha \frac{U}{6J}\right)$$

α~0.8

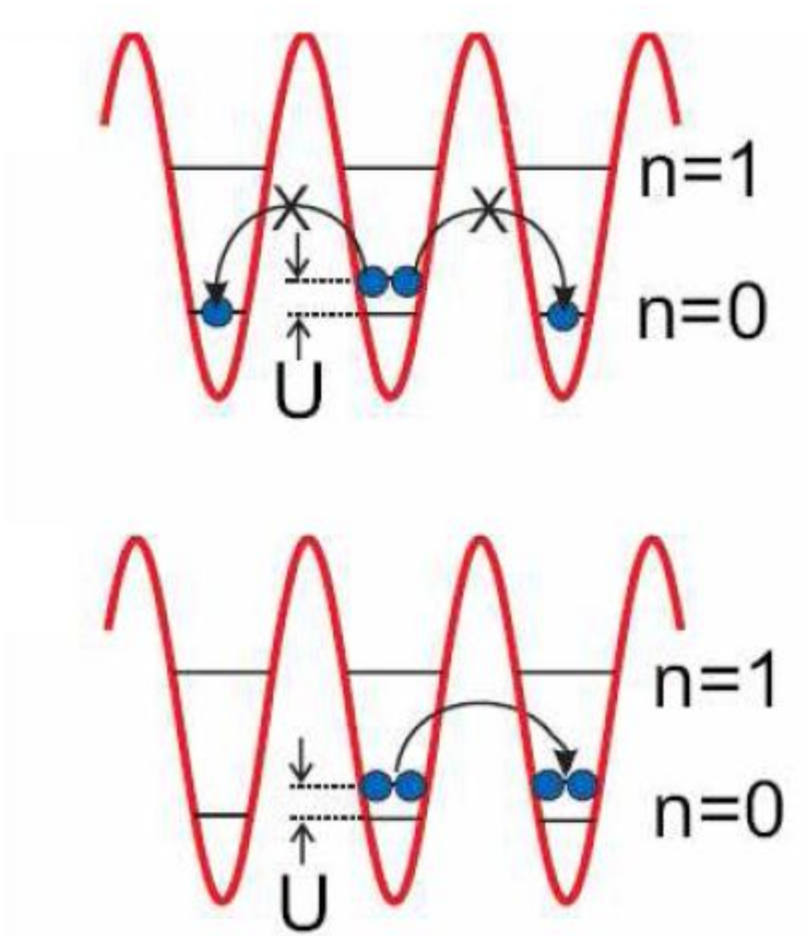
Isolated Pair

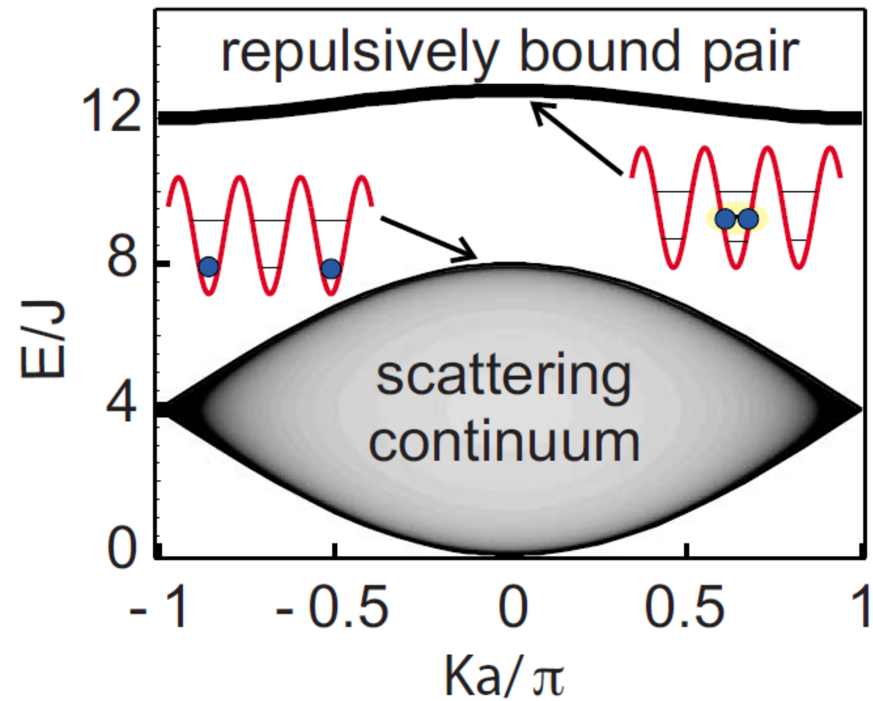


[K. Winkler *et. al.*, Nature **441**, 853 (2006)]

Repulsively Bound Pair in an Optical Lattice

[K. Winkler *et. al.*,   Nature **441**, 853 (2006)]

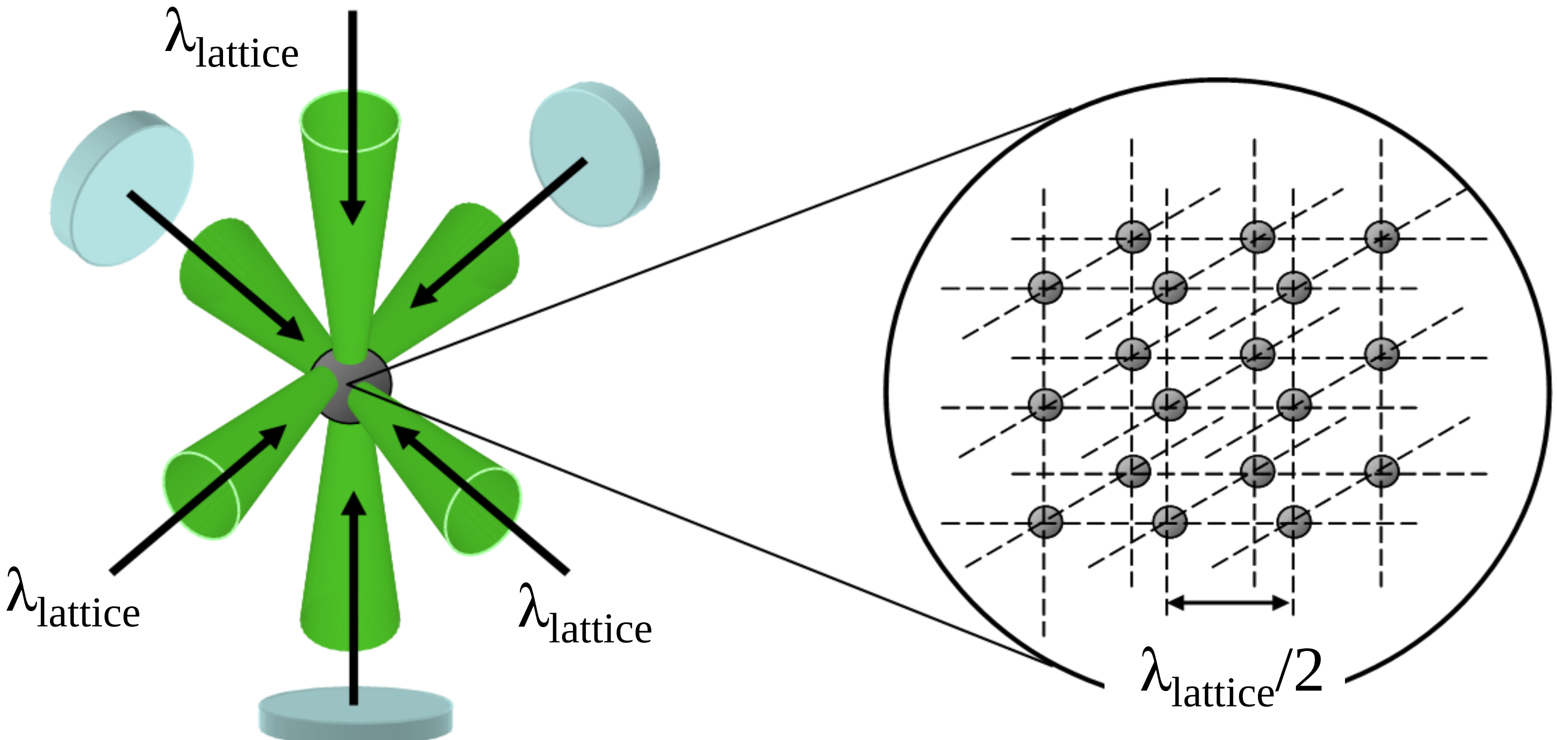




Bose-Fermi Mixture in a 3D optical lattice

$$H = -t_B \sum_{<i,j>} a_i^+ a_j + \frac{U_{BB}}{2} \sum_i n_{Bi}(n_{Bi} - 1) - t_F \sum_{<i,j>} c_i^+ c_j + U_{BF} \sum_i n_{Bi} n_{Fi}$$

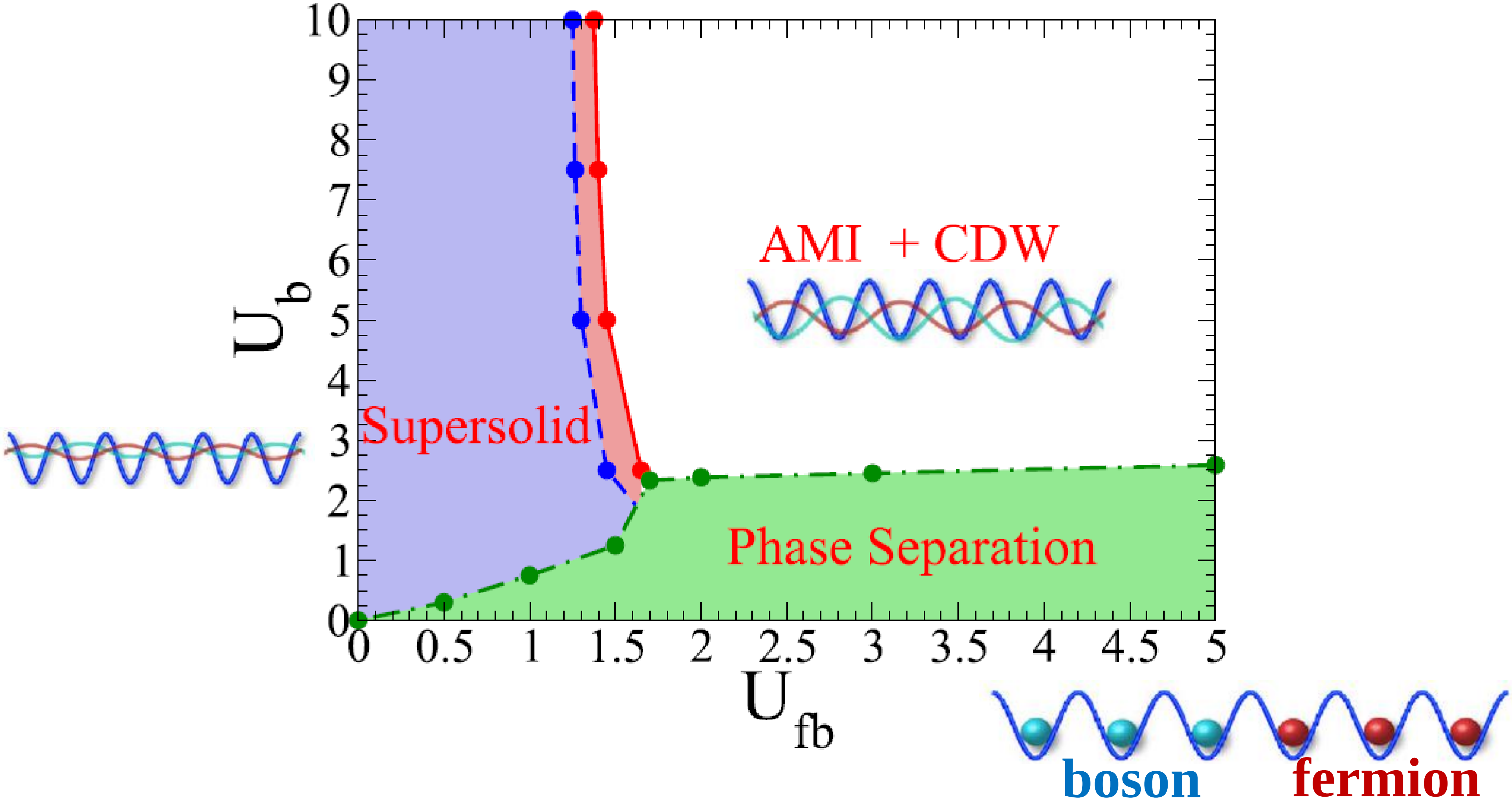
"Bose-Fermi Hubbard Model"



Phase Diagram of Bose-Fermi Mixture

[I. Titvinidze, *et al.*, . PRL 100, 100401(2008)]

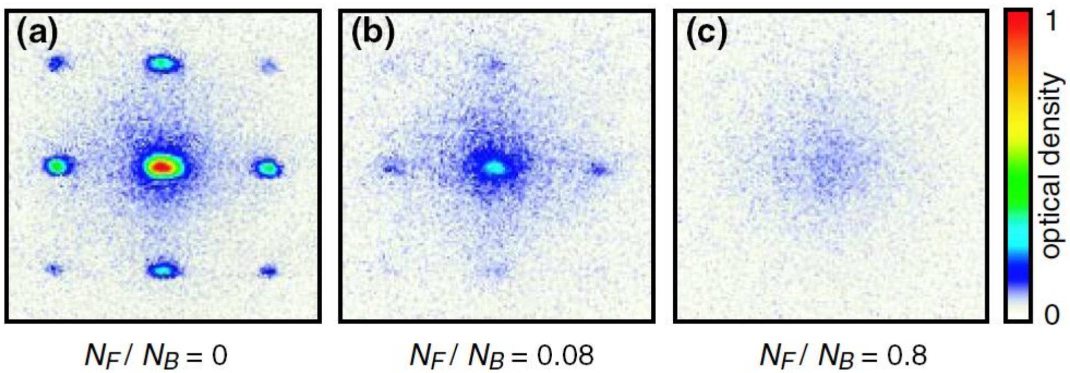
Spinless Fermion, Repulsive BF interaction, Half Filling, T=0



Bose-Fermi Mixture in a 3D optical lattice

$$H = -t_B \sum_{<i,j>} a_i^+ a_j + \frac{U_{BB}}{2} \sum_i n_{Bi}(n_{Bi} - 1) - t_F \sum_{<i,j>} c_i^+ c_j + U_{BF} \sum_i n_{Bi} n_{Fi}$$

“ ^{40}K(Fermion)-^{87}Rb(Boson)” **a_{BF} = -10.9 nm**

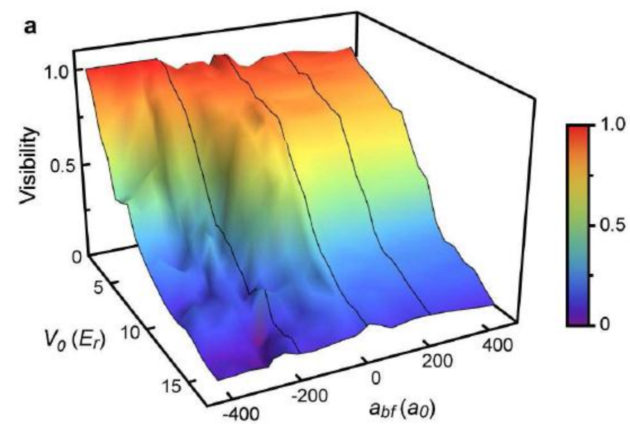


[K. Günter, et al, PRL**96**, 180402 (2006)]

[S. Ospelkaus, et al, PRL**96**, 180403 (2006)]

“Role of interactions in Rb-K Bose-Fermi mixtures in a 3D optical lattice”

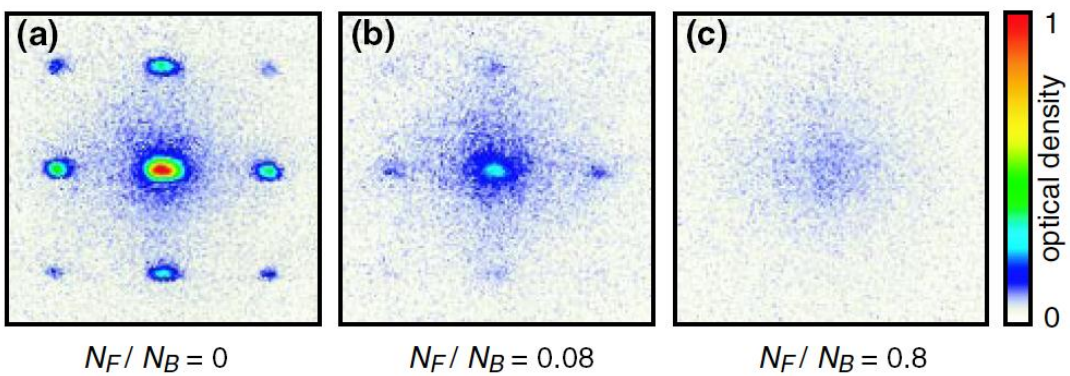
[Th. Best, *et al*, PRL**102**, 030408 (2008)]



Bose-Fermi Mixture in a 3D optical lattice

$$H = -t_B \sum_{<i,j>} a_i^+ a_j + \frac{U_{BB}}{2} \sum_i n_{Bi}(n_{Bi} - 1) - t_F \sum_{<i,j>} c_i^+ c_j + U_{BF} \sum_i n_{Bi} n_{Fi}$$

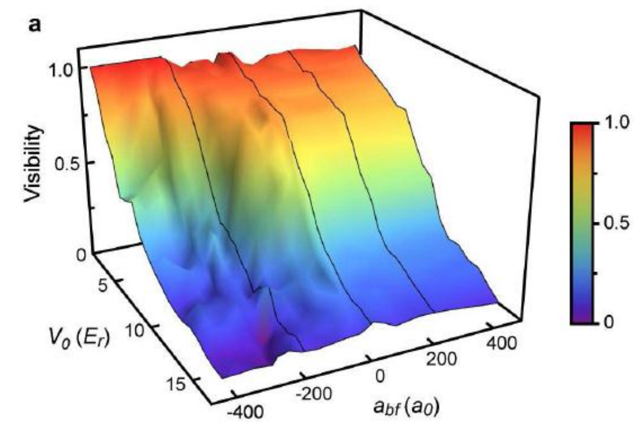
“ ^{40}K(Fermion)-^{87}Rb(Boson)” **a_{BF} = -10.9 nm**



[K. Günter, et al, PRL**96**, 180402 (2006)]

[S. Ospelkaus, et al, PRL**96**, 180403 (2006)]

“Role of interactions in Rb-K Bose-Fermi mixtures in a 3D optical lattice”
[Th. Best, *et al*, PRL**102**, 030408 (2008)]



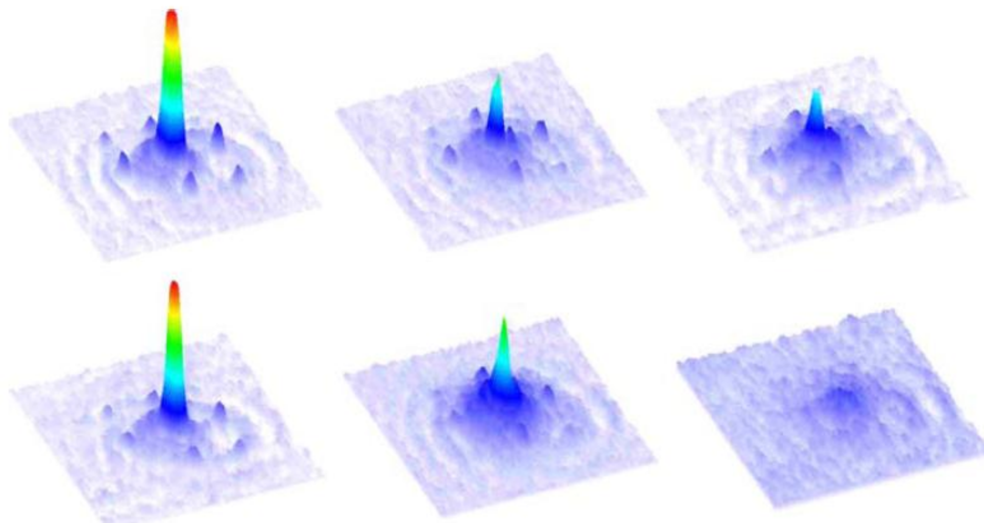
Bose-Bose Hubbard Model

[J. Catani, et al, PRA **77**, 011603(R) (2008)]

“ ^{41}K(Boson)-^{87}Rb(Boson)” **a_{BB} = +8.6 nm**

^{87}Rb only

^{87}Rb mixed with ^{41}K



[B. Gadway, et al, PRL **105**, 045303 (2010)]

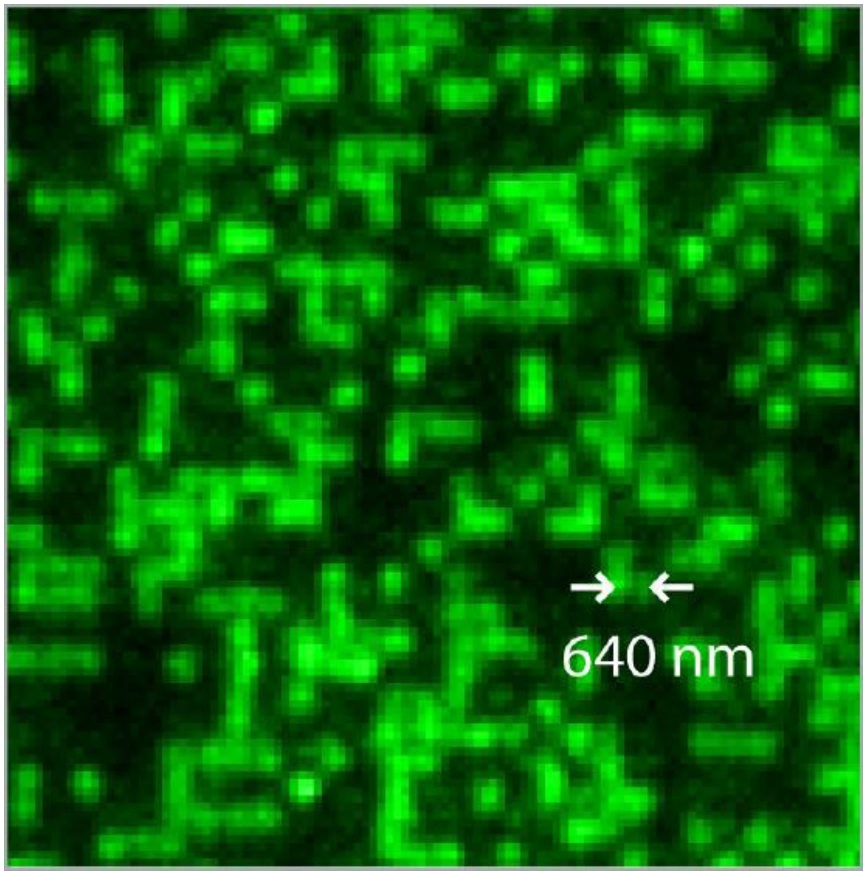
“^{87}Rb:F=1(Boson)-^{87}Rb:F=2(Boson)”

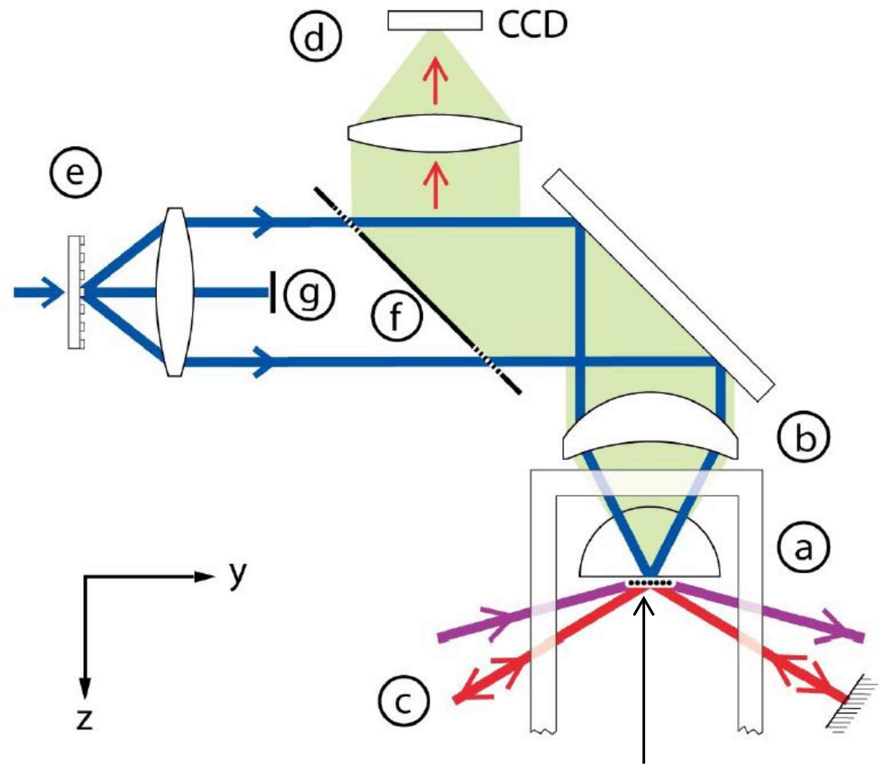
a_{BB} ~ +5.3 nm

New Technique: Single Site Observation

[WS. Bakr, I. Gillen, A. Peng, S. Folling, and M. Greiner, Nature 462(426), 74-77(2009)]

Fluorescence Imaging

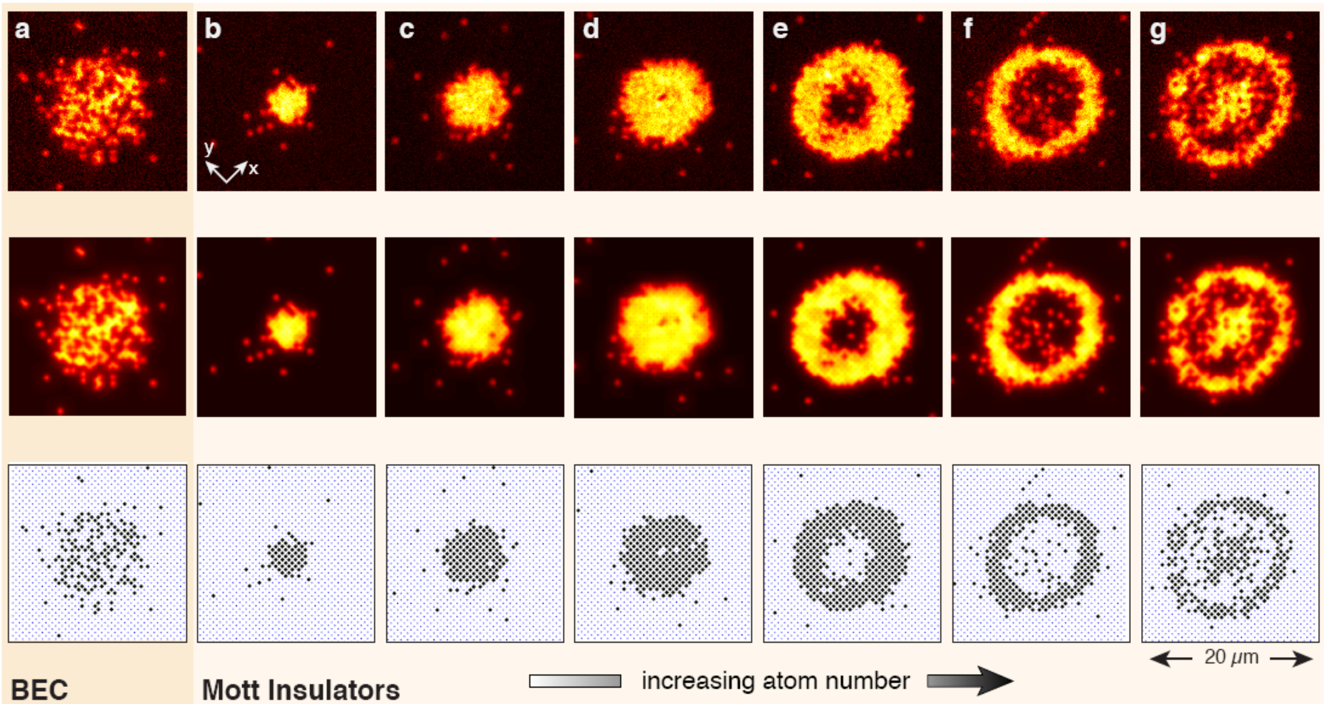


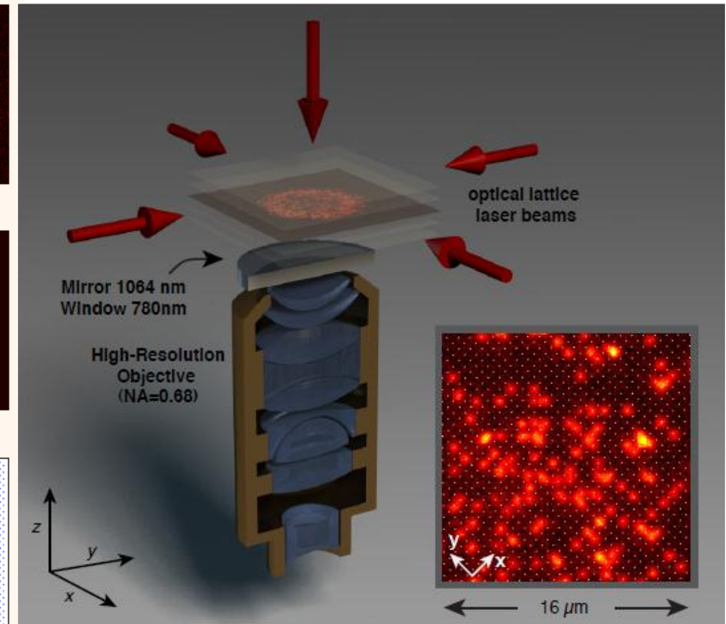


^{87}Rb

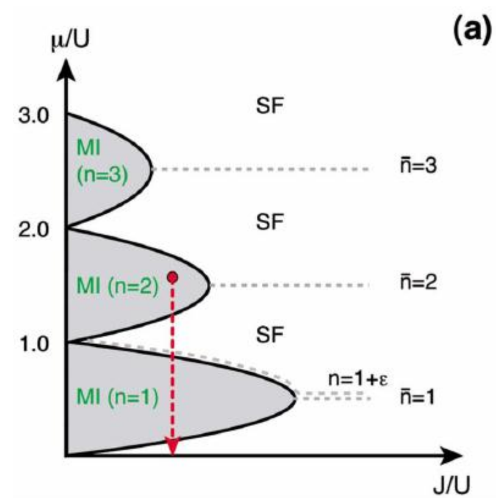
Single Site Resolved Detection of MI

[arXiv1006.3799v1 J. F. Sherson, et al.,]

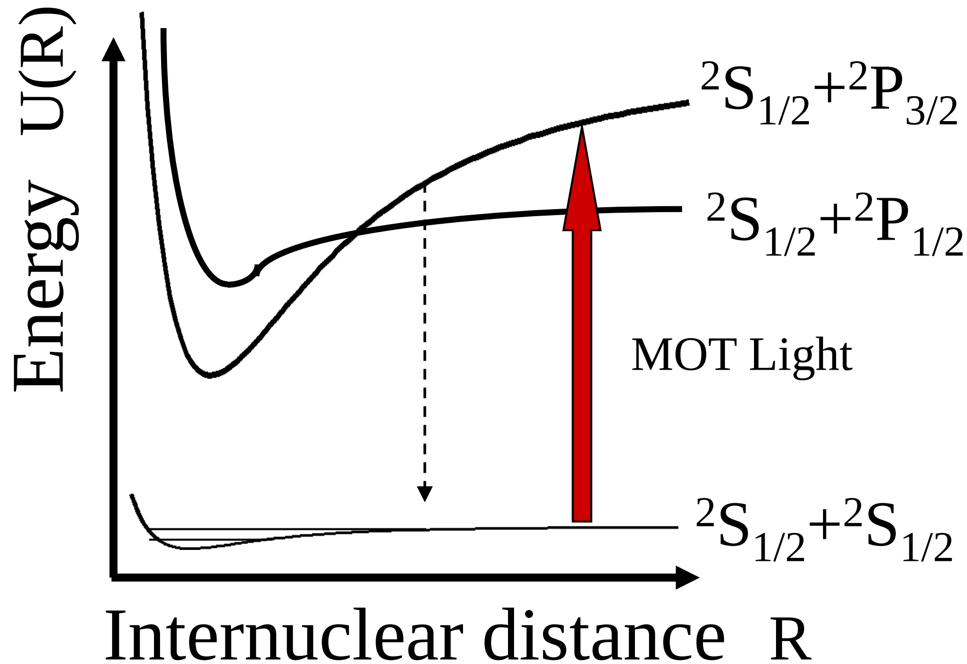




'IG. 2: **High resolution fluorescence images of a BEC and Mott insulators.** Top row: Experimentally obtained images f a BEC (a) and Mott insulators for increasing particle numbers (b-g) in the zero-tunneling limit. Middle row: Numerically econstructed atom distribution on the lattice. The images were convoluted with the point-spread function of our imaging ystem for comparison with the original images. Bottom row: Reconstructed atom number distribution. Each circle indicates single atom, the points mark the lattice sites.



Light-Assisted Collision



1) Fine-structure changing collision

$$^2S_{1/2}+^2P_{3/2} \longrightarrow {}^2S_{1/2}+^2P_{1/2} + \text{K.E.}$$

2) Radiative Escape

$$^2S_{1/2}+^2P_{3/2} \longrightarrow {}^2S_{1/2}+^2S_{1/2} + \text{K.E.}$$

Single Site Resolved Detection of MI

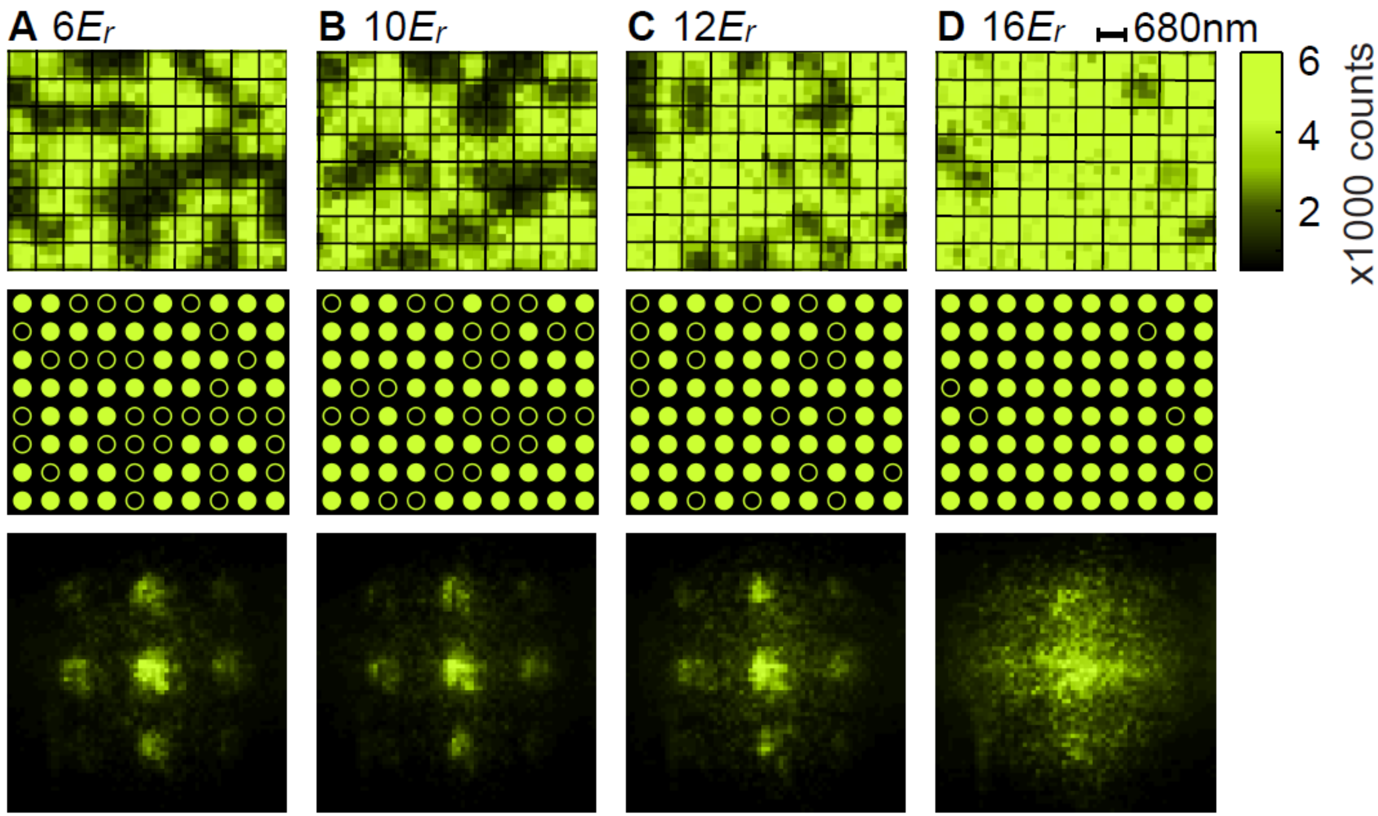
[arXiv1006.0754v1 WS Bakr, et al.,]

SF ⟷ MI

In Situ-image

after analysis

TOF-image



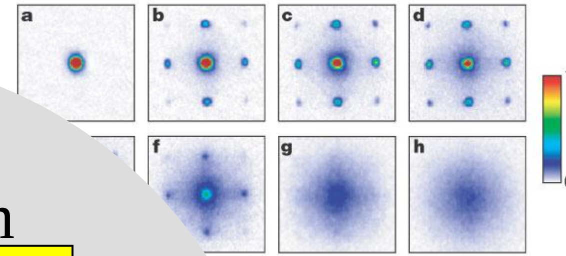
Quantum Simulators using Alkali Atoms

Bose-Hubbard Model:

"Superfluid - Mott-insulat

[M. Greiner, *et al.*, Nat

…



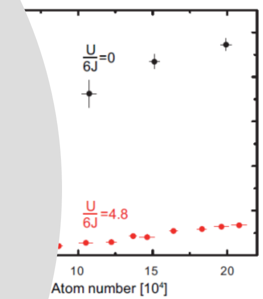
^{87}Rb

Fermi-Hubbard Mo

"Formation of M

[R. Jördens *et al.*,

[U. Schneider, *et*



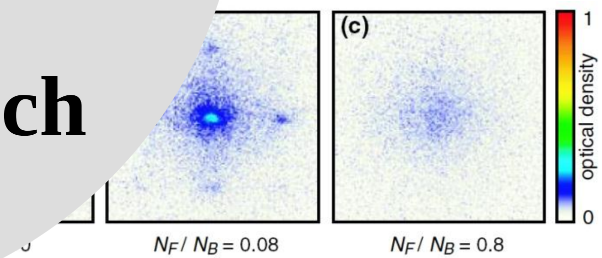
^{40}K

Bose-Fermi-Hubba

[K. Günter, et al, PRL

[S. Ospelkaus, et al, PR

[Th. Best, *et al*, PRL**102**, 0



^{87}Rb + ^{40}K

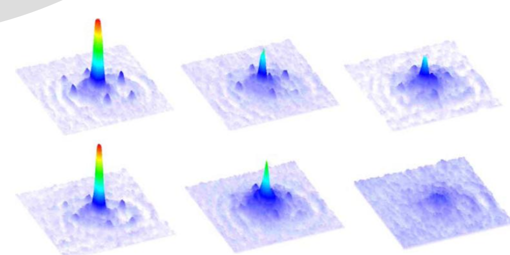
two-electron atom

Yb Atoms

Our Approach

Bose-Bose-Hubbard Model:

[J. Catani, et al, PRA**77**, 011603(R) (2008)]



^{87}Rb + ^{41}K

FIRST Quantum Information Processing Project
Summer School 2010

25 August 2010   Okinawa

Quantum Simulation of Hubbard Model Using Ultracold Two-Electron Atoms in an Optical Lattice

Kyoto University

Y. Takahashi

Unique Features of Ytterbium Atoms

Rich Variety of Isotopes

^{168}Yb (0.13%)	^{170}Yb (3.05%)	^{171}Yb (14.3%)	^{172}Yb (21.9%)	^{173}Yb (16.2%)	^{174}Yb (31.8%)	^{176}Yb (12.7%)
Boson	**Boson**	**Fermion**	**Boson**	**Fermion**	**Boson**	**Boson**

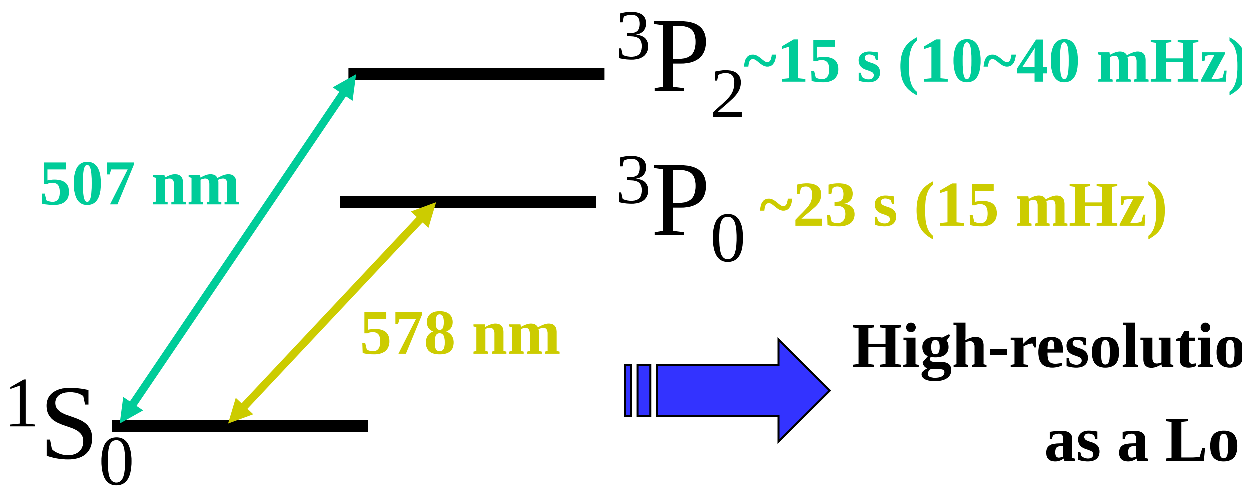
^{173}Yb (I=5/2) $$H_{\text{int}} = \frac{4\pi\hbar^2 a_s}{M}\delta(\vec{r}_1 - \vec{r}_2)$$ **SU(6) system**

→ novel magnetism

[M. A. Cazalilla, *et al.*, N. J. Phys **11**, 103033(2009), Hermele, et al., PRL 103, 130351 (2009); A. V. Gorshkov, *et al.*, Nat. Physics, 6, 289(2010)]

Unique Features of Ytterbium Atoms

Ultra-narrow Optical Transitions

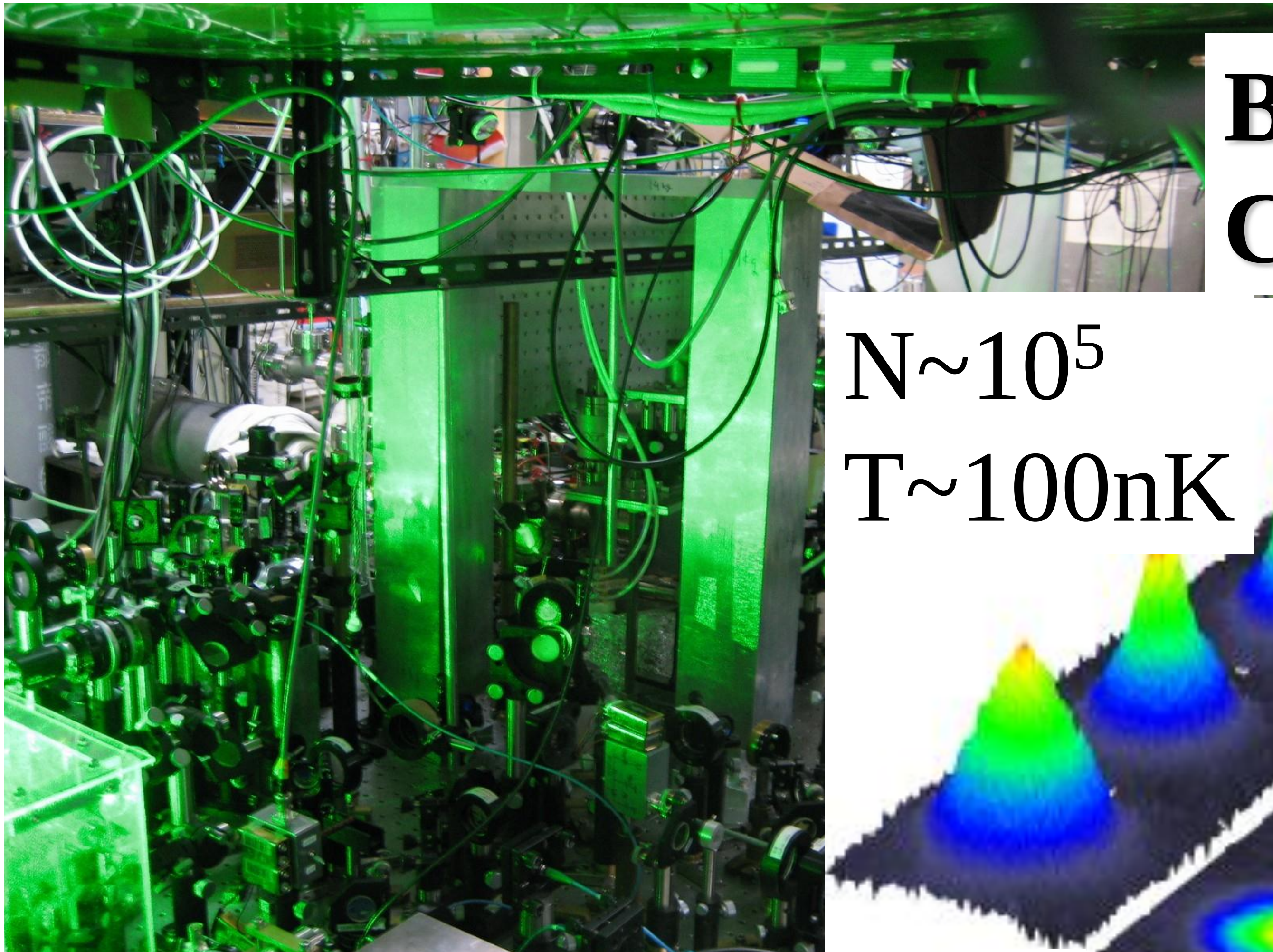


High-resolution laser spectroscopy as a Local Probe

High-spatial resolution Optical Magnetic Resonance Imaging

Another Useful Orbital States with Different Characters

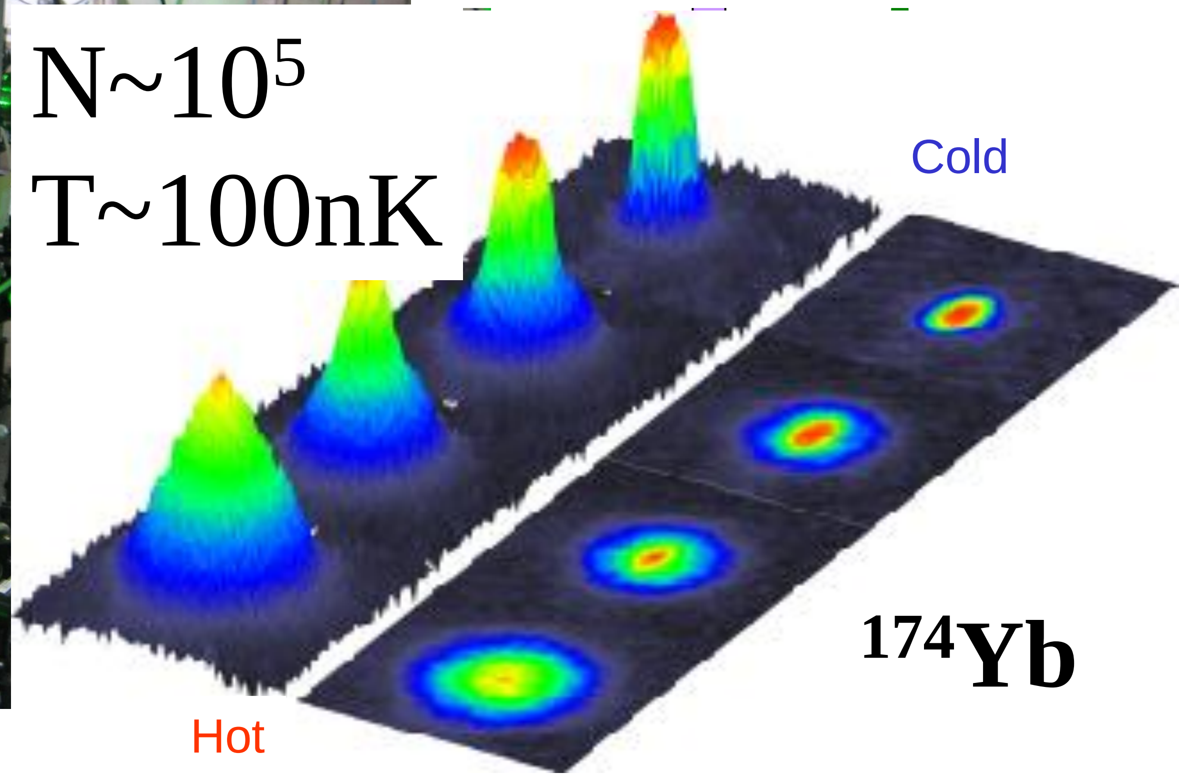
Preparation of Quantum Degenerate Gases



Bose-Einstein Condensation

N~10^5

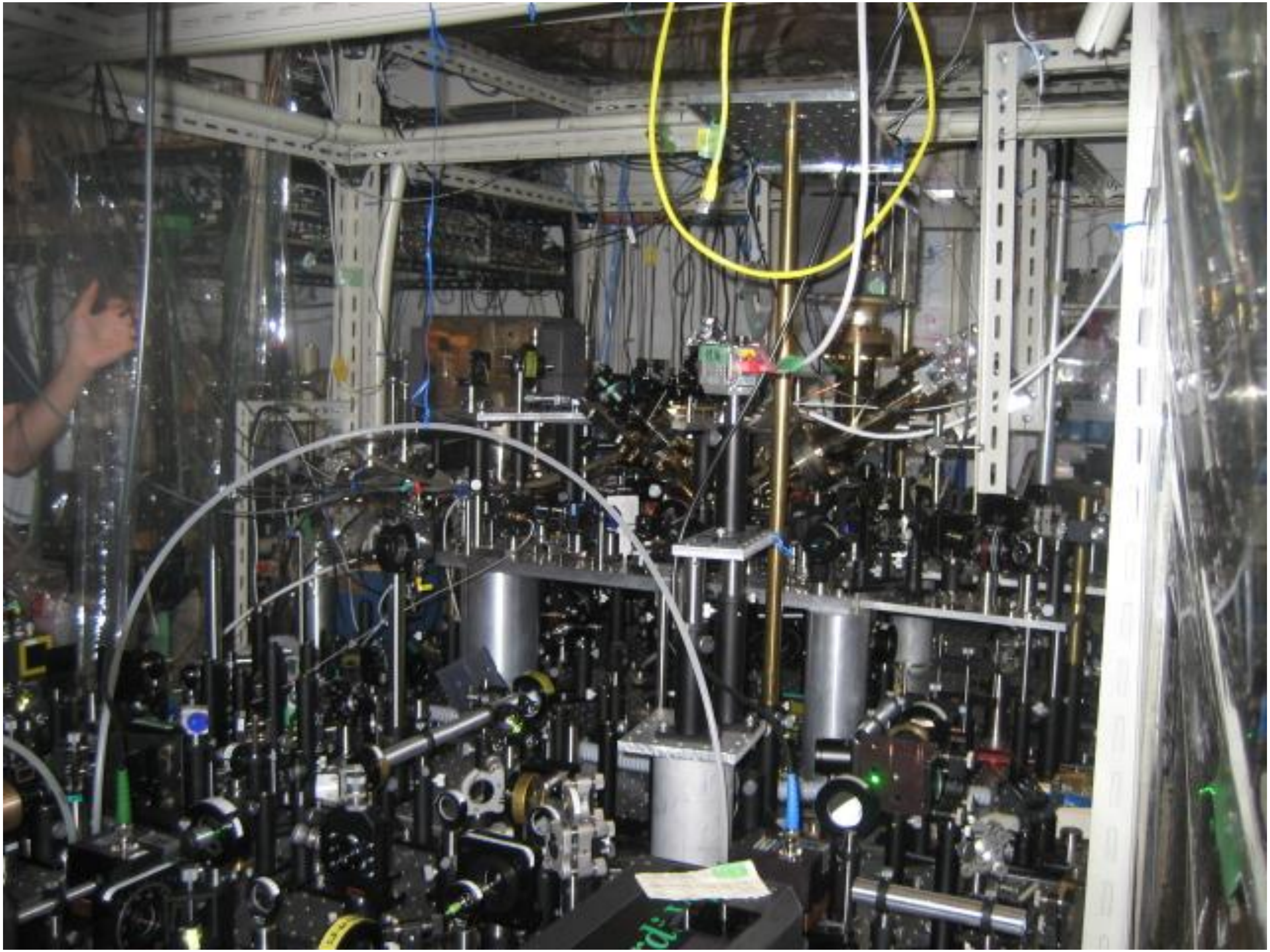
T~100nK



^{174}Yb

[Y. Takasu *et al.*, PRL **91**, 040404 (2003)]

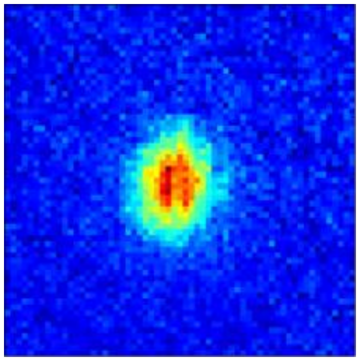
Current Experimental Setup



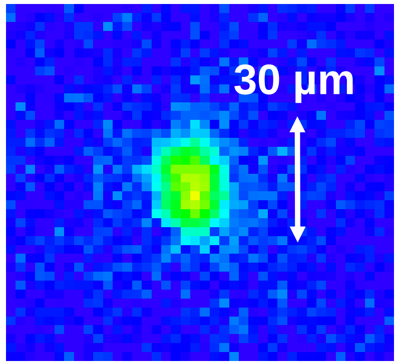
Quantum Degenerate Yb Gases

Boson [Y. Takasu *et al.*, PRL **91**, 040404 (2003)] [T. Fukuhara *et al.*, PRA **76**,051604(R)(2007)]

^{168}Yb(0.13%)



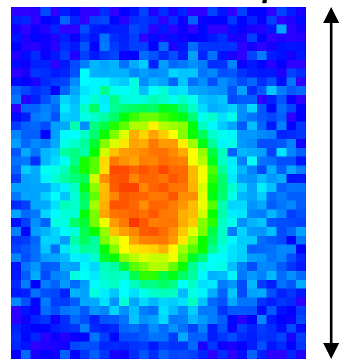
^{170}Yb



TOF: 10ms

^{174}Yb

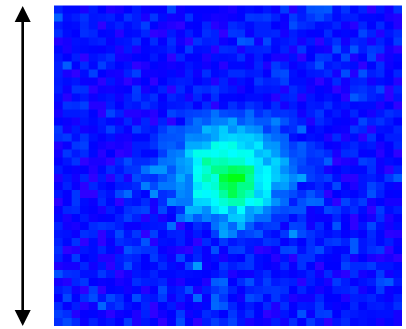
160µm



^{176}Yb

120 µm

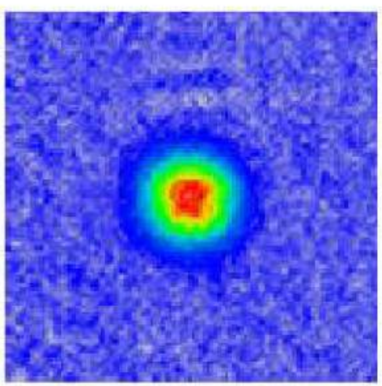
TOF: 8 ms

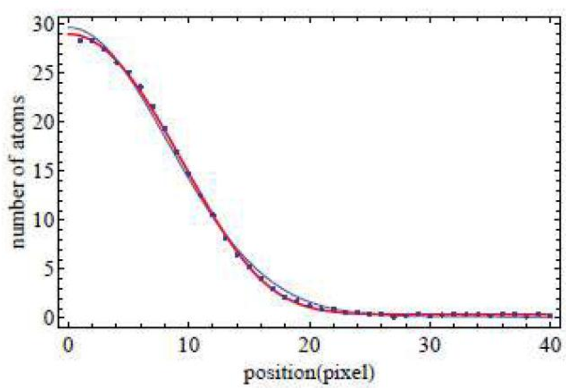


Fermion [T. Fukuhara *et al.*, PRL. **98**, 030401 (2007)]

^{171}Yb(I=1/2)

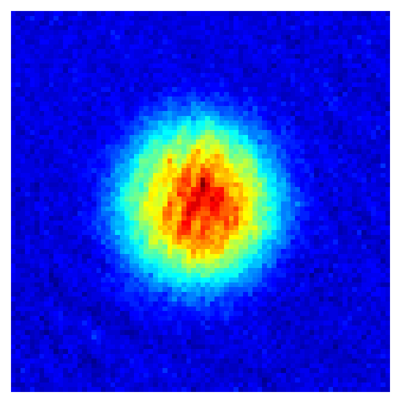
T/T_F =0.3 (2-component)

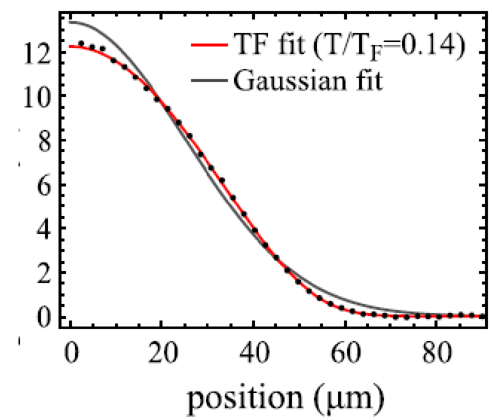




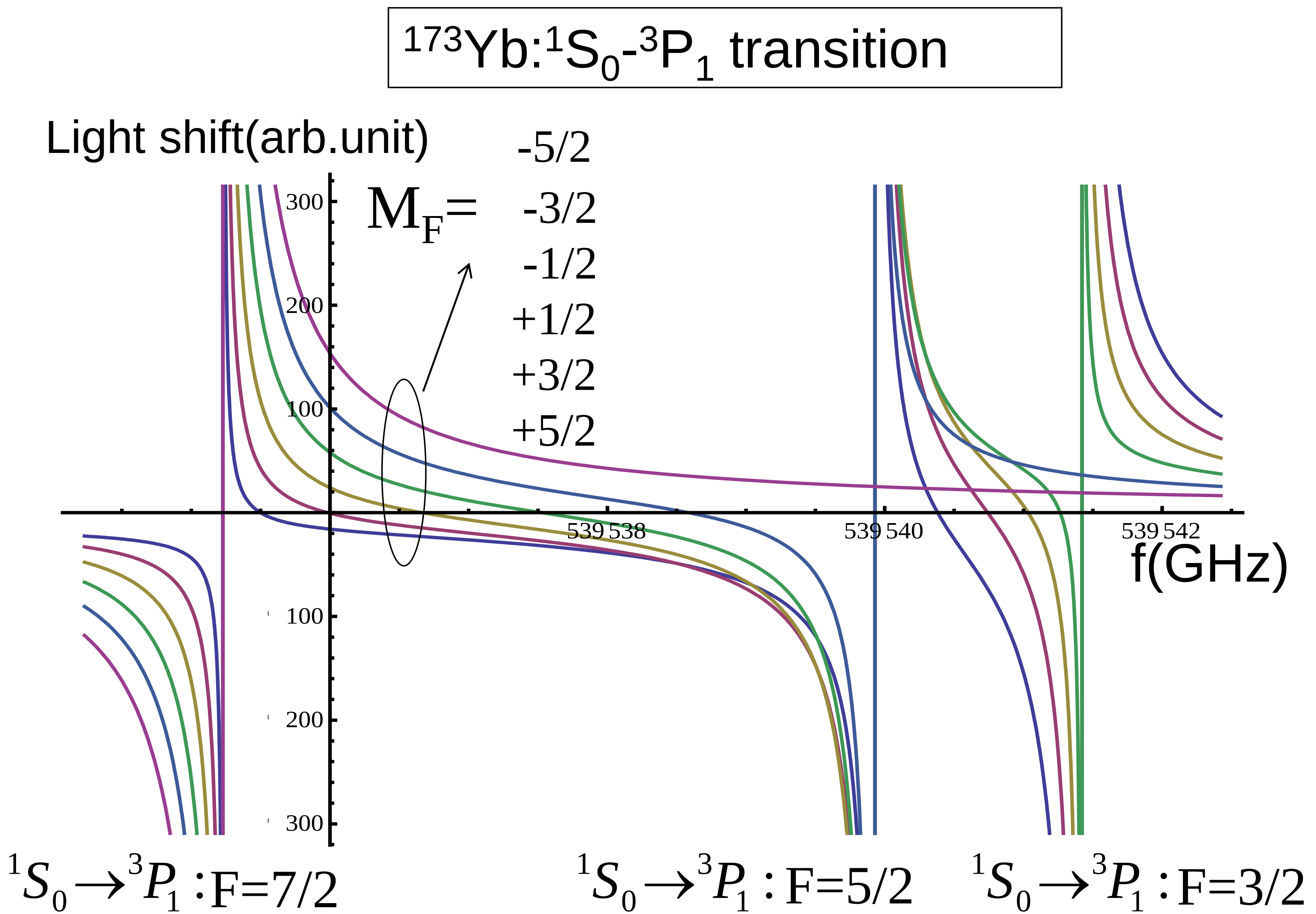
^{173}Yb(I=5/2)

T/T_F =0.14 (6-component)

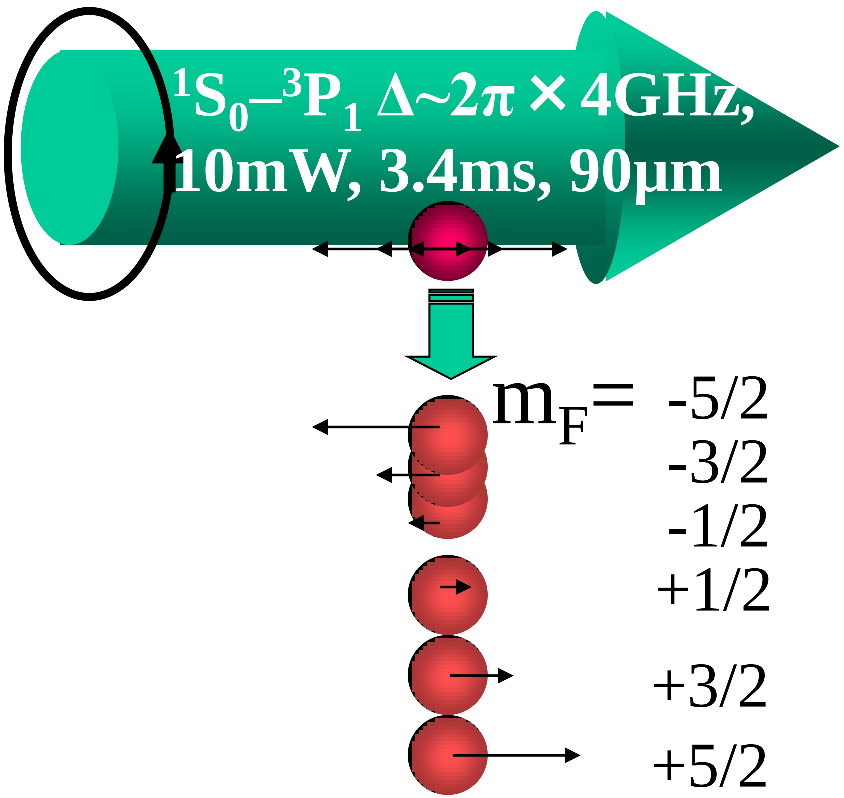




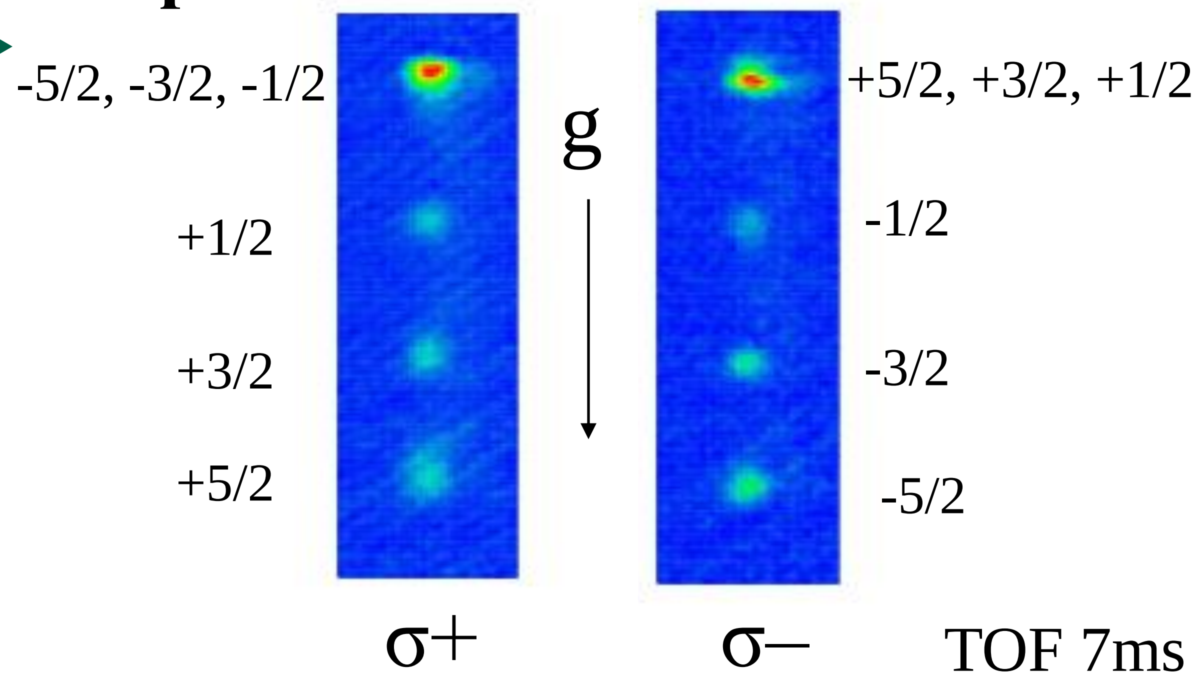
Nuclear Spin Dependent Light-Shift (calculation)



Ultracold ^{173}Yb: Fermi Gas with 6-spin components

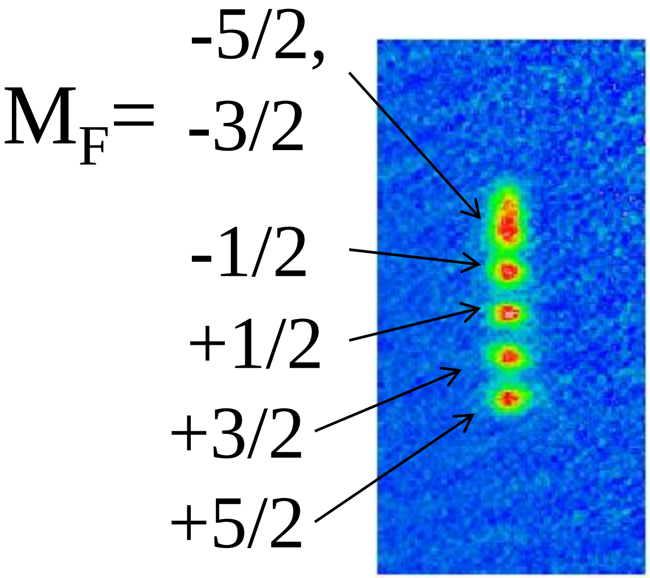


"Optical Stern-Gerlach Effect"

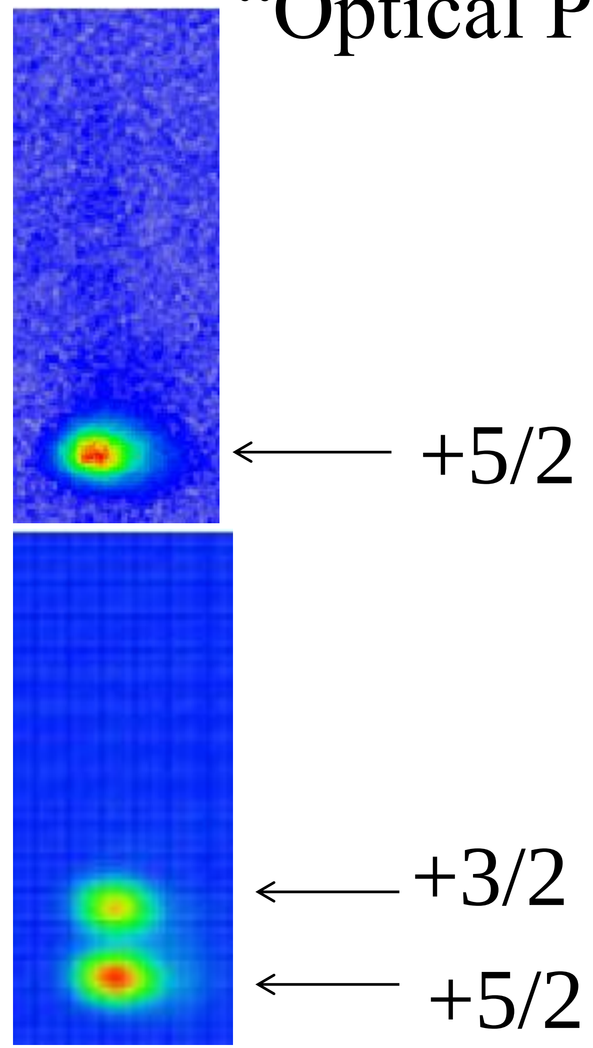


$$p_{m_F} = F_{m_F}\tau = -\frac{\partial E_{m_F}}{\partial z}\tau$$

Optical Stern Gerlach Separation: Optical Pumping Effect



"No Optical Pumping"



"Optical Pumping"

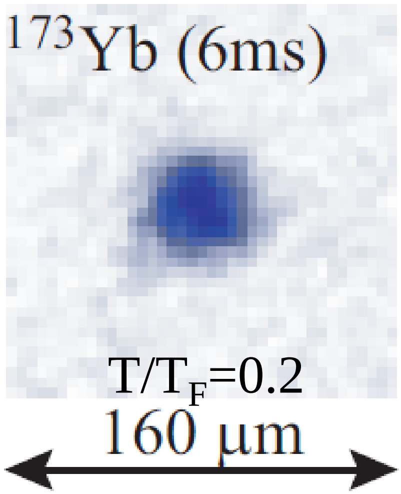
Other Quantum Gases of Two-Electron Atoms

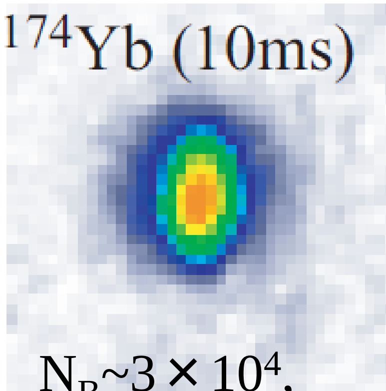
^{40}Ca:BEC (PTB, 2009)
^{84}Sr :BEC (Rice, Innsbruck, 2009)
^{87}Sr :Fermi-Degeneracy (Rice, 2010)

Quantum Degenerate Mixtures of Yb

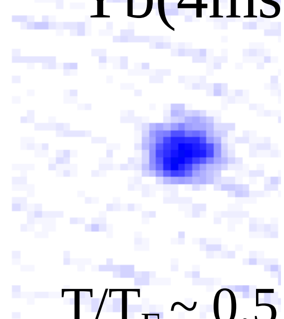
[T. Fukuhara *et al.*, Phys. Rev. A 79, 021601(R) (2008)]

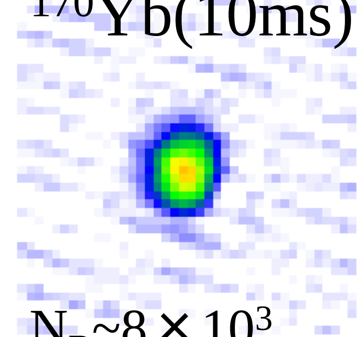
^{173}Yb(Fermion) + ^{174}Yb(Boson)



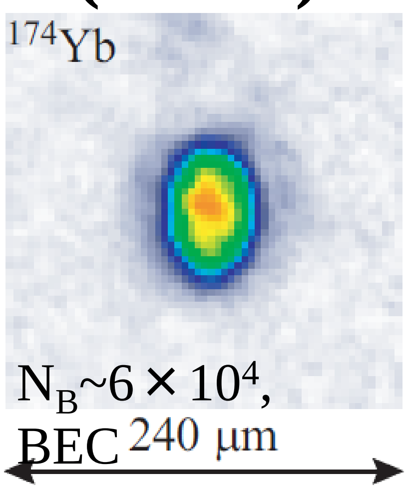


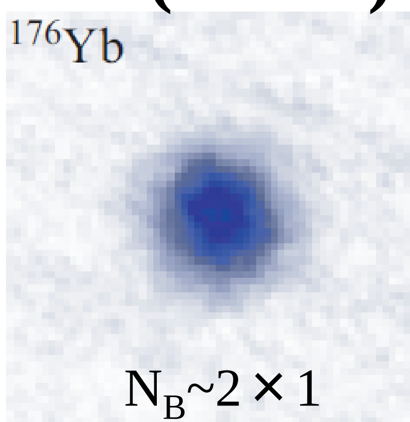
^{173}Yb(Fermion) + ^{170}Yb(Boson)



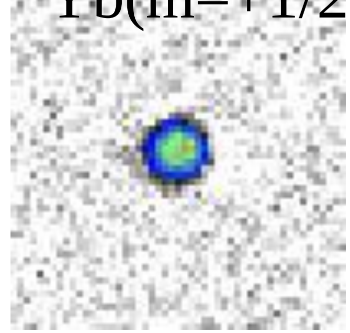


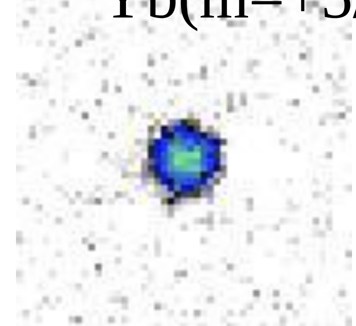
^{174}Yb(Boson)+ ^{176}Yb(Boson)



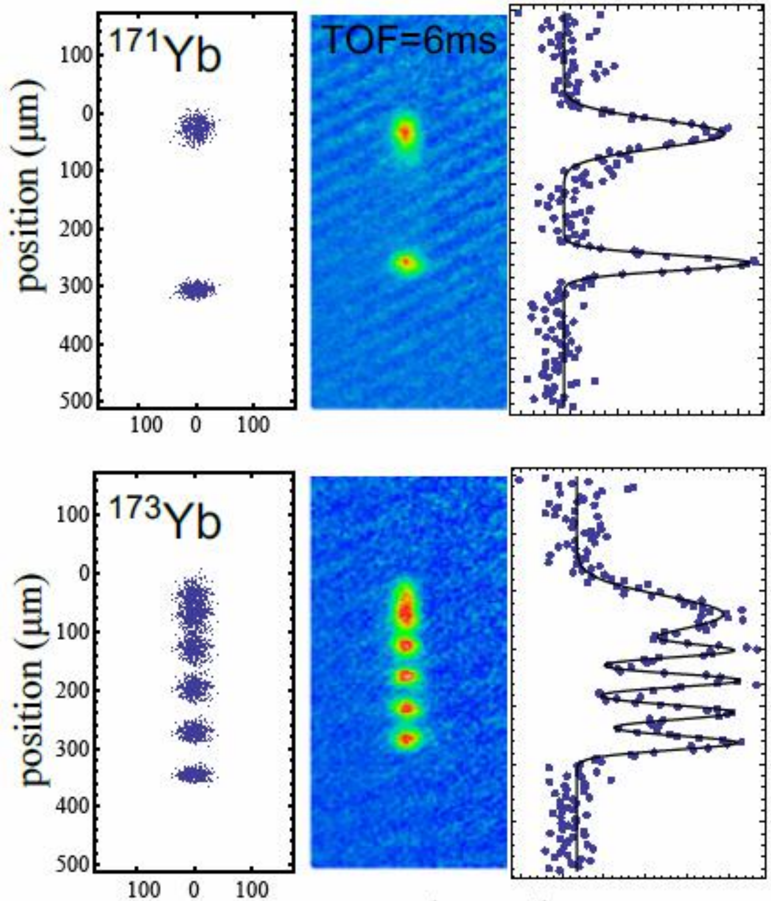


^{171}Yb(Fermion) + ^{173}Yb(Fermion)





SU(2) × SU(6) Symmetry



[S. Taie *et al* ., arxiv:1005.3710]

^{171}Yb: **N = 8.0×10^3**
T = 95 nK
$T/T_F = 0.46$ (2-component)

^{173}Yb: **N = 1.1×10^4**
T = 87 nK
$T/T_F = 0.54$ (6-component)

$$H_{171\text{-}173} = [W_0 + W_2 \vec{S}_{171} \cdot \vec{S}_{173}]\delta(\vec{r}_1 - \vec{r}_2)$$

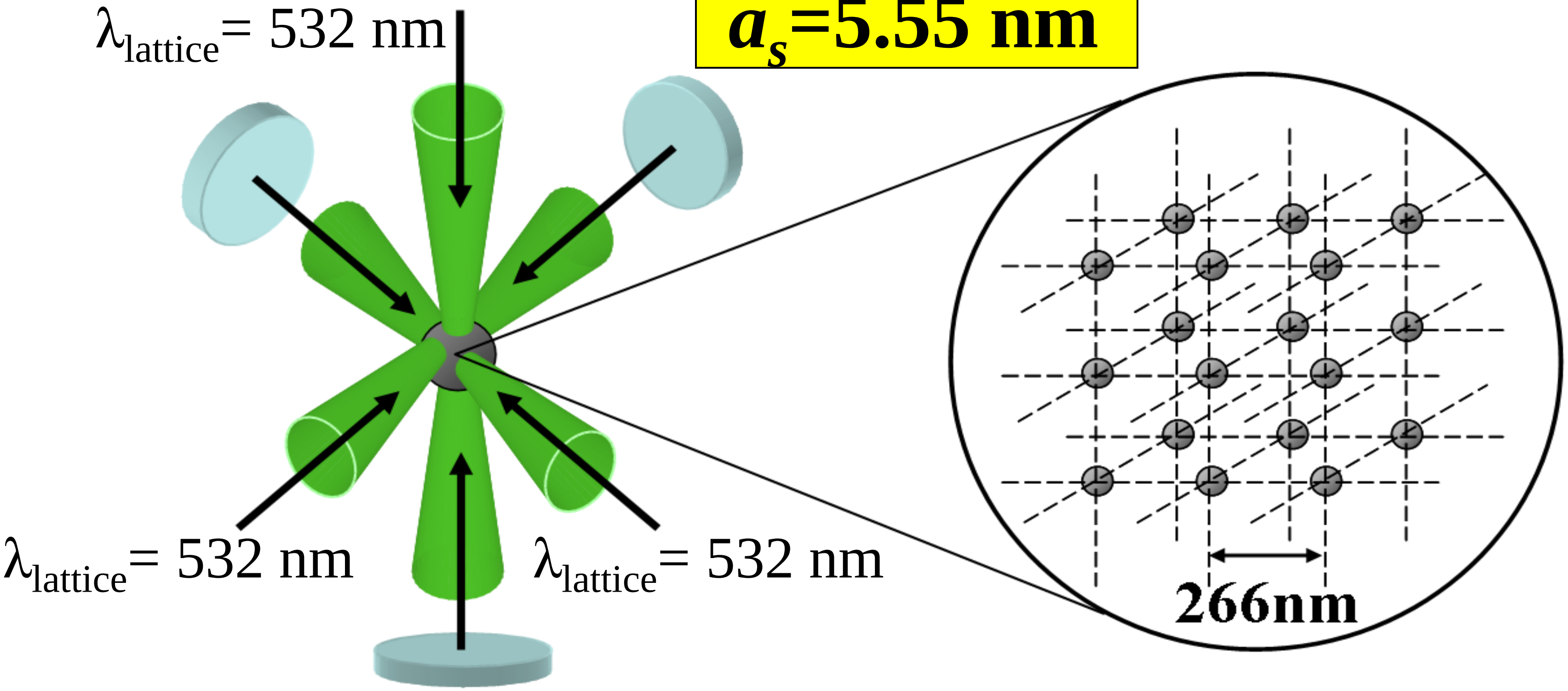
⟶ **"Spinor Superfluidity"**

[Theory: D. B. M. Dickerscheid *et al* ., Phys. Rev. A **77**, 053605 (2008)]

Boson ^{174}Yb in a 3D optical lattice

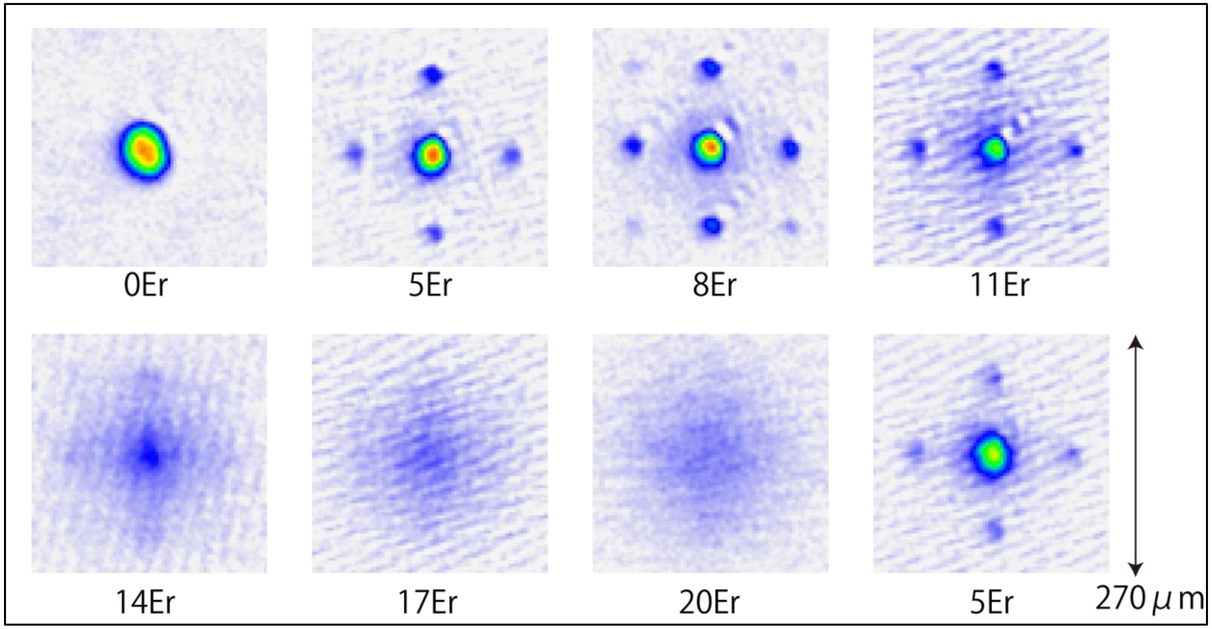
$$H = -J\sum_{<i,j>} a_i^{+} a_j + \frac{U}{2}\sum_i n_i(n_i - 1) + \sum_i \varepsilon_i n_i$$

^{174}Yb
a_s=5.55 nm



Superfluid-Mott Transition

T. Fukuhara, *et al.*, *PRA*. **79**, 041604R (2009);H. Moritz and T. Esslinger, Physics **2**,31(2009)(Viewpoint)



→ Unique Applications

K. Shibata *et al*, Appl. Phys. B **97**, 753(2009). Single-Atom Addressing by MRI

A. J. Daley *et al*, PRL. **101**, 170504(2008). Dual Lattice Configuration

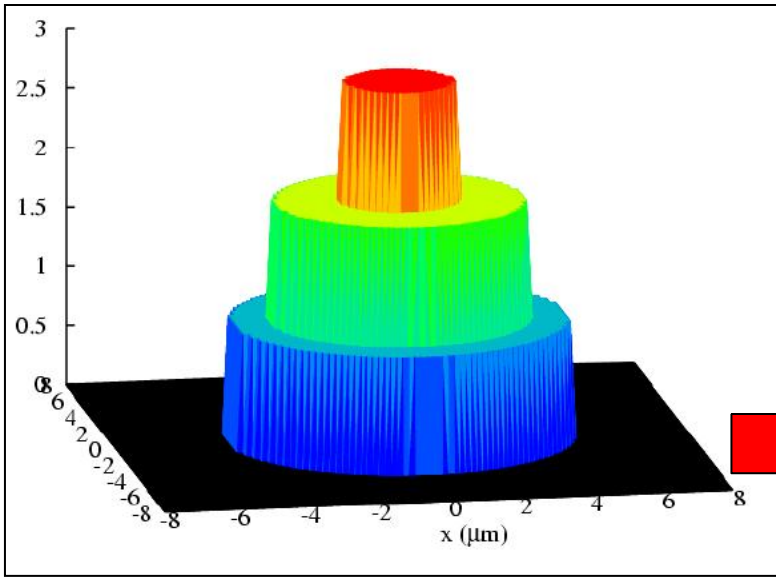
A. V. Gorshkov *et al*, PRL. **102**, 110503(2009). Few-Qubit Quantum Register

M. Hermele *et al*, PRL. **103**, 135301(2009). Chiral Spin Liquid

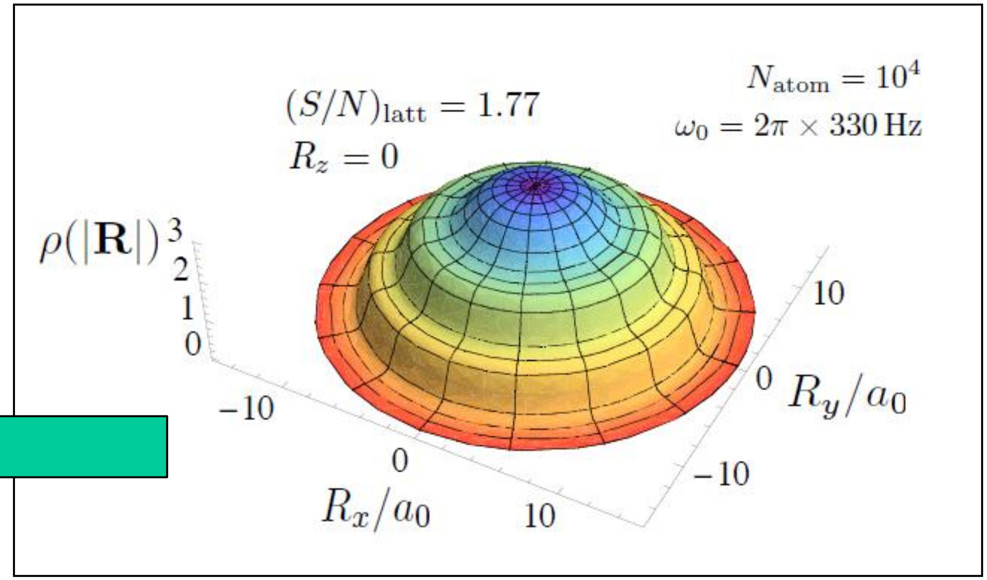
F. Gerbier and J. Dalibard, New J. Physics **12**, 033007(2010). Gauge fields

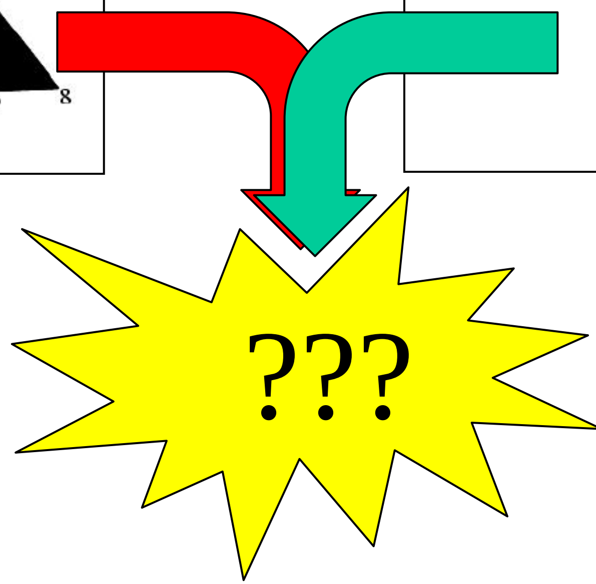
Strongly Interacting Two Different Mott Insulators

Bosonic MI



Fermionic MI





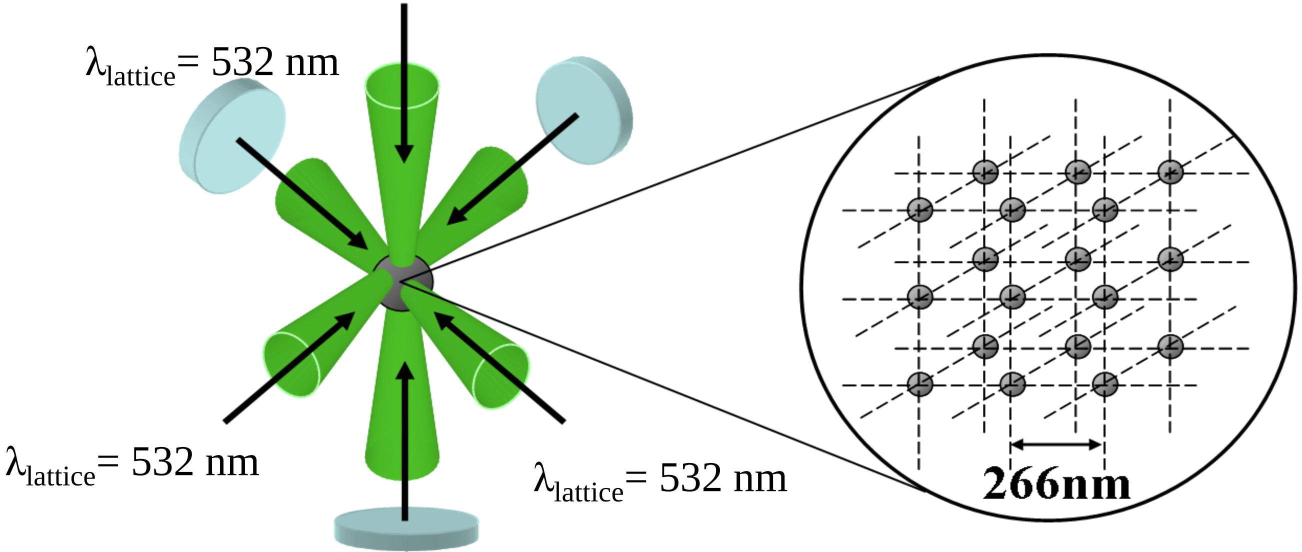
Bose-Fermi Mixture in a 3D optical lattice

- Repulsive Interaction: a_{BF} = +7.3 nm

^{174}Yb(Boson) +^{173}Yb(Fermion):
a_{BB} = +5.6 nm a_{FF} = +10.6 nm

- Attractive Interaction: a_{BF} = -4.3 nm

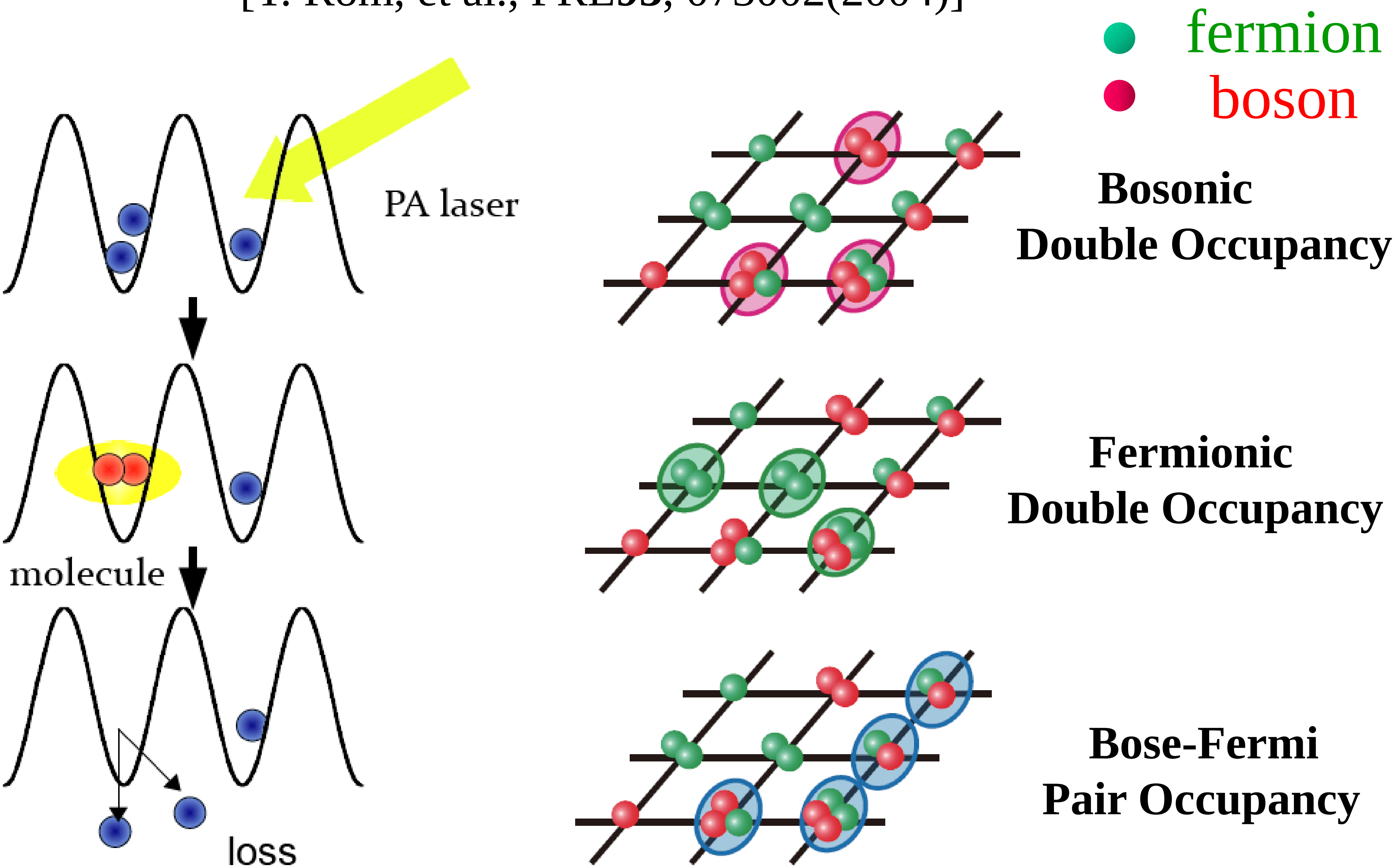
^{170}Yb(Boson) +^{173}Yb(Fermion):
a_{BB} = +3.4 nm a_{FF} = +10.6 nm



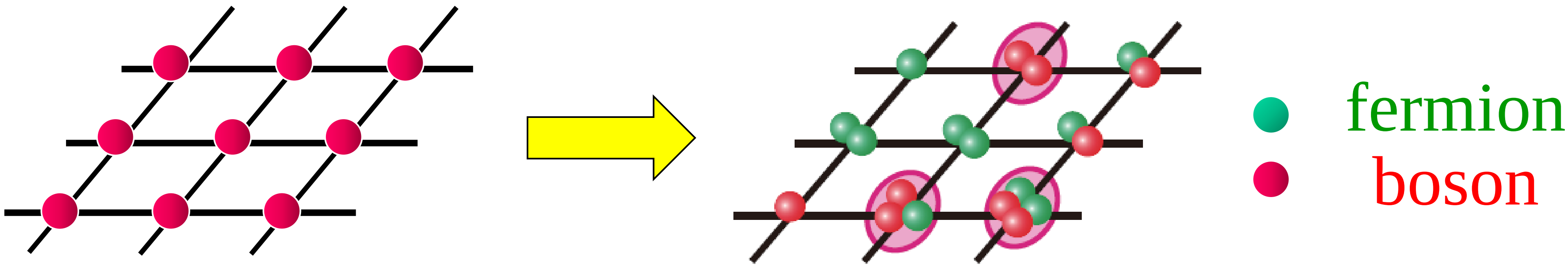
$$V_B \sim V_F$$
$$\omega_B \sim \omega_F$$
$$t_B \sim t_F$$
$$\Delta z_B \sim \Delta z_F$$

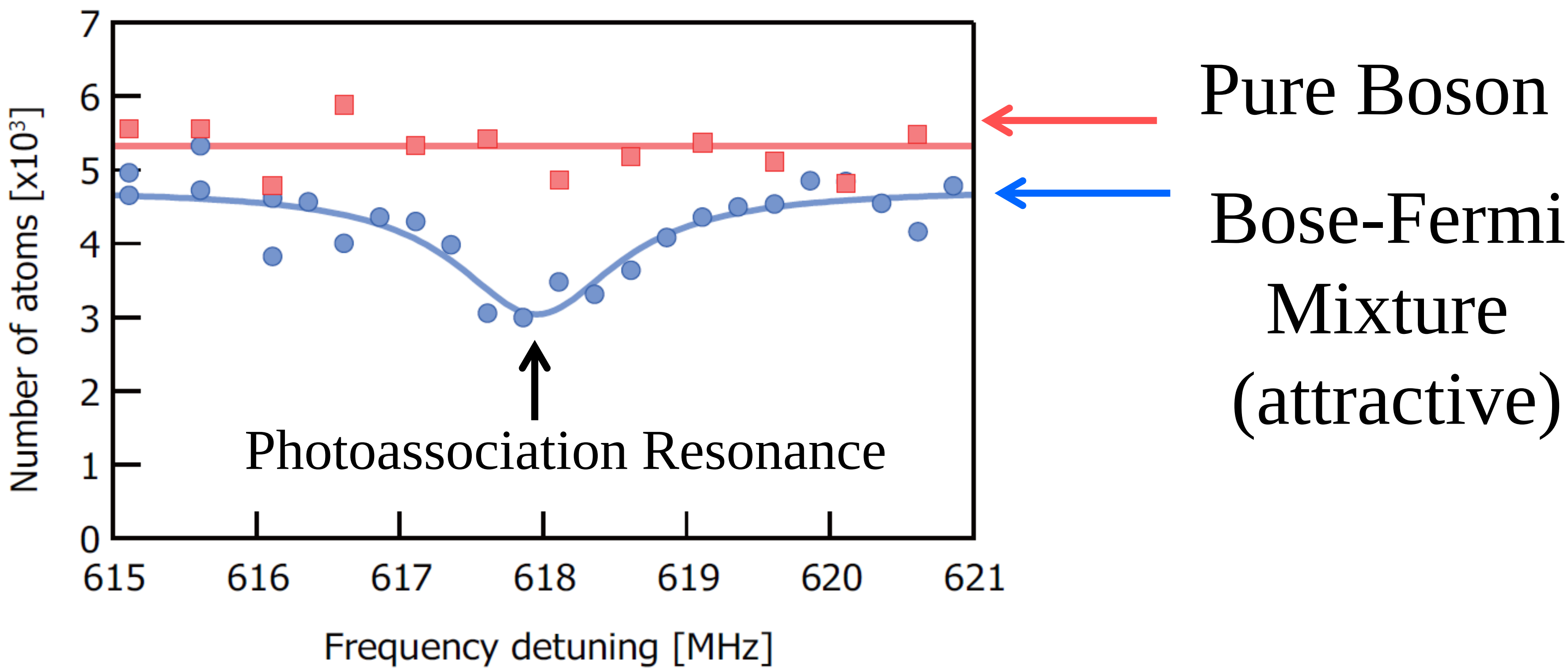
Measurement of Site Occupancy by Photoassociation

[T. Rom, et al., PRL**93**, 073002(2004)]



Example:
Fermion-Induced Bosonic Double Occupancy



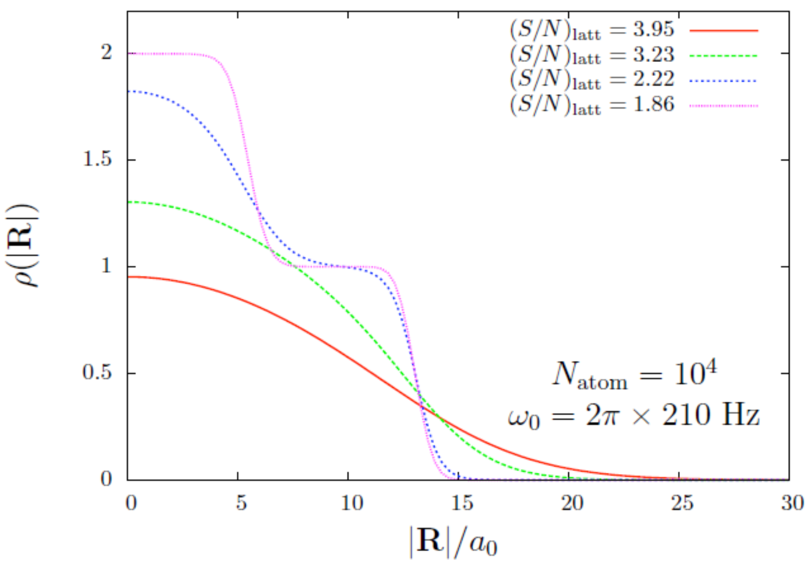


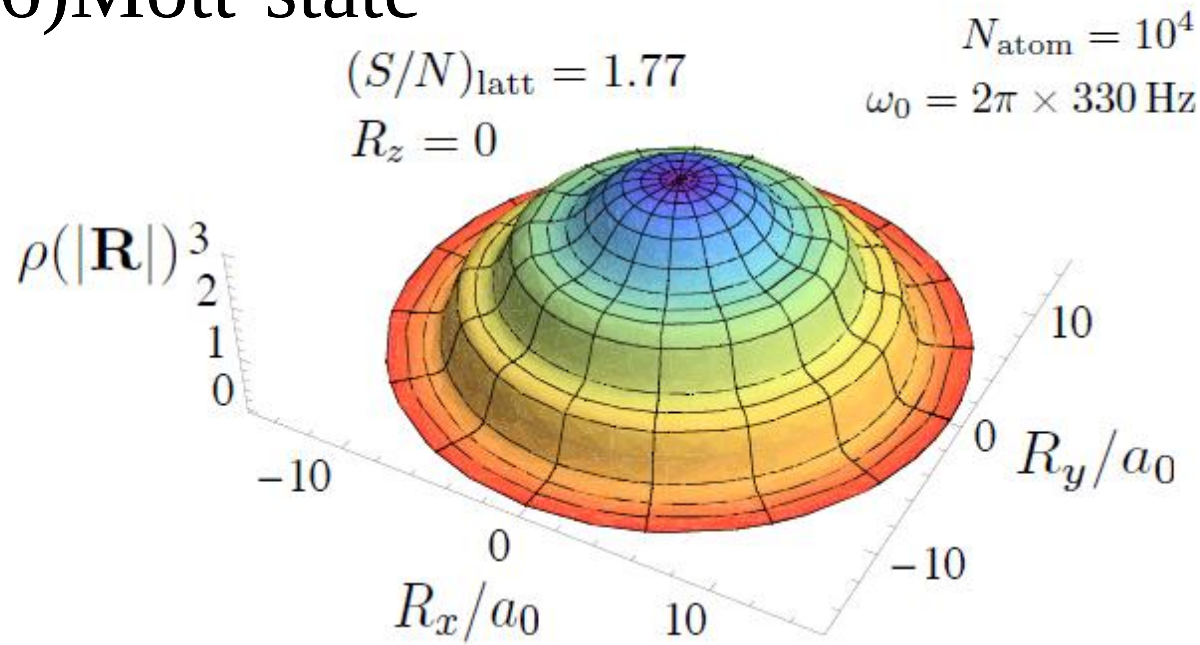
Fermion (^{173}Yb) in a 3D optical lattice

^{173}Yb(I=5/2)
a_s=10.5 nm

$$H = -t_F \sum_{<i,j>} c_i^{+} c_j + U_{FF} \sum_{i, m_F \neq m_F'} n_{m_F,i} n_{m_F',i}$$

SU(6)Mott-state

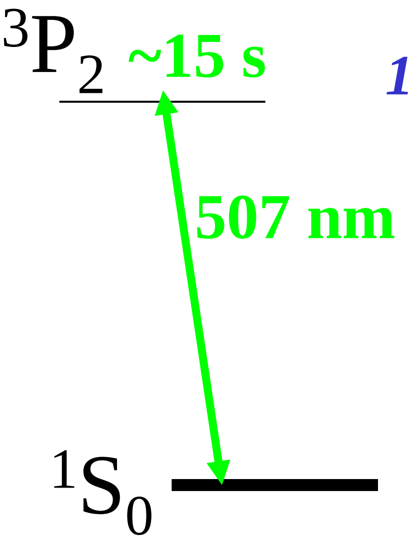




[M. A. Cazalilla, A. F. Ho, M. Ueda., N. J. Phys**11**, 103033(2009),]

Single Site Addressing: Optical Magnetic Resonance Imaging (MRI)

[K. Shibata *et al.*, App. Phys. B **97**, 753(2009)]



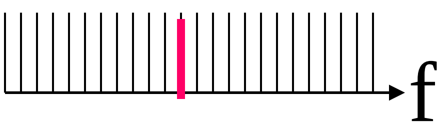
$^1S_0$-$^3P_2$: Optical absorption line of linewidth 15 mHz
$\mu = 3\mu_B$

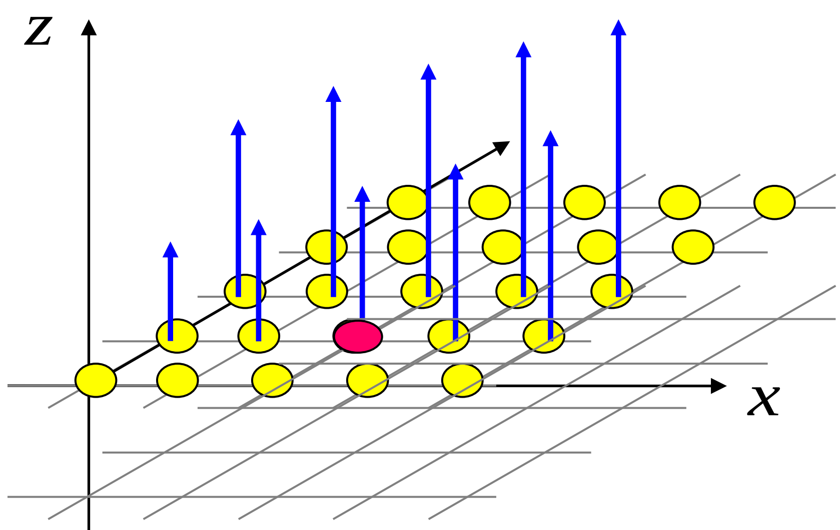
Magnetic field gradient $10G/cm$

Spectral Resolution $1kHz$

⟶ *Spatial resolution: 250 nm*

"Optical Spectrum of $^1S_0$-$^3P_2$ transition"



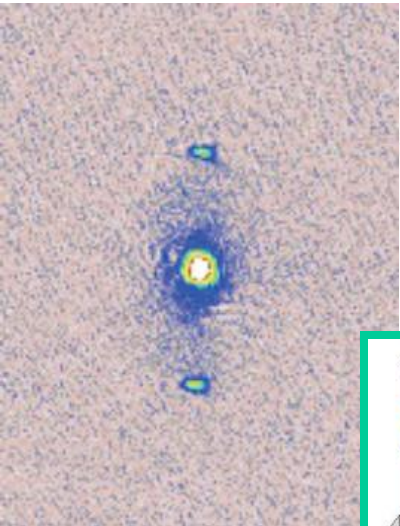


⇢ Nagaoka-ferro

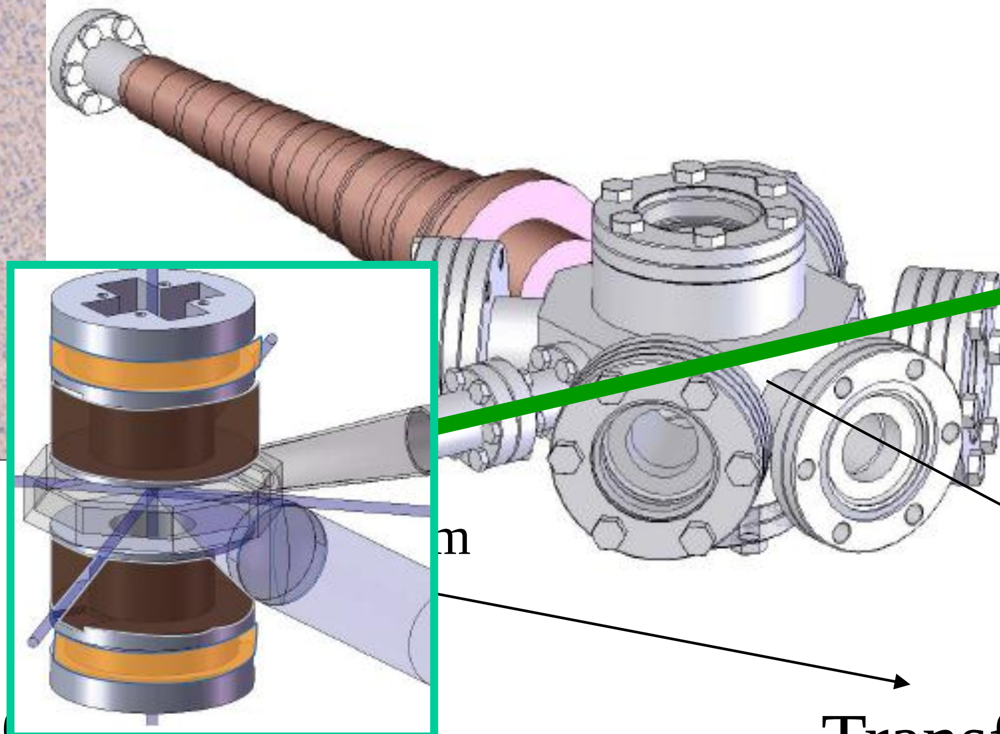
⇢ Quantum Computation

Cold Atoms in a Thin Glass Cell

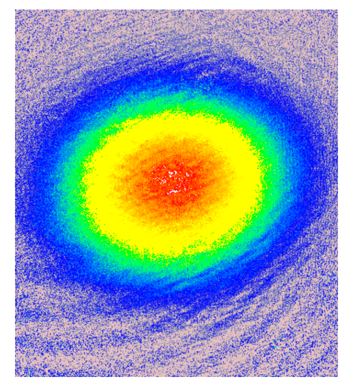
1D lattice



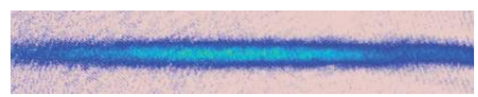
Optical Tweezer



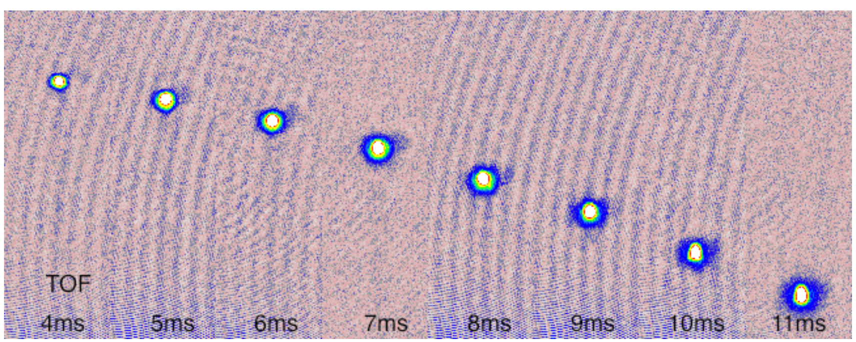
MOT



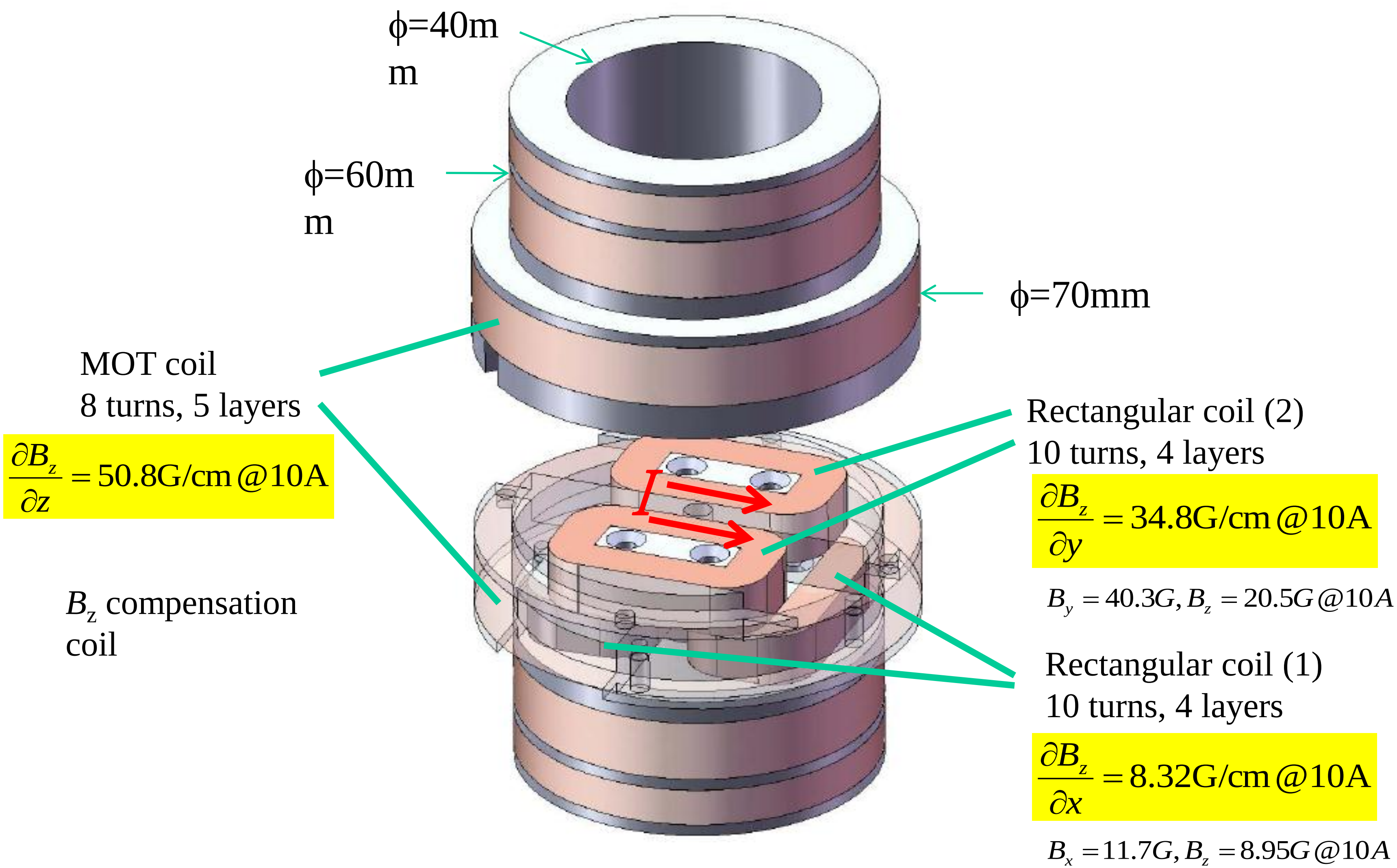
Transfered



BEC Formation



Towards Single Site Addressing in 2DLattice



Lattice-Spin Model Using Polar Molecule

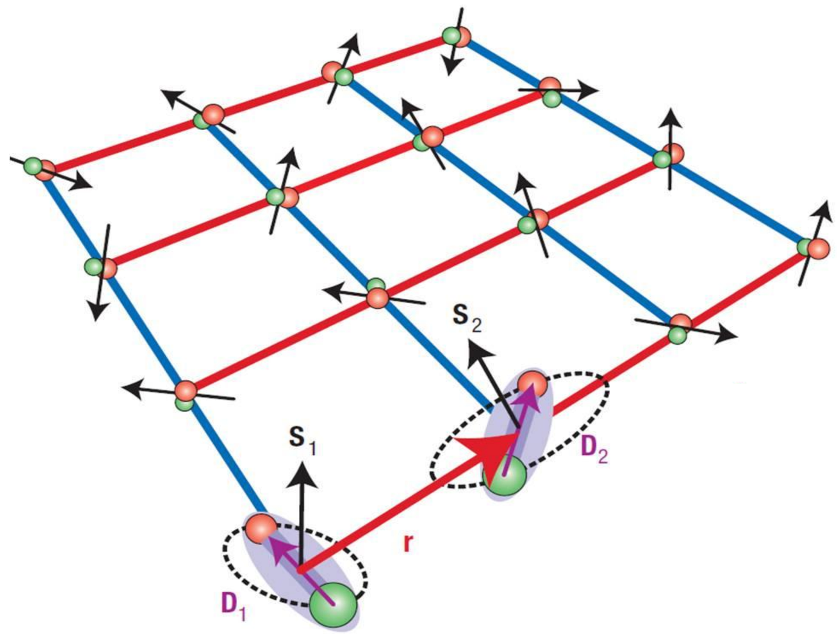
[A. Micheli, *et al.*, Nature Physics 2,341 (2006)]

Paramagnetic molecule $^2\Sigma_{1/2}$

$$H_m = B\mathbf{N}^2 + \gamma \mathbf{N}\cdot\mathbf{S}$$

N:rotation **S**:electron spin

$$H_{eff} = \frac{\hbar\Omega}{8}\sum_{\alpha,\beta=0}^{3}\sigma_1^{\alpha}A_{\alpha,\beta}\sigma_2^{\beta}$$



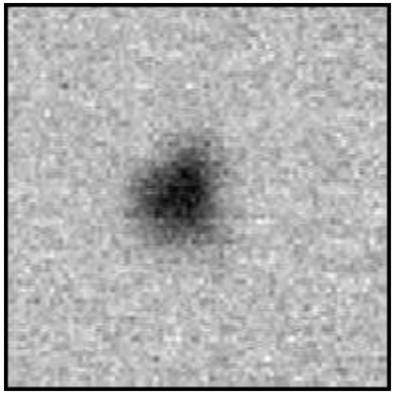
Current Status of YbLi Experiments

The First Yb-Li Simultaneous MOT [M. Okano *et al.*, Appl. Phys.B**98**,2(2009)]

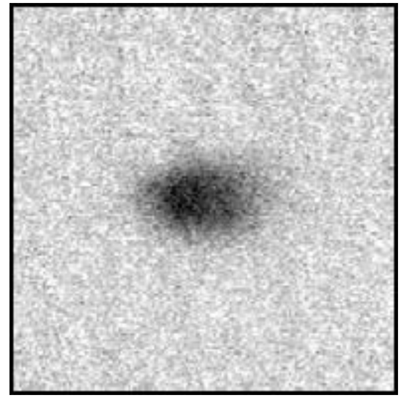
^{6}Li

N=1.5 × 10^{8}

T~ 280 μK



^{174}Yb



N=1.4 × 10^{7}

T=60 μK

Summary

Quantum Simulation of Hubbard Model Using Optical Lattice

Review of Experiments using Alkali Atoms

Superfluid-Mott Insulator Transition

Formation of Fermi Mott Insulator

Bose-Fermi and Bose-Bose Mixtures

Single Site Resolved Observation of SF-Mott Insulator Transition

Report of Experiments using Yb Atoms

Superfluid-Mott Insulator Transition

SU(6) Fermi Mott Insulator

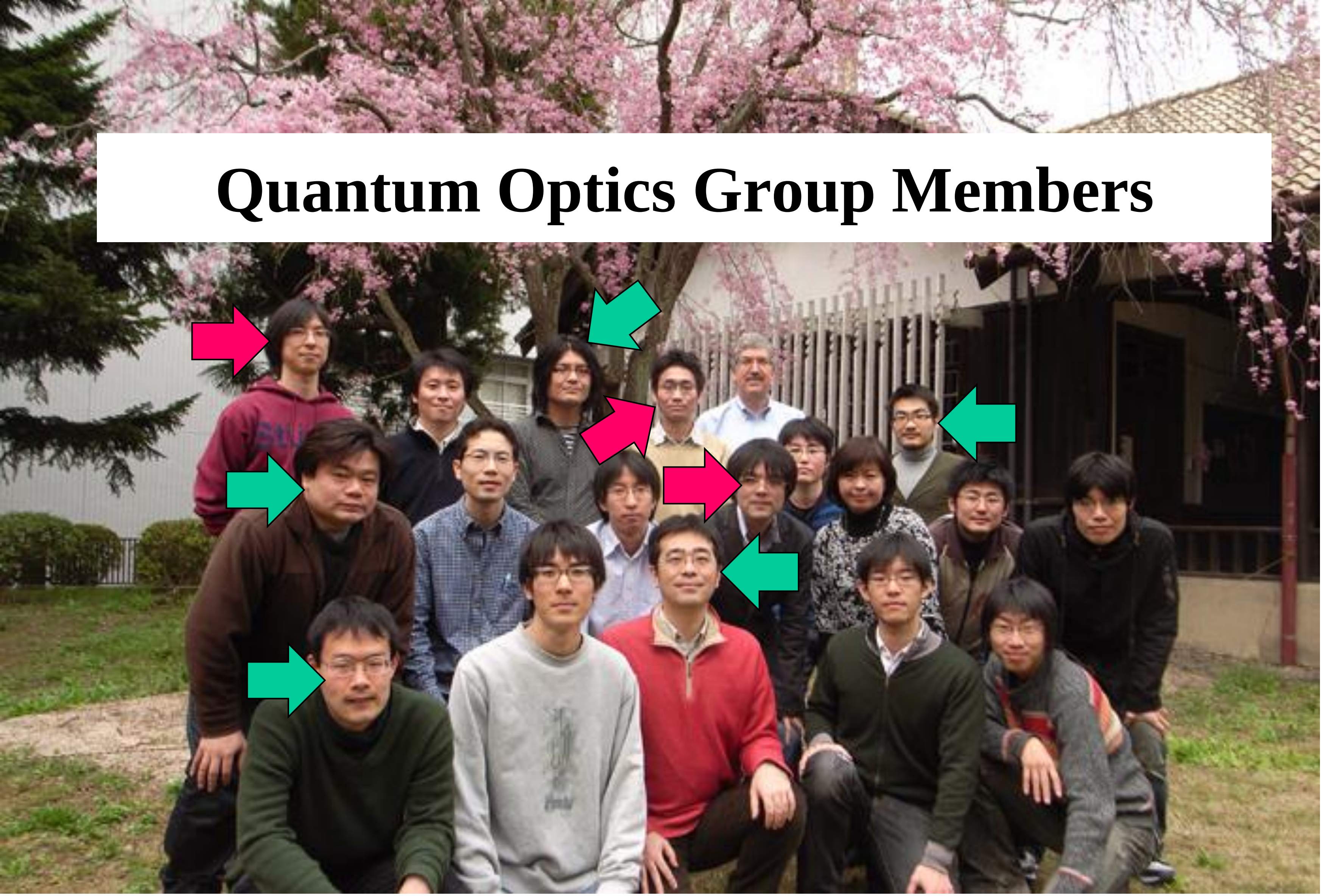
Strongly Interacting Bose-Fermi Mott Insulators

Nano-Scale Modulation of Interatomic Interaction in Bose Condenstate

Towards Single Site Addressing Using $^3P_2$ State

Towards Quantum Simulation Lattice Spin Model by YbLi polar Molecule

Quantum Optics Group Members



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Y. Yoshkawa, S. Uetake, S. Sugawa, S. Taie, H. Hara, H. Shimizu, R. Yamamoto, I. Takahashi
R. Namiki , H. Yamada, Y. Takasu, R. Murakami, S. Imai, (S. Tanaka. N. Hamaguchi)