Universal Thermodynamics of Strongly Interacting Fermi Gases

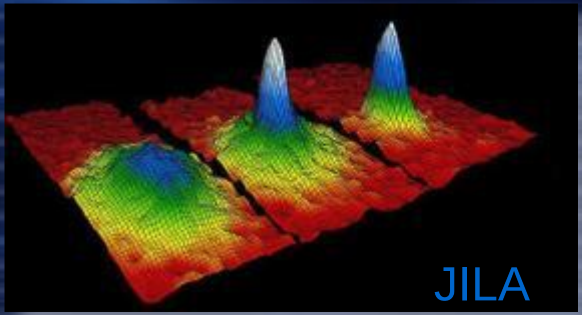
Takashi Mukaiyama

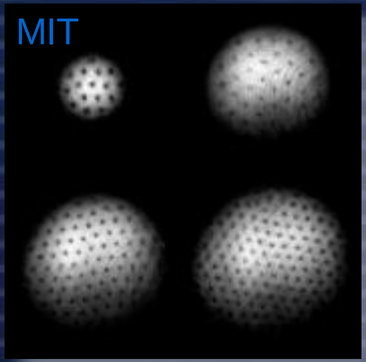
University of Electro-Communications

Japan Science Technology Agency, ERATO

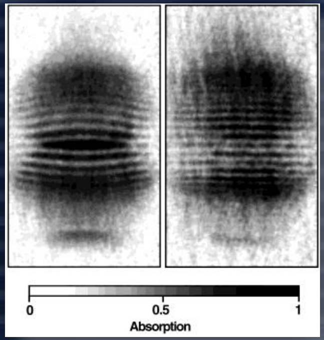
BEC in a cold atom system

1995 **<u>Realization of atomic gas Bose-Einstein condensation</u>**



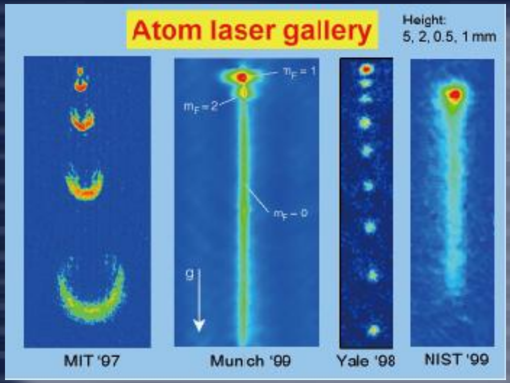


vortices

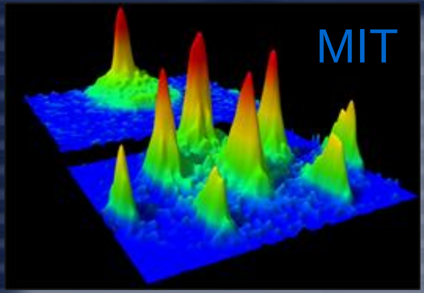


Science 275, 637 (1997).

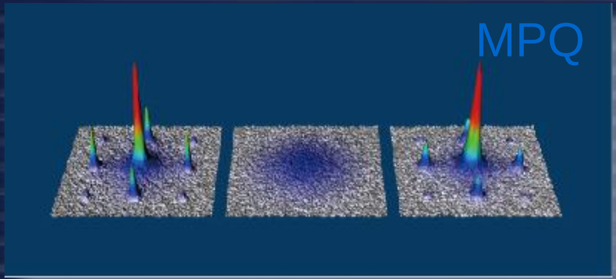
interference



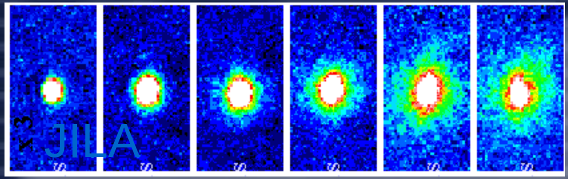
Atom laser



Super-radiance



Mott-insulator phase



Bose nova

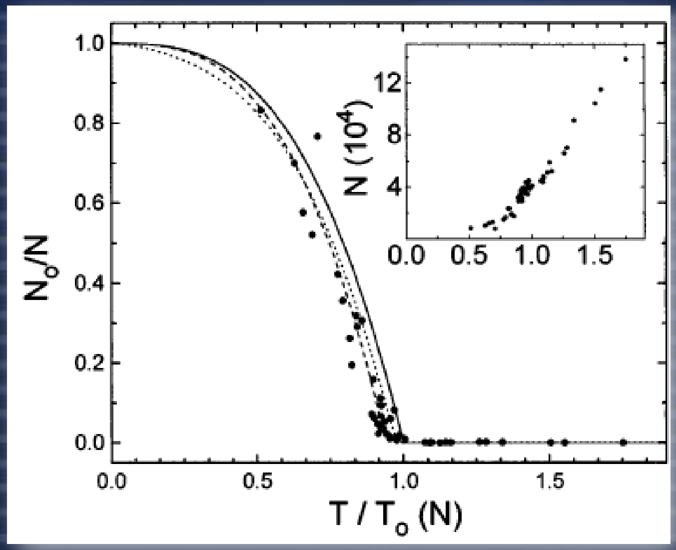
BEC in a cold atom system

Cold atoms are

- very dilute (10^{11}~10^{14} cm^{-3}),
- with no impurities, no defects.

→ Amenable to simple theoretical description

Condensate fraction



J. R. Ensher, et al.,
Phys. Rev. Lett. **77**, 4984 (1996).

5% deviation of critical temperature from theoretical predictions →

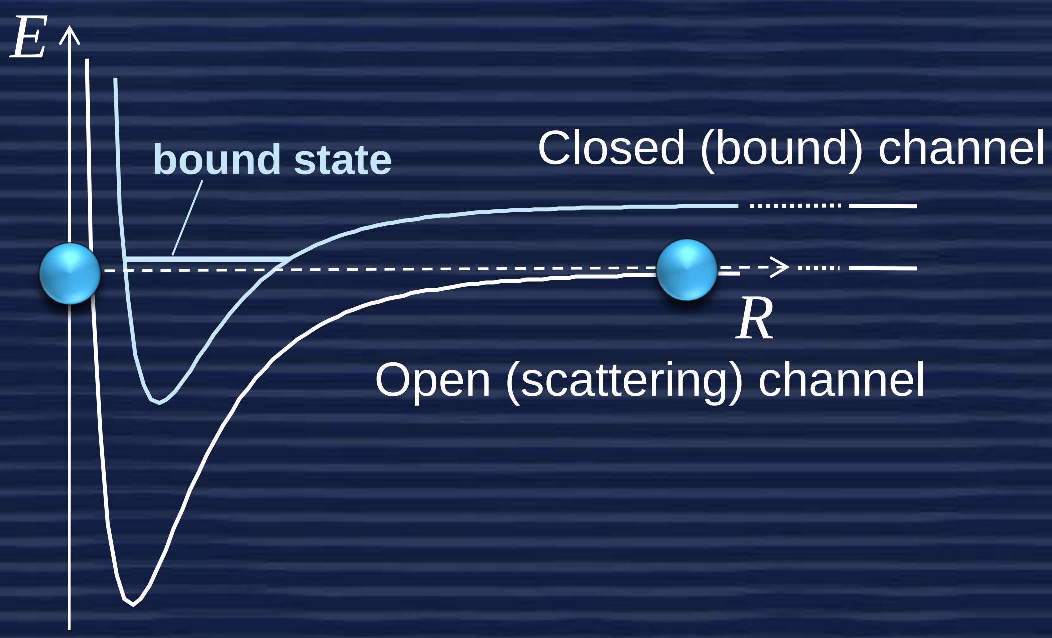
- 3% shift due to finite number correction
- 2% shift due to interaction

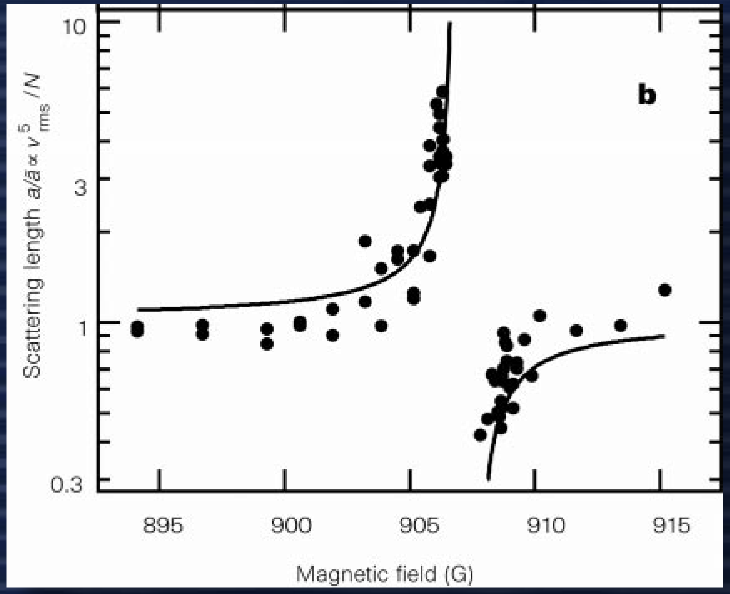
Inter-atomic interaction is tunable !!

Feshbach resonance

There are two channels corresponding to different spin states.

Resonance occurs when open and closed channel are energetically degenerate.

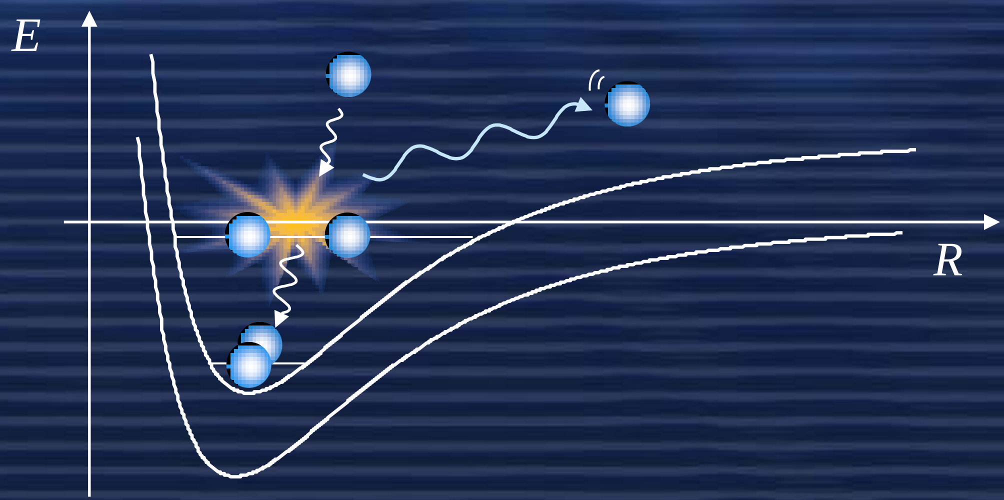


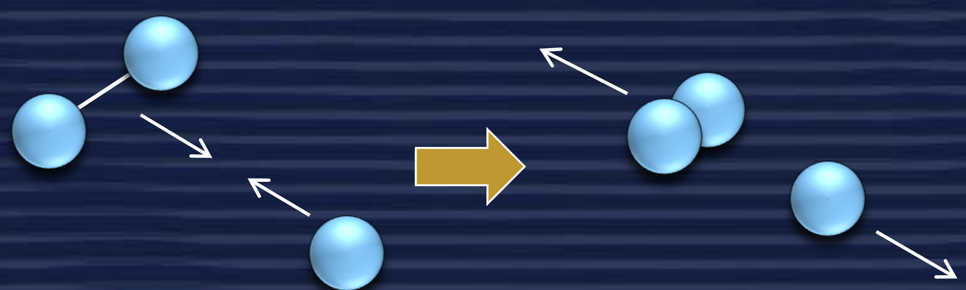


S. Inouye, et al.,
Nature **392**, 151 (1998).

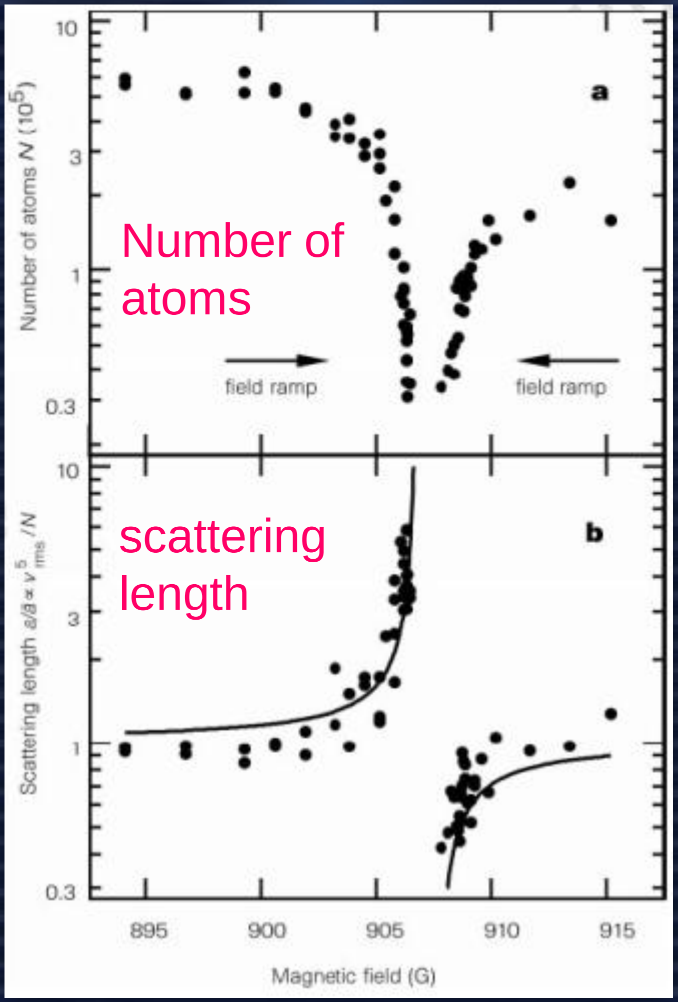
Loss near Feshbach resonance

loss due to
vibrational quenching





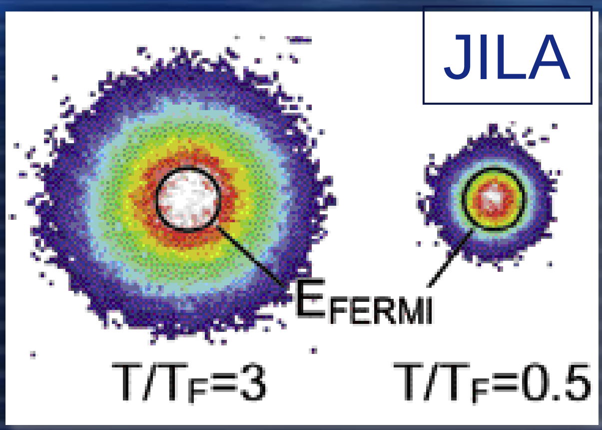
Vibrational quenching



S. Inouye et al.,
Nature, **392**, 151 (1998).

ultracold fermionic atoms

1999 Fermi degenerate gas



Collision channel

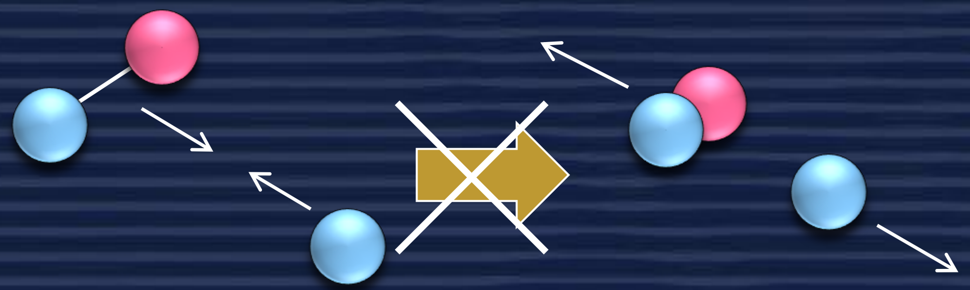
Identical bosons : l=0(s-wave), l=2 (d-wave), …
Identical fermions: l=1 (p-wave), l=3 (f-wave), …

ultracold : s-wave is the dominant collision channel.

⟹ Identical fermions do not collide.

Think about two-component fermions 

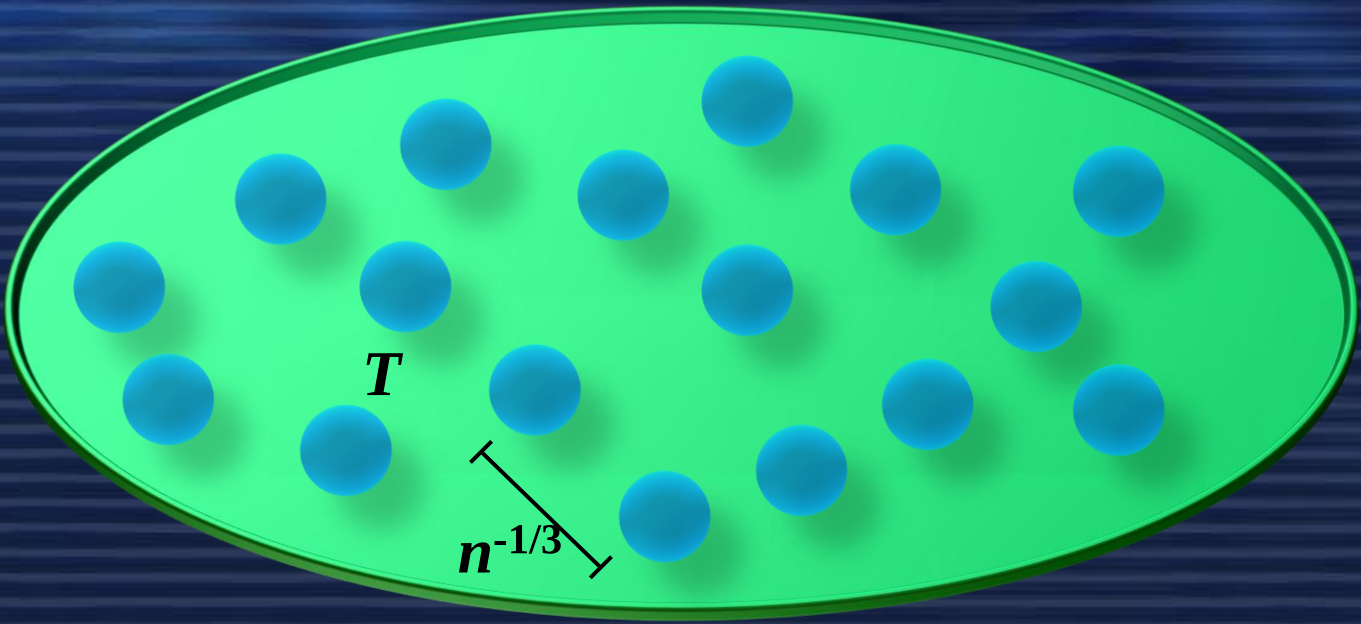
At the Feshbach resonance for  and  ,
no loss occurs due to Pauli exclusion principle.



Therefore two-component fermions are stable even at
a Feshbach resonance.

So, we are able to prepare an interacting (reasonably stable) two-component Fermi gas of atoms with an arbitrary interaction strength !!

Ideal Fermi gas



Thermodynamic behavior of an ideal Fermi gas is described by its temperature T and density n.

Thermodynamic of an ideal Fermi gas

Fermi-Dirac distribution

$$n(\varepsilon)=\frac{1}{e^{\beta(\varepsilon-\mu)}+1}=\frac{1}{z^{-1}e^{\beta\varepsilon}+1}\qquad\left(z\equiv e^{\beta\mu},\ \beta=(k_{\mathrm{B}}T)^{-1}\right)$$

$$\begin{cases} D(\varepsilon)=\dfrac{V}{4\pi^2}\left(\dfrac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon} \\ -Li_s(-z)=\dfrac{1}{\Gamma(s)}\displaystyle\int_0^\infty\frac{t^{s-1}}{z^{-1}e^{t}+1}dt \end{cases}$$

$$N=\int_0^\infty\frac{D(\varepsilon)}{z^{-1}e^{\beta\varepsilon}+1}d\varepsilon$$

$$=\ \cdots$$

$$=\frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\beta^{-3/2}\frac{\sqrt{\pi}}{2}\left(-Li_{3/2}(-z)\right)$$

$$n=\frac{N}{V}=\frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(k_{\mathrm{B}}T)^{3/2}\frac{\sqrt{\pi}}{2}\left(-Li_{3/2}(-z)\right)$$

$$=\frac{1}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E_{\mathrm{F}}^{3/2}\qquad E_{\mathrm{F}}=\frac{\hbar^2}{2m}\left(6\pi^2 n\right)^{2/3}$$

Thermodynamic of an ideal Fermi gas

$$\left(\frac{k_B T}{E_F}\right)^{-3/2} = -\frac{3\sqrt{\pi}}{4} Li_{3/2}(-z)$$

$$z = e^{\beta\mu} = \exp\left(\frac{\mu}{k_B T}\right) = \exp\left(\frac{\mu/E_F}{k_B T/E_F}\right) = \exp\left(\frac{\mu/E_F}{k_B T/E_F}\right)$$

$$\left(\frac{k_B T}{E_F}\right)^{-3/2} = -\frac{3\sqrt{\pi}}{4} Li_{3/2}\left[-\exp\left(\frac{\mu/E_F}{k_B T/E_F}\right)\right]$$

$$\boxed{\frac{\mu}{E_F} = f_\mu\left(\frac{k_B T}{E_F}\right)}$$

Thermodynamic of an ideal Fermi gas

$$E = \int_0^\infty \frac{\varepsilon D(\varepsilon)}{z^{-1}e^{\beta\varepsilon}+1} d\varepsilon$$

$$\frac{E}{NE_{\mathrm{F}}} = -\frac{3\sqrt{\pi}}{4}\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)^{5/2} Li_{5/2}(-z)$$

$$\boxed{\frac{E}{NE_{\mathrm{F}}} = f_E\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)}$$

Other thermodynamic functions also have this similarity.

$$\frac{S}{k_{\mathrm{B}}} = f_S\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

$$\frac{F}{NE_{\mathrm{F}}} = f_F\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Thermodynamic of an ideal Fermi gas

Internal energy : $\dfrac{E}{NE_{\mathrm{F}}} = f_{E,ideal}\left(\dfrac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$

Helmholtz free energy : $\dfrac{F}{NE_{\mathrm{F}}} = f_{F,ideal}\left(\dfrac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$

Chemical potential : $\dfrac{\mu}{E_{\mathrm{F}}} = f_{\mu,ideal}\left(\dfrac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$

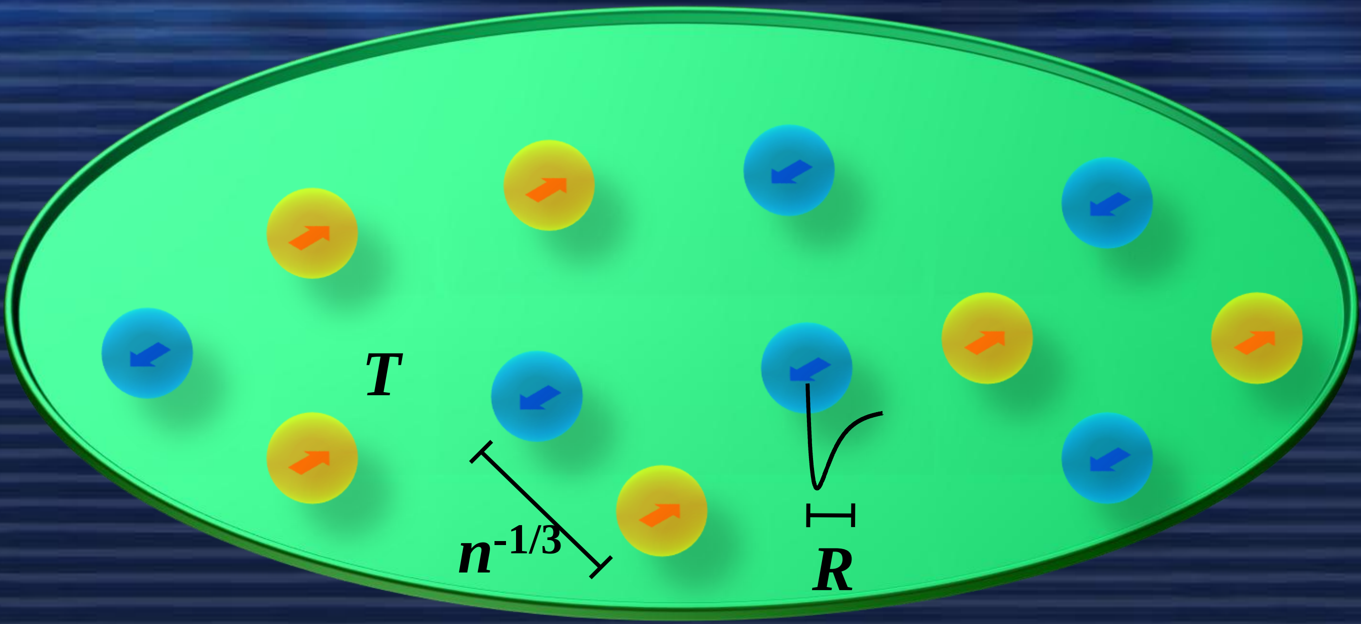
Entropy : $\dfrac{S}{Nk_{\mathrm{B}}} = f_{S,ideal}\left(\dfrac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$

Dimensionless functions

Material specific parameter, such as m, is taken up by E_{F} (T_{F}).
(Shape of the functions do not depend on the particle's nature.)

⇒ Universal thermodynamics

Ultracold, dilute, interacting Fermi gases



- <u>ultracold</u> : s-wave is the dominant channel.

  ⇒  collide only with 

- <u>dilute</u> : details of the potential is much smaller than $n^{-1/3}$

  ⇒ The collision process can be described by a single parameter, so-called scattering length a_s.

Thermodynamic of an interacting Fermions

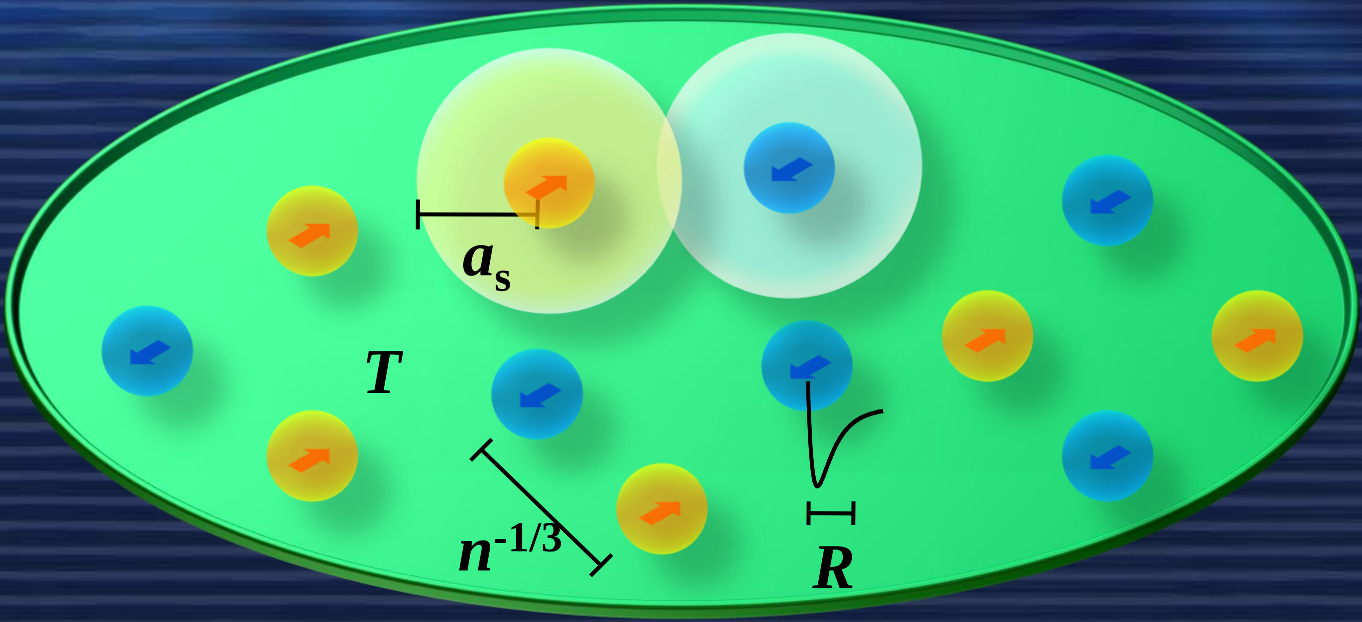
Ideal Fermi gas

$$\frac{E}{NE_{\mathrm{F}}} = f_{E,ideal}\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Fermi gas with interaction

$$\frac{E}{NE_{\mathrm{F}}} = f_{E}\left(k_{\mathrm{B}}T,\, E_{\mathrm{F}},\, E_{\mathrm{int}}\left(a_{\mathrm{s}}\right)\right)$$

Ultracold dilute Fermi gas



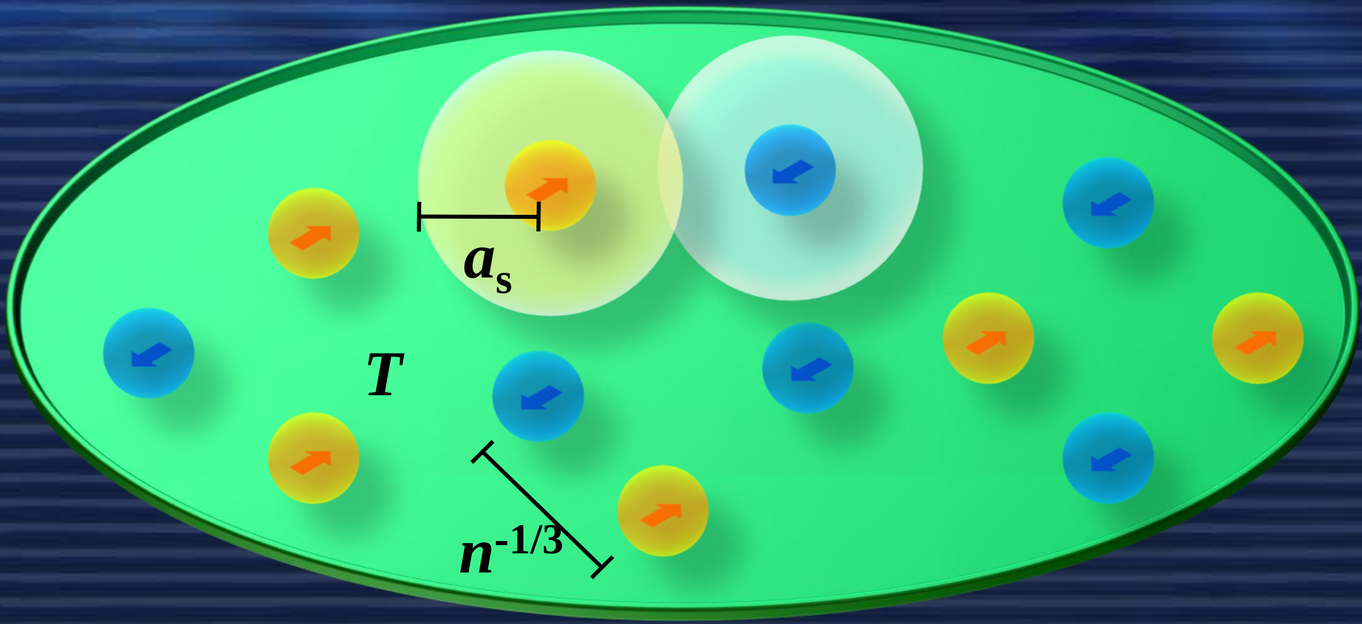
Remember the fact that a_s ***is tunable!!***

Then, what happens when…

$$|a_s| \longrightarrow \infty$$

This situation is called unitarity limit.

Unitarity limit and Universality

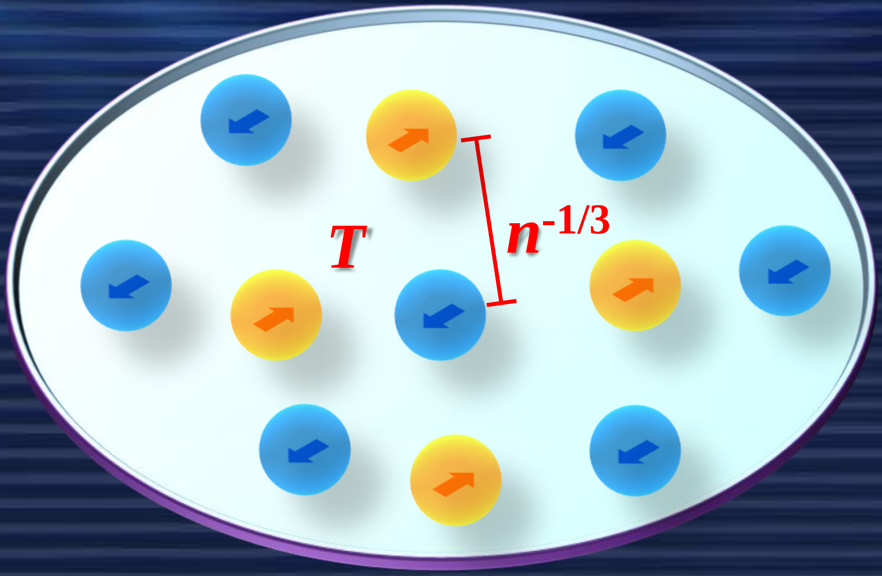


a_s drops out of the description of the thermodynamics.

Thermodynamics depends only by the density n and temperature T.

Universal thermodynamics holds again…?

Unitarity limit and Universality



a_s drops out of the description of the thermodynamics.

Thermodynamics depends only by the density n and temperature T.

Universal thermodynamics holds again…?

Thermodynamic of an interacting Fermions

Ideal Fermi gas

$$\frac{E}{NE_F} = f_{E,ideal}\left(\frac{k_B T}{E_F}\right)$$

Fermi gas with interaction

$$\frac{E}{NE_F} = f_E\left(k_B T, k_B T_F, U(a)\right)$$

When the scattering length diverges…

$$\frac{E}{NE_F} = f_E\left(k_B T, k_B T_F, \cancel{U(a)}\right) \Rightarrow f_{E,|a|=\infty}\left(k_B T, k_B T_F\right) = f_{E,|a|=\infty}\left(\frac{k_B T}{E_F}\right)$$

There is a hypothesis that the thermodynamic functions again have the universal form.

Universal hypothesis

Universal thermodynamics

According to universal hypothesis, all thermodynamics should obey the universal functions:

Internal energy : $$\frac{E}{NE_{\mathrm{F}}} = f_E\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Helmholtz free energy : $$\frac{F}{NE_{\mathrm{F}}} = f_F\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Chemical potential : $$\frac{\mu}{E_{\mathrm{F}}} = f_\mu\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Entropy : $$\frac{s}{Nk_{\mathrm{B}}} = f_S\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Dimensionless universal functions, (shape of the function is different from those for an ideal gas)

System looks like a non-interacting Fermi gas.
(no other dimensional parameters involved in the problem)

Universal thermodynamics

Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

$$E_{gs} = f\left(N, V, m\right) = N \cdot E_{\mathrm{F}} \times \xi$$

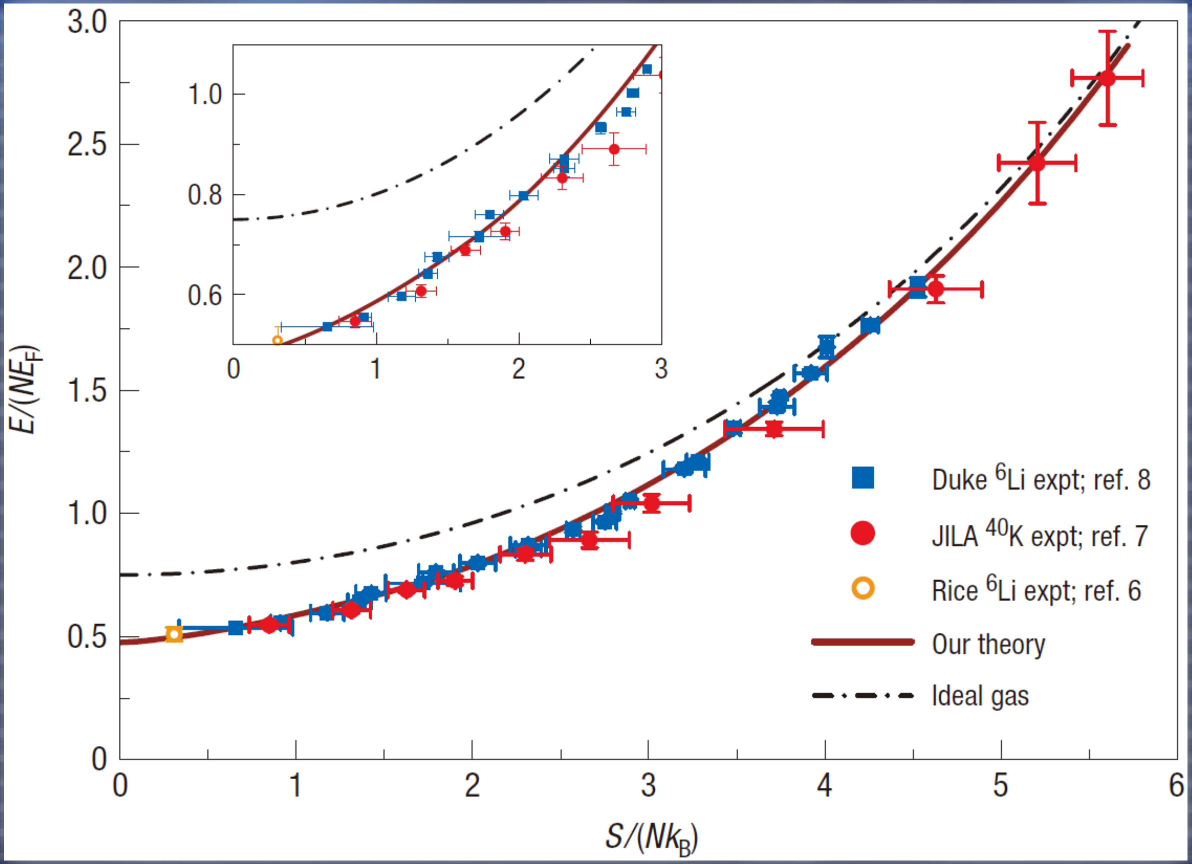
pure number

Besides pure theoretical curiosity, this problem is relevant to neutron stars!

Neutron star

Universal thermodynamics

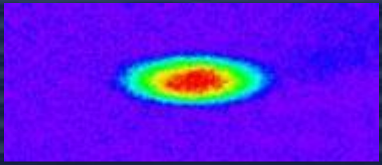
H. Hu, P. D. Drummond & X.-J. Liu,
Nature Physics **3**, 469 - 472 (2007)

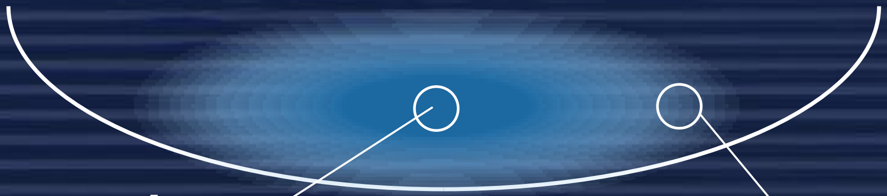


$f_E\left(\frac{k_B T}{E_F}\right)$ is still not known…

T is constant over the cloud (thermal equilibrium).
E_F depends on the position (local density).

$\Rightarrow$ $\frac{k_B T}{E_F}$ is position-dependent.





$$\frac{k_B T}{E_F(n_0)}$$

$$\frac{k_B T}{E_F\left[n(\mathbf{r})\right]} \geq \frac{k_B T}{E_F(n_0)}$$

Global measurement only gives the integration of all the different phases.

Goal of this experiment

Measurement of <u>local</u> thermodynamic quantities

and

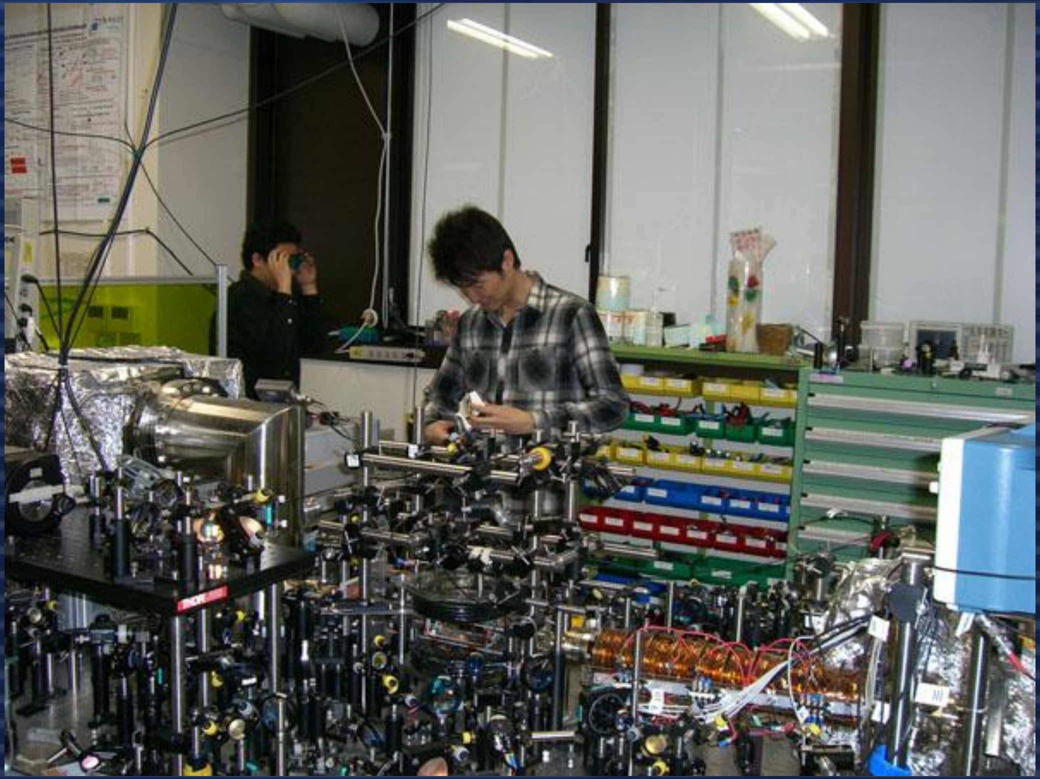
the determination of the universal thermodynamic function.

$$\frac{E}{NE_{\mathrm{F}}} = f_E\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right) \Rightarrow \frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})E_{\mathrm{F}}\left[n(\mathbf{r})\right]} = f_E\left(\frac{T}{T_{\mathrm{F}}\left[n(\mathbf{r})\right]}\right)$$

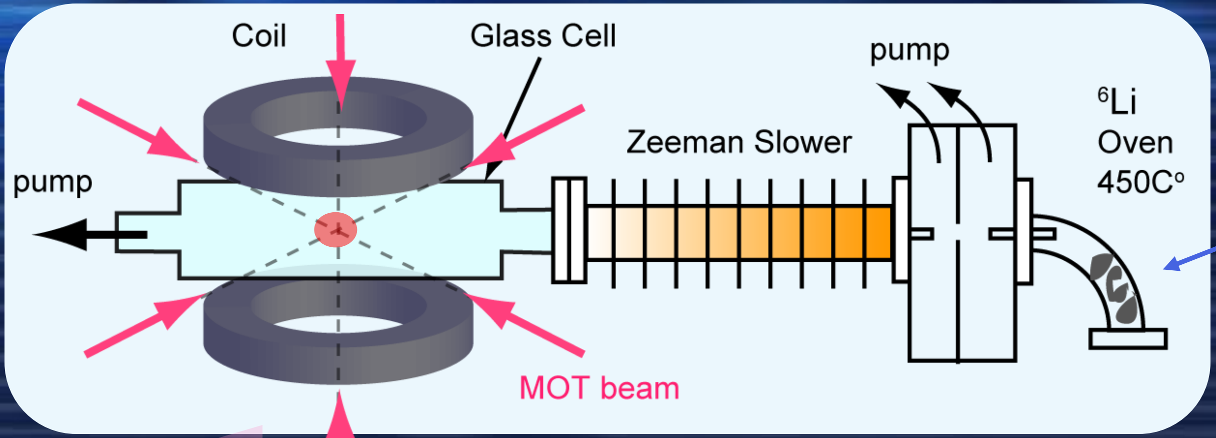
ε : local energy density

$$E_{\mathrm{F}} = k_{\mathrm{B}}T_{\mathrm{F}}$$

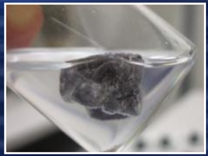
Experiment setup



原子の冷却(レーザー冷却)



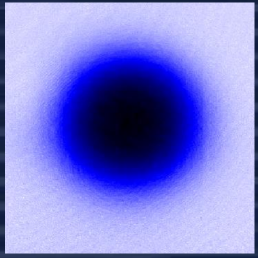
金属Li



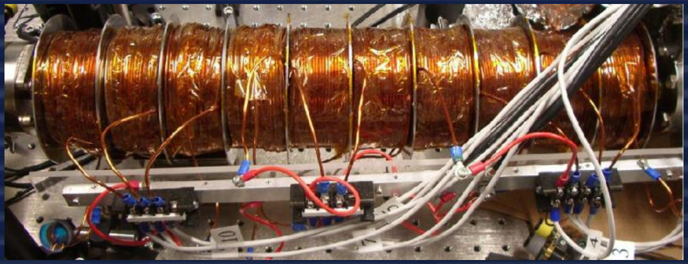
MOT　T = 200 μK　N > 10^8　Zeeman Slower　Liの蒸気(450℃)

MOT(磁気光学トラップ)
四重極磁場と共鳴光を組み合わせた冷却

吸収イメージ



Zeeman Slower
共鳴光による減速
ドップラーシフトをゼーマンシフトでキャンセル



共振器光トラップ

Optical dipole trap

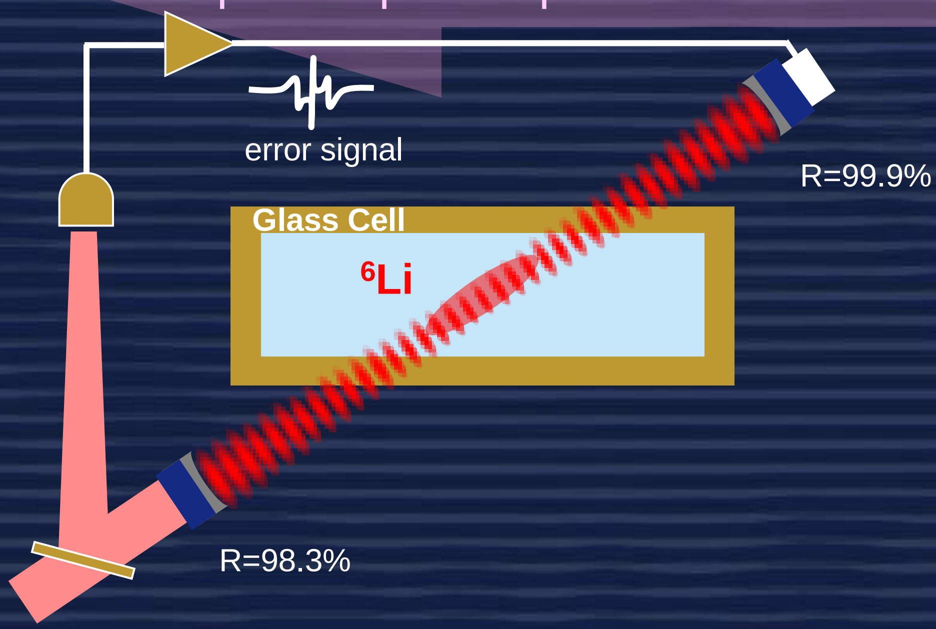
- No need to use multiple-coil configuration as used in a magnetic trap
- wide optical access
- trap can be turned off very quickly

Cavity enhanced optical dipole trap

T = 200 μK
N = 5 × 10⁷

MOT

T = 200 μK
N > 10⁸



λ = 1064nm, P = 10 W, w = 250 μm

⇒ トラップ深さ U = k_Bx5 μK

×

共振器で増幅 ×100

||

w = 260 μm トラップ深さ k_Bx2 mK

シングルビーム光トラップ

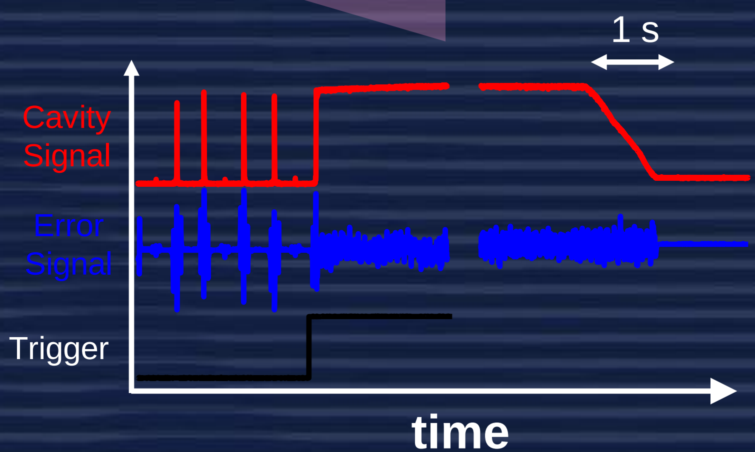
共振器光トラップ：定在波によるトラップ

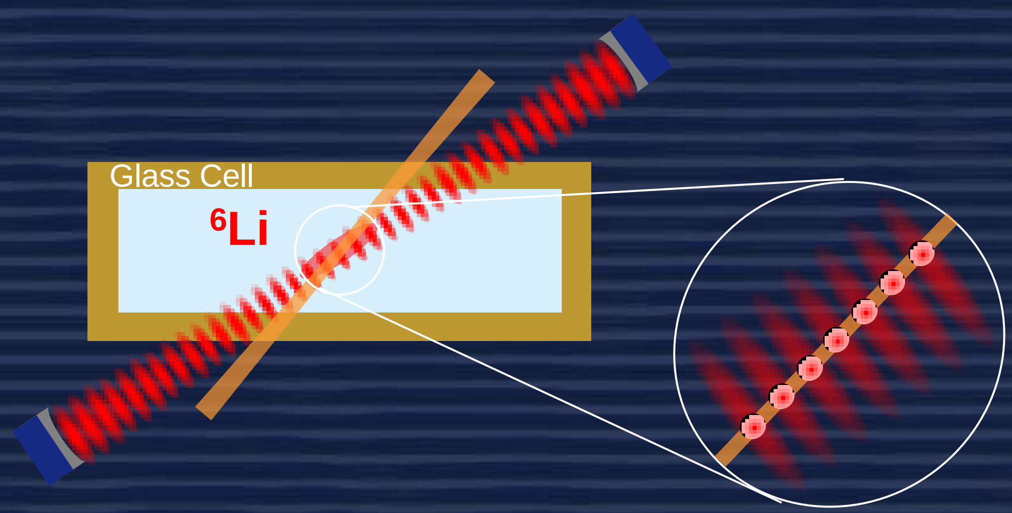
トラップした原子全体が熱緩和しない
調和型のトラップではない

→ 単純に絞ったビームにトラップしなおす

Single beam optical trap　$T = 80\ \mu K$　$N = 1 \times 10^7$

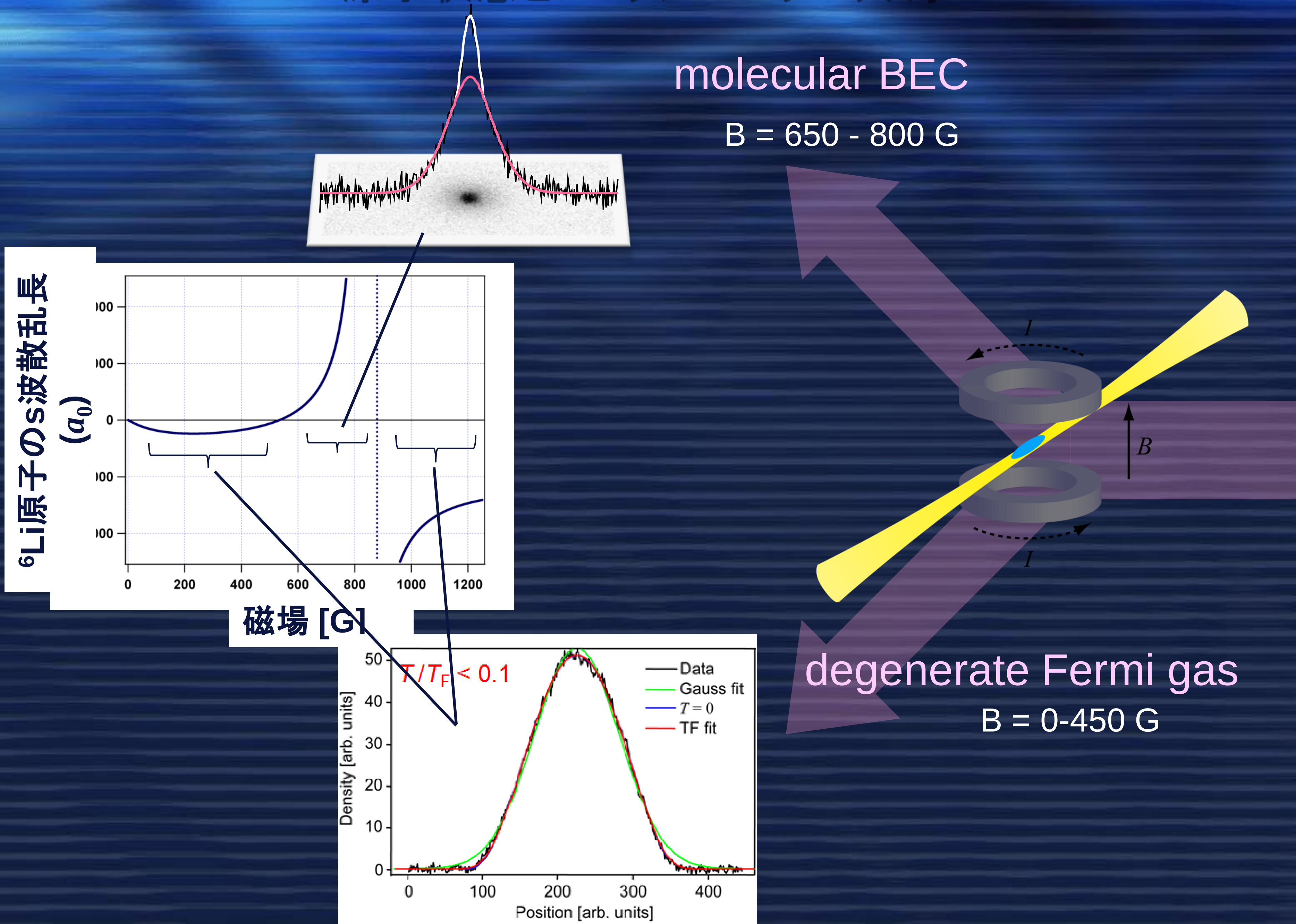
Cavity enhanced optical dipole trap　$T = 200\ \mu K$　$N = 5 \times 10^7$





・共振器の光強度を断熱的に下げる
・フェッシュバッハ共鳴を使った散乱断面積の増
共振器トラップに重なるようにビームを入射

原子状態とフェッシュバッハ共鳴

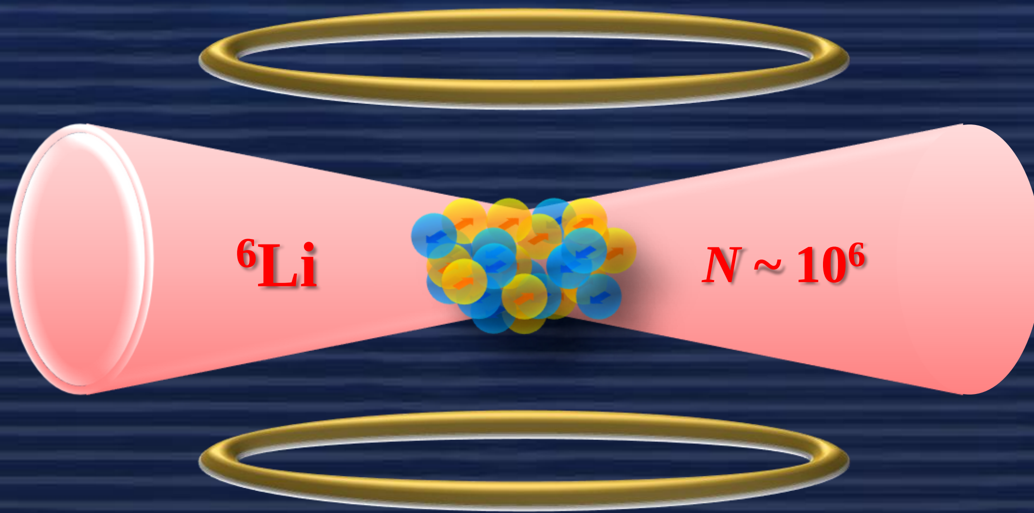


Experimental scheme

834Gauss

(Resonance magnetic field of Feshbach resonance)

Optical dipole trap
(1064nm)



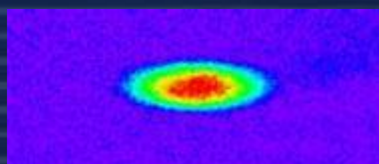
Equal mixture of

: $|F=1/2, m_F=+1/2>$

: $|F=1/2, m_F=-1/2>$

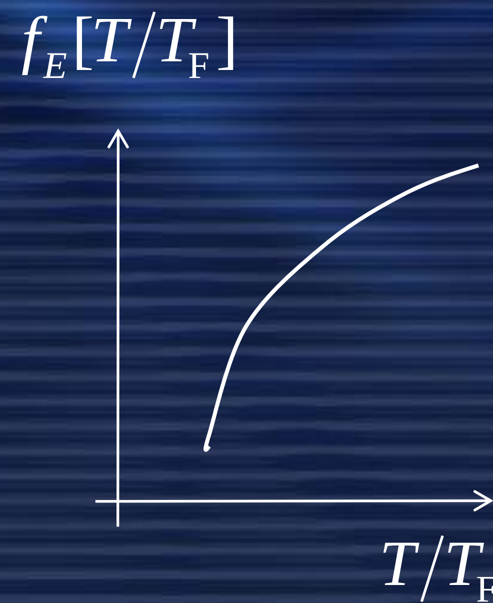
Determination of local energy $\varepsilon(\mathbf{r})$

$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_F\left[n(\mathbf{r})\right]} = f_E\left[T/T_F\right]$$



density profile

$$n(\mathbf{r})$$



Useful equations :

- **Equation of state of unitary gas :** $p(\mathbf{r}) = \frac{2}{3}\varepsilon(\mathbf{r})$
- **mechanical equilibrium (eq. of force balance) :**

$$\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{\mathrm{Trap}}(\mathbf{r}) = 0$$

$$n(\mathbf{r}) \Rightarrow p(\mathbf{r}) \Rightarrow \varepsilon(\mathbf{r})$$

Determination of temperature T

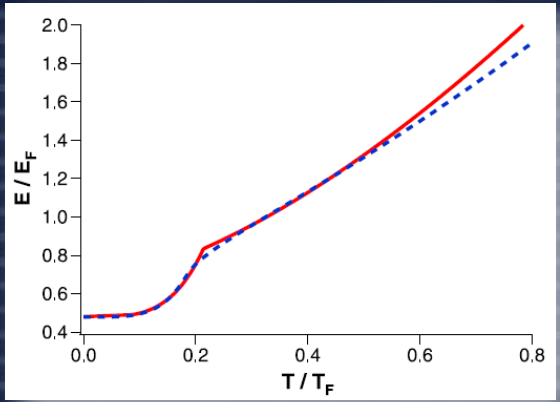
$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_F\left[n(\mathbf{r})\right]} = f_E[T/T_F]$$

$p(\mathbf{r}) = \frac{2}{3}\varepsilon(\mathbf{r})$ and $\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{\mathrm{Trap}}(\mathbf{r}) = 0$ ⟹ $E_{\mathrm{total}} = 2 \times E_{\mathrm{potential}}$

Adiabatic B-field sweep to turn off the interaction ⟹ entropy S

E_{total} vs S ⟹ E_{total} vs T

$$1/T = \partial S/\partial E$$



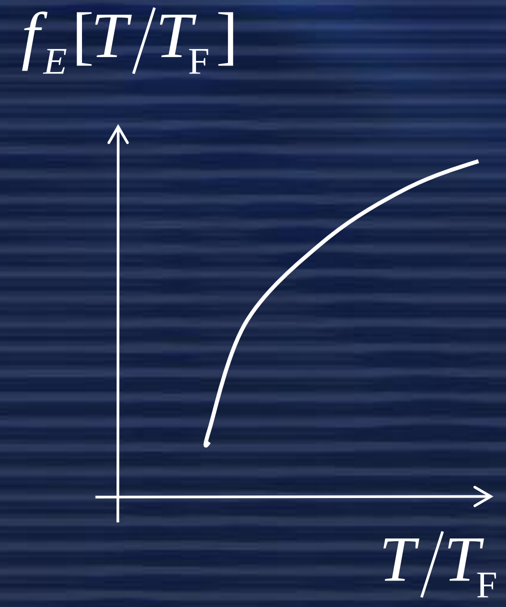
Le Luo and J.E. Thomas,
J Low Temp Phys **154,** 1 (2009).

Our scheme

$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_{\mathrm{F}}\left[n(\mathbf{r})\right]} = f_E[T/T_{\mathrm{F}}]$$

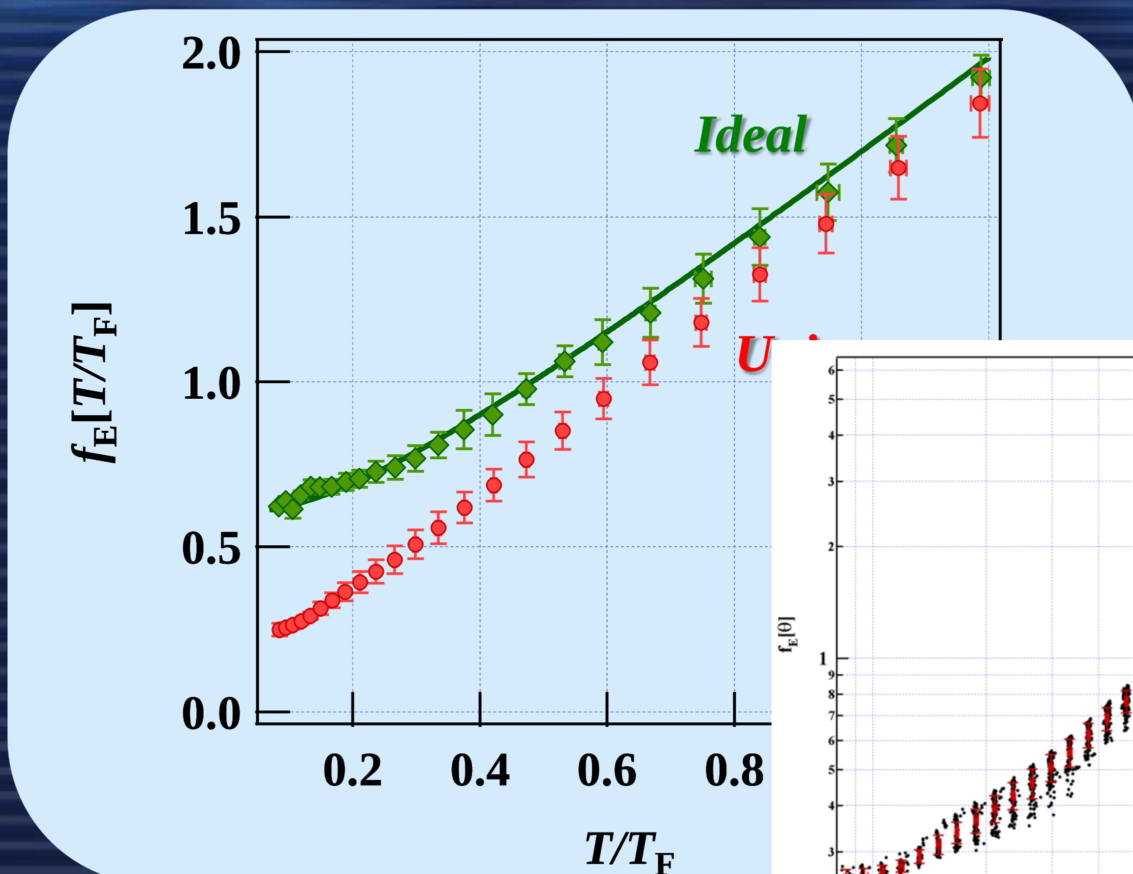
$$p = \frac{2}{3}\varepsilon$$

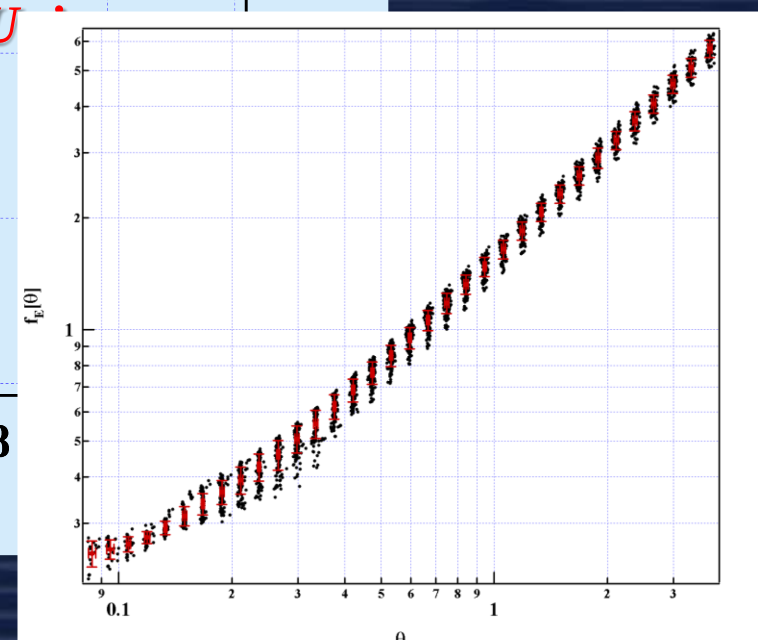
$$\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{\mathrm{Trap}}(\mathbf{r}) = 0$$



Experimental determination of $f_E[T/T_F]$

$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_F\left[n(\mathbf{r})\right]} = f_E[T/T_F]$$





M. Horikoshi, S. Nakajima,
M. Ueda and T. Mukaiyama,
Science, **327**, 442 (2010).

About 800 images are analyzed.

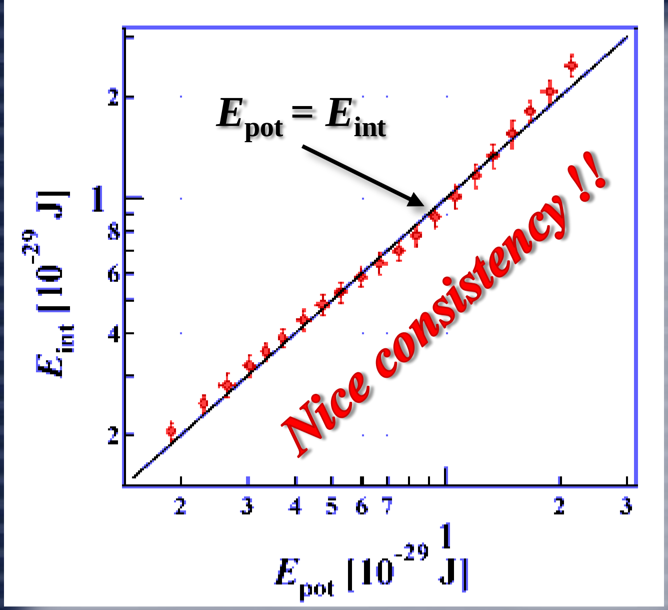
Verification of the determined $f_E[T/T_F]$

1. Energy comparison

$$E_{total} = 2\times E_{potential} \Rightarrow E_{pot} = E_{internal}$$

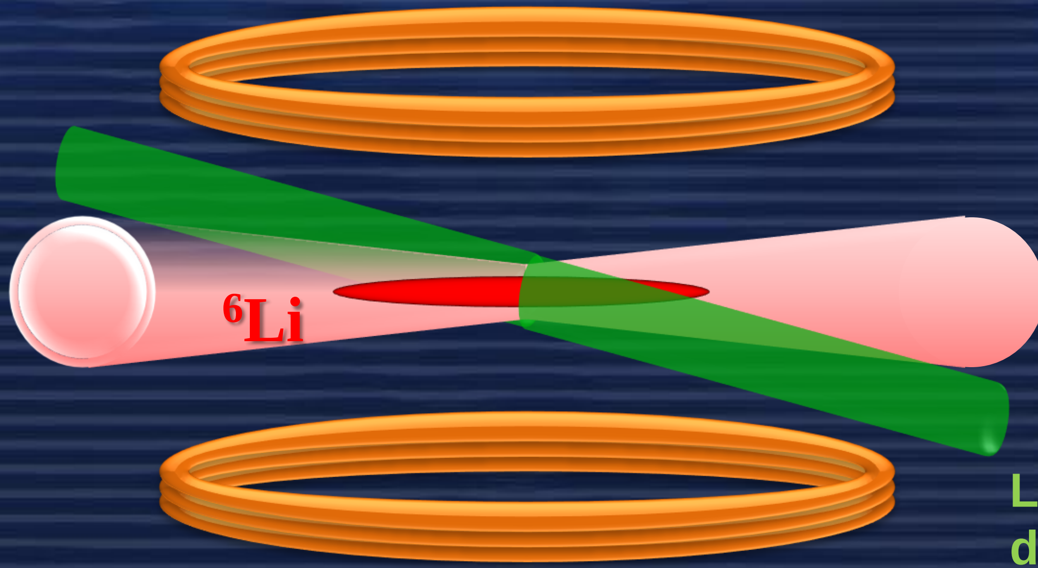
Comparison

- ✓ **Potential energy par particle :** $E_{pot} = \frac{3}{2} m\omega_z^2 < z^2 >$
- ✓ **Internal energy par particle :** $E_{internal} = \int n\varepsilon_F(n)\, \underline{\underline{f_E[\theta]}}\, \mathrm{d}V \big/ N$

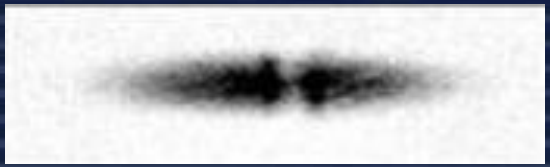


Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound



Light pulse to make density perturbation



Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound

Propagation time

0.1ms

1.1ms

2.1ms

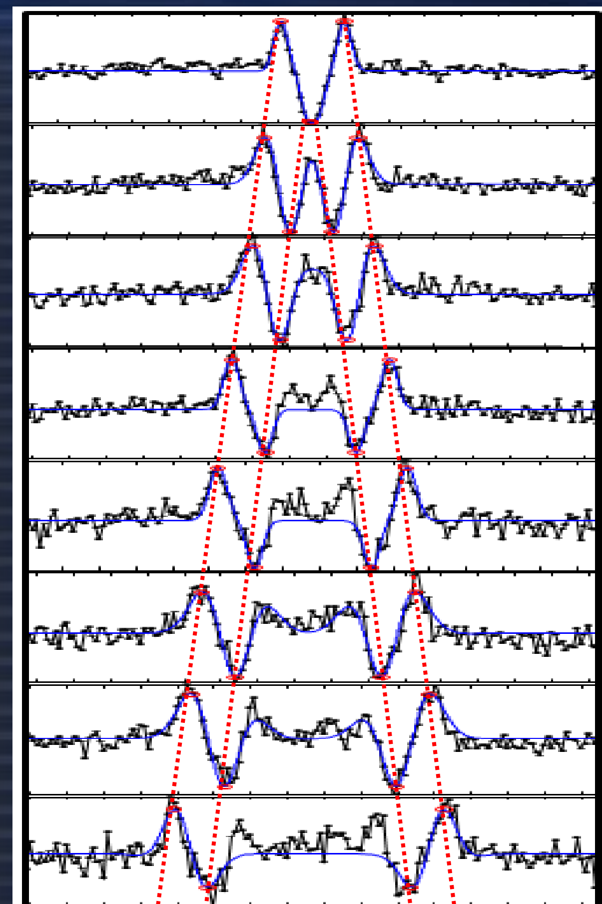
3.1ms

4.1ms

5.1ms

6.1ms

7.1ms



Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound

Unitary gas shows hydrodynamic behavior due to the large collision rate

Effective speed of the first sound : $$\bar{u}_1^2[n,\theta] = \left. \frac{\iint n\,\mathrm{d}x\mathrm{d}y}{m\iint n\left(\frac{\partial p}{\partial n}\right)^{-1}\mathrm{d}x\mathrm{d}y} \right|_{z=0}$$

$$p = \frac{2}{3}\varepsilon \propto f_E[T/T_F]$$

[P. Capuzzi, PRA **73**, 021603(R) (2006)]

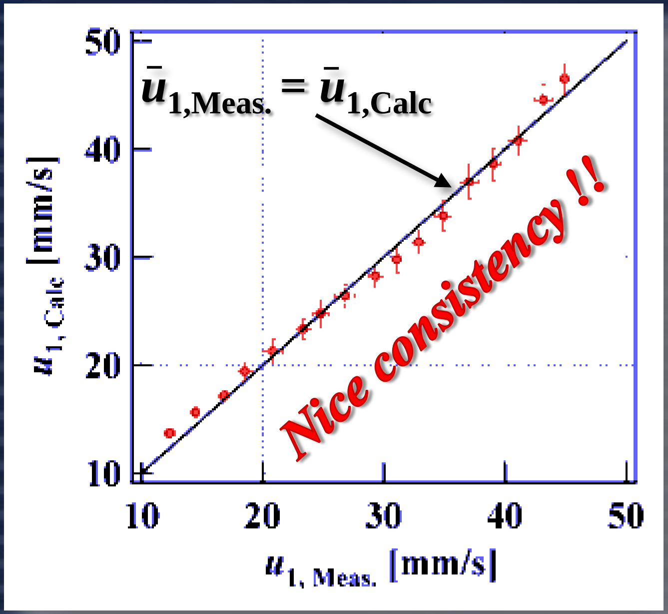
Comparison

Experiment

Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound

Experimental values vs. calculated values from $f_E[\theta]$

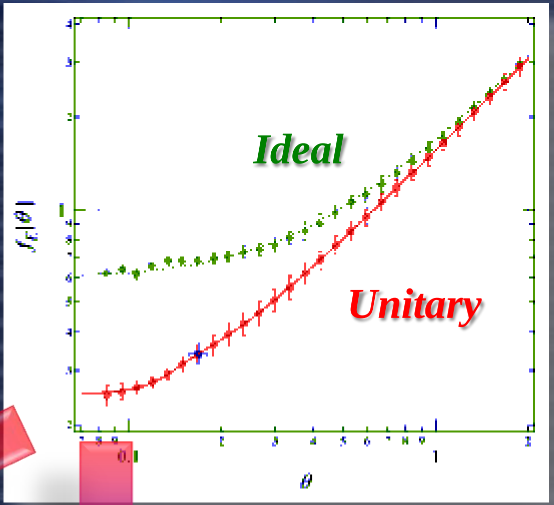


The universal function of the internal energy $f_E[T/T_F]$

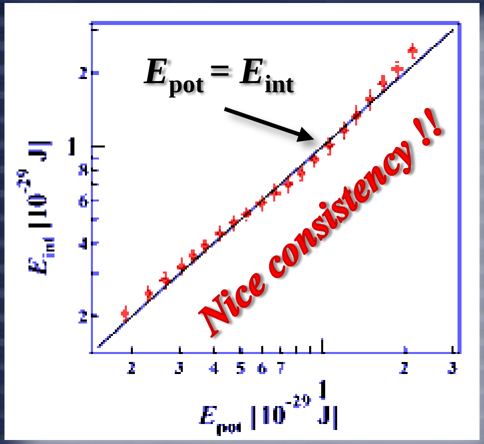
Universal hypothesis : $\dfrac{\varepsilon}{nE_F} = f_E[T/T_F]$

Equation of state : $p = \dfrac{2}{3}\varepsilon$

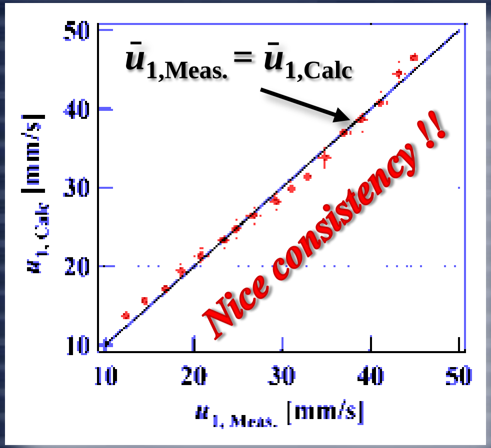
Mechanical equilibrium : $\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{Trap}(\mathbf{r}) = 0$



Energy comparison

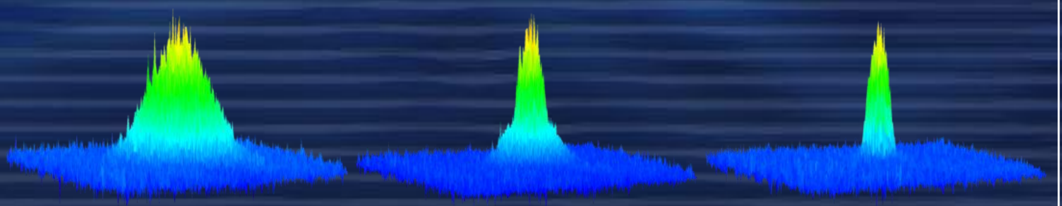


Speed of the first sound

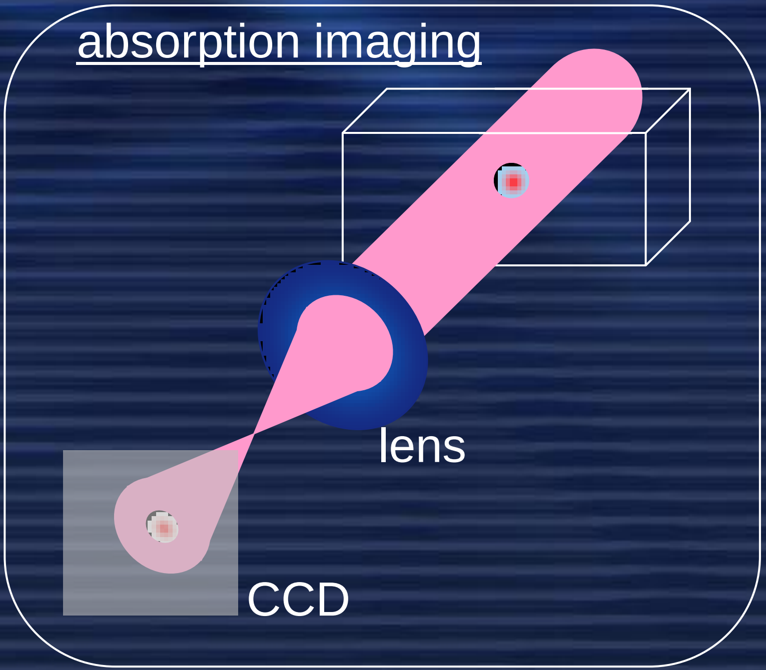


momentum distribution measurement

Bosonic case



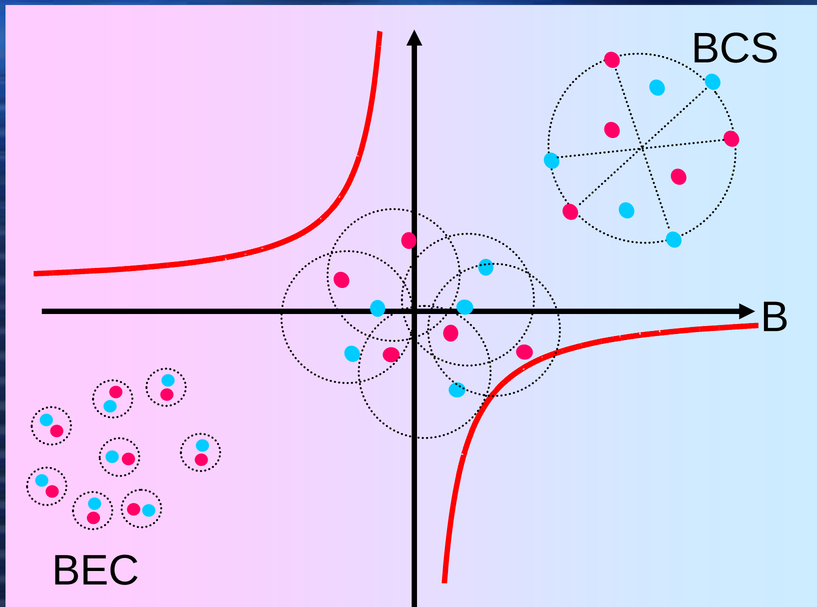
thermal　　bimodal　　pure BEC

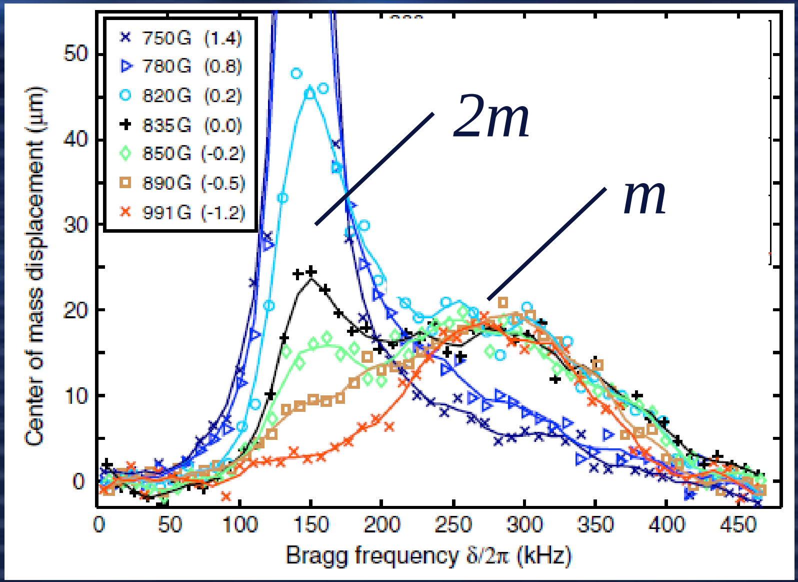


Is it condensed or not?
See the bimodal profile !!

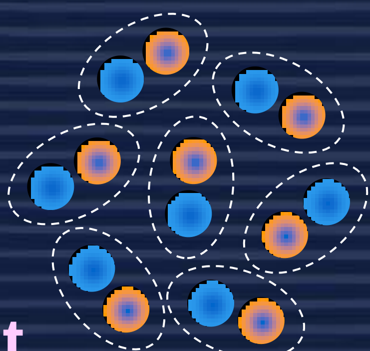
… unfortunately this scheme does not work.

Fermion pair condensate



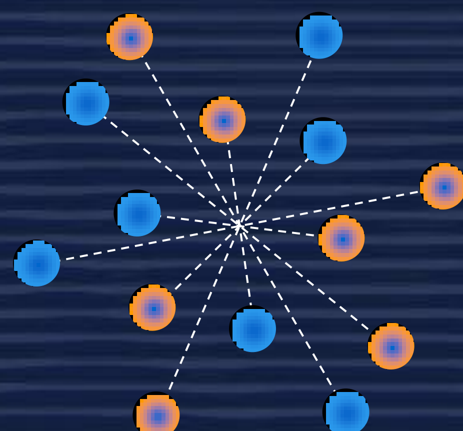


G. Veeravalli et al. Phys. Rev. Lett. **101**, 250403 (2008)



BEC limit

spatially correlated pair



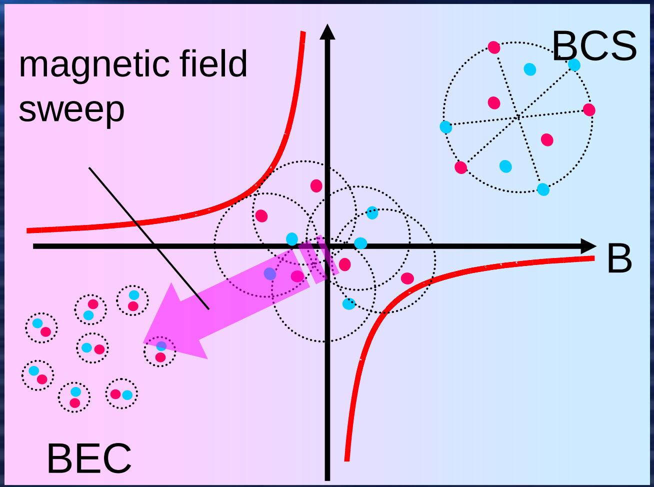
momentum correlated pair

BCS limit

"projection"

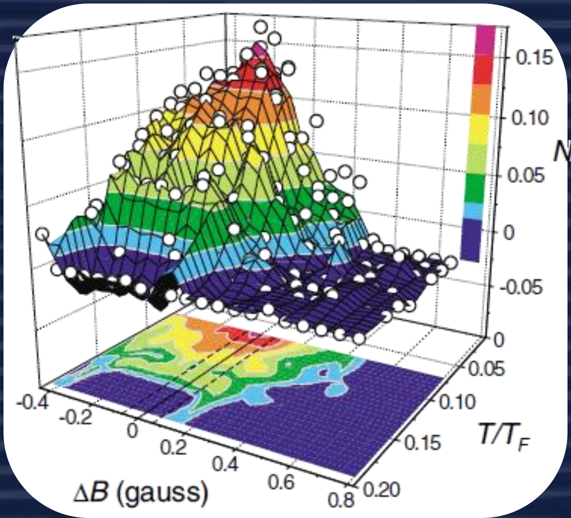
If we sweep the magnetic field

- slow enough to convert atom pairs into molecules
- fast enough such that the momentum distribution of the projected molecules reflects that of pairs prior to the sweep



C. A. Regal et al.
Phys. Rev. Lett., **92**, 040403 (2004)

We can convert correlated pairs into tightly-bound molecules.

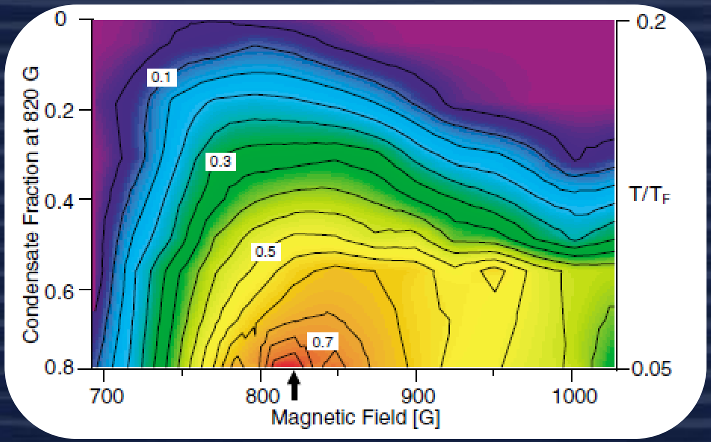


JILA

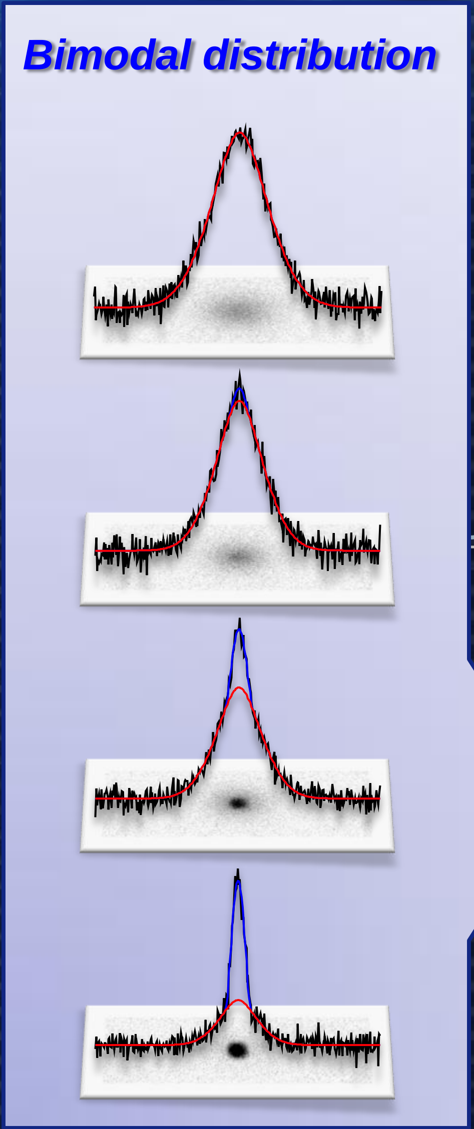
C. A. Regal *et al.*, PRL 92, 040403 (2004)

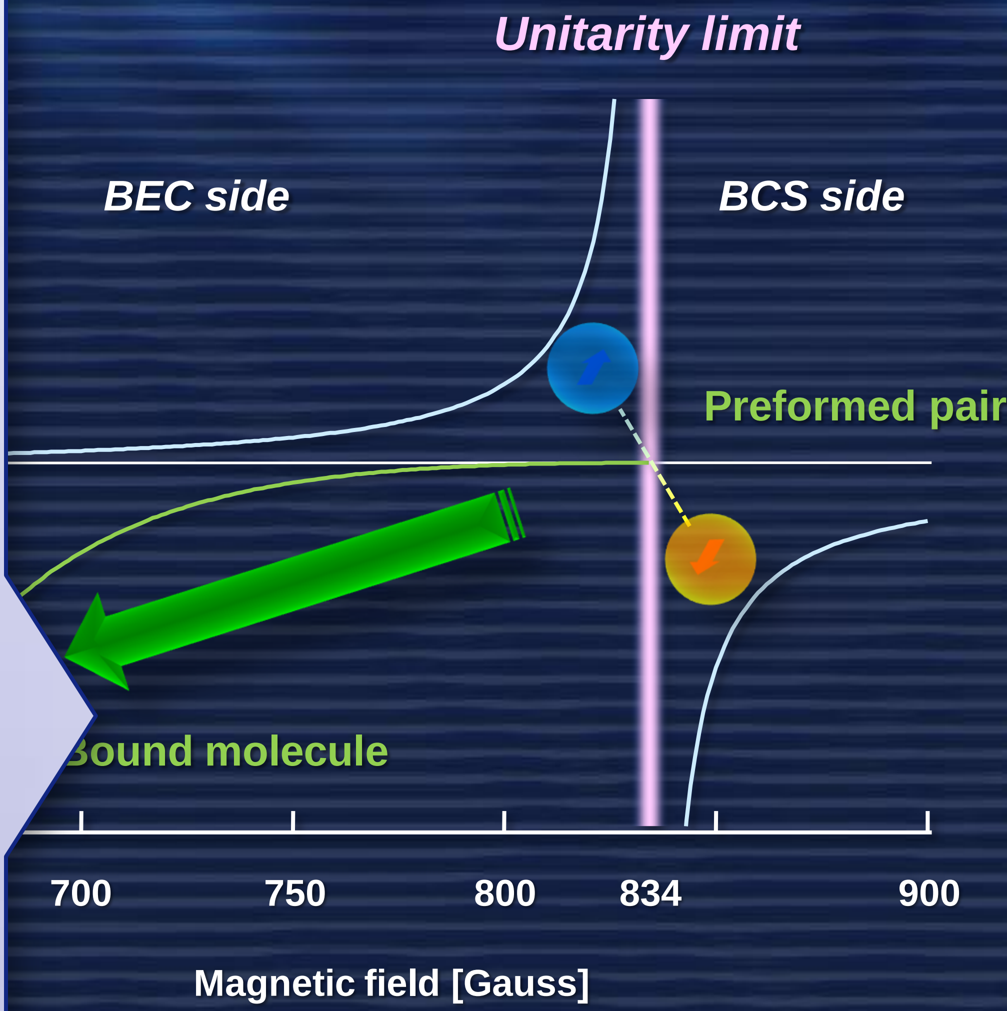
MIT

M. W. Zwierlein *et al.*, PRL 92, 120403 (2004)

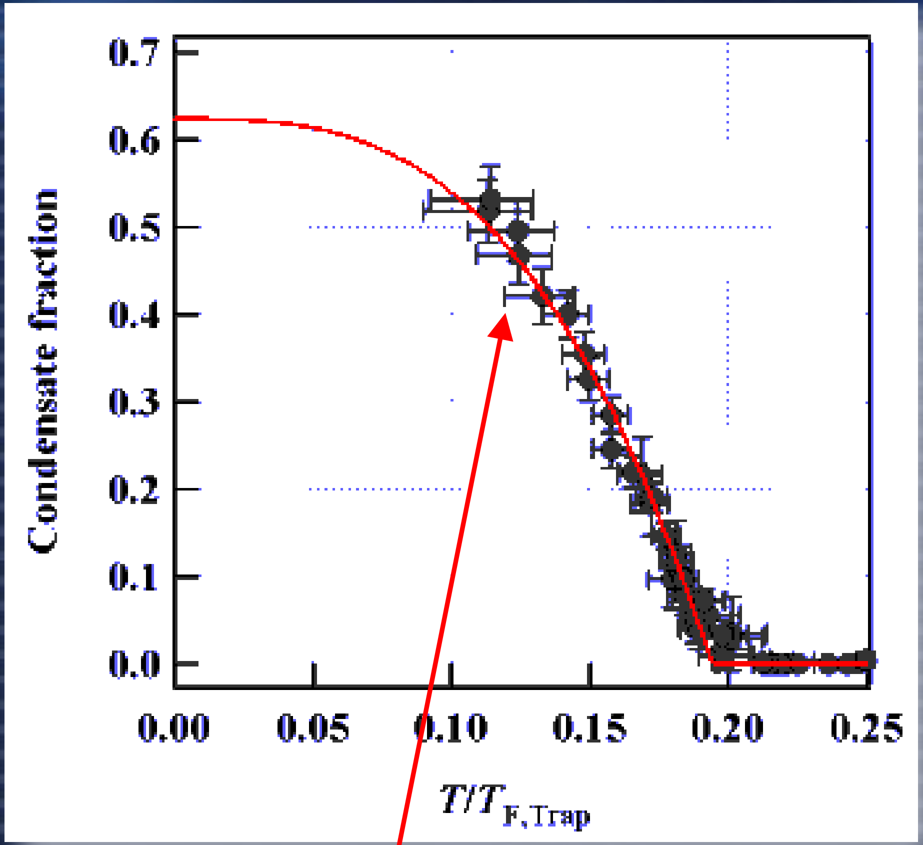


Bimodal distribution of a fermion pair condensate





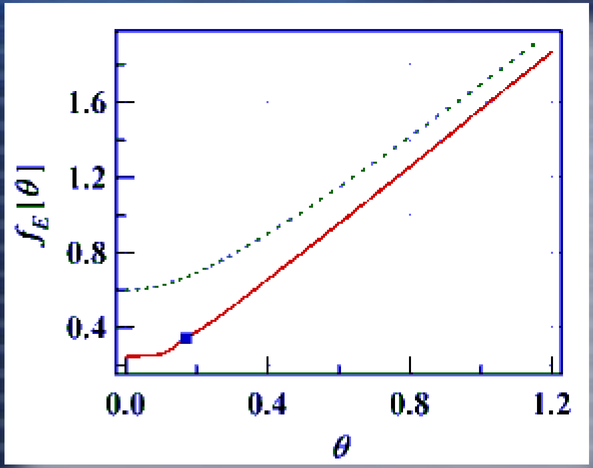
Condensate fraction vs Temperature



$$f(x) = CF_{\mathrm{Max}} \cdot \left\{ 1 - \left(\frac{T}{T_C} \right)^{\mathbf{3.0(1)}} \right\}$$

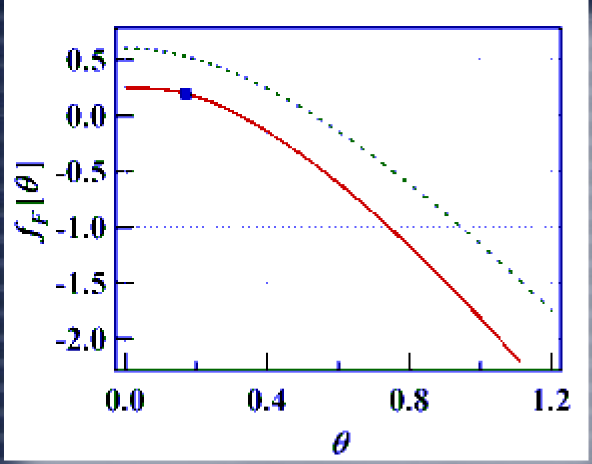
Universal thermodynamic functions

Internal energy

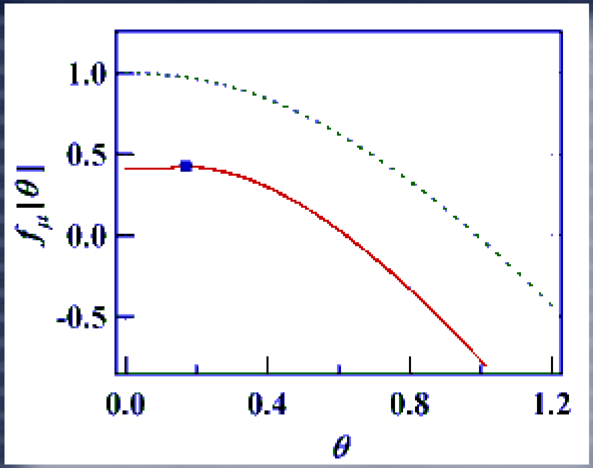


$$f_E = f_F - \theta f_F'$$
$$f_\mu = \left(5 f_E - 2\theta f_F'\right)/3$$
$$f_S = -f_F'$$

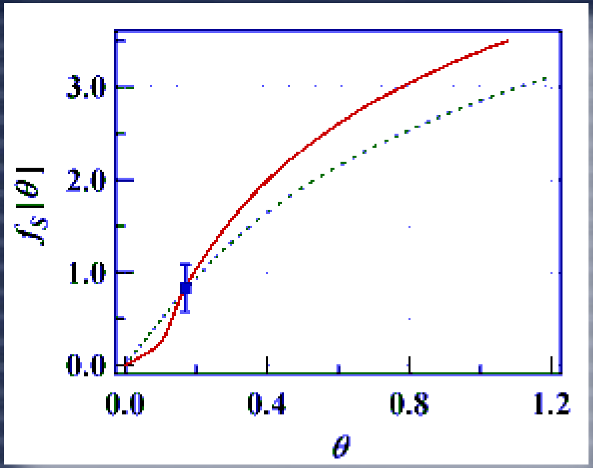
Helmholtz free energy



Chemical potential



Entropy

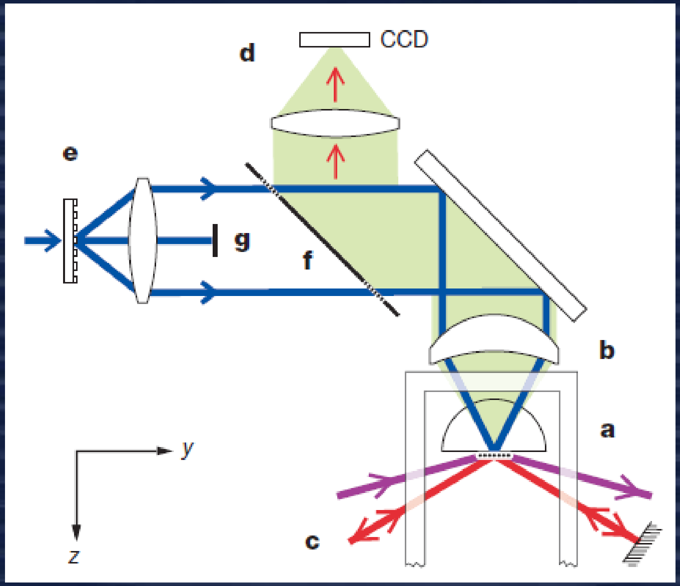


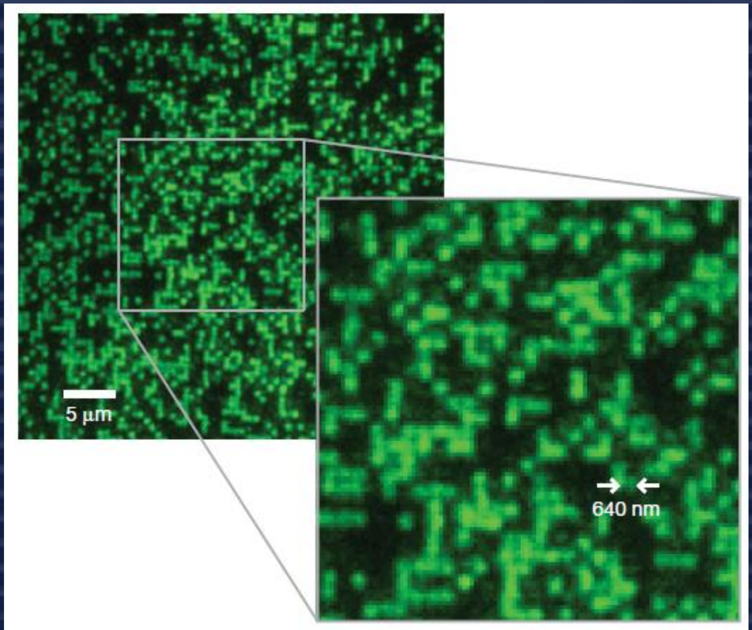
In the case of unitary gas, equation of state $p(\mathbf{r}) = 2\varepsilon(\mathbf{r})/3$ is available (exceptional case !!) which enable us to measure local thermodynamic quantities.

$$\varepsilon(\mathbf{r}) = n(\mathbf{r})\, E_{\mathrm{F}}(\mathbf{r})\, f_E[T/T_{\mathrm{F}}(\mathbf{r})]$$

Then, how can we determine local thermodynamic quantities without help of equation of state ?

→ High resolution local probe



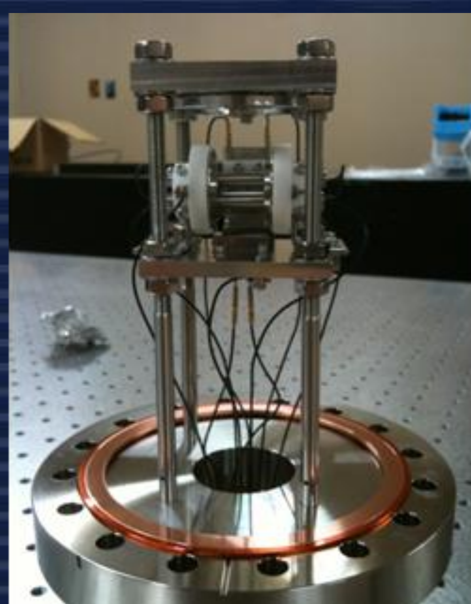


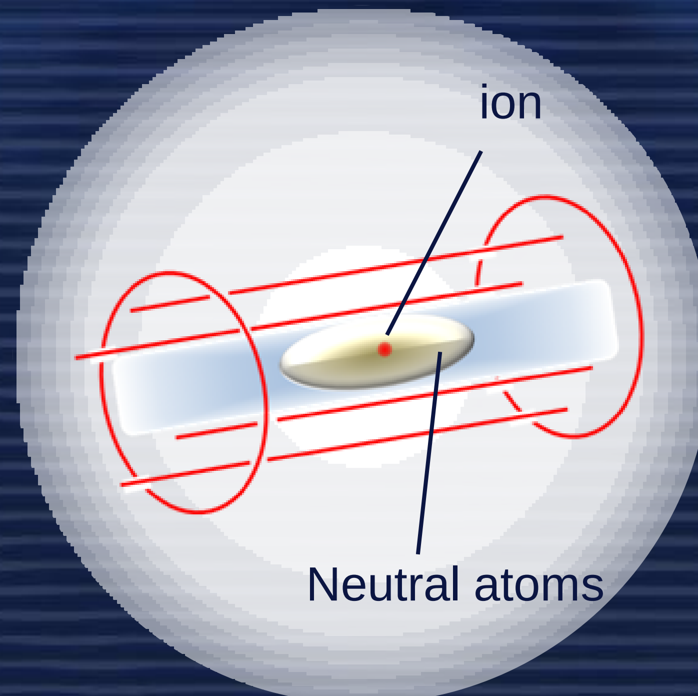
W. S. Bakr et al.Nature 462, 74 (2009).

The system that I'm setting up in University of Electro-Communications

Co-trapping system of ions and neutral atoms

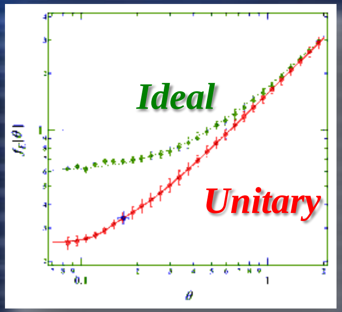
ion ＝
- **local probe**
- **control by electric field**



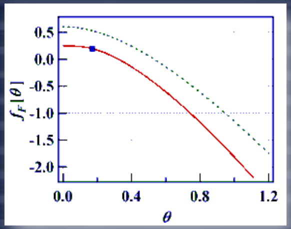


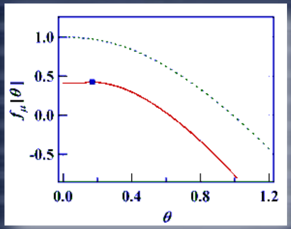
Summary

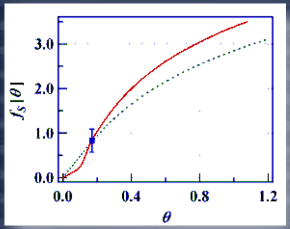
- The universal function of the internal energy was determined at the unitarity limit



- The other thermodynamic functions were derived from the thermodynamic relationship
- The critical parameters were determined at the superfluid transition temperature

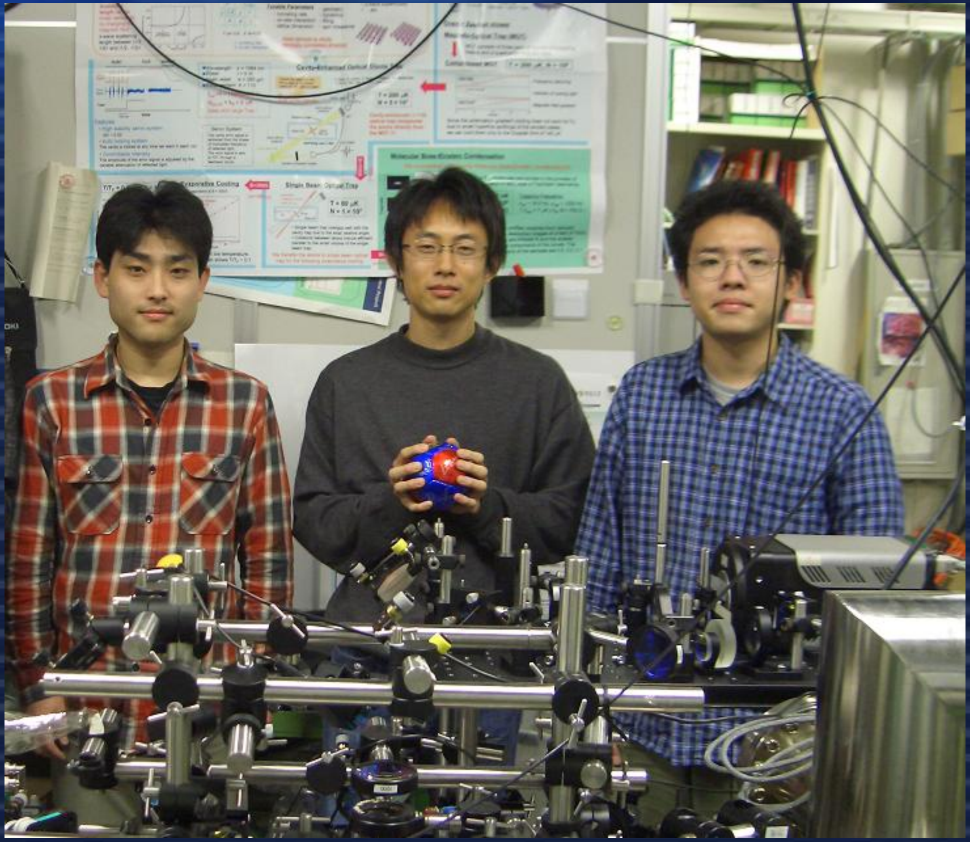






M. Horikoshi, S. Nakajima, M. Ueda and T. Mukaiyama, Science, **327**, 442 (2010).

The team (ERATO project)



T. Mukaiyama
(Group leader)

M. Horikoshi
(Postdoc)

S. Nakajima
(Ph.D student)

Unitary gas

Efimov physics

Masahito Ueda
(project leader)