Microwave quantum optics in superconducting quantum circuits

Yasunobu Nakamura
RIKEN Advanced Science Institute
NEC Green Innovation Research Laboratories
E-mail: yasunobu@ce.jp.nec.com

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Farruh Abdumalikov
Yuri Pashkin
Fumiki Yoshihara
Kunihiro Inomata
Toshiyuki Miyazaki
Tsuyoshi Yamamoto (→UCSB '09-'10)
Pierre Billangeon
Zuihui Peng
JawShen Tsai

<u>Collaborators:</u>
Alexander Zagoskin (Loughborough Univ.)
Khalil Harrabi (→King Fahed Univ., Saudi Arabia)
Jonas Bylander (MIT)
Simon Gustavsson (MIT)
Will Oliver (MIT)
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Satoshi Ishizaka (NEC→Hiroshima Univ.)

Quantum optics

Quantization of electromagnetic field
$\rightarrow$ Ensemble of harmonic oscillators or bosonic modes

Quantum properties of light

Interaction with atom(s)

Cavities

Laser, single photon source

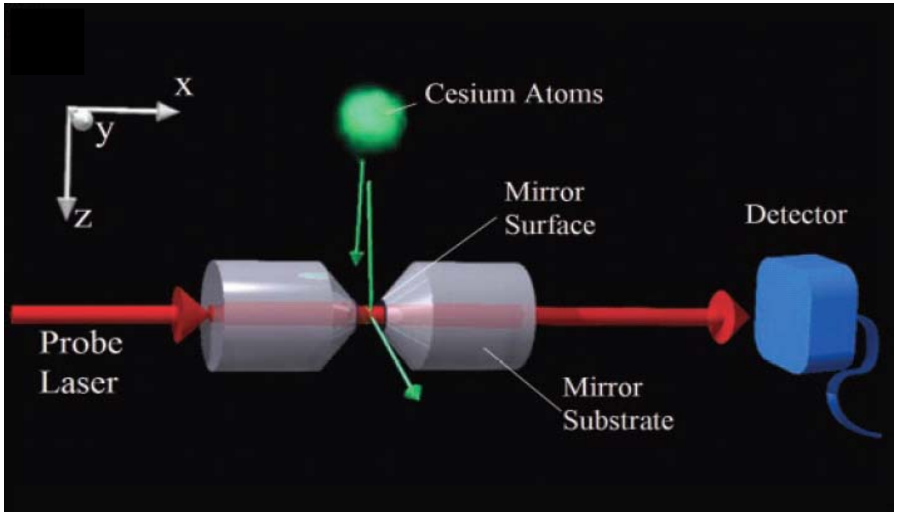
Single photon counting, homodyne detection

Optical domain and microwave domain

Optics	Microwave
Frequency 100-1000 THz Wavelength 3 μm – 300 nm	Frequency 1-10 GHz Wavelength 30 cm – 3 cm
Free space Optical fiber, low loss ~ 0.2 dB/km	Free space Waveguides Coaxial cables
Mirrors, beam splitters, etc. Cavities	Mirrors, couplers, etc. Cavities
Atom (orbital)	Atom (hyperfine), Rydberg atom
Laser	Maser Generator
photon counting homodyne detection	No photon counting existing homodyne detection

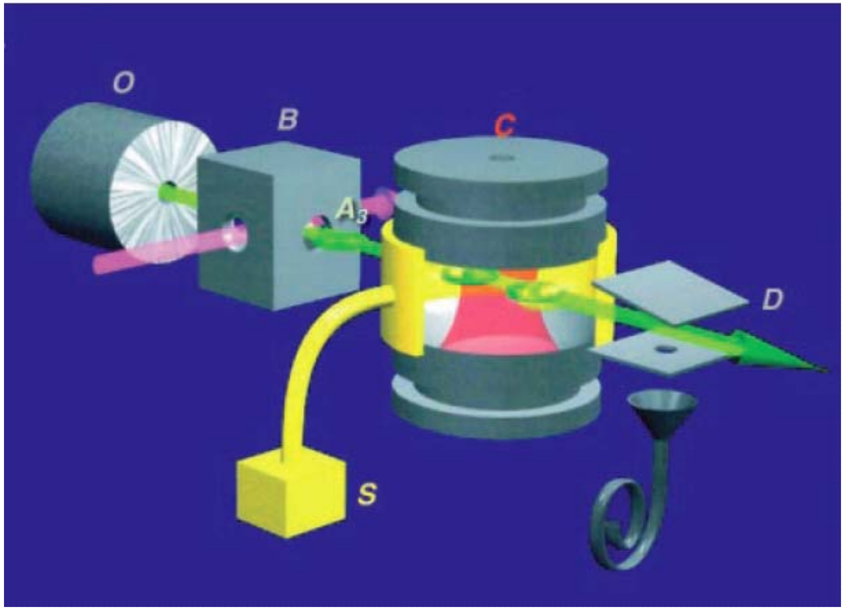
Cavity QED

Optics



Kimble group, Caltech

Microwave



Haroche group, ENS, Paris

Microwave quantum optics in circuits

Low-dissipation superconducting planar waveguide ~ 0.3 dB/km(?)
Confined electromagnetic modes in 1D

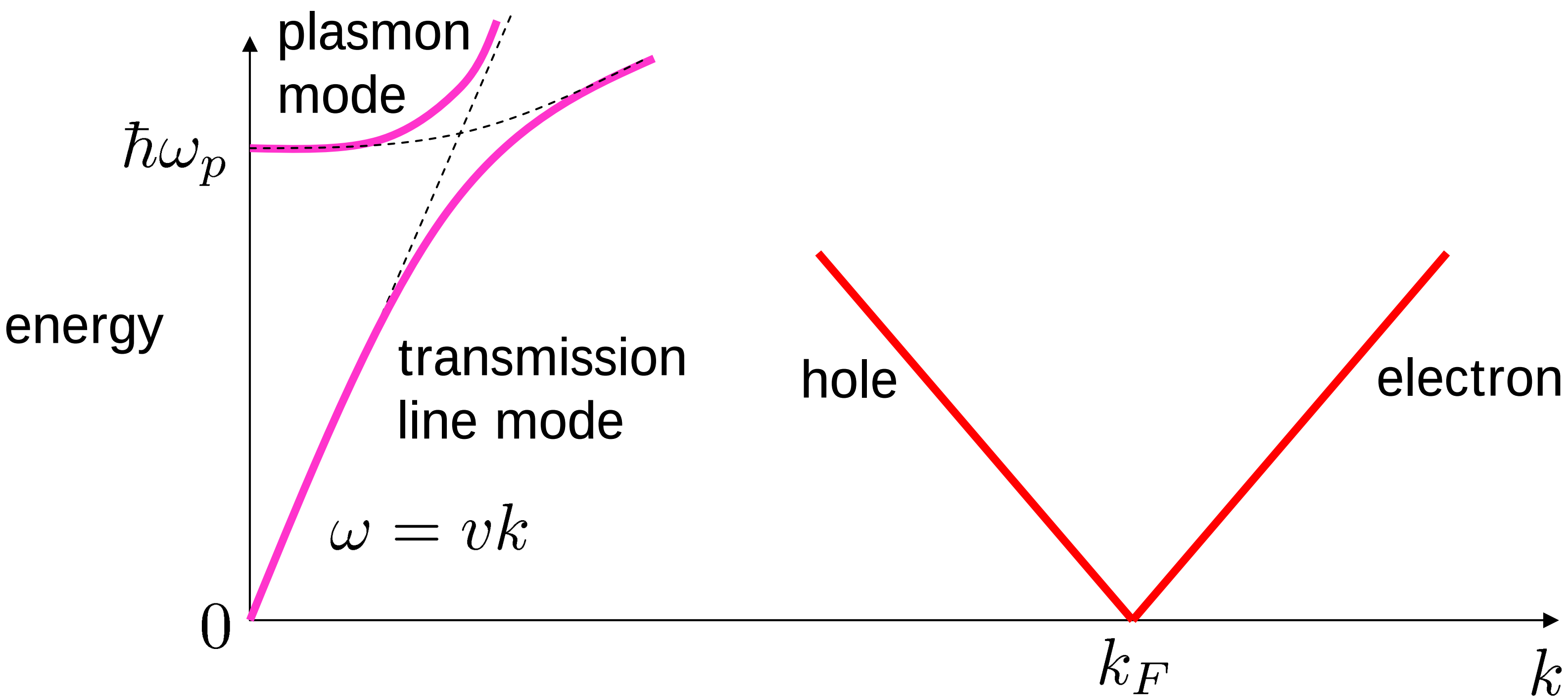
Fixed single artificial atoms with large dipole moment
at designed locations

Controllability of the parameters
In-situ (dynamical) tunability of parameters

Strong coupling

Strong nonlinearity

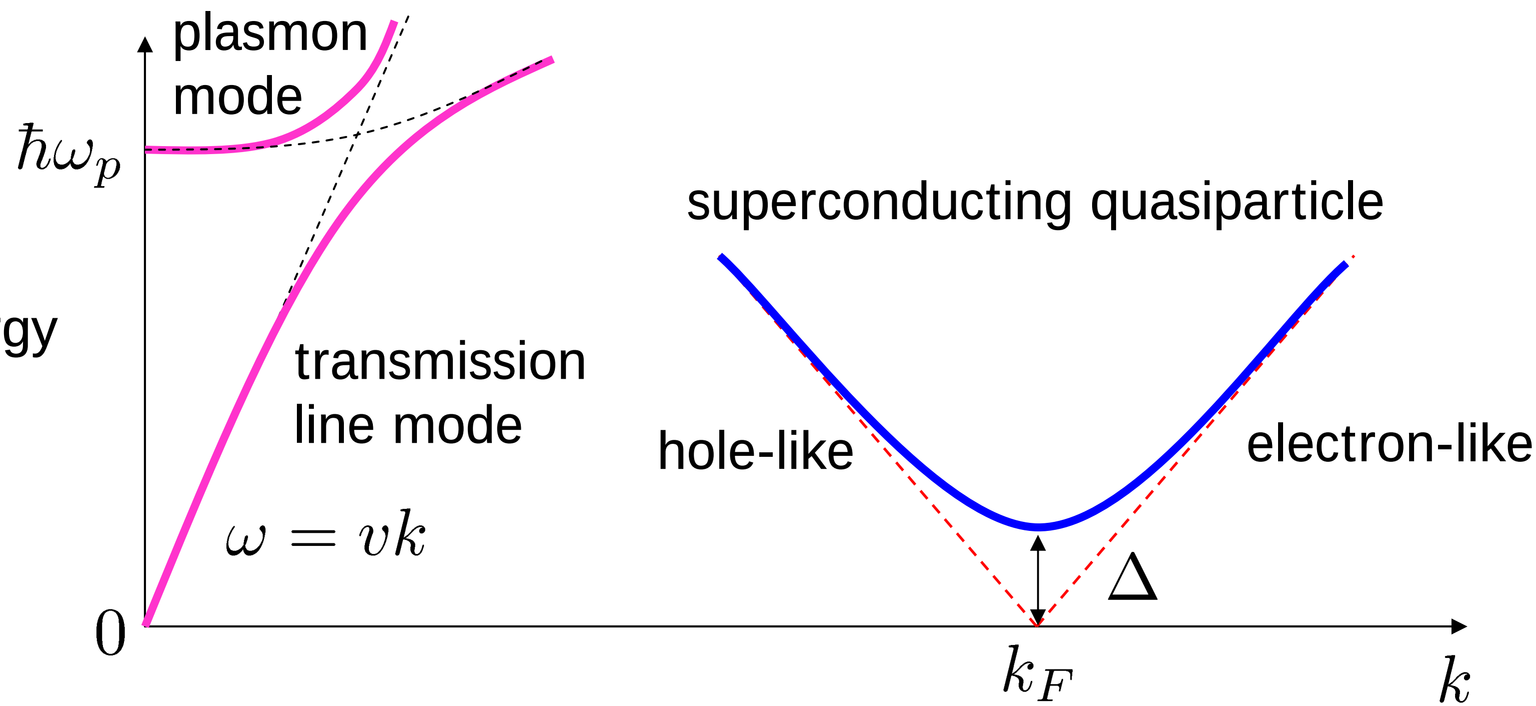
Elementary excitations in metallic electrodes



bosonic collective mode
with linear dispersion

Circuits and quantum optics

Elementary excitations in metallic electrodes



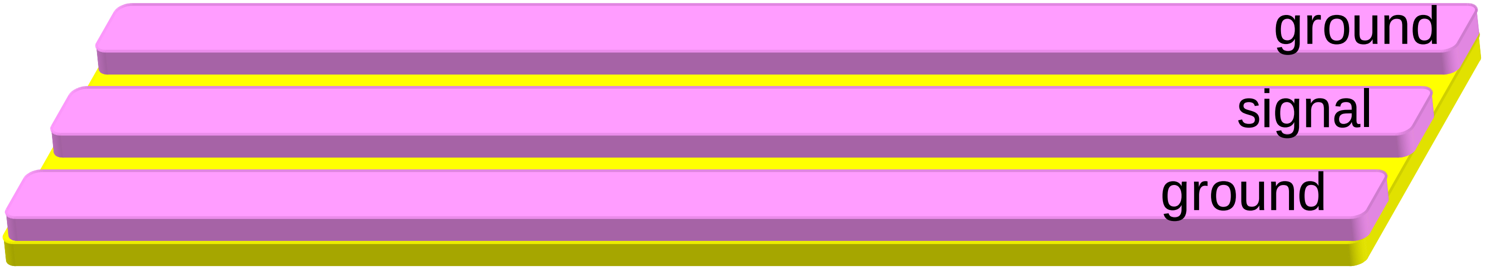
bosonic collective mode
with linear dispersion

Superconducting transmission line

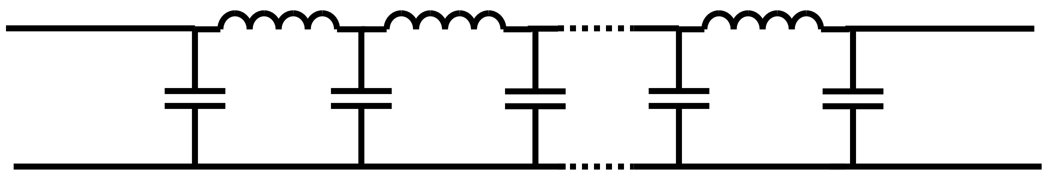
$Z = \sqrt{l/c}$

$v = 1/\sqrt{lc}$

$\omega = vk$



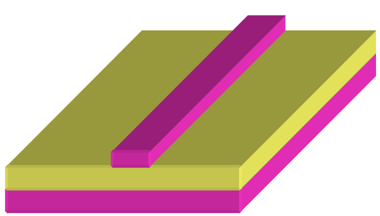
distributed element
↕
lumped element



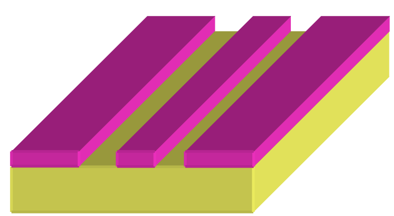
- small dissipation for $\hbar\omega_r, k_B T \ll \Delta$
- 1D transmission mode
- Photon life time ~ 100 μs, 10 km (?)

Variety of transmission lines:

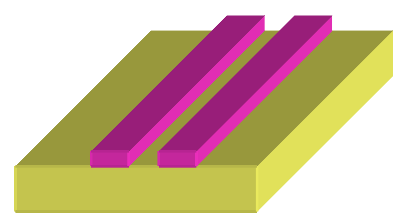
microstrip line



coplanar waveguide

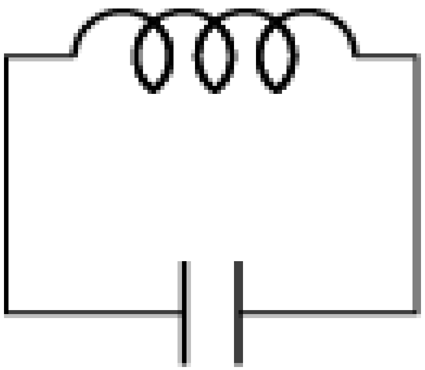


slot line



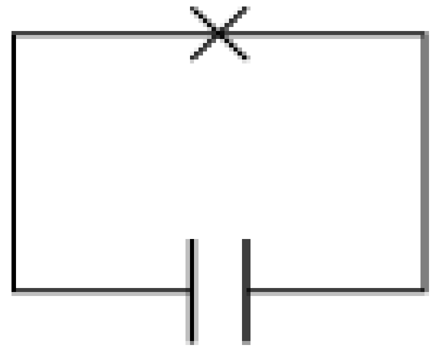
Superconducting qubit – nonlinear resonator

LC resonator

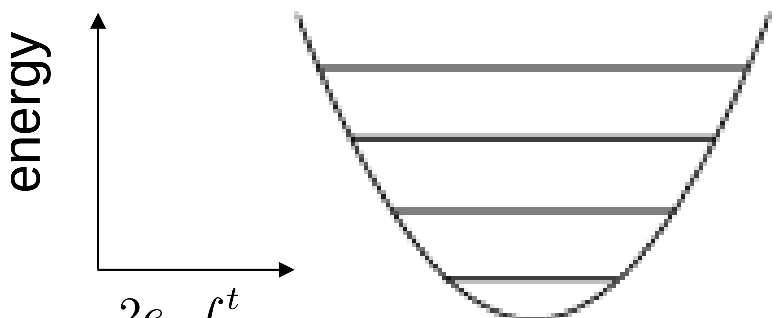


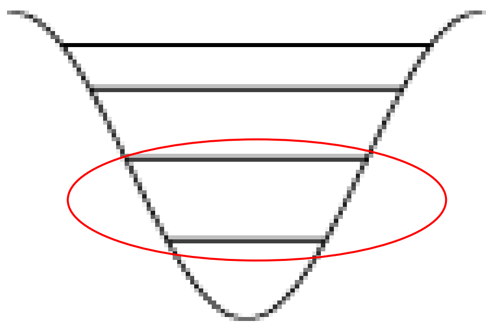
Josephson junction resonator

Josephson junction = nonlinear inductor



anharmonicity ⇒ effective two-level system





$$\theta(t) = \frac{2e}{\hbar}\int_{-\infty}^{t} dt' V(t')$$

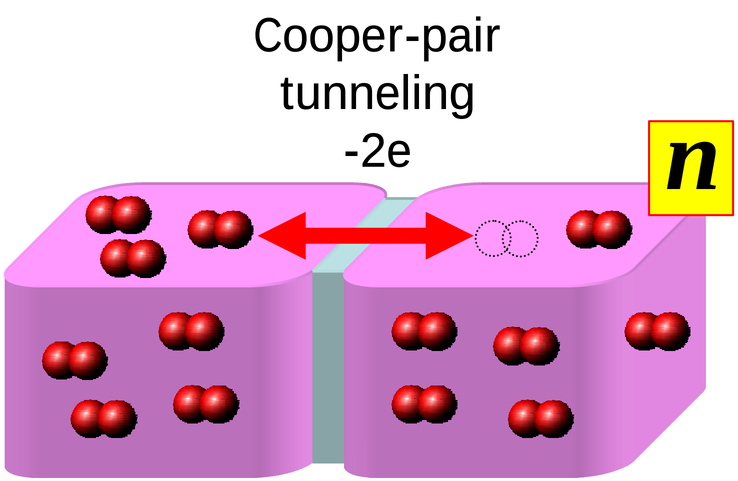
inductive energy = confinement potential
charging energy = kinetic energy ⇒ quantized states

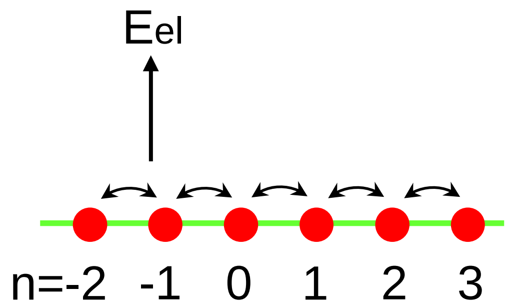
Josephson junction — nonlinear inductance

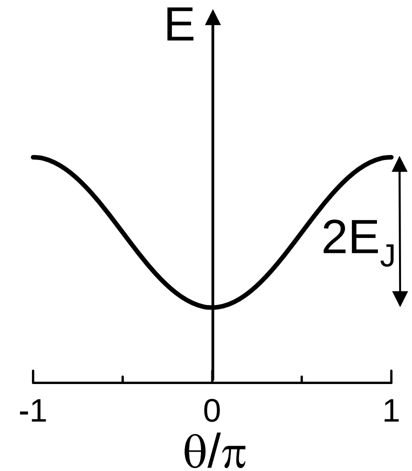
B.D. Josephson 1962

number n ⇔ phase difference θ

$$[n, \theta] = -i$$





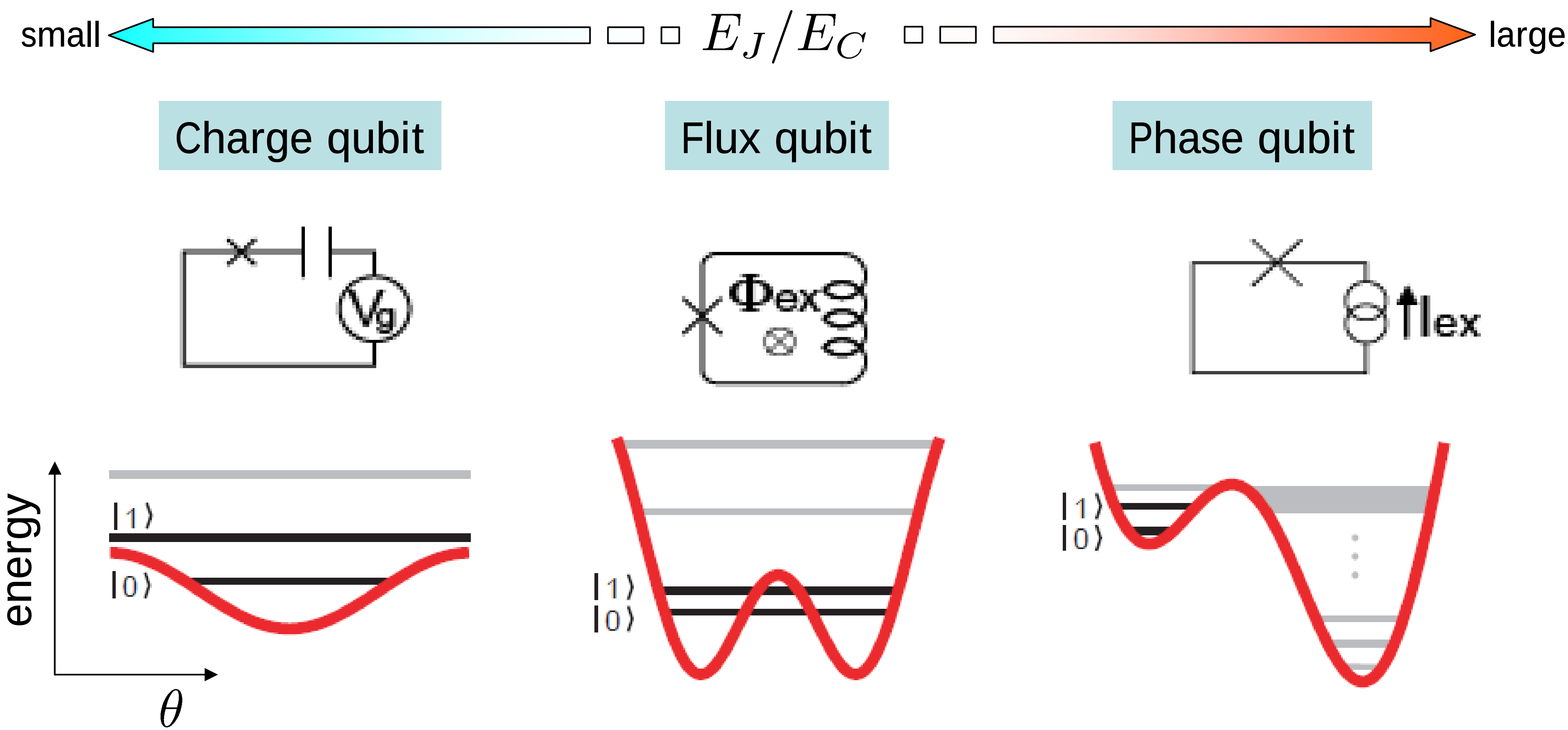


$$H = -\frac{E_J}{2}\sum_n \{|n\rangle\langle n+1| + |n+1\rangle\langle n|\} = -\int_0^{2\pi} d\theta E_J \cos\theta |\theta\rangle\langle\theta|$$

Tight-binding model in 1d lattice ⇒ Bloch band

$$|\theta\rangle = \sum_n e^{in\theta}|n\rangle$$

Superconducting qubits – artificial atoms in electric circuit



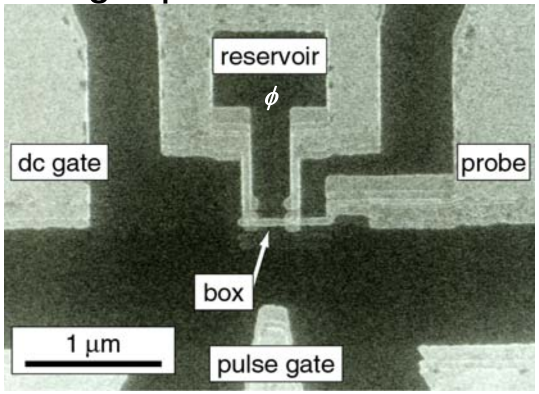
Josephson energy E_J = confinement potential
charging energy E_C = kinetic energy ⇒ quantized states

typical qubit energy $E_{01} \sim 10\,\mathrm{GHz} \sim 0.5\,\mathrm{K}$

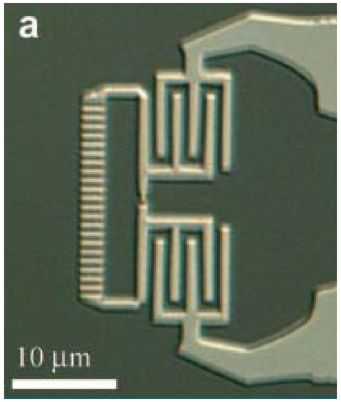
typical experimental temperature $T \sim 0.02\,\mathrm{K}$

Superconducting qubits – macroscopic artificial atom in circuits

charge qubit/NEC $E_J/E_C \sim 0.3$

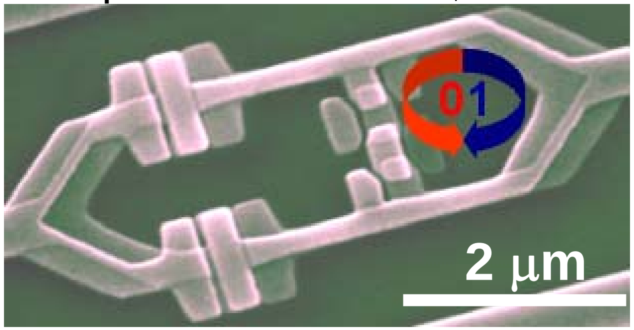


"fluxonium"/Yale

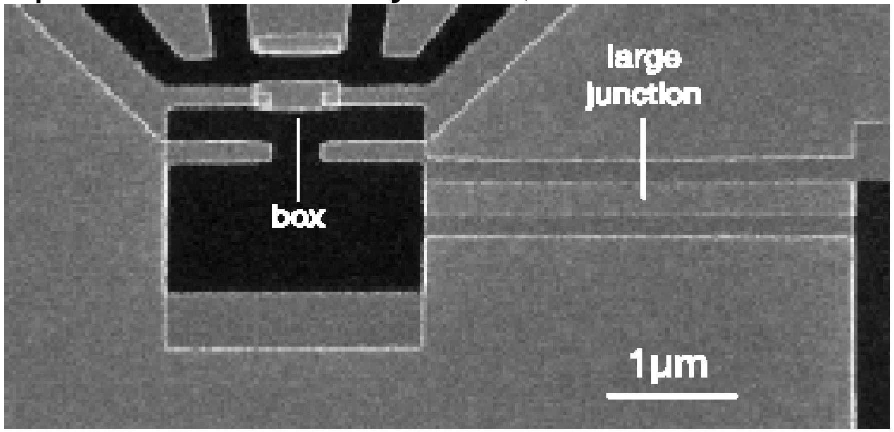


$E_J/E_C \sim 3$

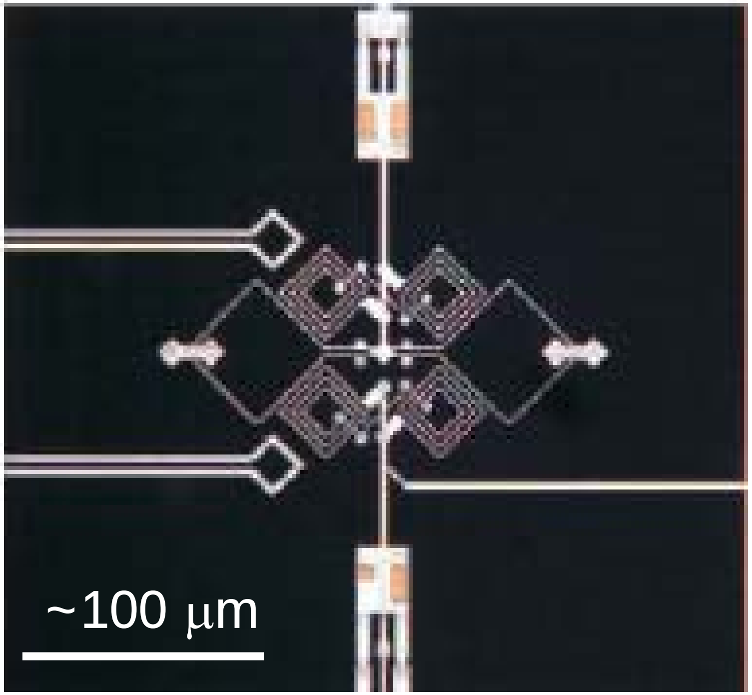
flux qubit/Delft $E_J/E_C \sim 40$



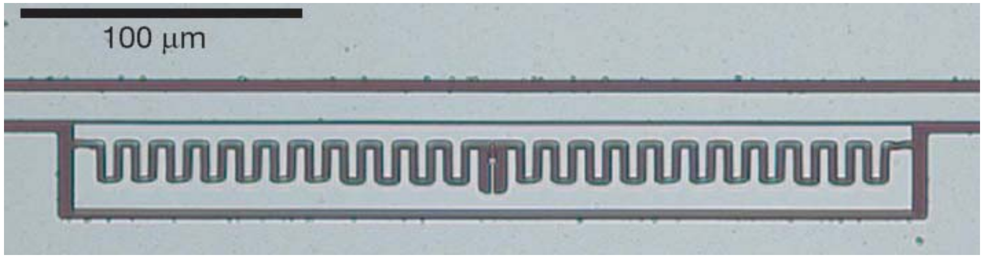
"quantronium"/Saclay $E_J/E_C \sim 5$



phase qubit/NIST/UCSB $E_J/E_C \sim 10^4$



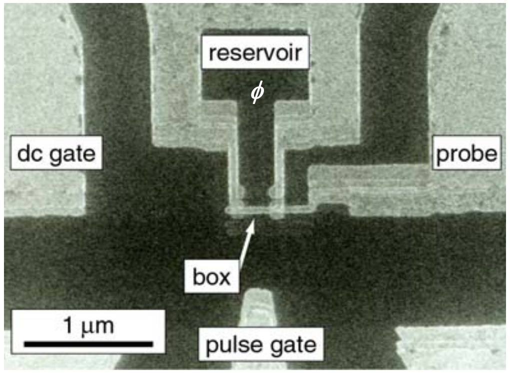
"transmon"/Yale $E_J/E_C \sim 50$

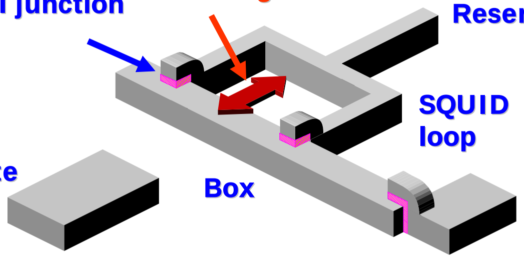


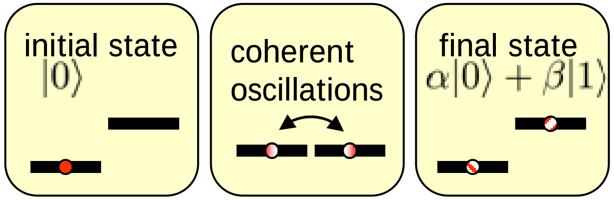
Charge qubit, 1998

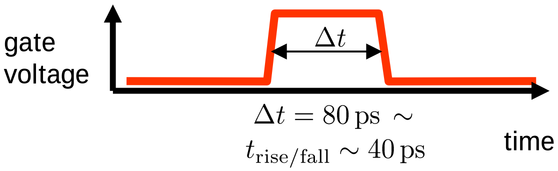
Coherent control of macroscopic quantum states in a single-Cooper-pair box

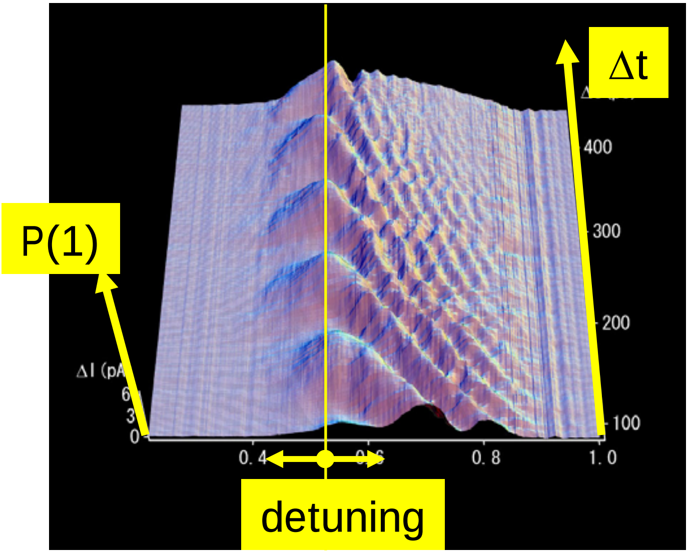
Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*









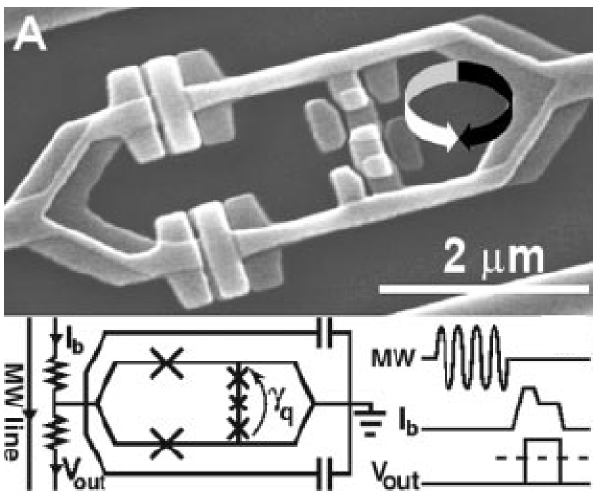


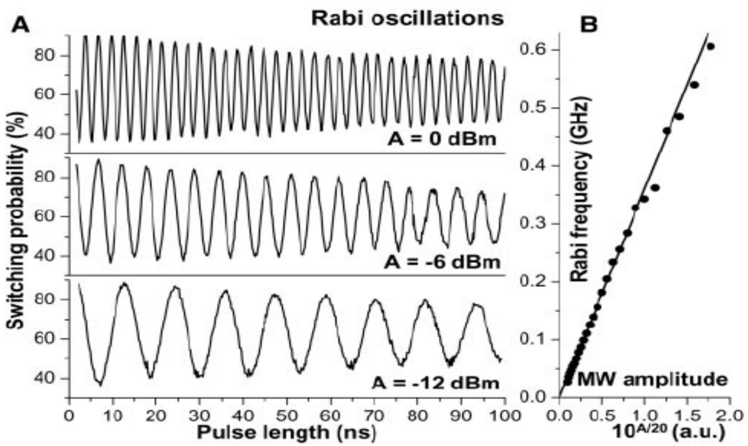
Nature 398, 786 (1999).

Flux qubit, 2002

Coherent Quantum Dynamics of a Superconducting Flux Qubit

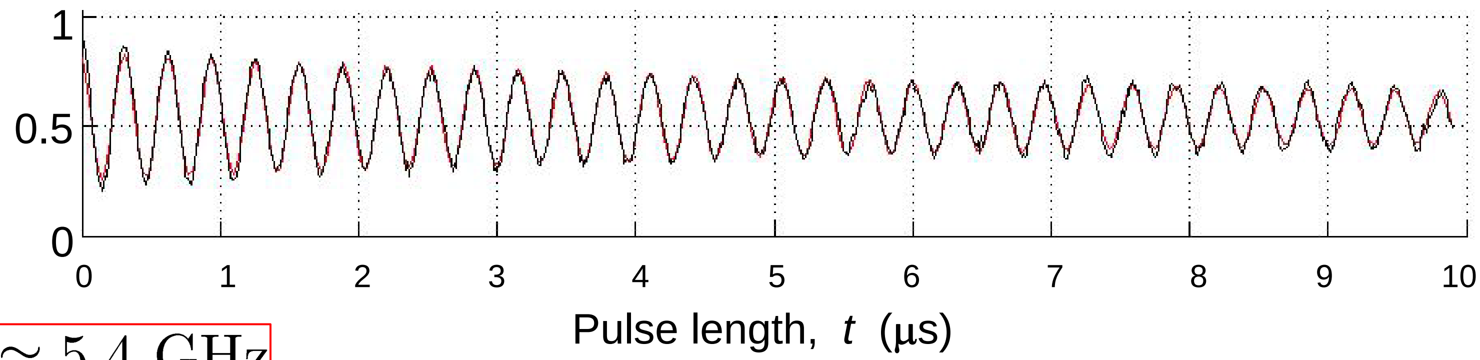
I. Chiorescu,[1*] Y. Nakamura,[1,2] C. J. P. M. Harmans,[1] J. E. Mooij[1]



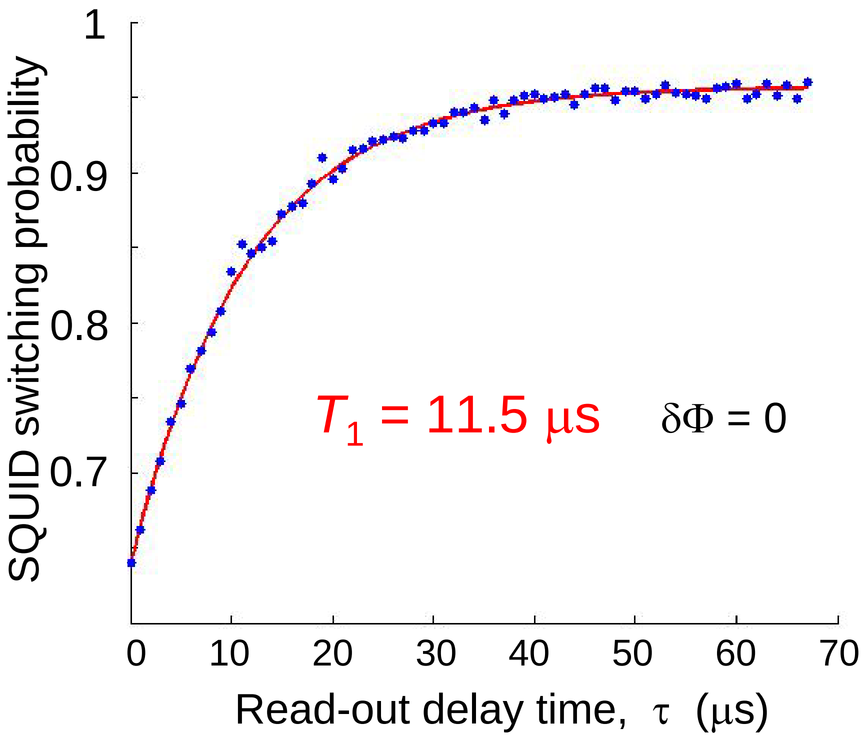


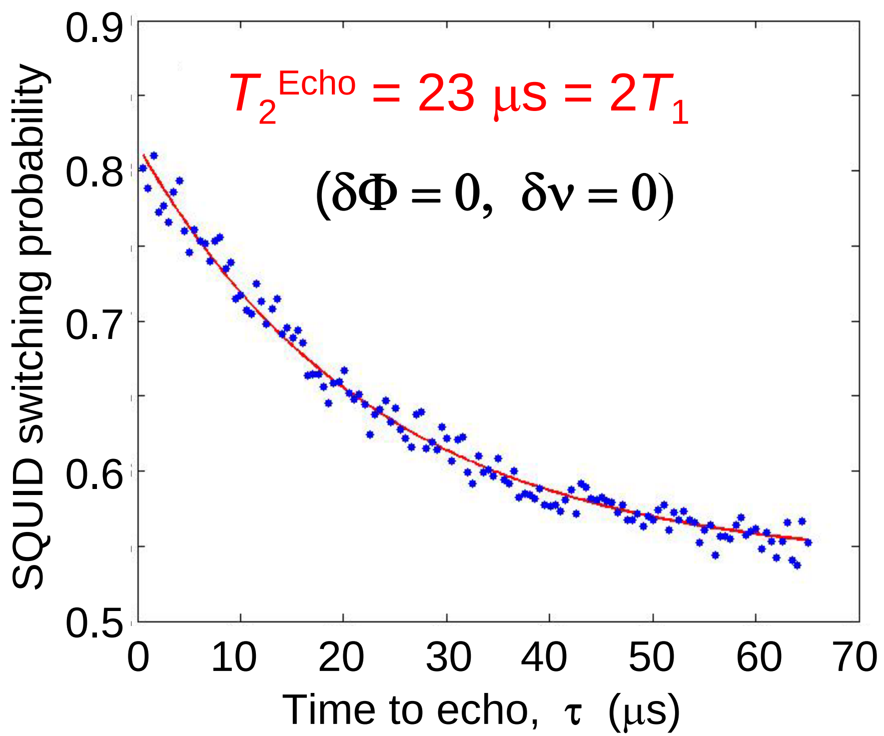
Science 299, 1869 (2003) Delft

How long could the qubit lifetime be?



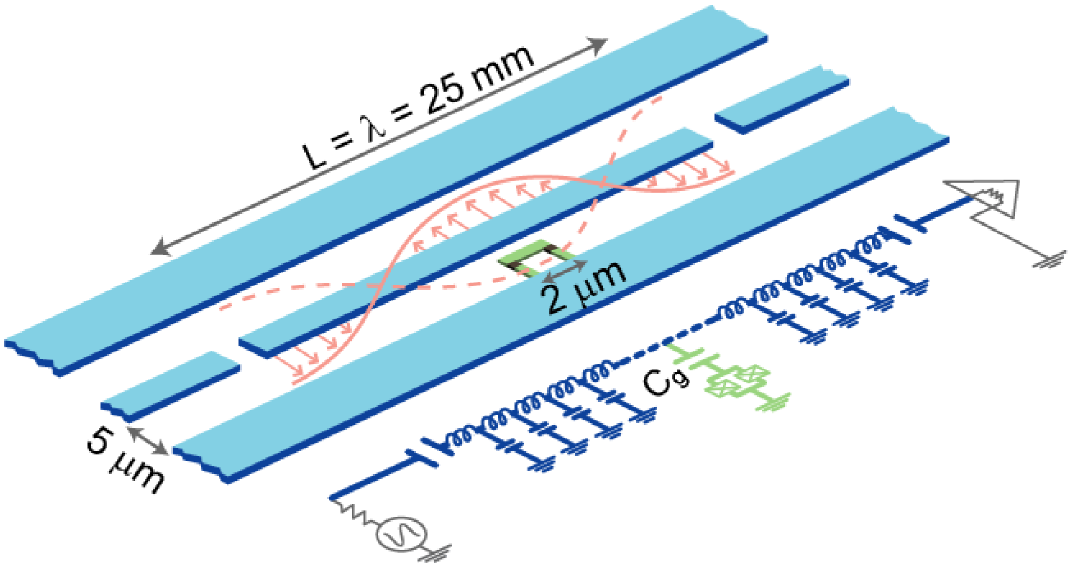
$\omega_{01}/2\pi \approx 5.4\ \mathrm{GHz}$

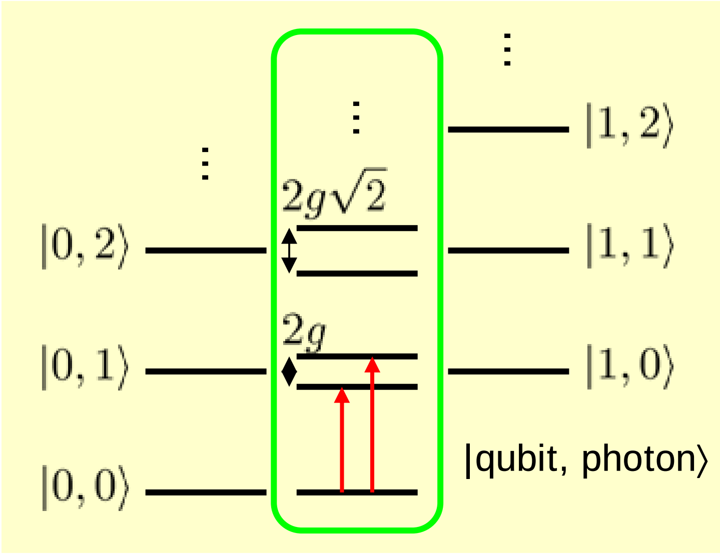




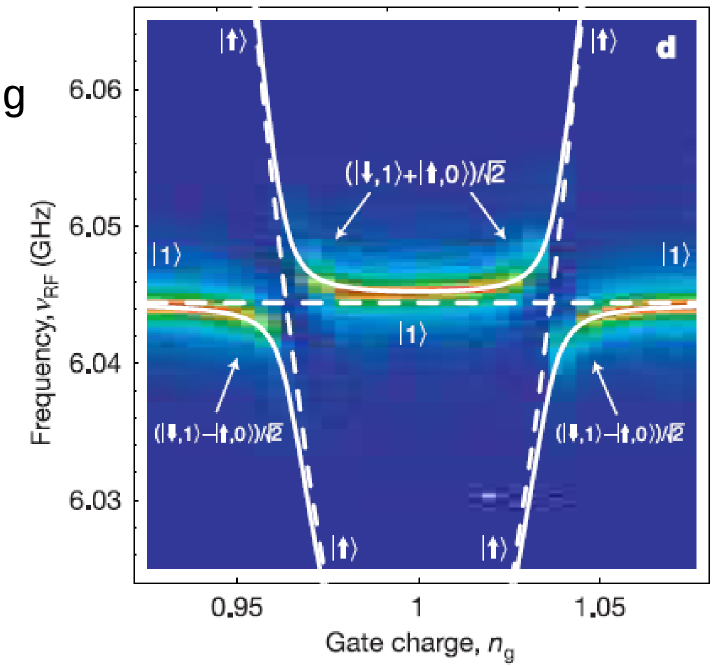
Jonas Bylander, Simon Gustavsson, Will Oliver (MIT)

Circuit quantum electrodynamics (circuit QED)





vacuum Rabi splitting



Jaynes-Cummings Hamiltonian:

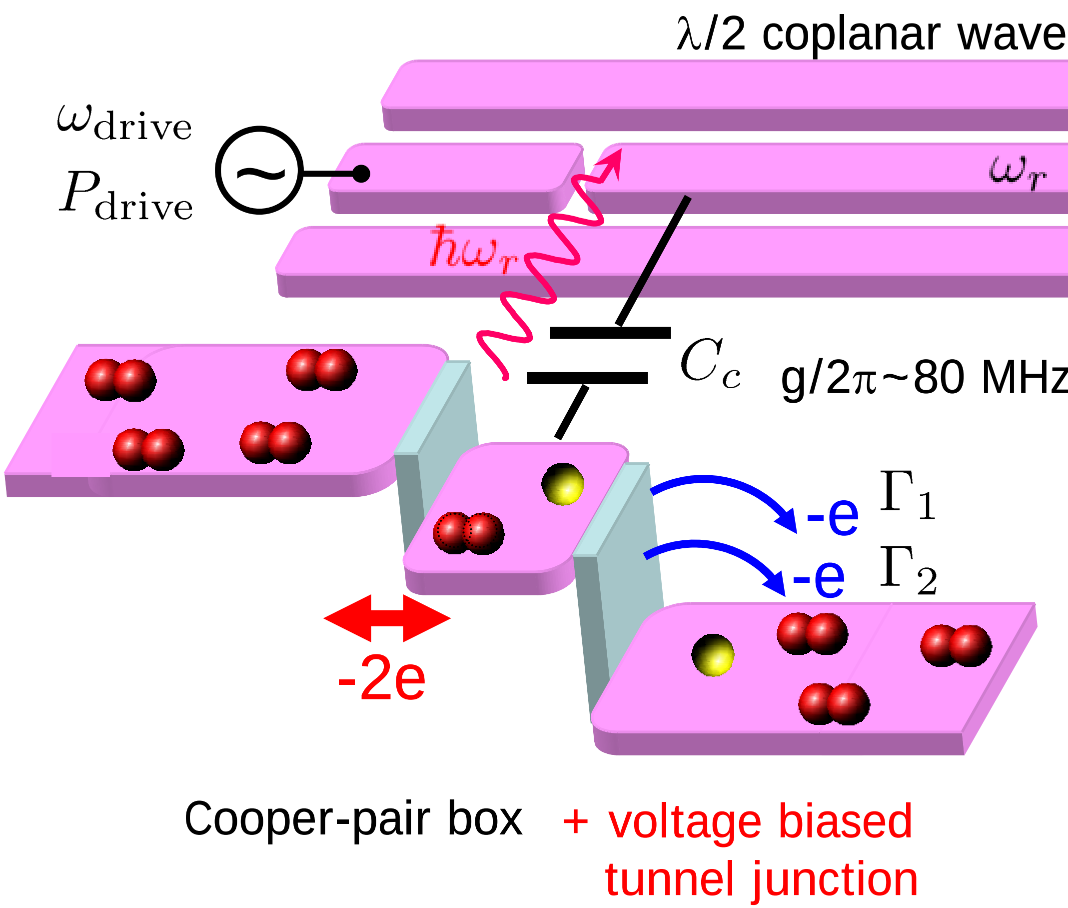
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+)$$

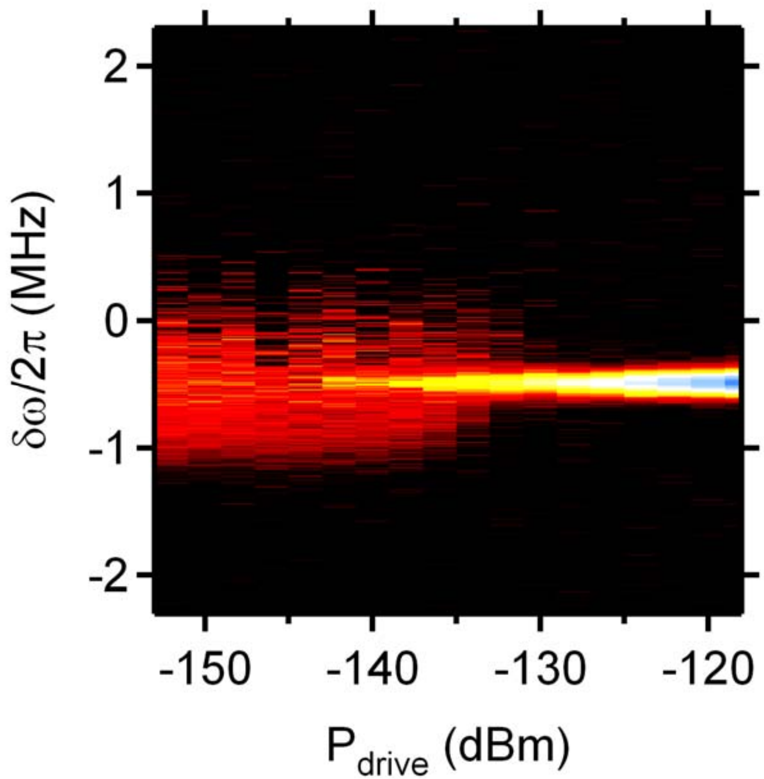
Strong coupling: $g \gg \gamma, \kappa$

g/2π~10 MHz (γ,κ)/2π~1 MHz

A. Blais et al. PRA 69, 062320 (2004); A. Wallraff et al. Nature 431, 162 (2004) Yale

Single artificial-atom maser





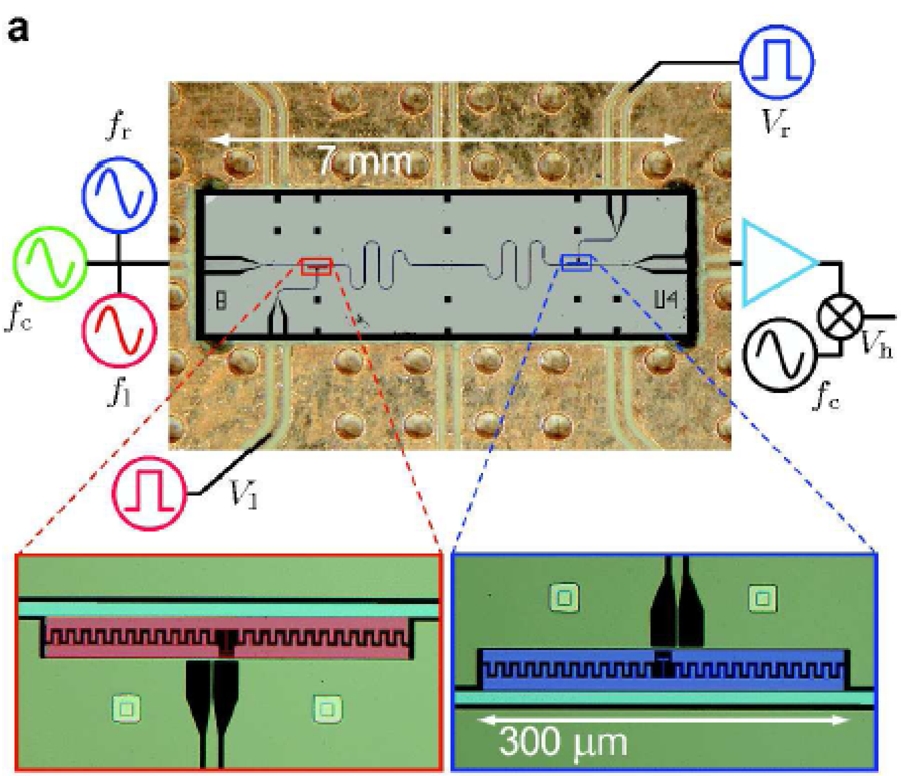
- Population inversion generated by current injection
- Capacitive coupling with cavity mode

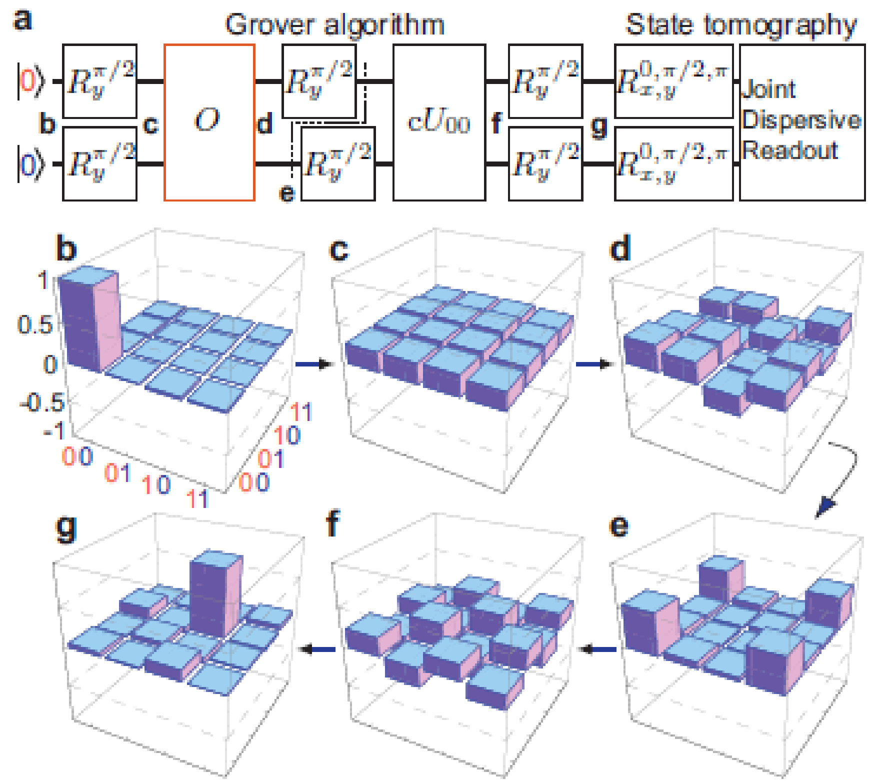
固体物理2008年9月号;O. Astafiev et al. Nature 449, 588 (2007)

Quantum algorithms implemented

Demonstration of Two-Qubit Algorithms with a Superconducting Quantum Processor

L. DiCarlo,[1] J. M. Chow,[2] J. M. Gambetta,[3] Lev S. Bishop,[2] D. I. Schuster,[1]
J. Majer,[4] A. Blais,[5] L. Frunzio,[1] S. M. Girvin,[6] and R. J. Schoelkopf[6]



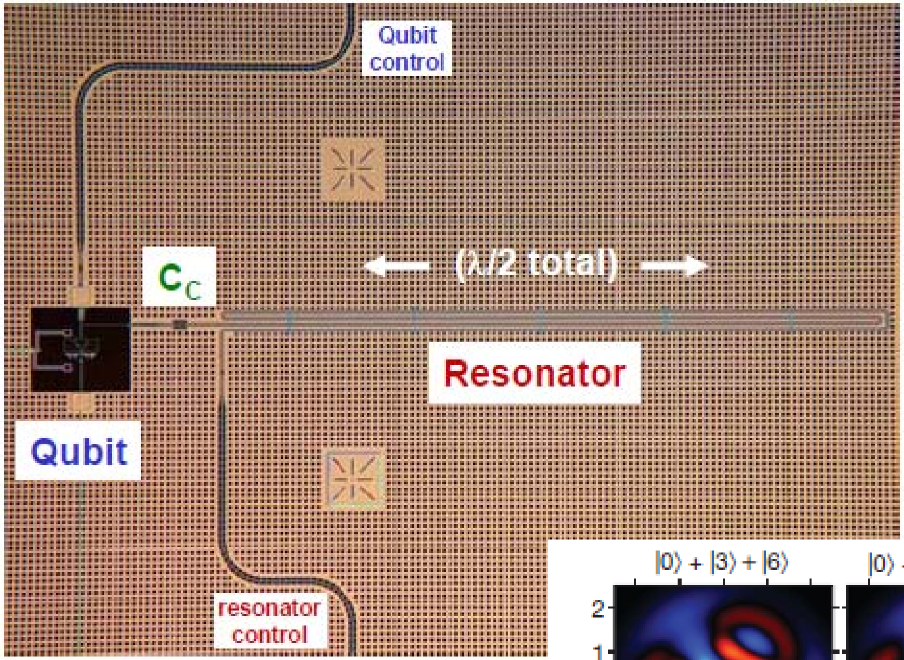


g/2π~200-300 MHz

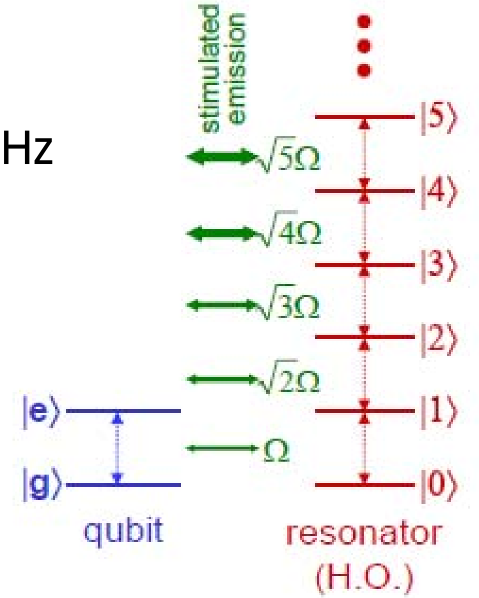
Gate fidelity ~ 99 % (1-qubit)
~ 90 % (2-qubit)

L. DiCarlo et al. Nature 460, 260 (2009) Yale; see also arXiv:1004.4324 for three-qubit entanglement

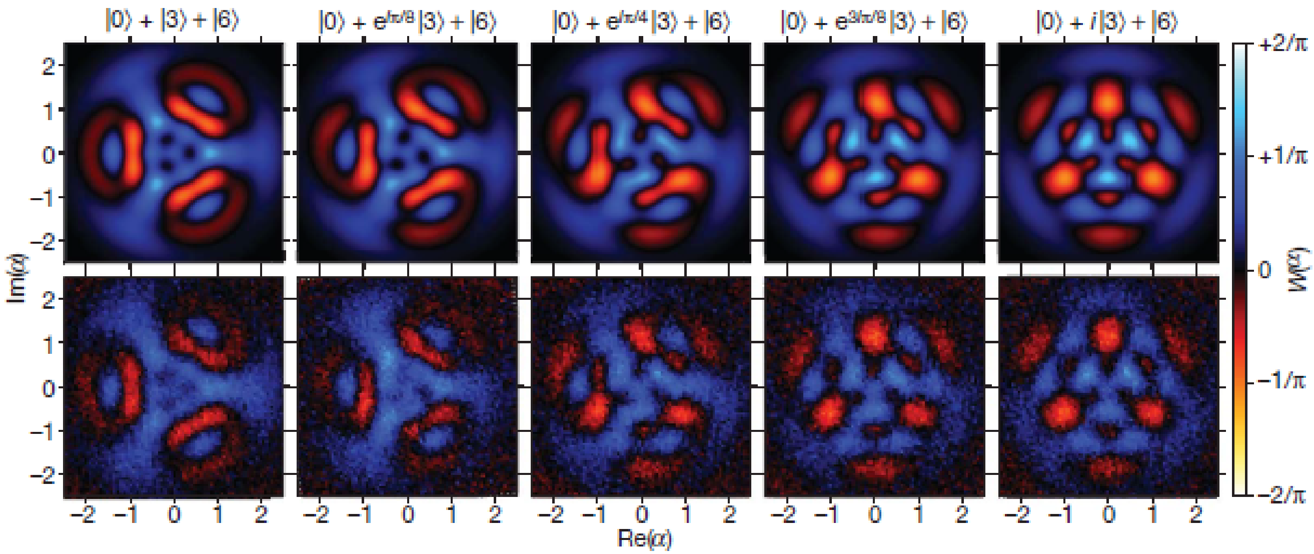
Synthesizing arbitrary quantum states in a superconducting resonator



$g/2\pi \sim 20$ MHz

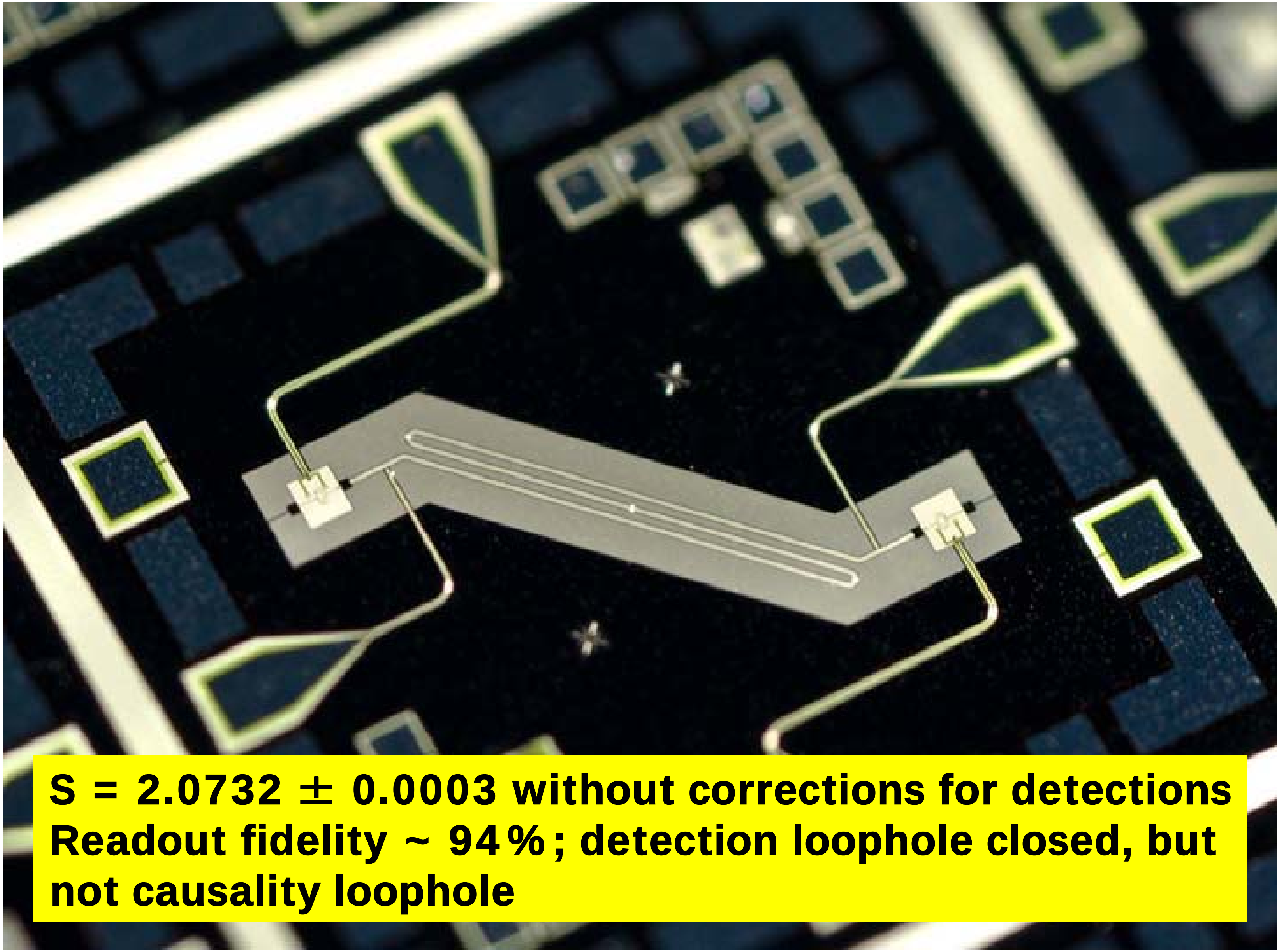


Generation of arbitrary intra-cavity photon states



M. Hofheinz et al. Nature 459, 546 (2009). UCSB

Violation of Bell's inequality



S = 2.0732 ± 0.0003 without corrections for detections
Readout fidelity ~ 94%; detection loophole closed, but not causality loophole

M. Ansmann et al. Nature 461, 504 (2009) UCSB

Photon-number discrimination in a resonator

ac Stark + Lamb shift

$$H = \hbar\omega_r a^\dagger a - \frac{\hbar}{2}\left(\omega_q + \underline{\frac{2g^2}{\delta}a^\dagger a} + \frac{g^2}{\delta}\right)\sigma_z$$

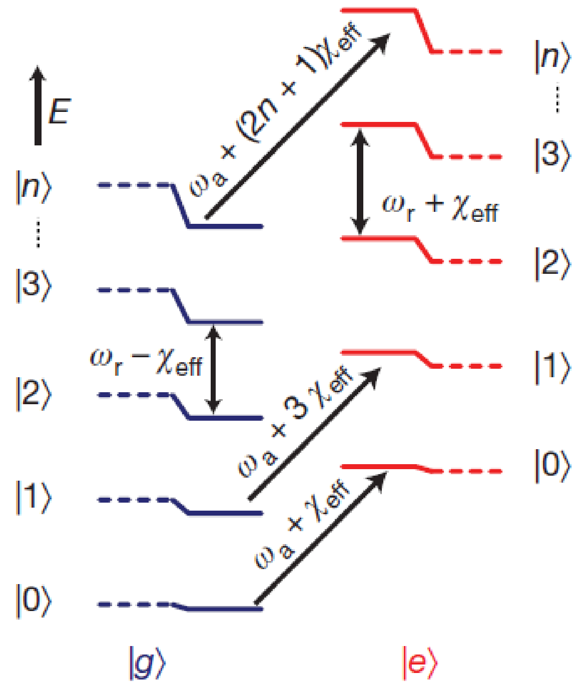
dispersive strong coupling

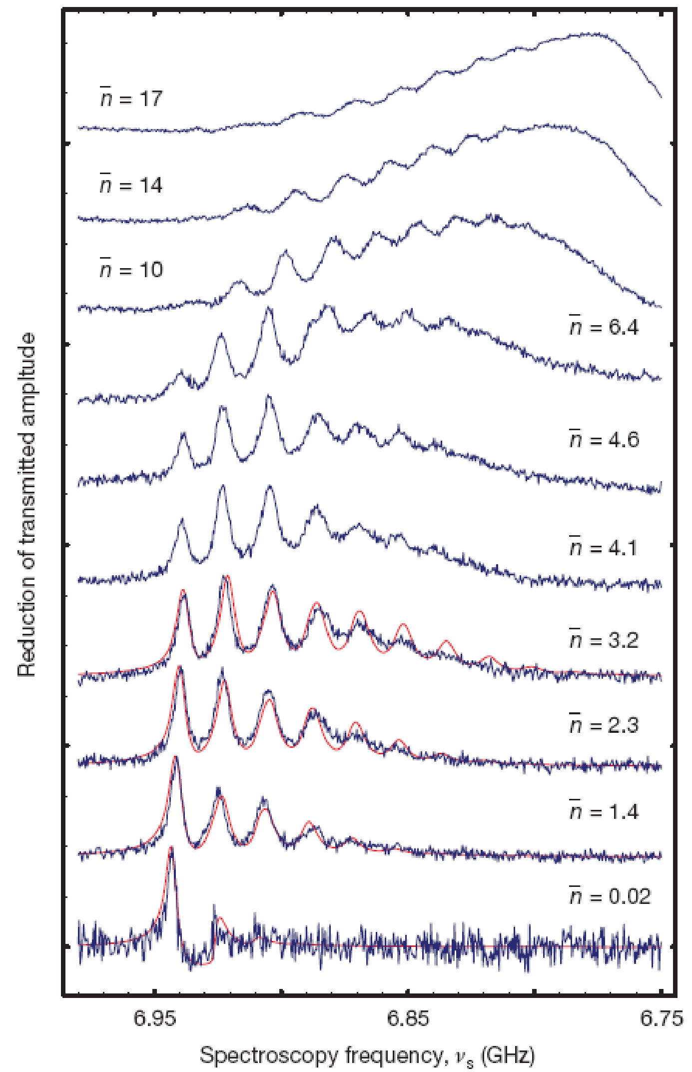
$g/\delta \ll 1$ ⇨ non-demolition readout

$g^2/\delta > \gamma,\ \kappa$ ⇨ single-photon Stark shift > line width

$$\chi_{\rm eff} = g^2/\delta$$

g/2π~100 MHz





D.I. Schuster et al. Nature 445, 515 (2007). Yale

Ultrastrong coupling regime

Dipole interaction Hamiltonian $$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger + a)\sigma_x$$

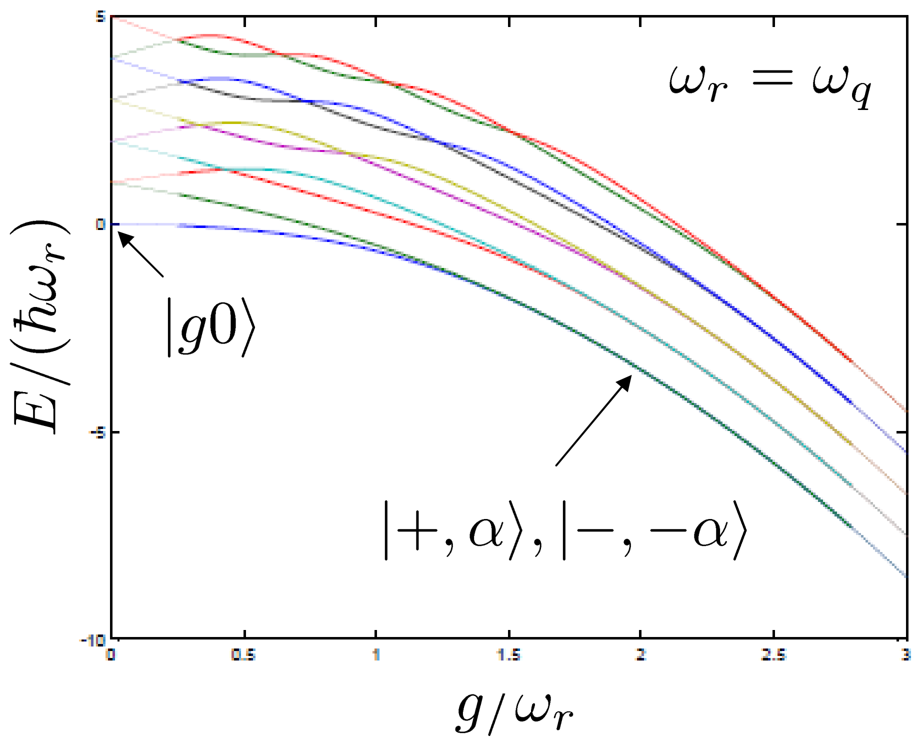
rotating-wave approx. $g \ll \omega_r \approx \omega_q$

Jaynes-Cummings model $$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger \sigma_- + a\sigma_+)$$

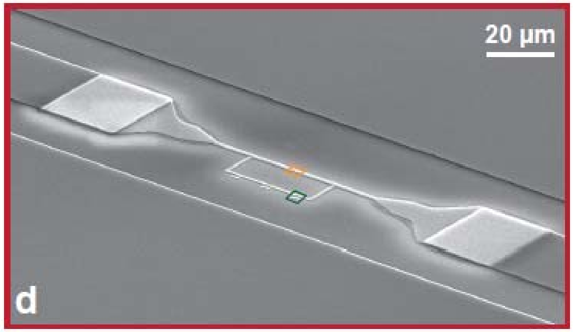
$g \sim \omega_r$

Breakdown of Jaynes-Cummings model

$$|\pm\rangle \propto |g\rangle \pm |e\rangle$$

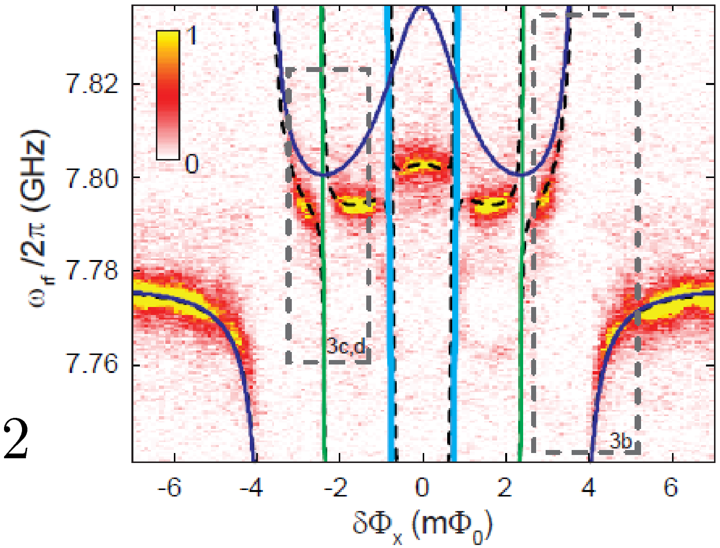


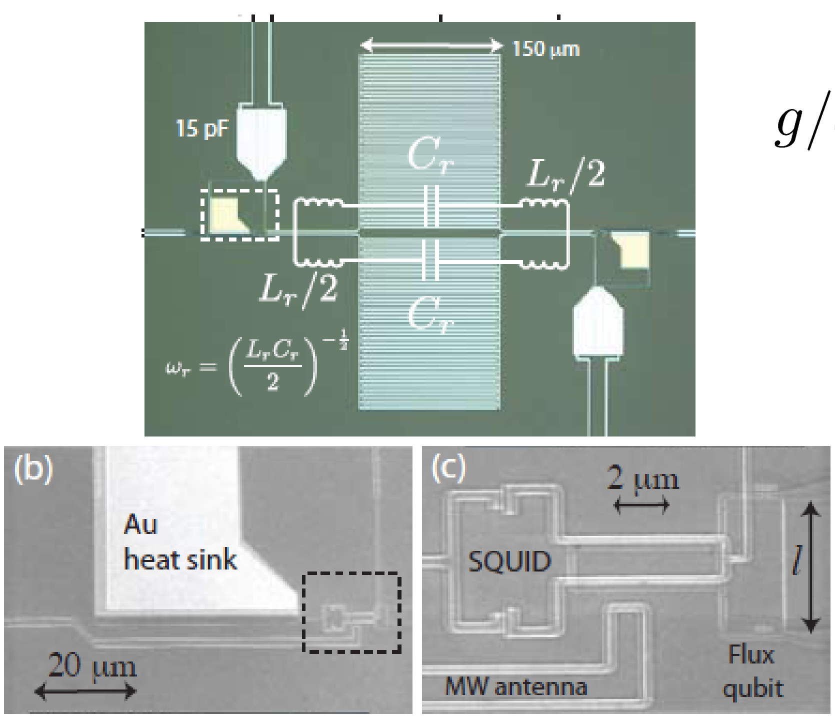
Towards ultrastrong coupling regime



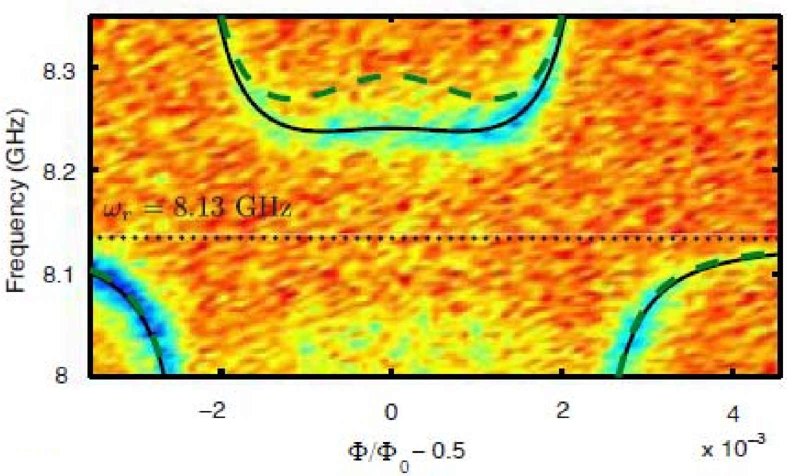
T. Niemczyk et al. arXiv:1003.2376 (WMI)

$g/\omega_r \sim 0.12$





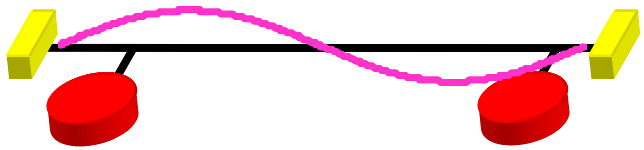
$g/\omega_r \sim 0.1$



P. Forn-Diaz et al. arXiv:1005.1559 (Delft)

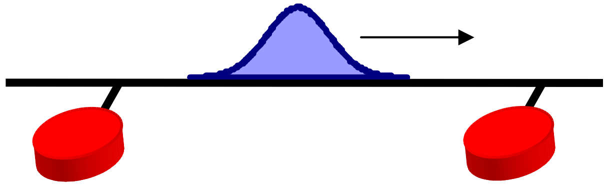
Confined photon and flying photon

- in resonator (confined "0D" photon; single-mode)



$$H = \hbar\omega_r a^\dagger a$$

- through transmission line (flying photon; multi-mode; continuum)

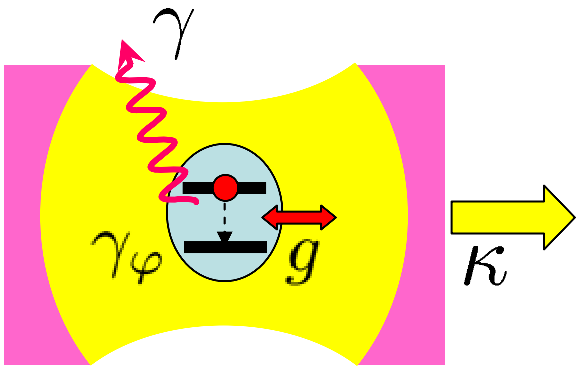


$$H = \hbar \int_0^\infty d\omega\, \omega a^\dagger(\omega) a(\omega)$$

Strong coupling conditions

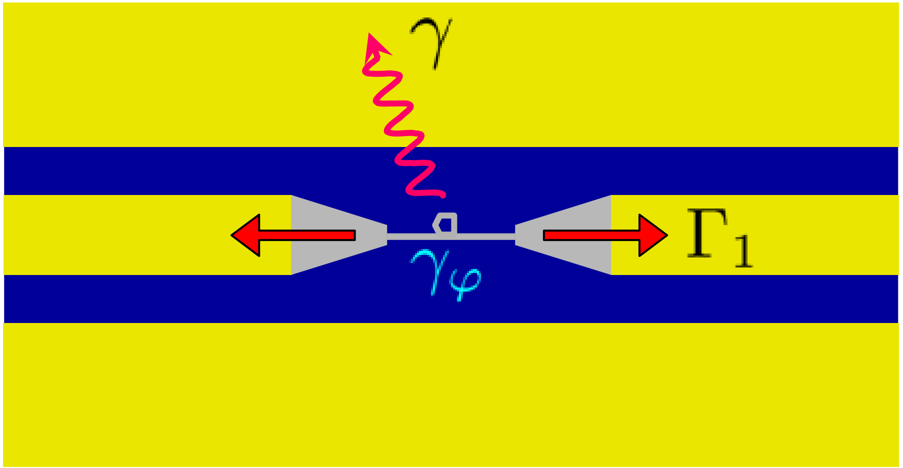
Strong coupling in cavity QED

$$g \gg \kappa, \gamma, \gamma_\varphi$$



"Strong coupling" in 1D open space

$$\Gamma_1 \gg \gamma, \gamma_\varphi$$



Superconducting qubits coupled to a transmission line

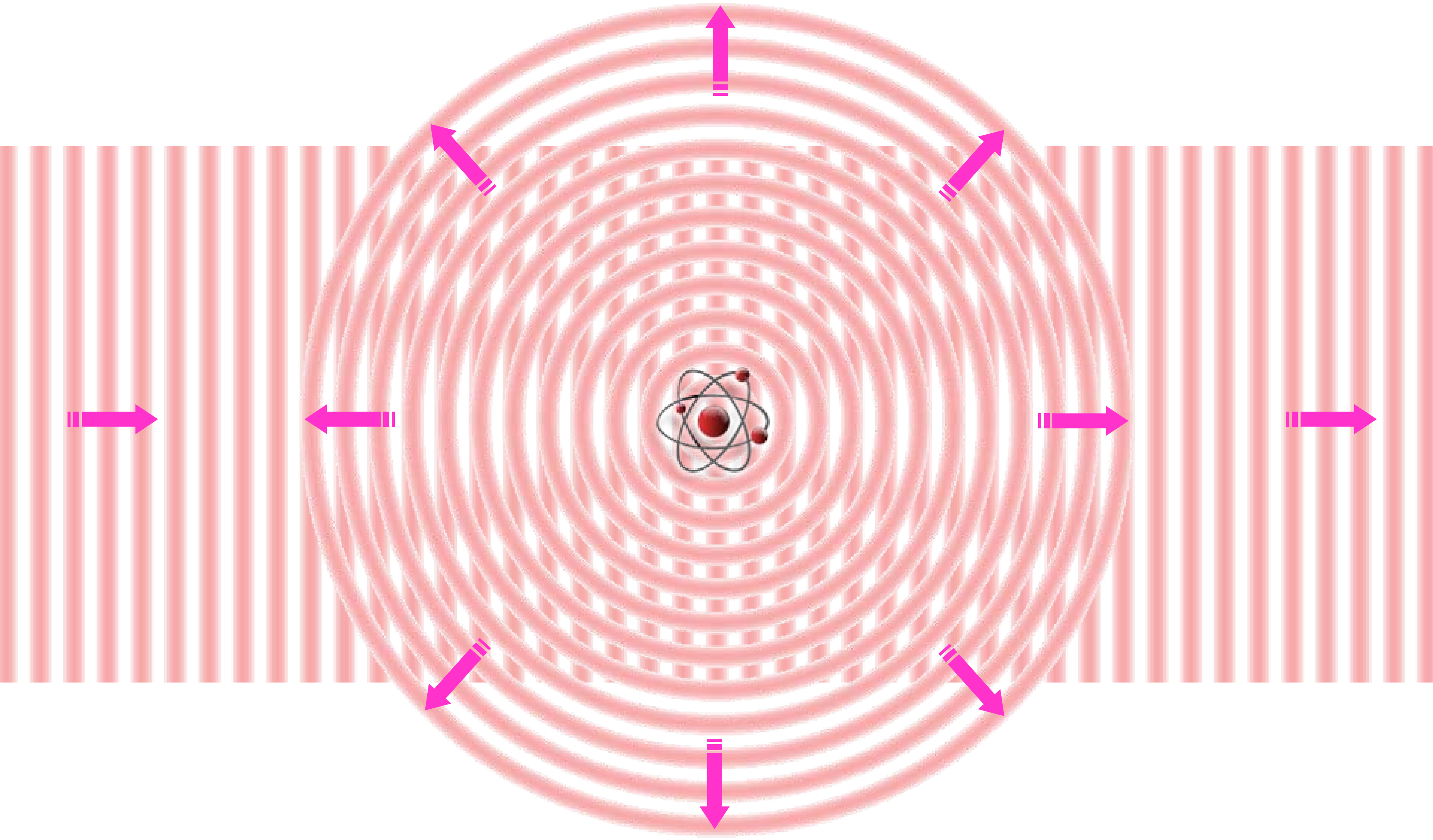
- Beauty of 1D
  - Microwave transmission line as 1D channel
  - Perfect spatial mode matching

- Superconducting qubits as artificial atoms
  - Fixed on chip
  - Strong coupling
  - Multi levels, selection rules

- Spontaneous emission – coherent process

- Use of interference
  - Importance of temporal modes
  - Limitation with bandwidth

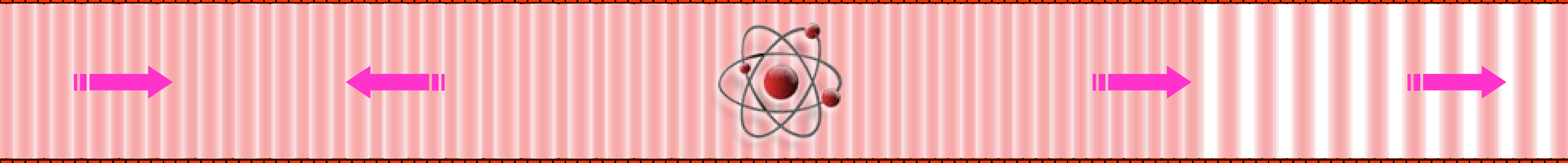
Resonant scattering in 3D space

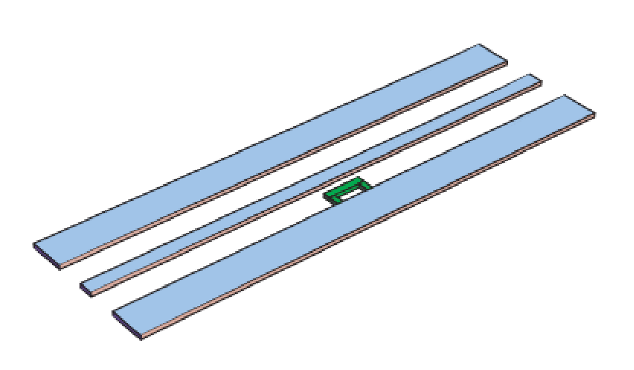


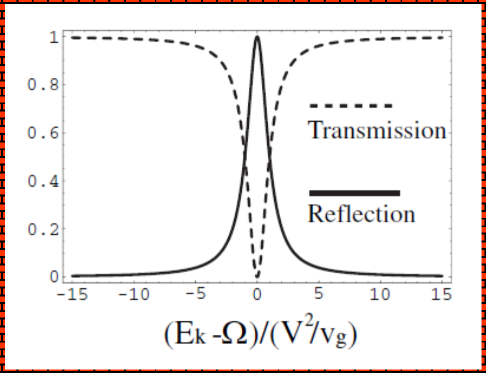
- Small scattering cross section
- Spatial mode mismatch between incident and radiated waves

Resonant scattering in 1D waveguide

Destructive interference of transmitted wave
⇒ Extinction of transmittance
⇒ Perfect reflection



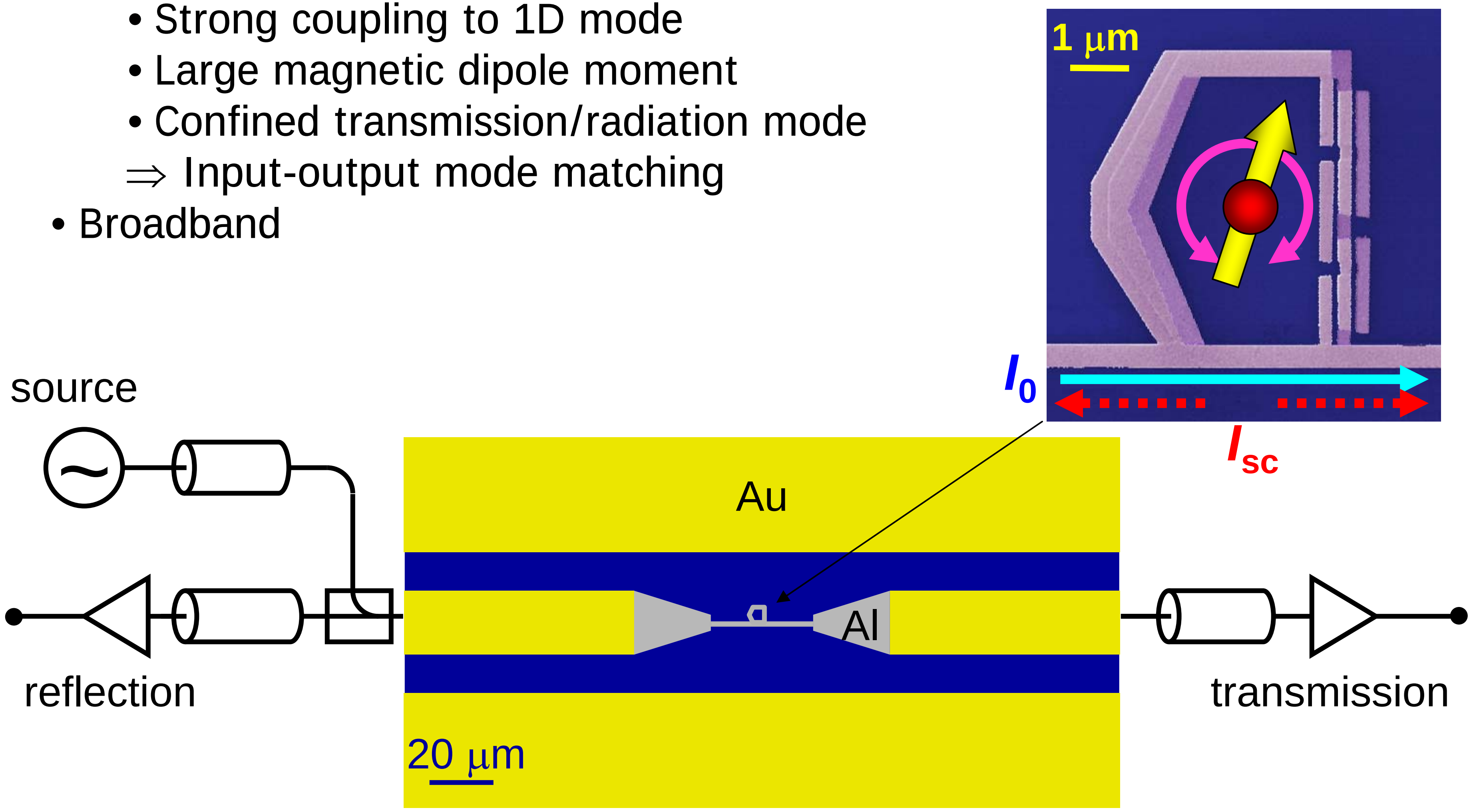




Shen and Fan, PRL 95, 213001 (2005) Stanford; Chang et al. PRL 97, 053002 (2006) Harvard

Artificial atom in 1D open space

- Flux qubit coupled to transmission line via kinetic inductance
  - Strong coupling to 1D mode
  - Large magnetic dipole moment
  - Confined transmission/radiation mode
  - ⇒ Input-output mode matching
- Broadband



Similar work using transmon: C. Wilson et al. unpublished (Chalmers)

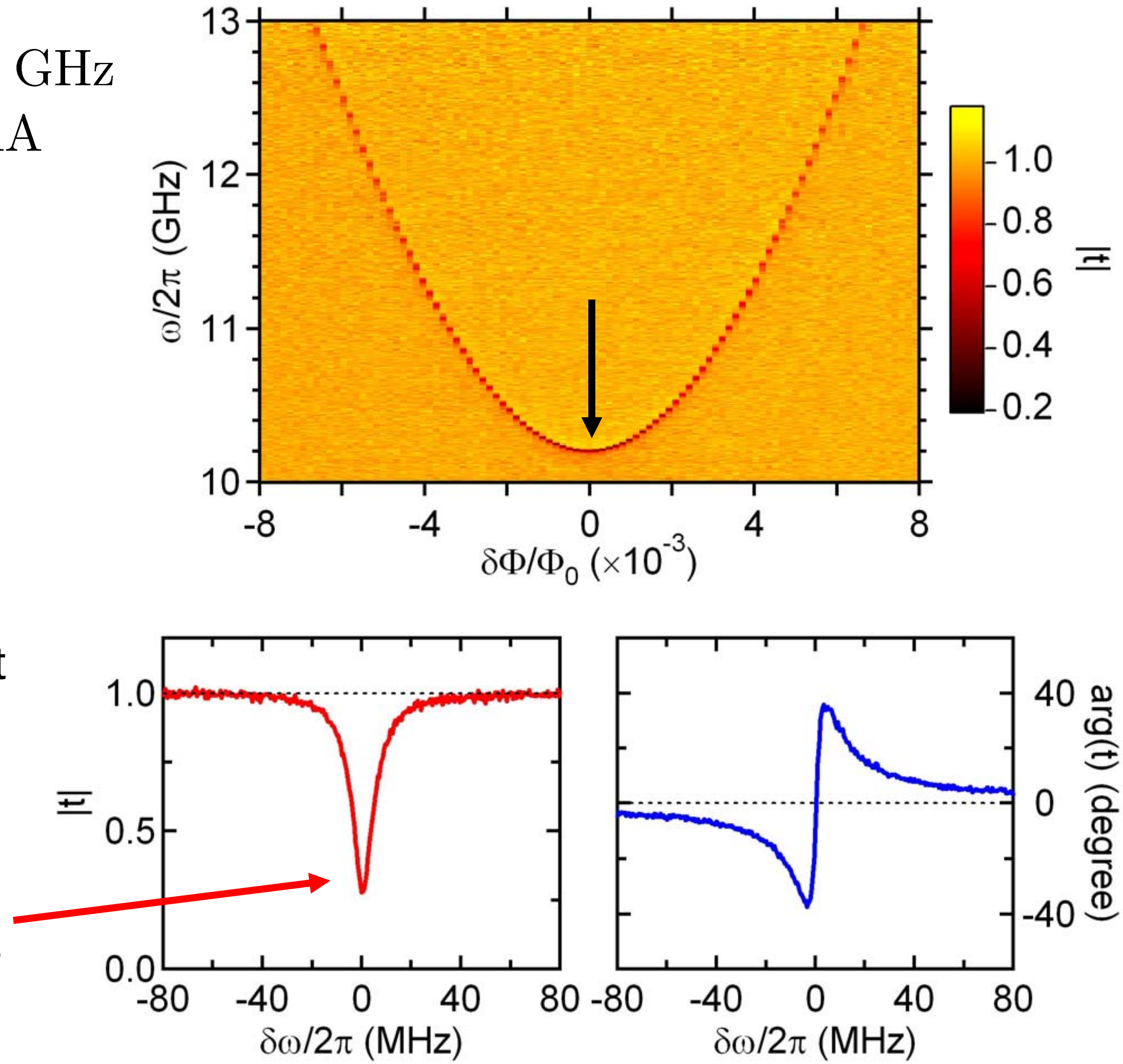
Transmission spectroscopy — elastic scattering

$$\Delta/h = 10.20 \text{ GHz}$$
$$I_p = 195 \text{ nA}$$

At degeneracy point

$$\delta\Phi/\Phi_0 = 0$$

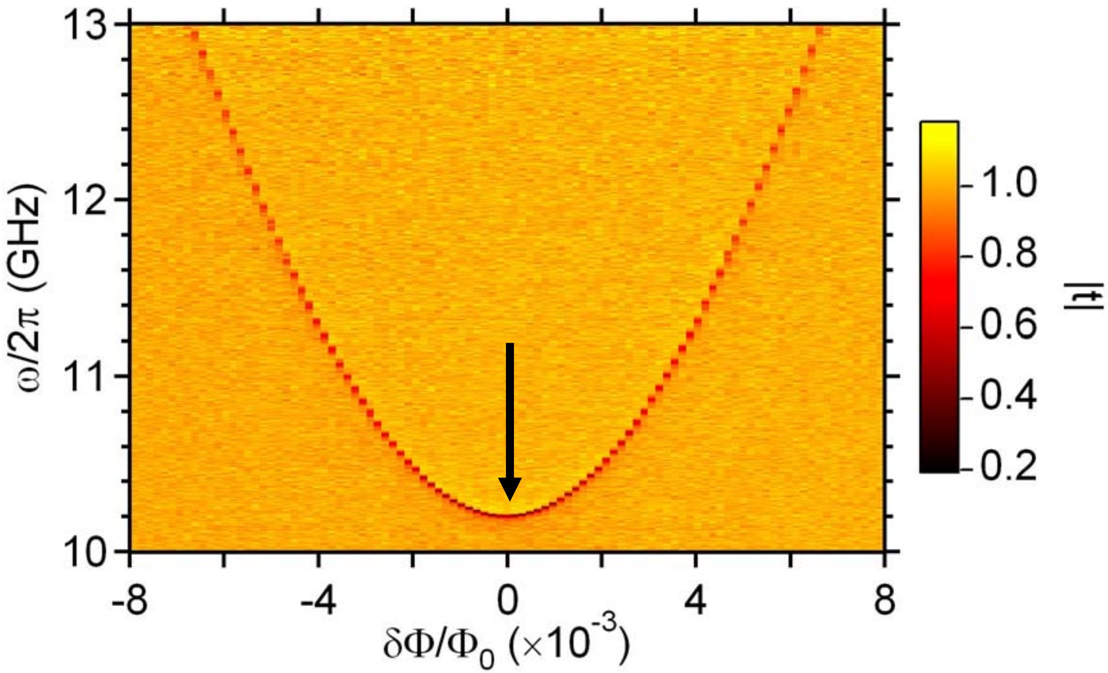
Strong extinction of transmitted wave

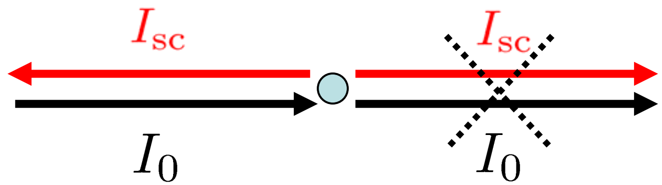


O. Astafiev et al. Science 327, 840 (2010)

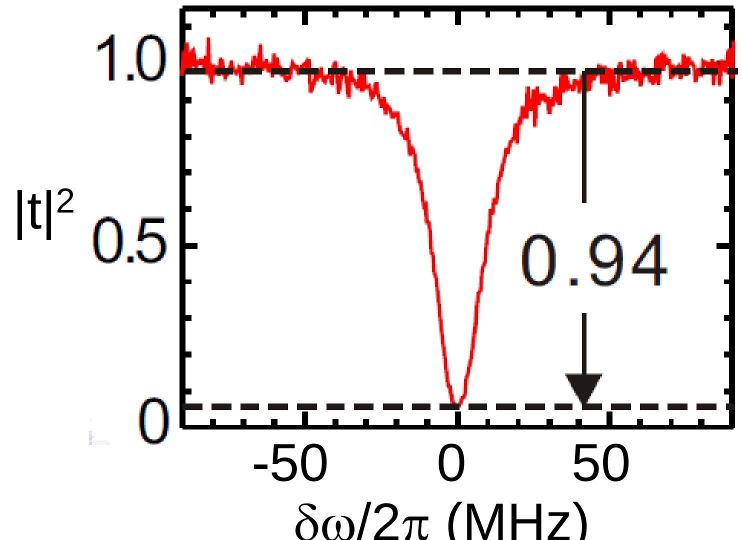
Transmission spectroscopy — elastic scattering

$$\Delta/h = 10.20\ \text{GHz}$$
$$I_p = 195\ \text{nA}$$





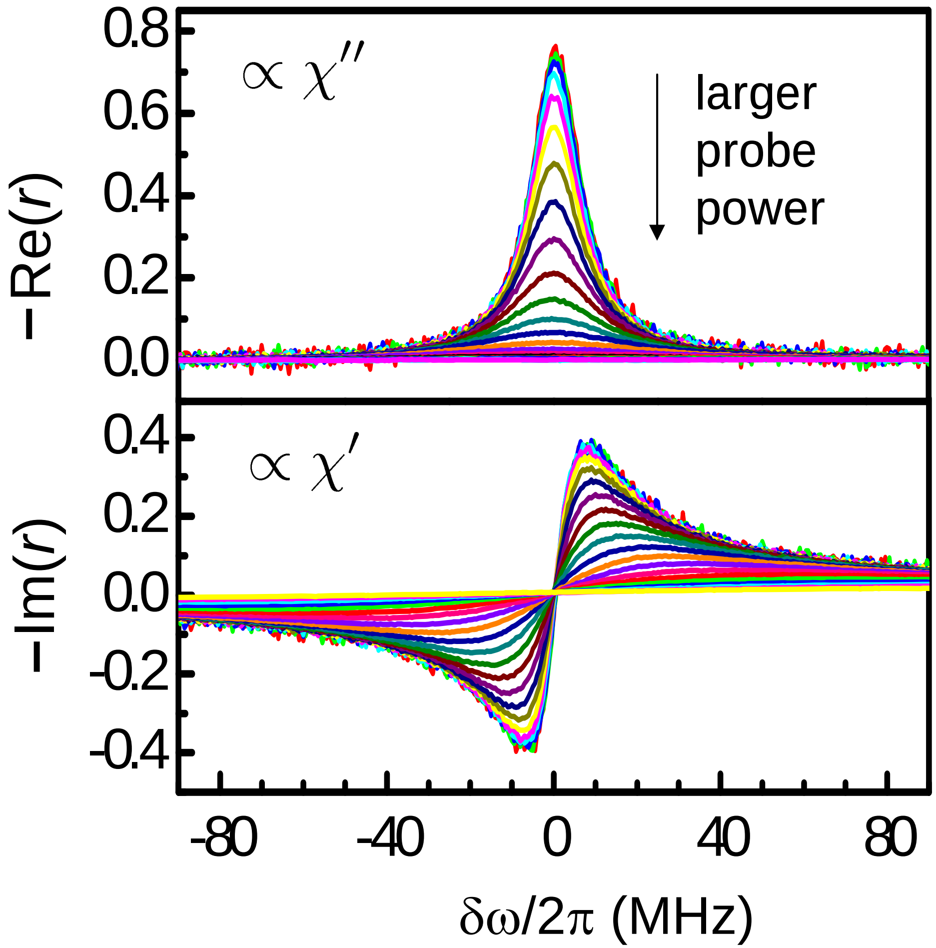
Perfect reflection

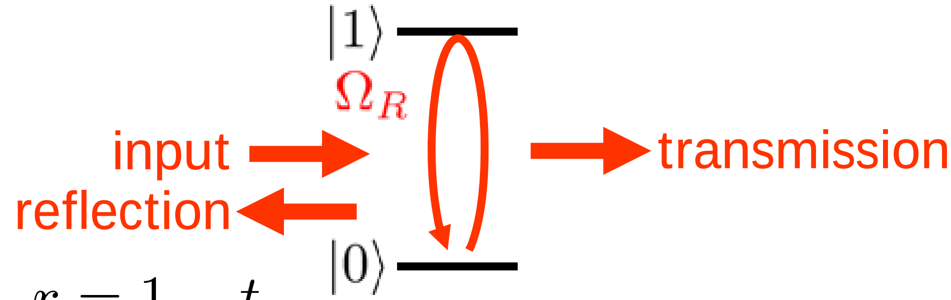


$$|r_0|^2 = \left(\frac{\Gamma_1}{2\Gamma_2}\right)^2$$

O. Astafiev et al. Science 327, 840 (2010)

Power dependence — saturation of atom





$$-r = 1 - t$$

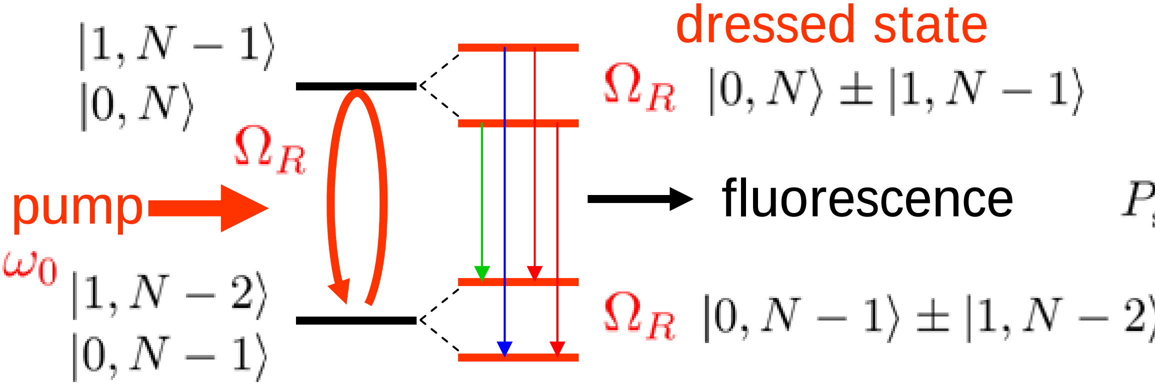
$$r = -r_0 \frac{1 + i\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega_R^2/\Gamma_1\Gamma_2}$$

$$r_0 = \frac{\Gamma_1}{2\Gamma_2} = \frac{\Gamma_1}{\Gamma_1 + 2\Gamma_\varphi}$$

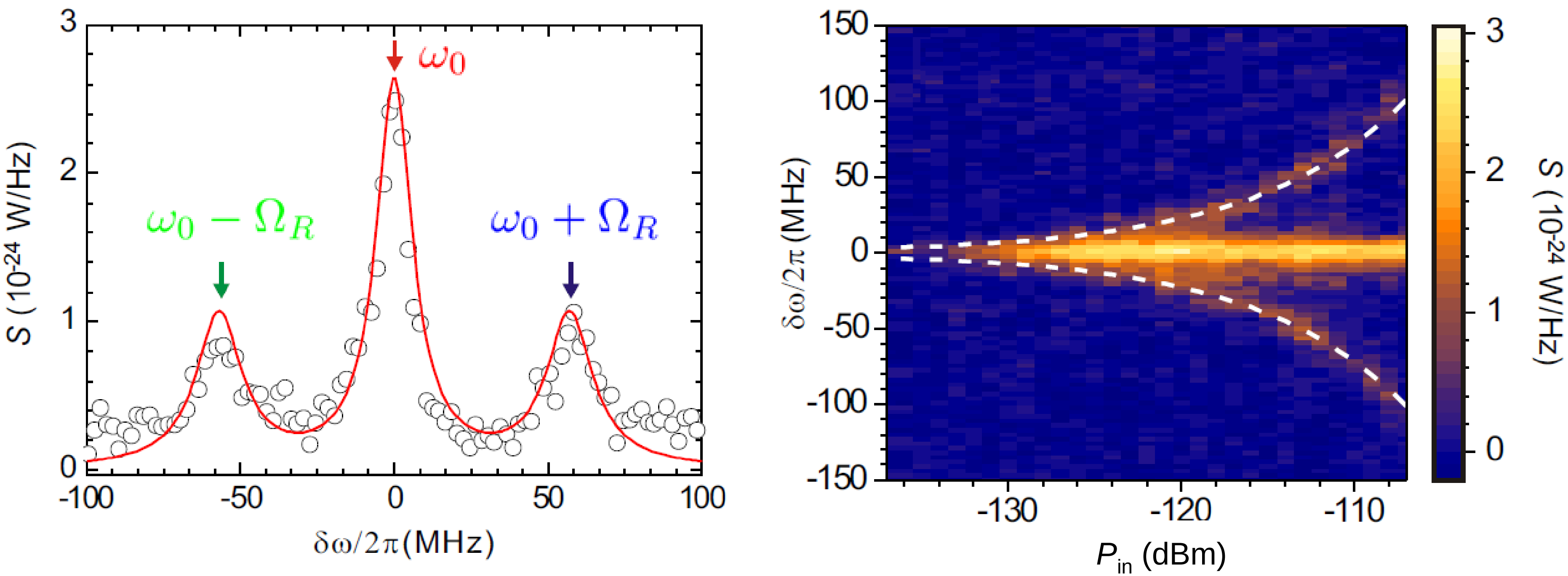
$$\Gamma_1/2\pi = 11.0 \text{ MHz}$$
$$\Gamma_\varphi/2\pi = 1.7 \text{ MHz}$$
$$M = 12.2 \text{ pH}$$

Inherent nonlinearity of the two-level atom

Resonance fluorescence: inelastic scattering



$$P_{sc}^{\mathrm{inel.}}/P_{\mathrm{in}} \approx 1 - |r|^2 - |t|^2$$

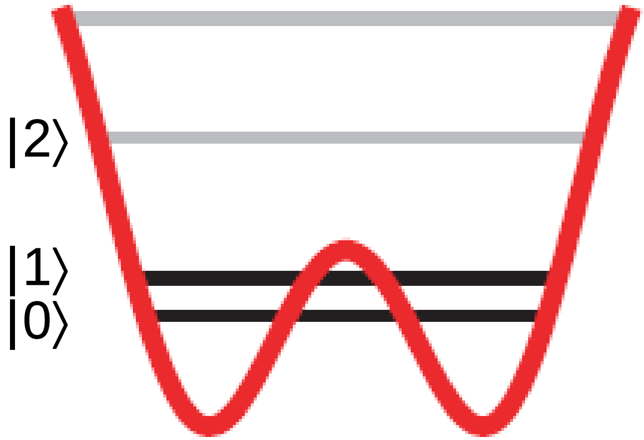


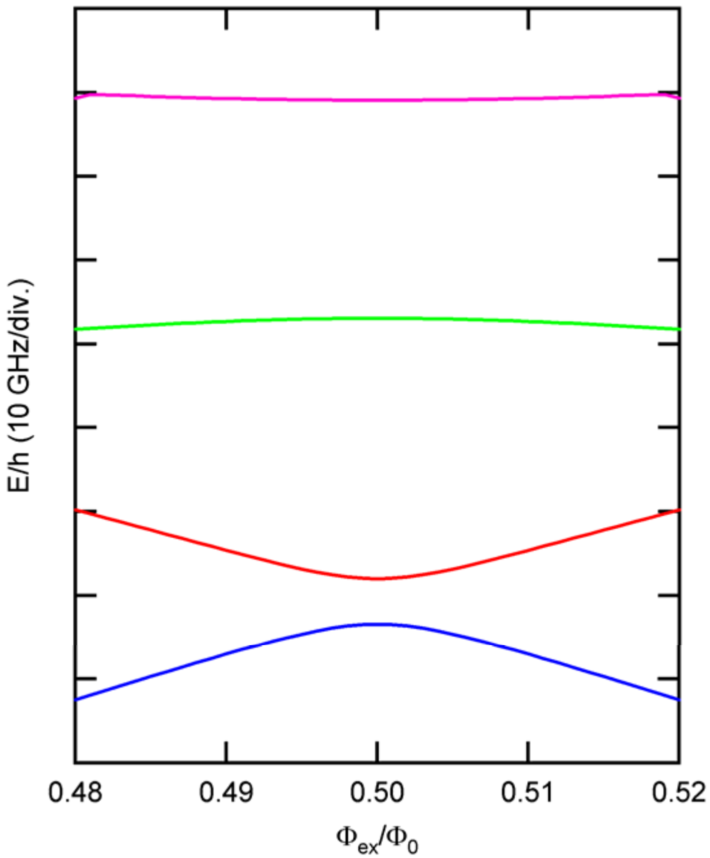
Mollow triplet: $S(\omega) \approx \frac{1}{2\pi}\frac{\hbar\omega\Gamma_1}{8}\left(\frac{\gamma_s}{(\delta\omega+\Omega)^2+\gamma_s^2} + \frac{2\gamma_c}{\delta\omega^2+\gamma_c^2} + \frac{\gamma_s}{(\delta\omega-\Omega)^2+\gamma_s^2}\right)$

O. Astafiev et al. Science 327, 840 (2010)

$\gamma_s = (\Gamma_1 + \Gamma_2)/2 \qquad \gamma_c = \Gamma_2 = \Gamma_1/2 + \Gamma_\varphi$

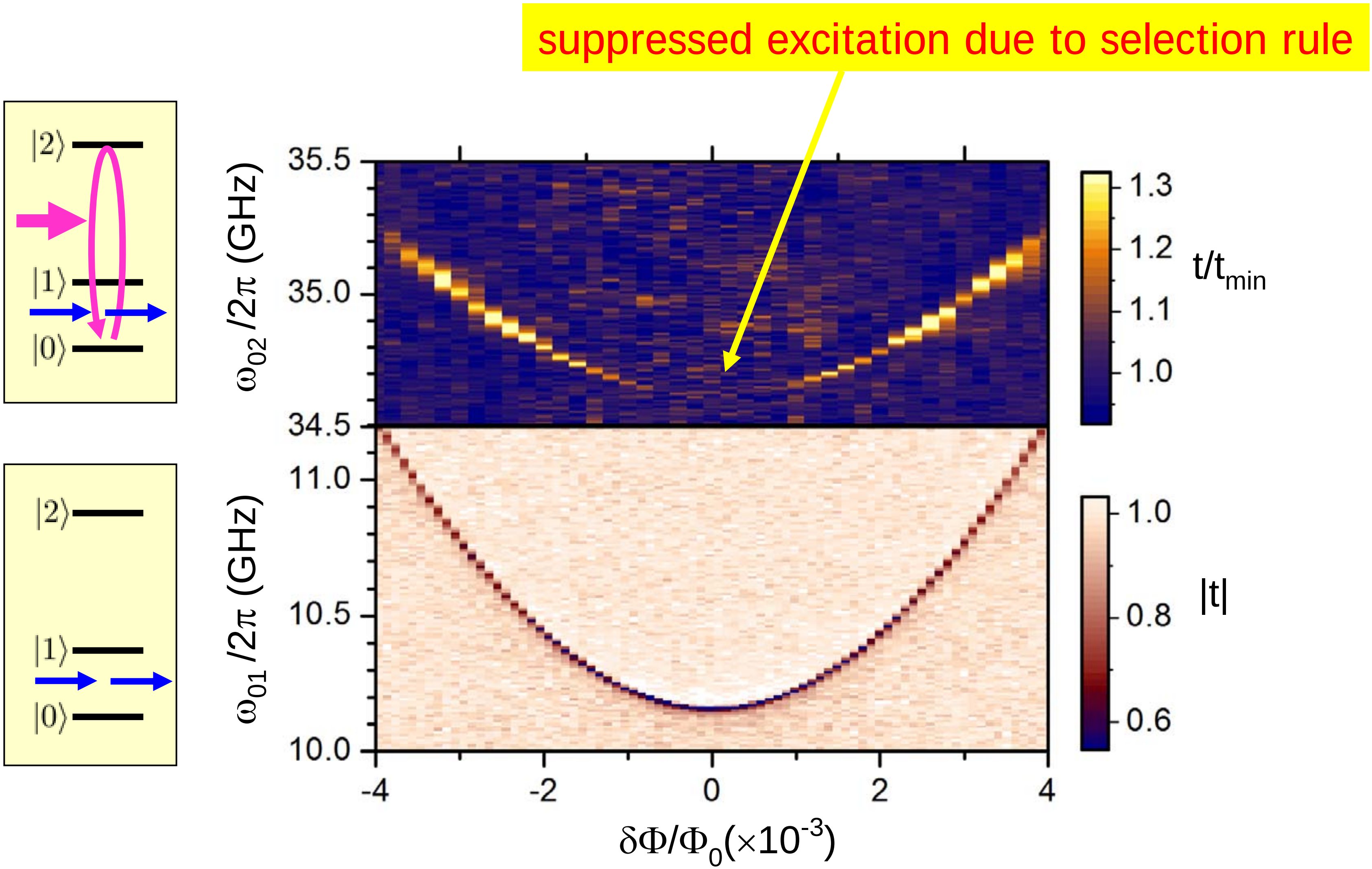
Flux qubit as a three-level artificial atom



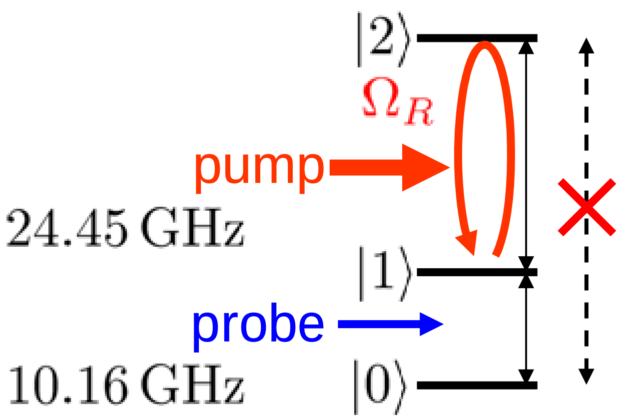


- Josephson junction qubits = effective two-level system
- presence of auxiliary states
- strong anharmonicity/nonlinearity
- selection rule due to symmetry when flux bias $\delta\Phi=0$

Spectroscopy of three-level atoms



Ladder system at degeneracy point: EIT

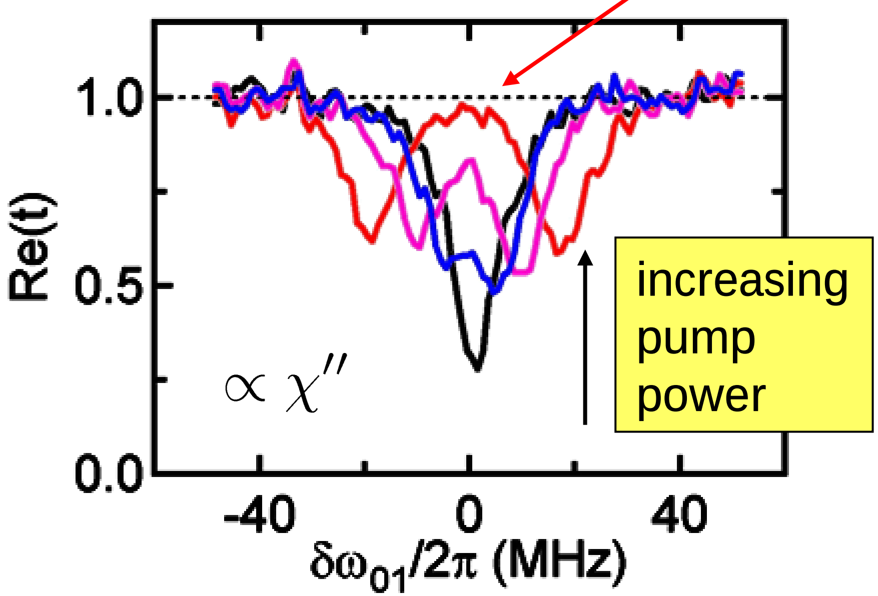


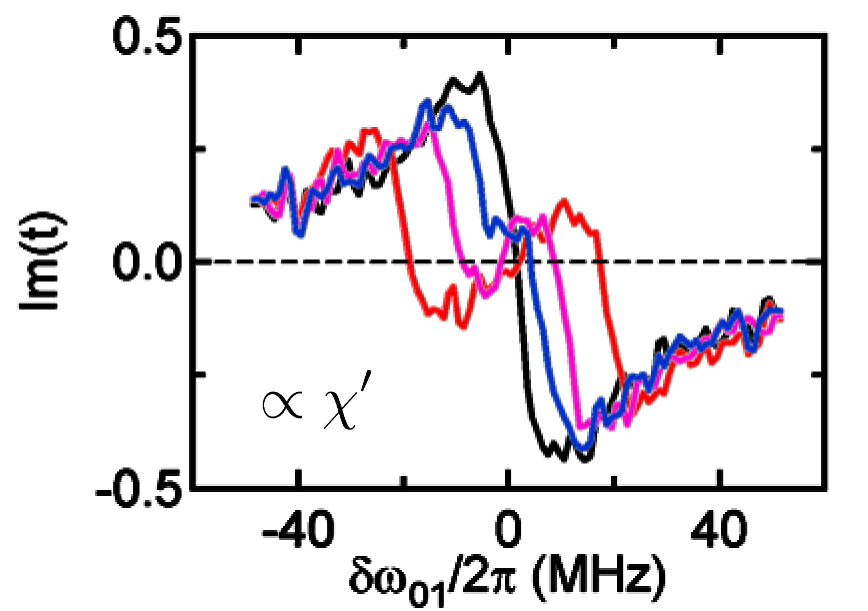
Biased at degeneracy point
Transition 0 ↔ 2 not allowed

ladder-type

"Electromagnetically-induced transparency"

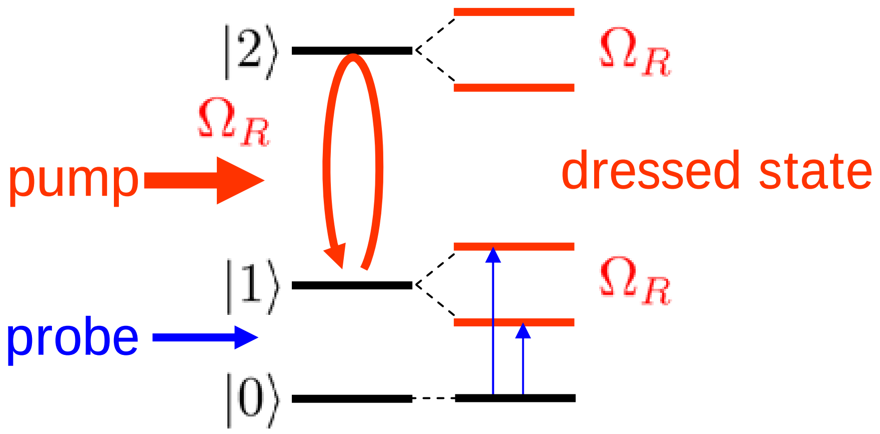
$\Gamma_{10} < \Gamma_{21} < \Omega_R$





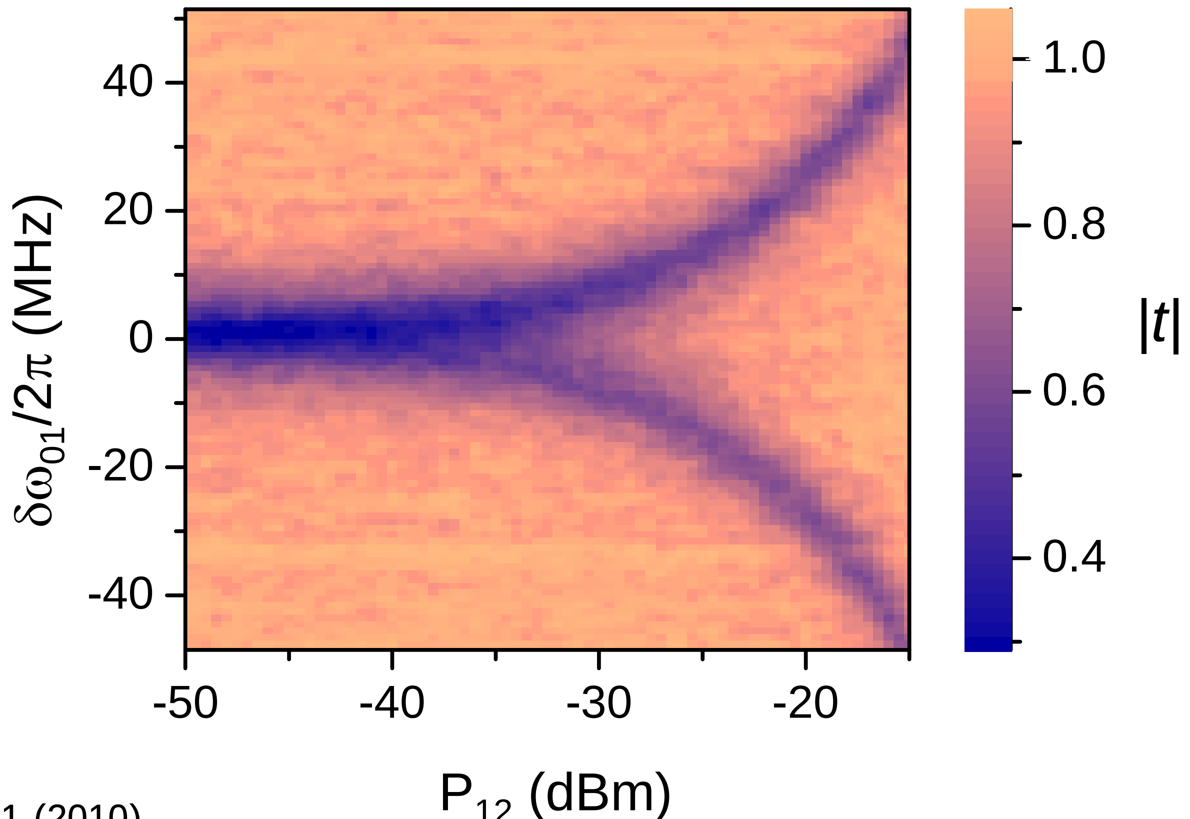
A. A. Abdumalikov et al. PRL 104, 193601 (2010)

Ladder system at degeneracy point: EIT



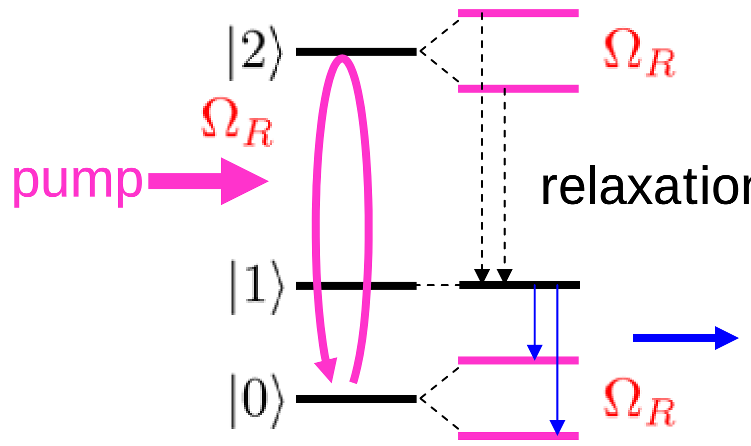
Autler-Townes doublet

Transmission of probe signal



A. A. Abdumalikov et al. PRL 104, 193601 (2010)

Optical pumping and spontaneous emission

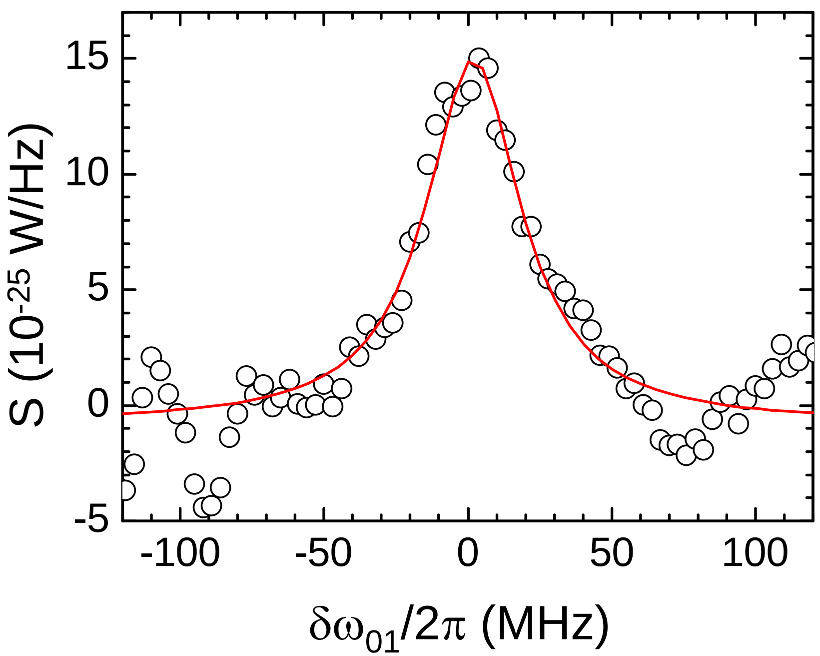


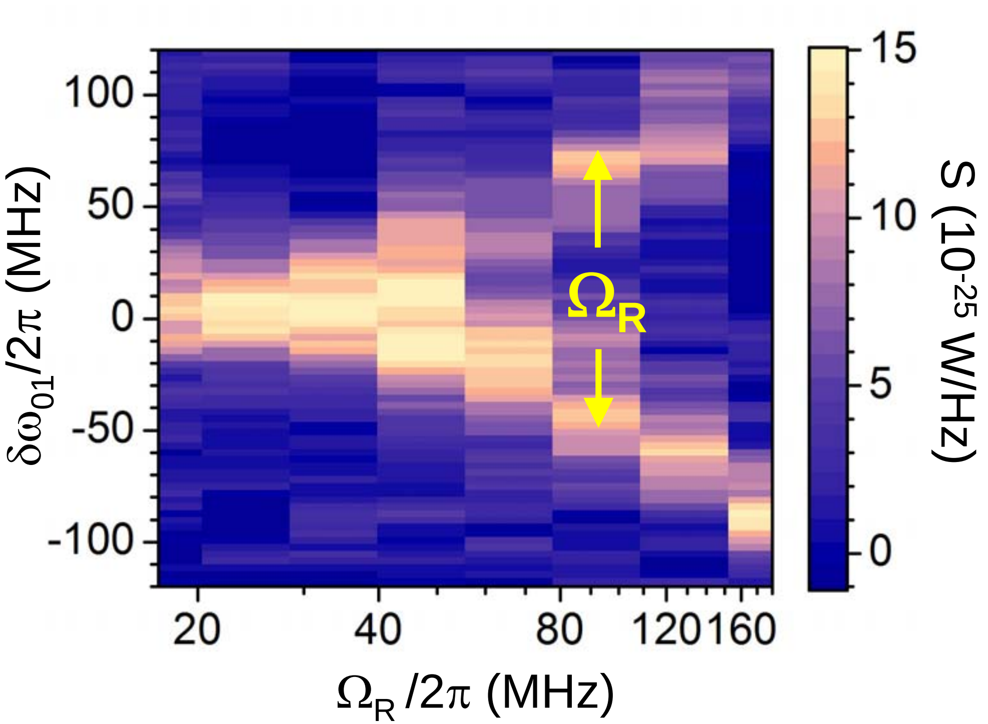
Biased away from degeneracy point
Transition 0 ↔ 2 allowed

relaxation → excited-state population

spontaneous emission

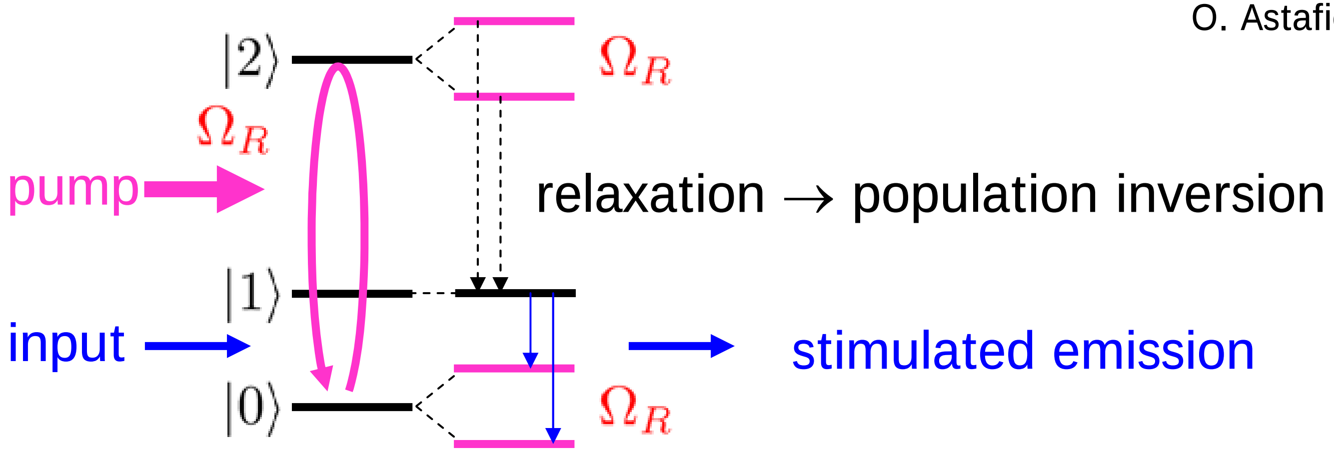
$\Omega_R/2\pi$ = 24 MHz

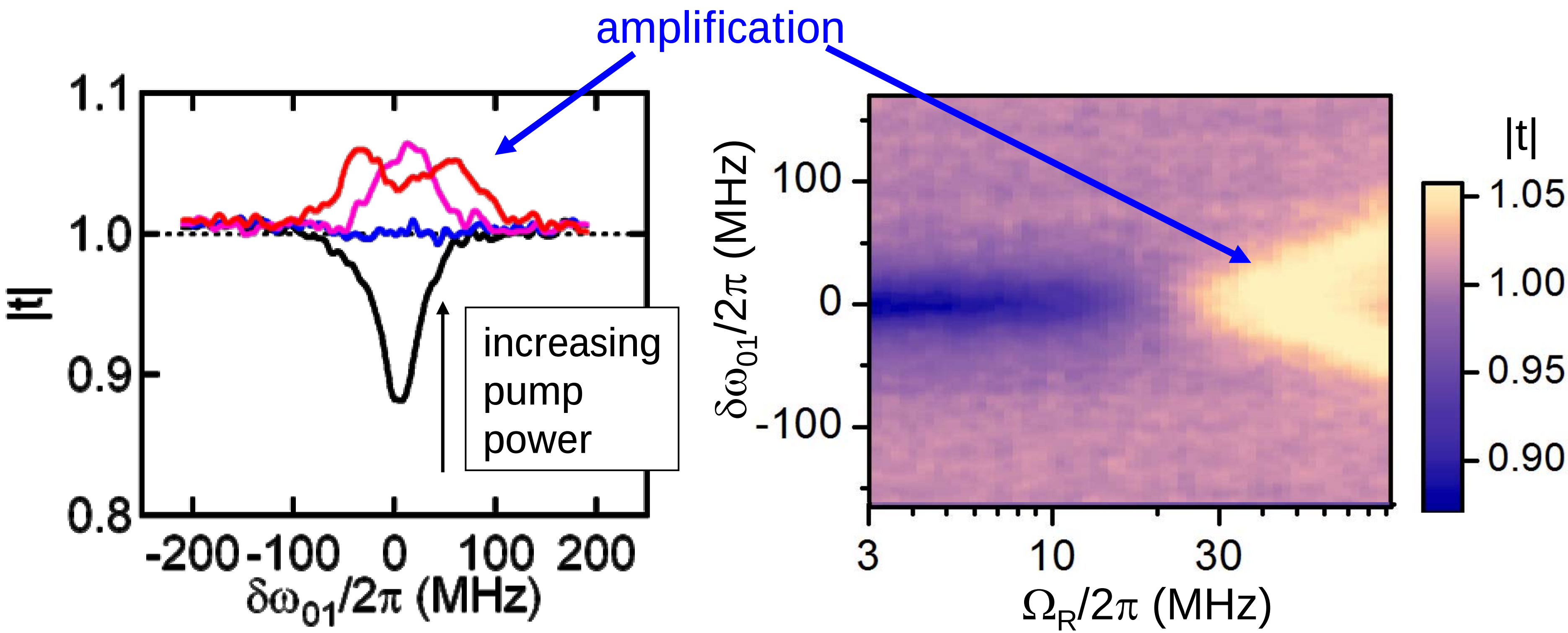




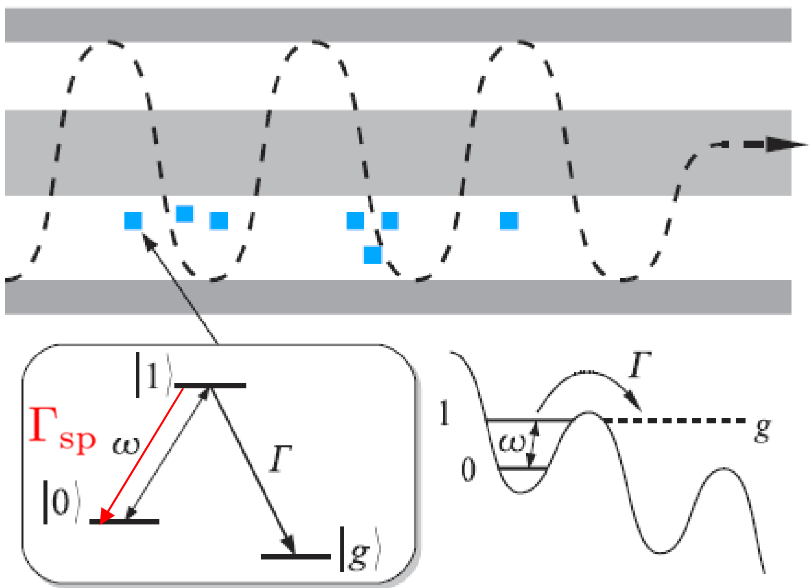
Stimulated emission and amplification

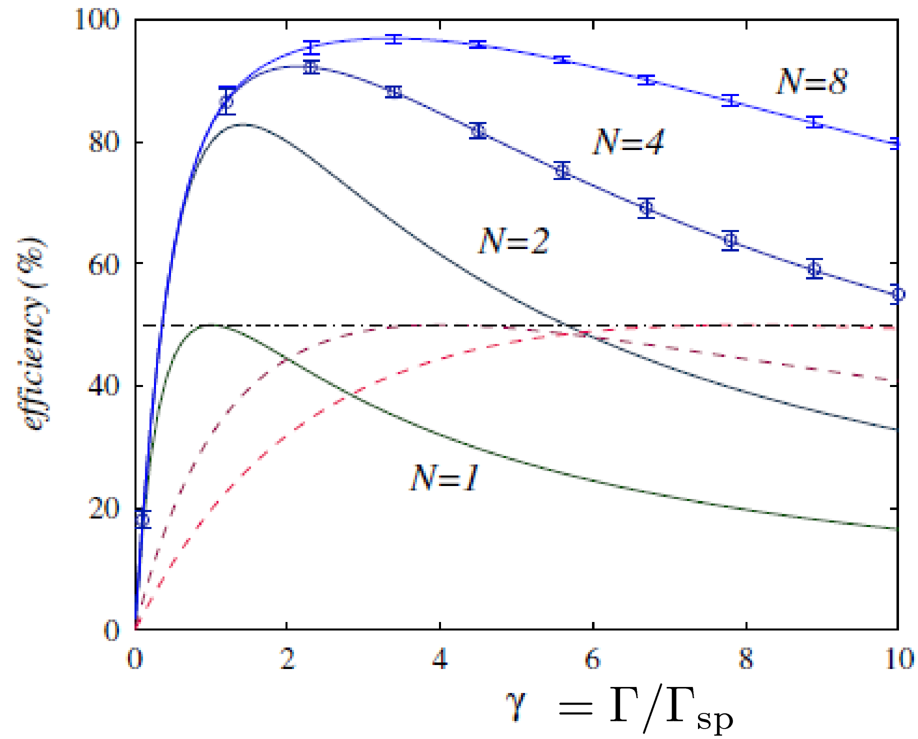
O. Astafiev et al. PRL 104, 183603 (2010)



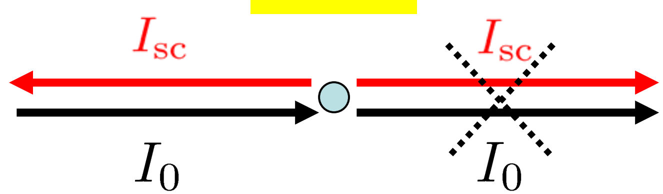


Single-photon detector



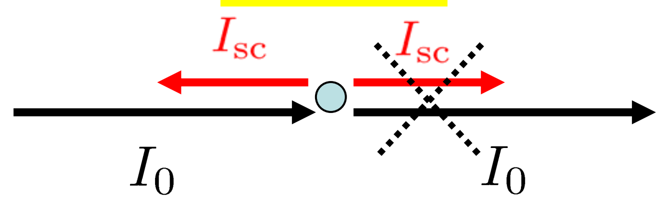


$\Gamma = 0$



Perfect reflection

$\Gamma = \Gamma_{sp}$

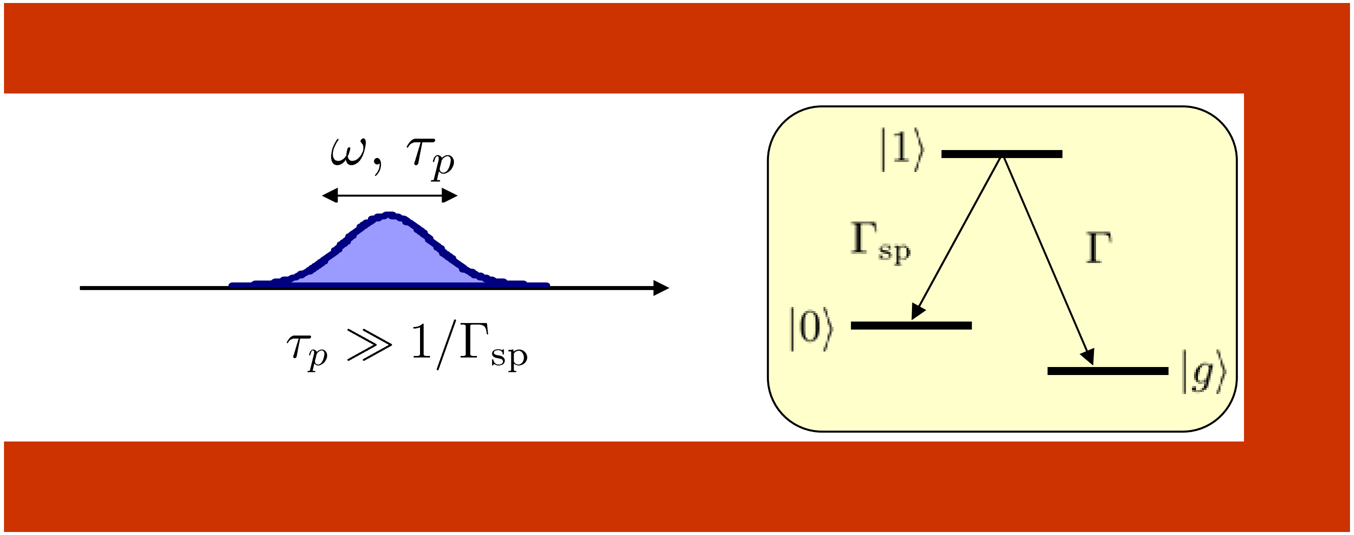


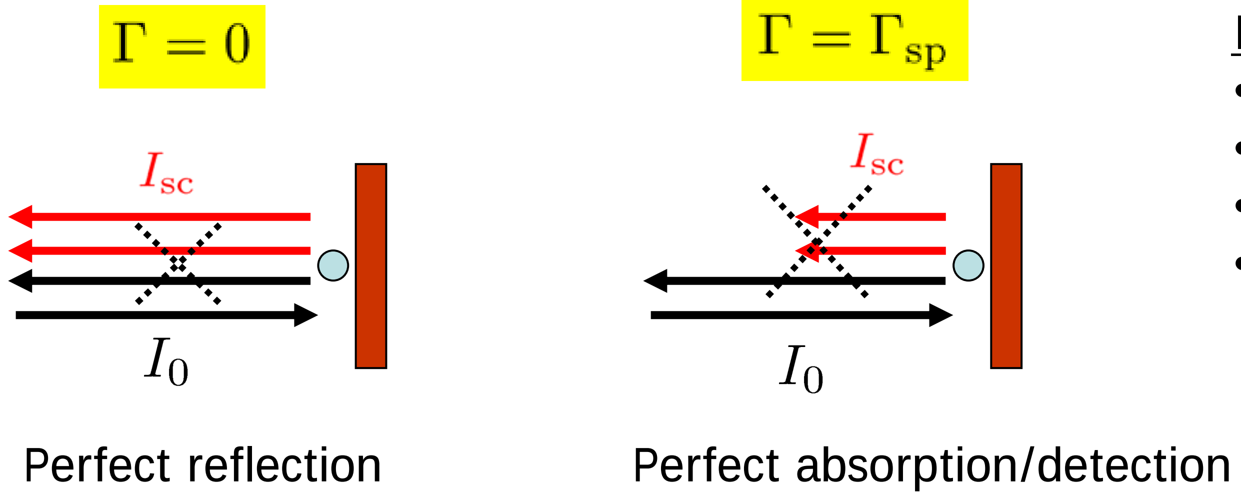
50% absorption

G. Romero et al. PRL 102, 173602 (2009) Santiago

Single-photon detector: improved design

~100% efficiency with a single atom

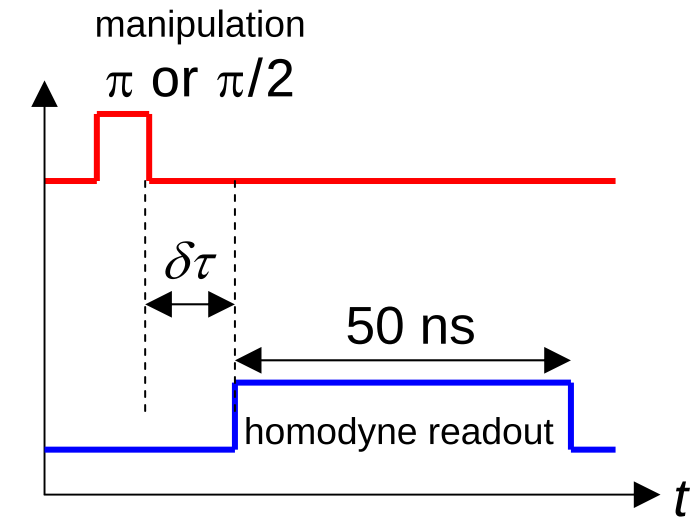


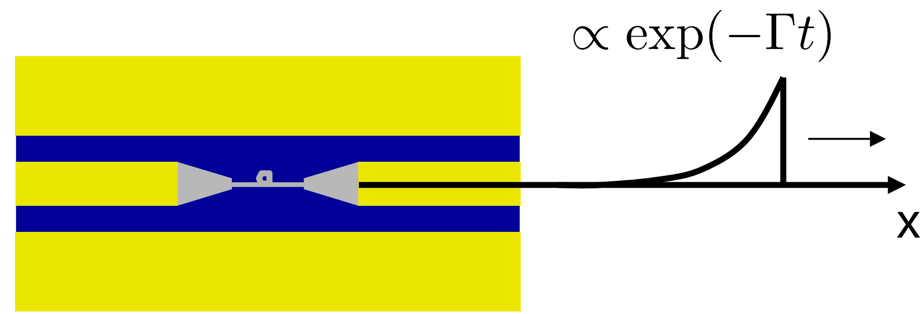


Issues

- efficiency
- dark count
- dead time
- bandwidth

Time-domain measurement of decoherence time

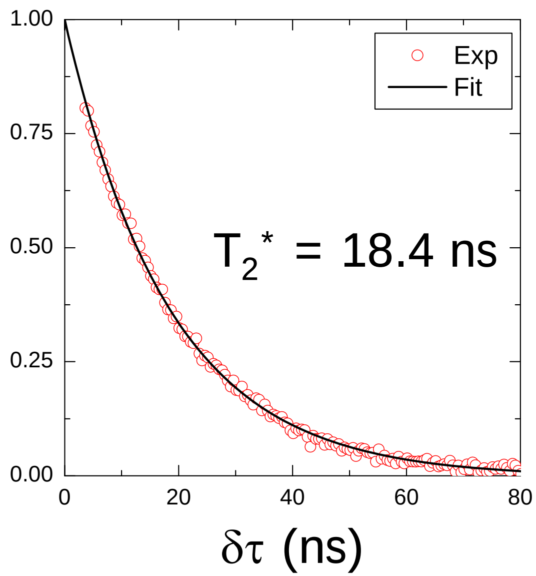




$(\pi/2)_x$ - $(\delta\tau)$ - readout

$\langle \sigma_y(t) \rangle$

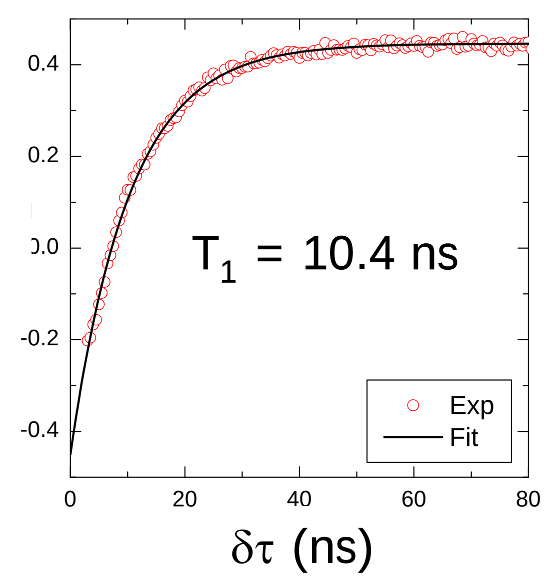
$\propto \langle a - a^\dagger \rangle$



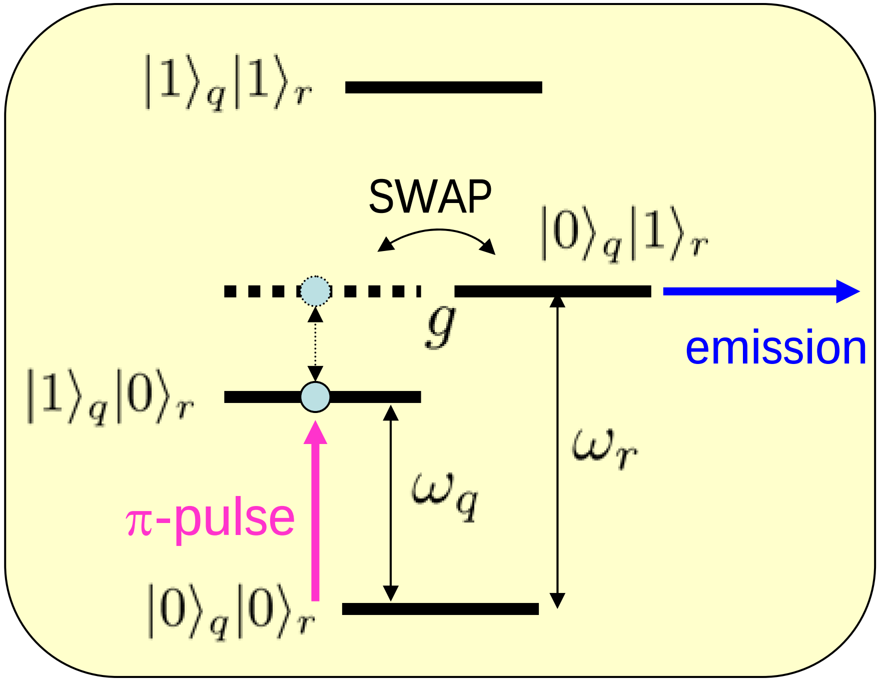
π_x - $(\delta\tau)$ – $(\pi/2)_x$ - readout

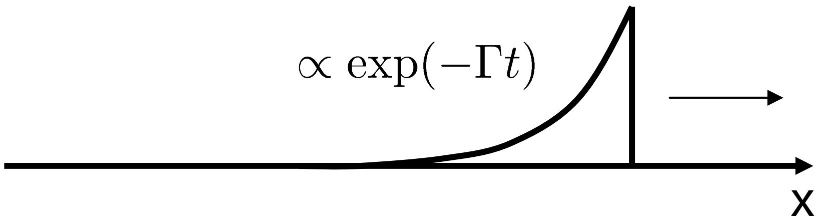
$\langle \sigma_z(t) \rangle$

$\propto \langle a^\dagger a \rangle - \frac{1}{2}$

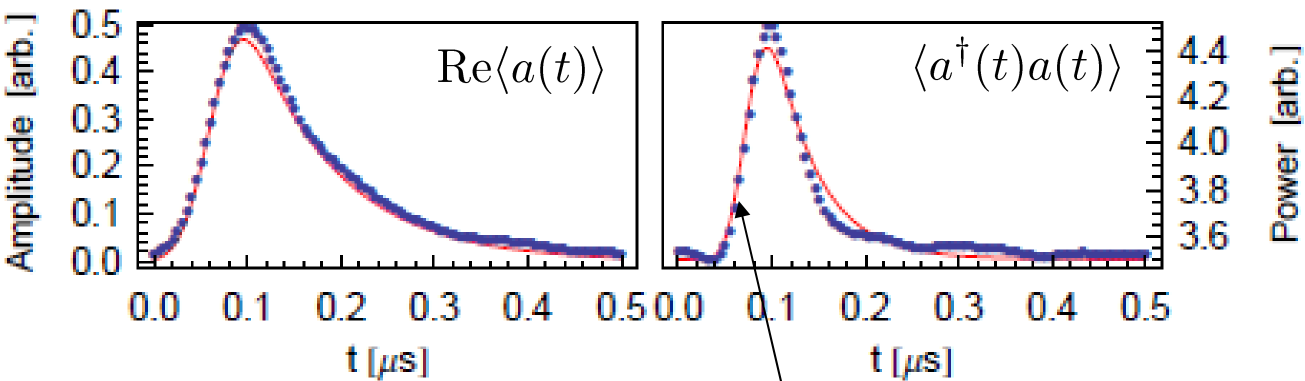


Improved single-photon source





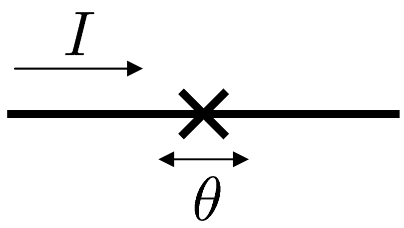
Emission spectrally separated from excitation

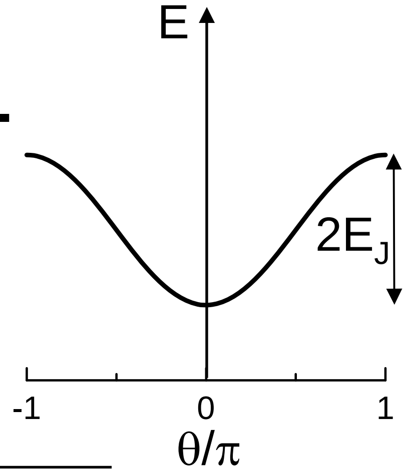


Slope limited by measurement bandwidth

D. Bozyigit et al. arXiv:1002.3738 (2010) ETH Zurich

Strong nonlinearity of Josephson junction circuits





Energy $$E = -E_J \cos\theta$$

Current $$I = \frac{2e}{\hbar}\frac{\partial E}{\partial \theta} = I_c \sin\theta$$

Inductance

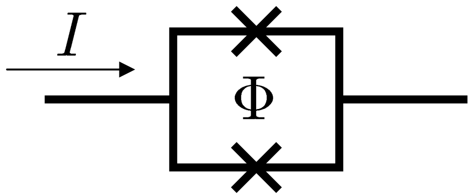
$$L_J = \left(\frac{2e}{\hbar}\frac{\partial I}{\partial \theta}\right)^{-1} = \frac{\Phi_0}{2\pi I_c \cos\theta} = \frac{\Phi_0}{2\pi I_c \cos(\arcsin I/I_c)}$$

$$\Phi_0 \equiv \frac{h}{2e}$$

SQUID

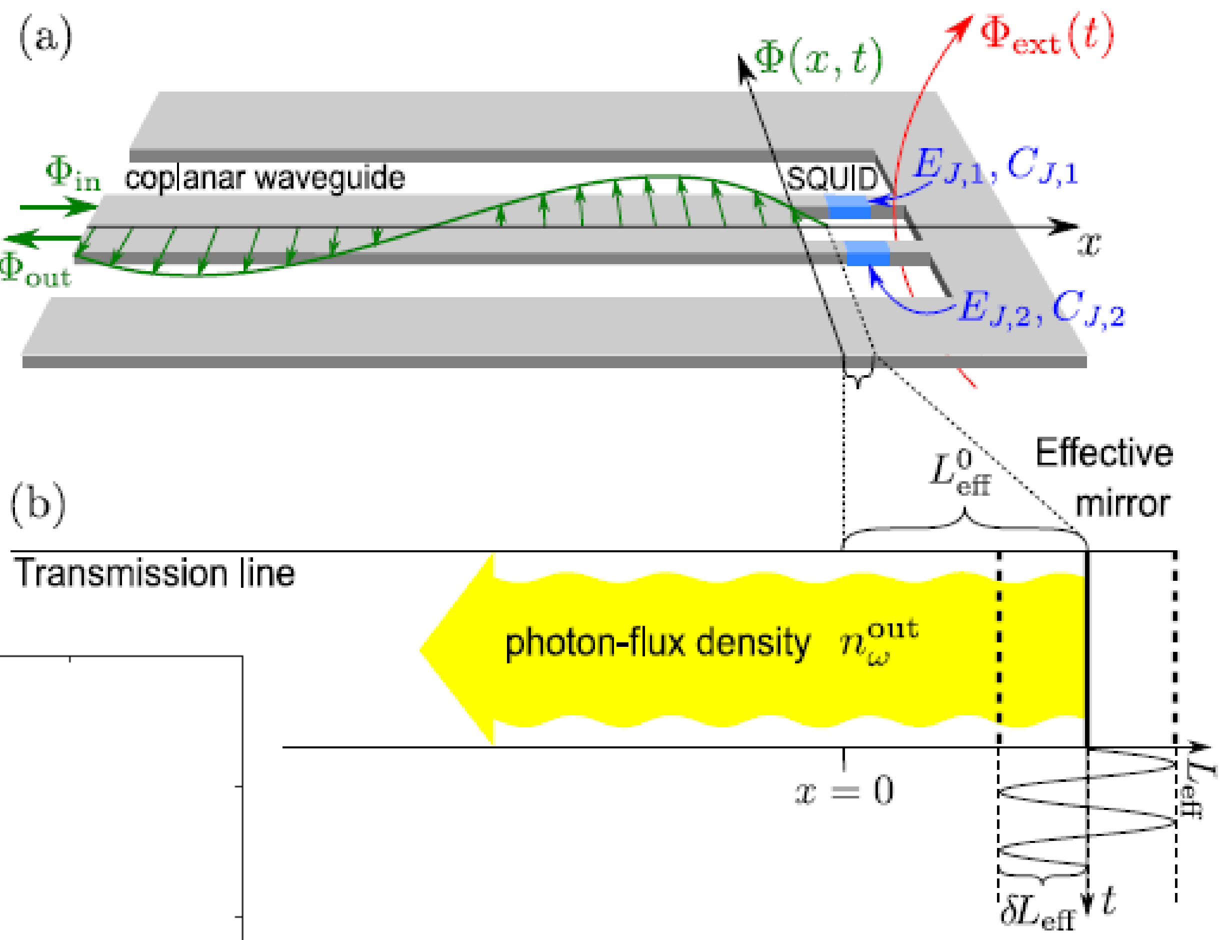
$$I_c^{\mathrm{SQ}}(\Phi) = 2I_c \left|\cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|$$

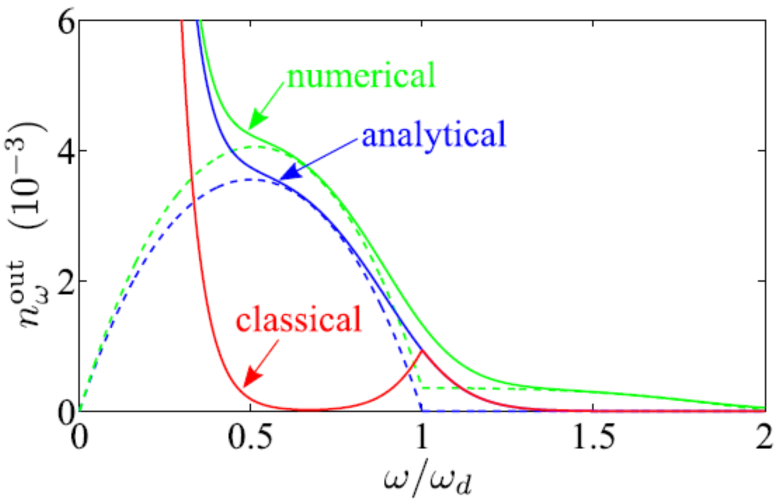
$$L_J^{\mathrm{SQ}} = L_J^{\mathrm{SQ}}(\Phi(t)) = \frac{\Phi_0}{2\pi I_c^{\mathrm{SQ}}(\Phi(t))}$$



Dynamical Casimir effect

$$L_J^{SQ} = L_J^{SQ}(\Phi(t))$$



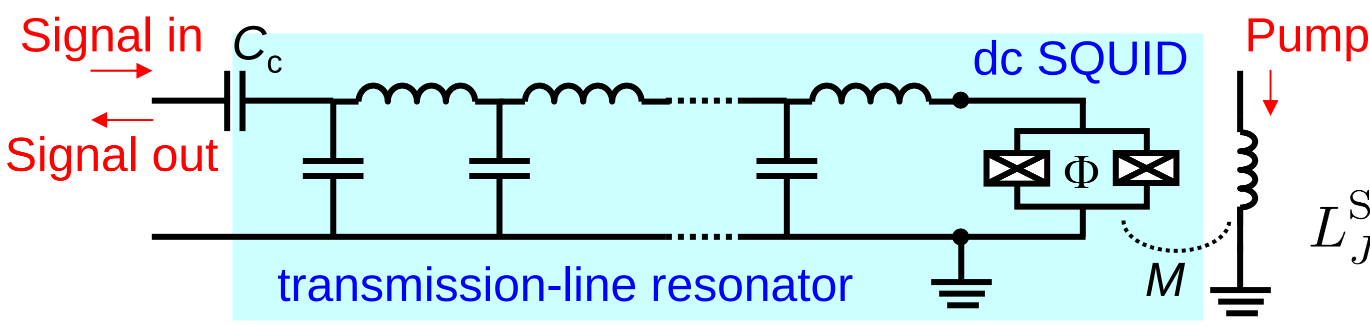


Dynamical tuning of the boundary condition

J. R. Johansson et al. PRL 103, 147003 (2009). (RIKEN & Chalmers)

Flux-driven Josephson parametric amplifier

SQUID = Superconducting Quantum Interfering Device
⇒ flux dependent variable nonlinear inductance



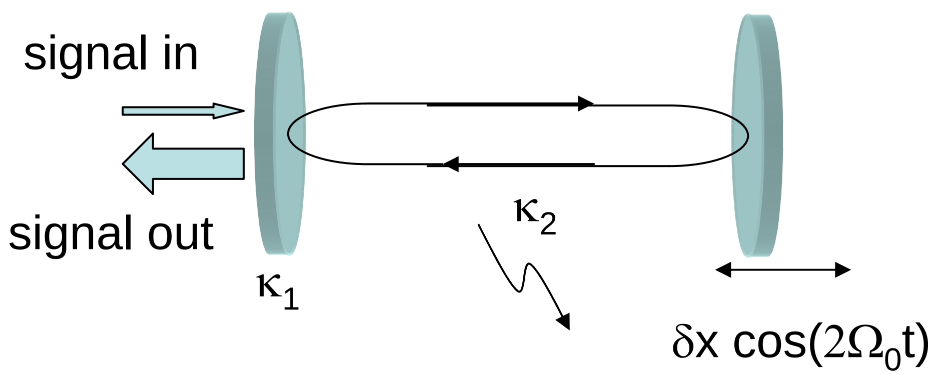
$$L_J^{SQ} = L_J^{SQ}(\Phi(t))$$

Degenerate parametric amplifier
∙∙∙ phase sensitive

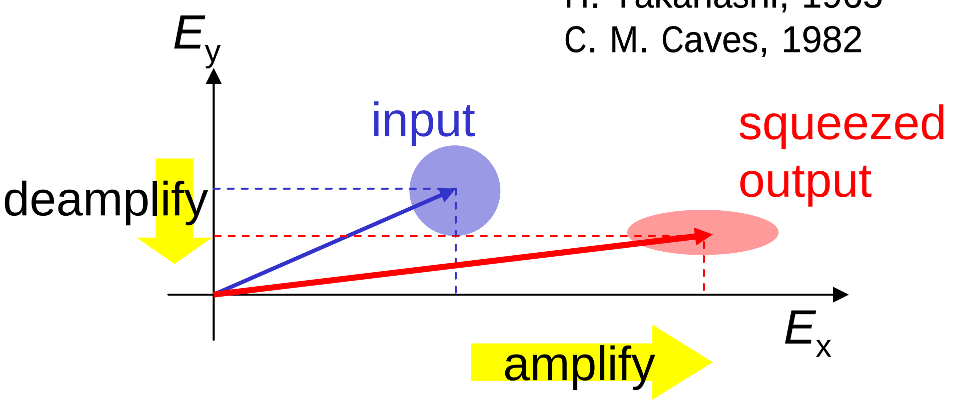
⇨ noiseless amplification

H. Takahashi, 1965
C. M. Caves, 1982

$\omega_r = \Omega_0[1 + \delta \cos(2\Omega_0 t)]$



Opto-mechanical analogue

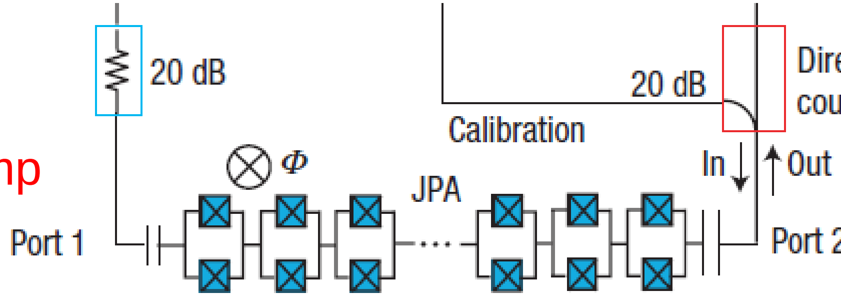


T. Yamamoto et al. APL 93, 042510 (2008)

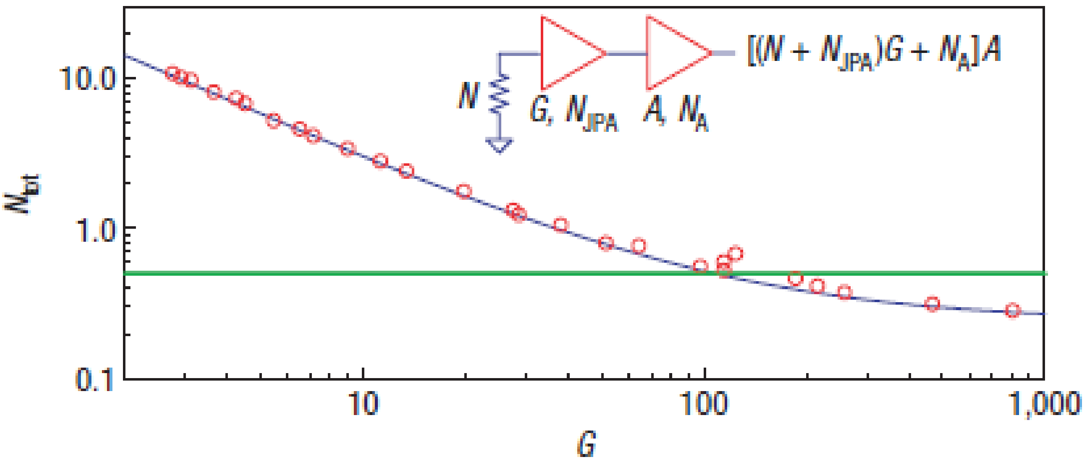
Current-driven Josephson parametric amplifier

$$L_J = L_J(I)$$

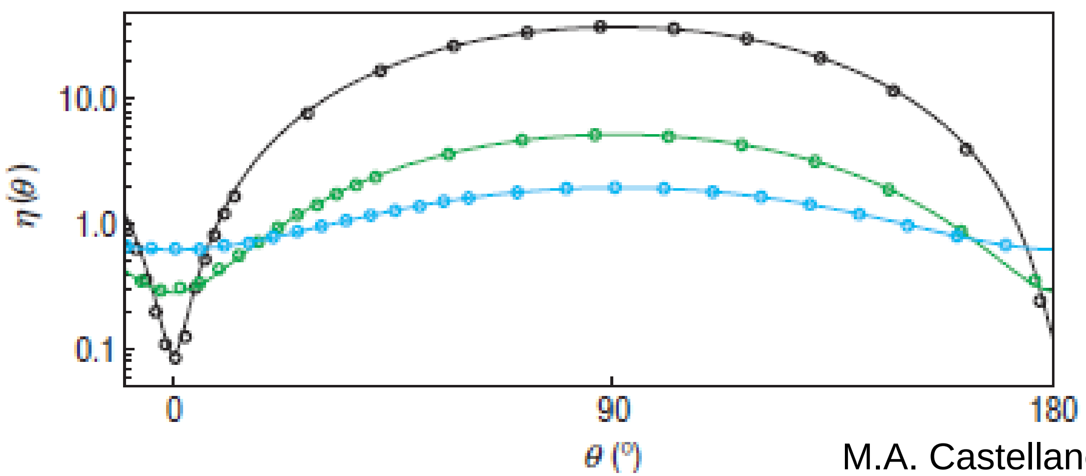
pump



signal



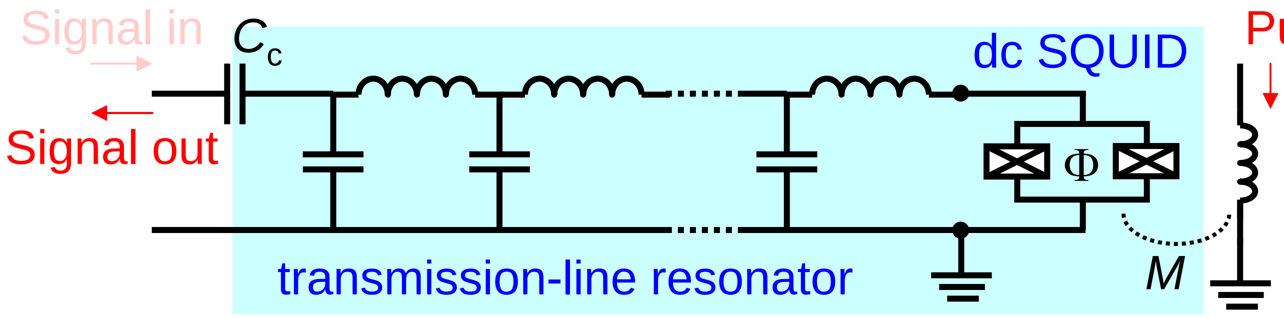
Added noise below standard quantum limit (SQL)



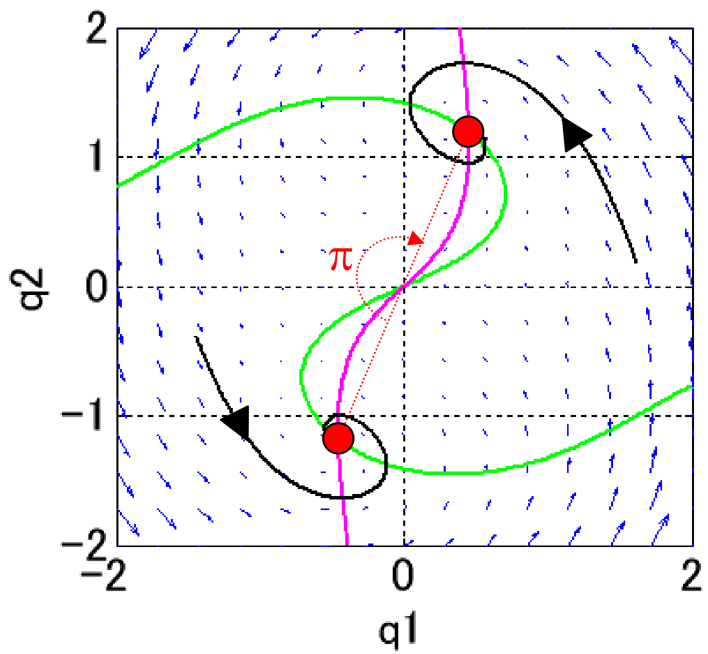
>10dB vacuum squeezing

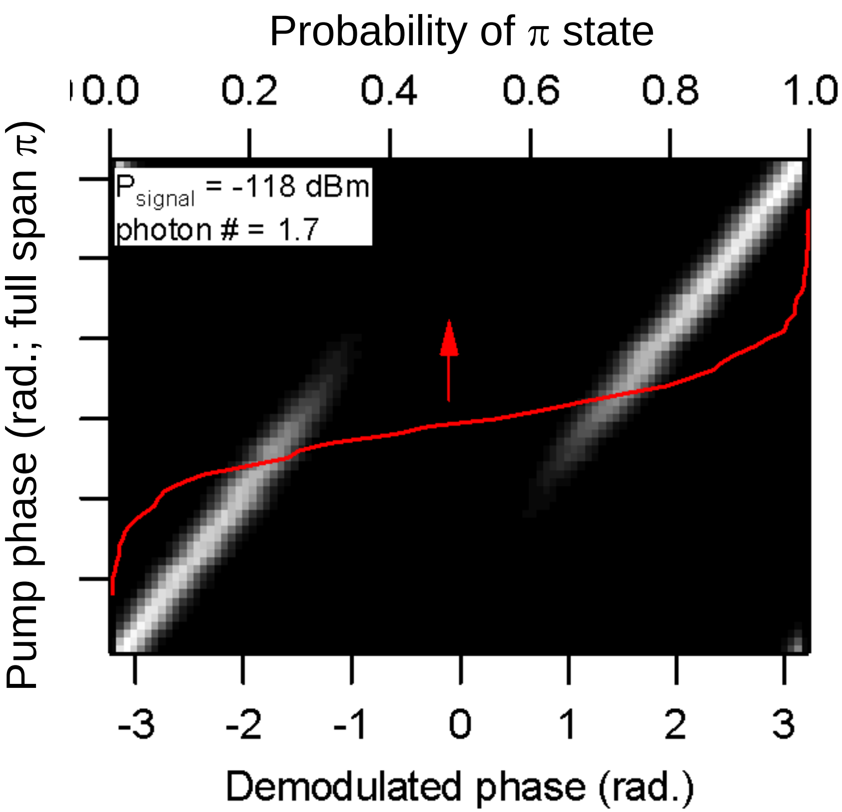
M.A. Castellanos-Beltran et al. Nature Phys. 4, 928 (2009) JILA

Parametric oscillator as a binary detector



Flux drive above threshold





T. Yamamoto et al. unpublished

Summary

Superconducting quantum circuits offer unique and versatile systems in microwave domain to investigate unprecedented parameter regimes of quantum optics.

The keywords are:

Strong coupling
- confined electromagnetic field modes
- large dipole of artificial atoms

Strong nonlinearity
- Josephson effect

Weak dissipation
- superconductivity
- confined electromagnetic field modes

Fascinating results have been obtained. But there remain a number of things to be developed and demonstrated.