THE UNIVERSITY OF TOKYO

量子情報処理プロジェクト　夏期研修会

沖縄、19 August 2010

強相関系の物理と光格子

東大 理
青木 秀夫

- 電子相関の物理
  - 磁性、超伝導
- 相関現象のplaygroundとしての光格子
  - 強磁性（梯子、籠目）
  - 超流動（多成分格子系での集団励起）
  - 超流動・モット絶縁体転移（Kibble-Zurek）
  - 格子系でフェルミオン間相互作用を斥力 ⇔ 引力変換可？

- 強磁性（梯子、籠目）

M Okumura（RIKEN）, M Machida（JAEA）

- 超流動（多成分格子系での集団励起）

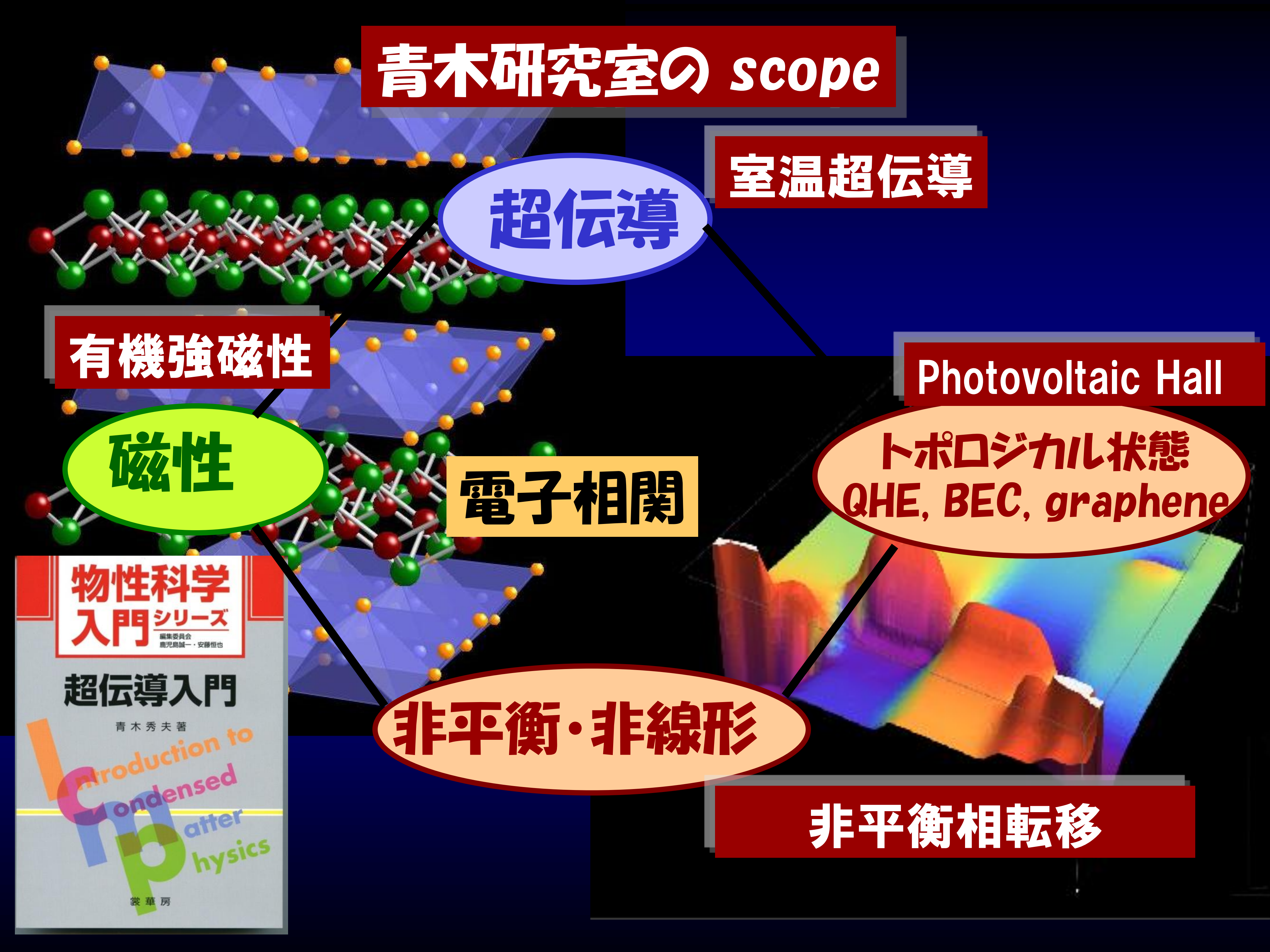
Y Ota, M Machida, S Yamada（JAEA）

- 超流動・モット絶縁体転移

N Horiguchi（UT, now at NEC）, T Oka（UT）

- 格子系でフェルミオン間の相互作用を斥力⇔引力変換可？

N Tsuji, T Oka（UT）, P Werner（ETH）



青木研究室の scope
室温超伝導
超伝導
有機強磁性
磁性
電子相関
Photovoltaic Hall
トポロジカル状態
QHE, BEC, graphene
非平衡・非線形
非平衡相転移
物性科学入門シリーズ
編集委員会
鹿児島誠一・安藤恒也
超伝導入門
青木秀夫 著
Introduction to Condensed Matter Physics
裳華房

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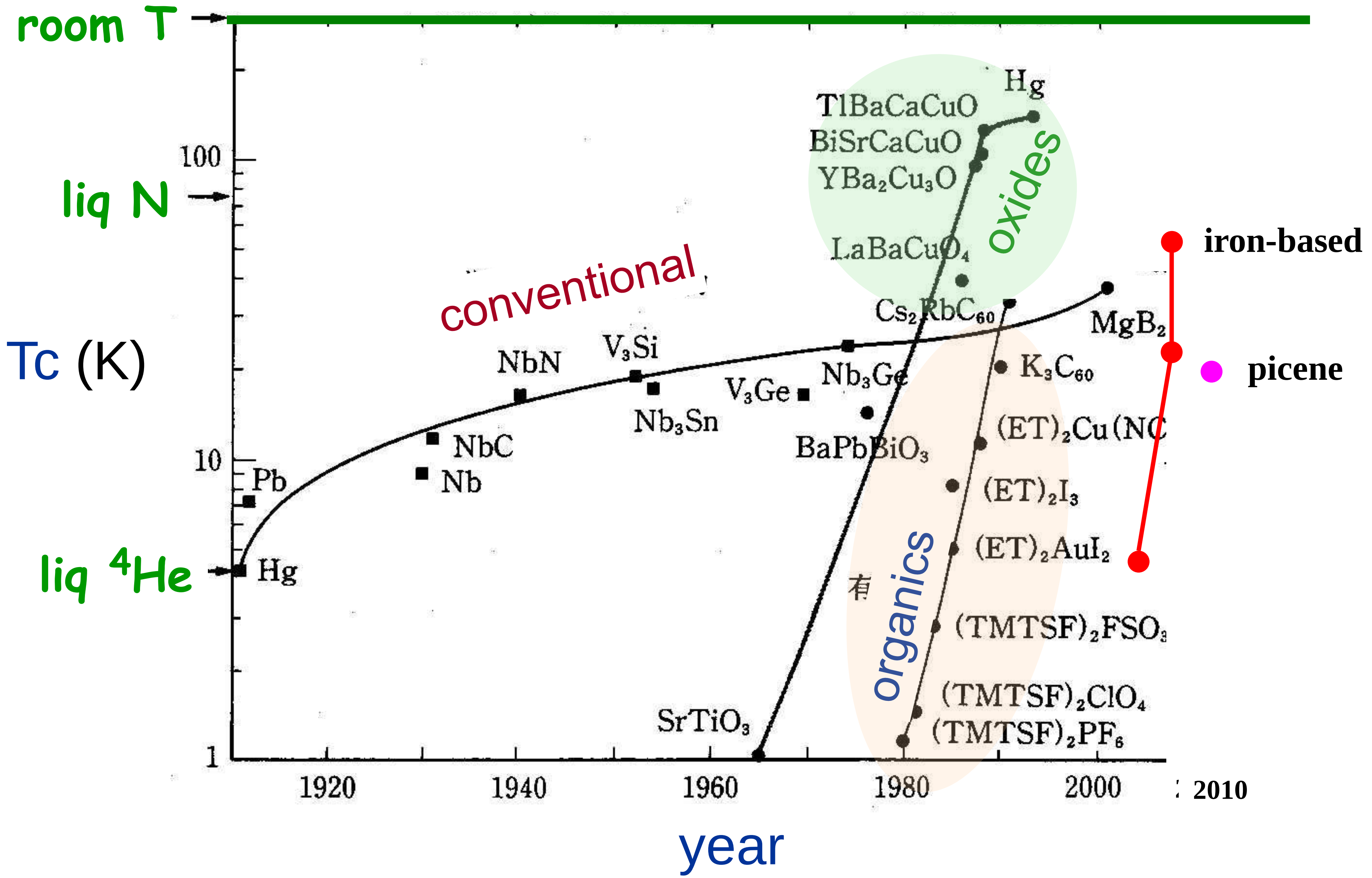
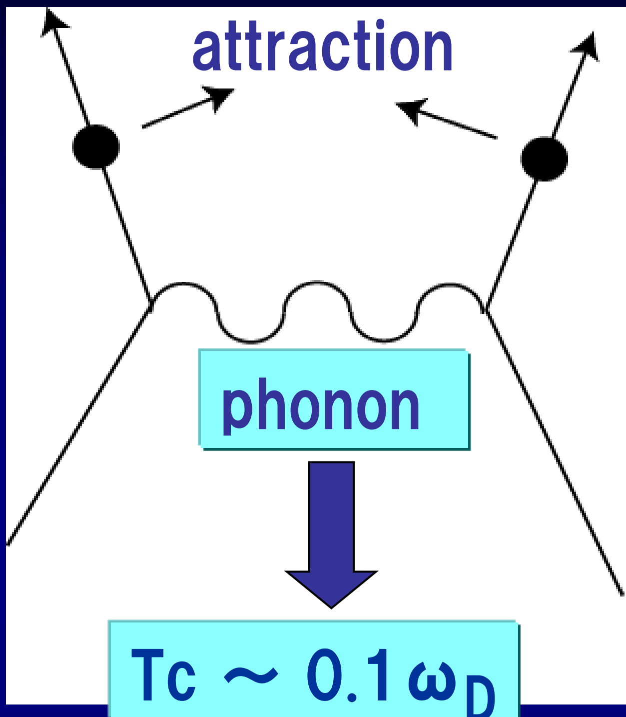
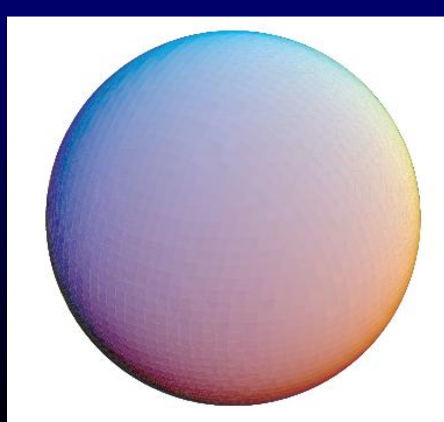
Evolution of Tc in superconductivity



* phonon mechanism

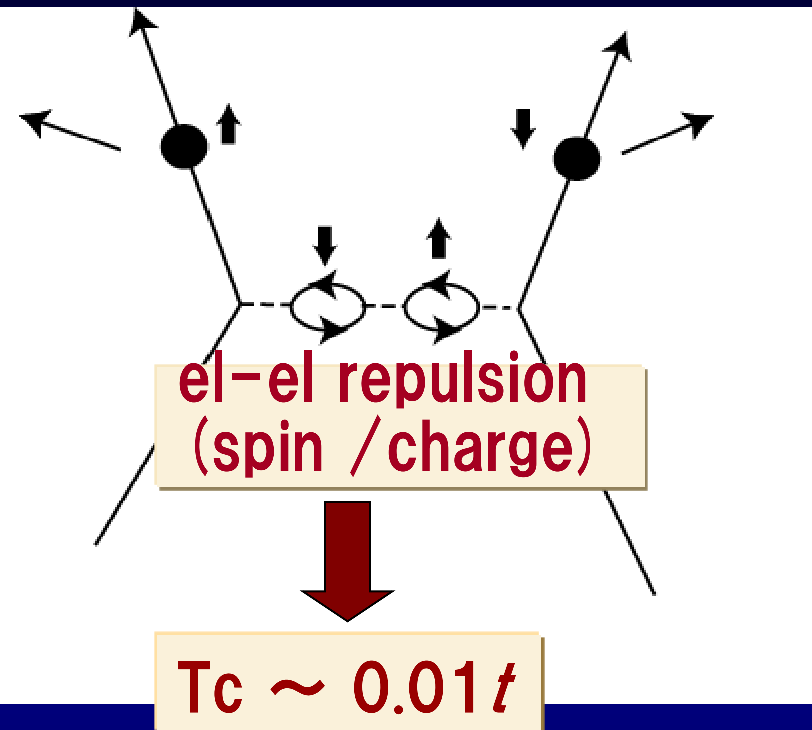


10K ← 100K

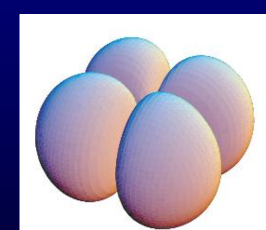


isotropic pairing

* electron mechanism

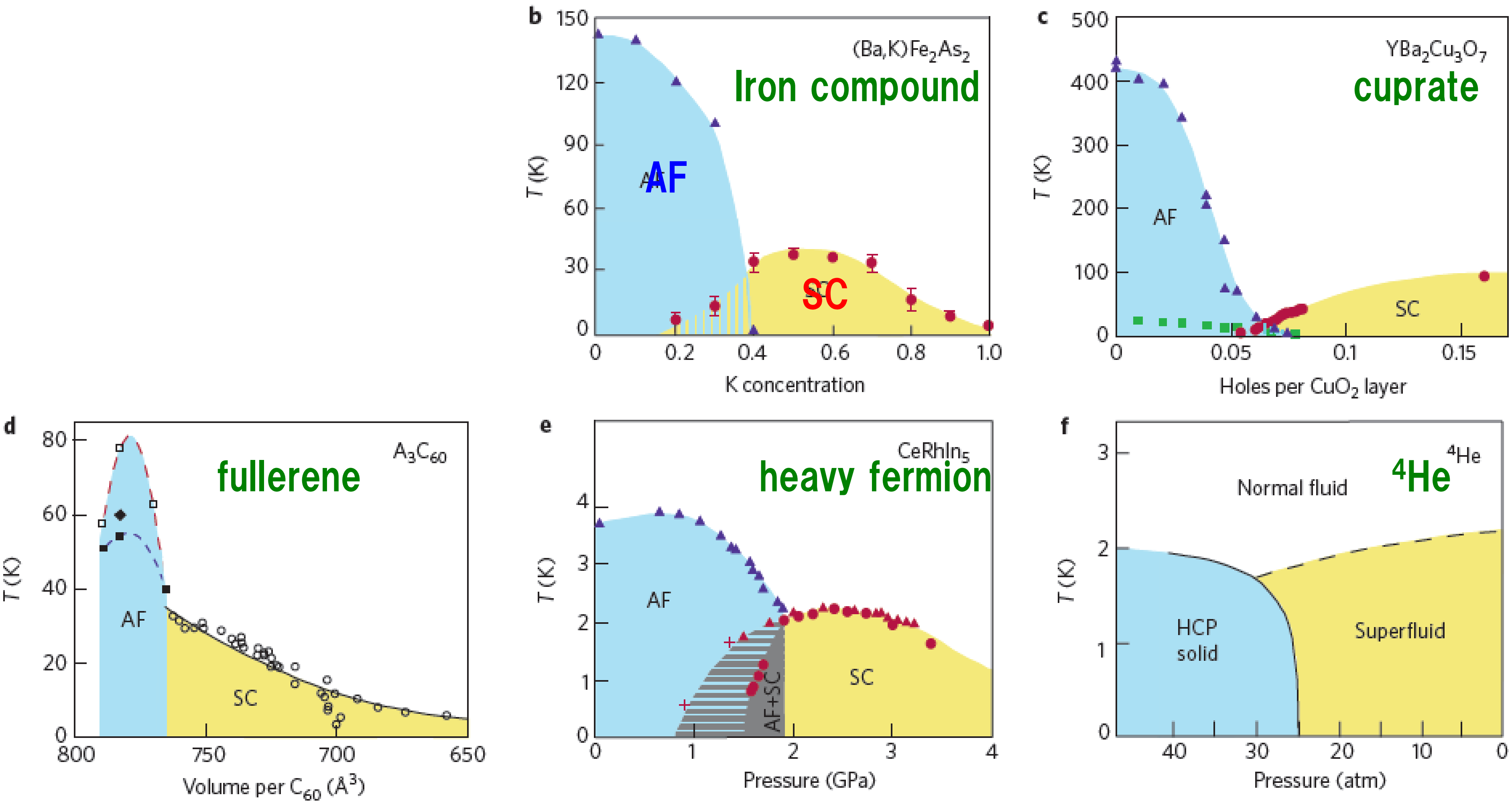


100K ← 10000K



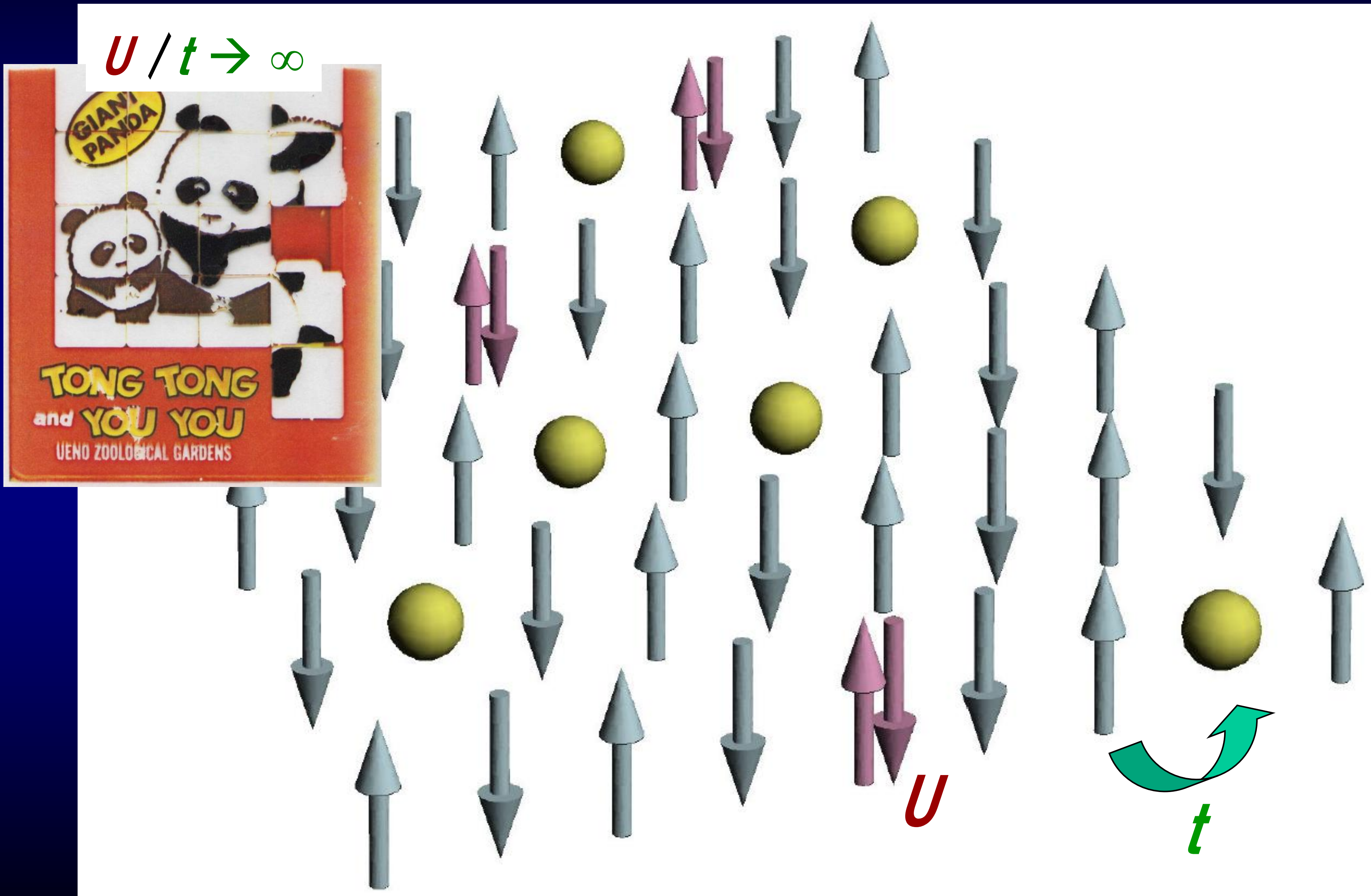
anisotropic pairing

Phase diagrams for various classes of materials



Uemura, nature mat, news&views 2009

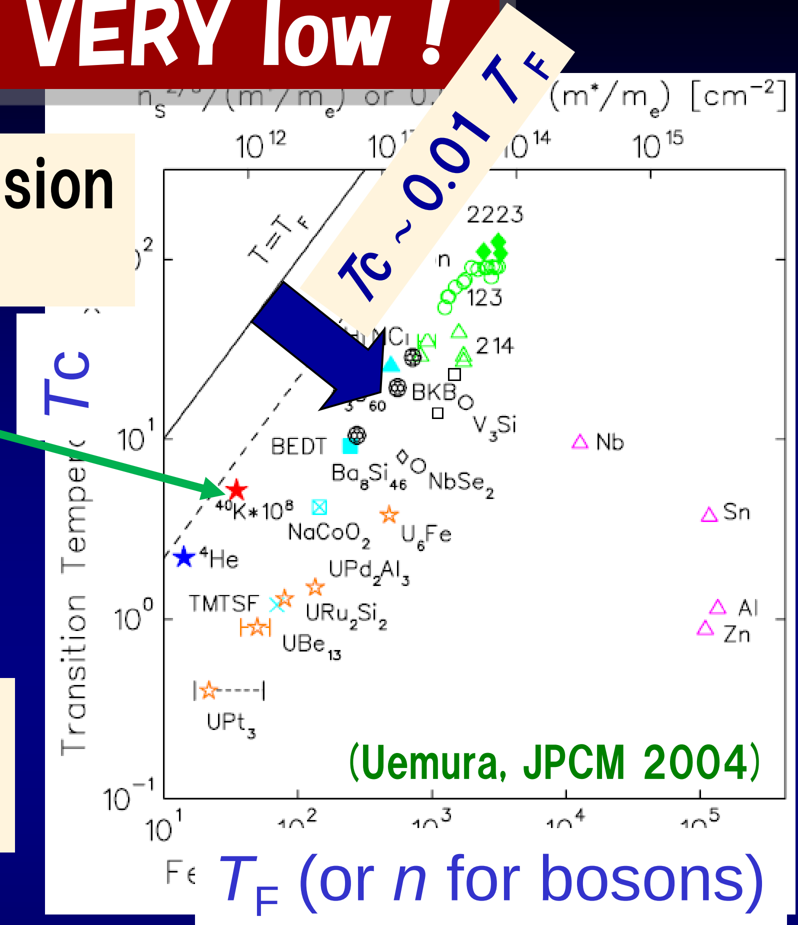
Hubbard model (a generic model)



Tc ~ T_F/ 100 is VERY low !

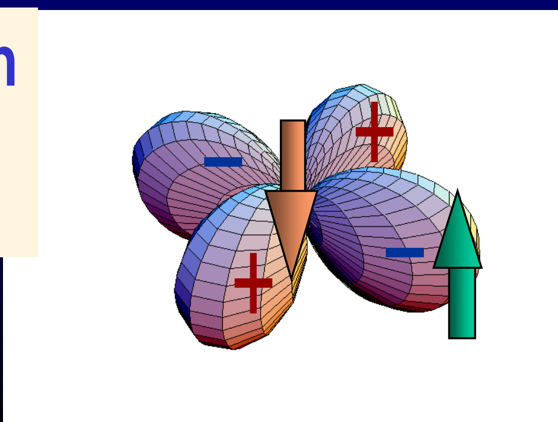
(1) Pairing int'action from el-el repulsion
= weak

Cf. Laser-cooled Fermi gas (2004)
← Tc ~ 0.1 T_F
← attractive int'action
↑ Feshbach resonance

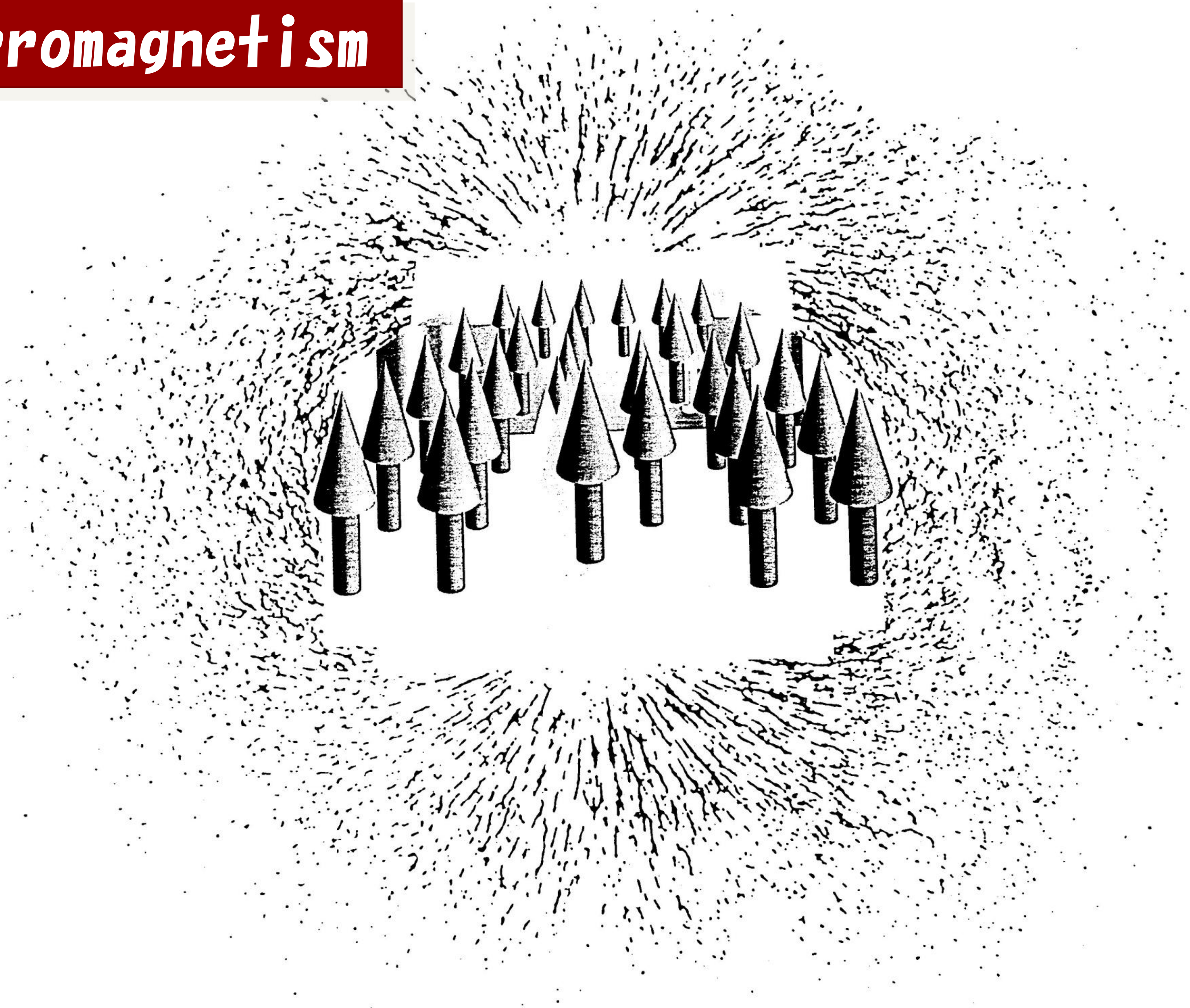


(2) Self-energy correction
→ quasi-particles short-lived

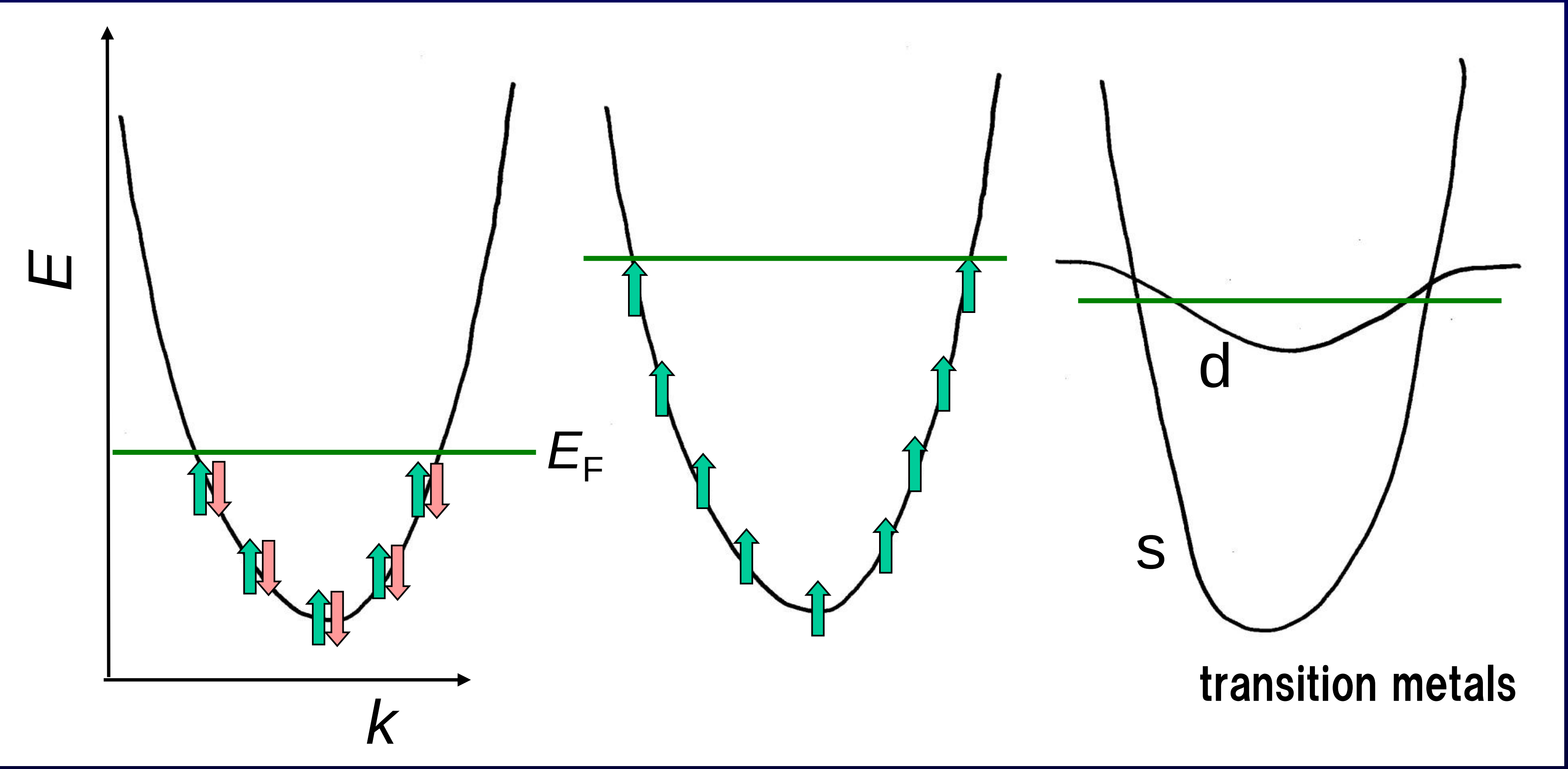
(3) Pairing from el-el repulsion
= anisotropic
(i.e., nodes in Δ_{BCS})



Ferromagnetism

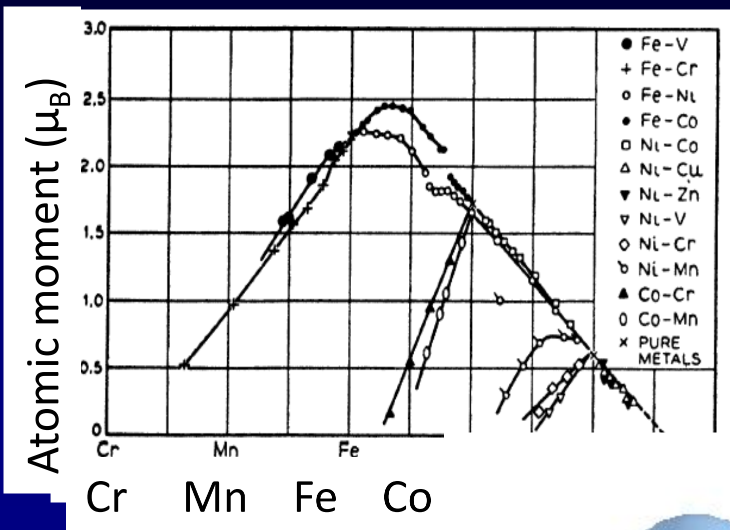


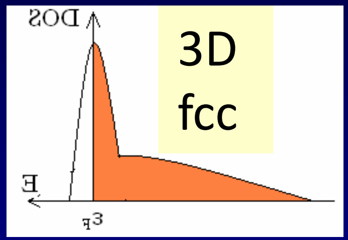
Ferromagnetism ← very difficult to realise



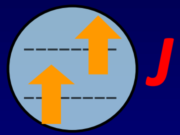
Itinerant ferromagnetism in transition metals

Sakai, Arita & Aoki, PRL 2007

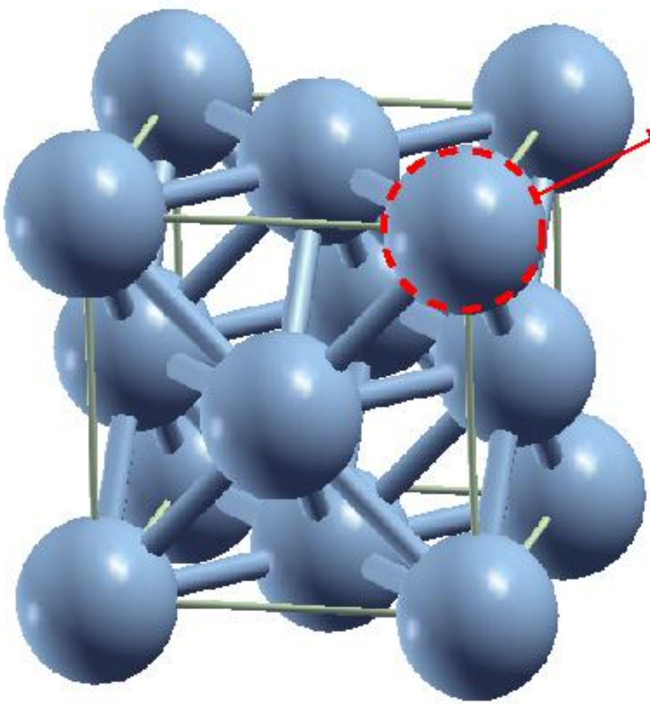


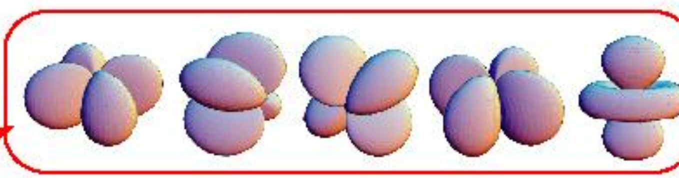


Lattice structure



Multi-band → Hund's coupling



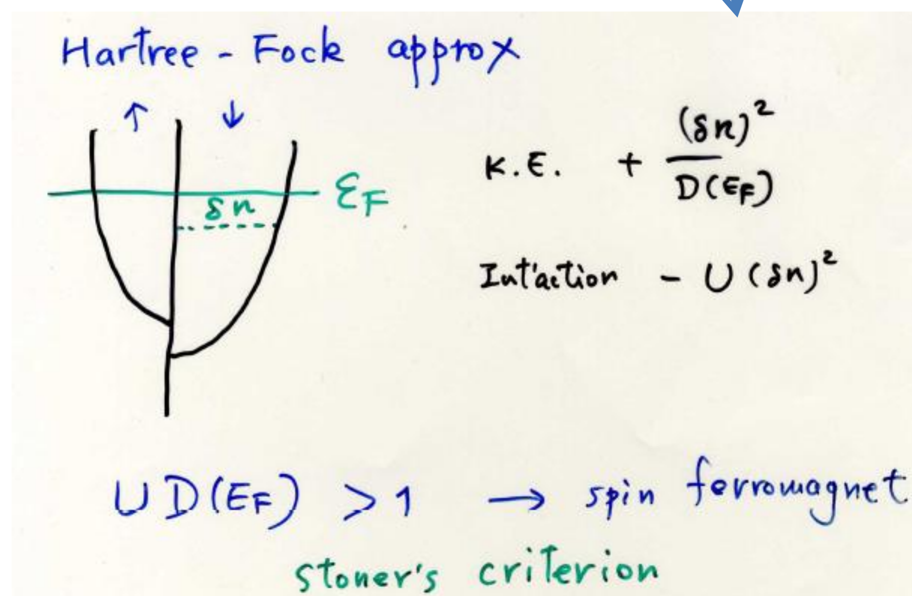


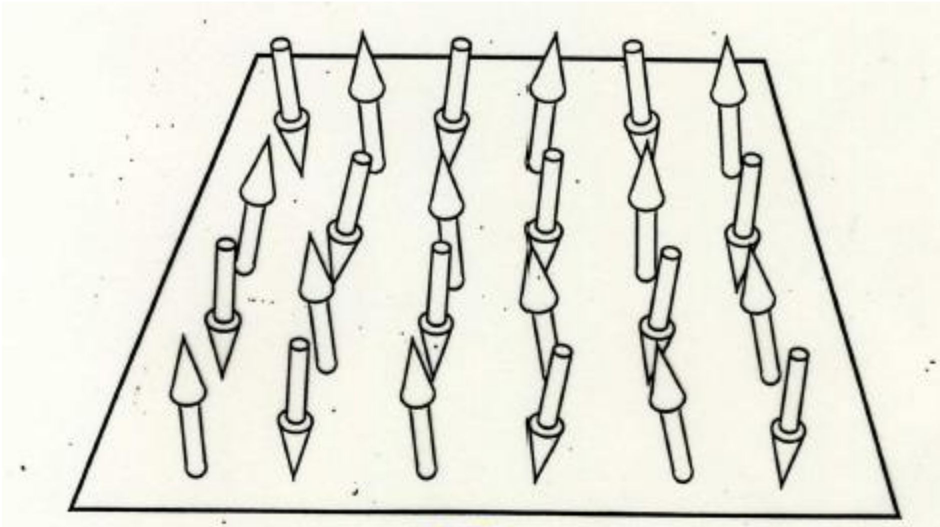
Repulsively interacting one-band (no Hund) systems
→ Ferromagnetism ?

Stoner $U D(E_F) > 1$ too crude a criterion (mean-field)

$U \to \infty$ → Nagaoka F

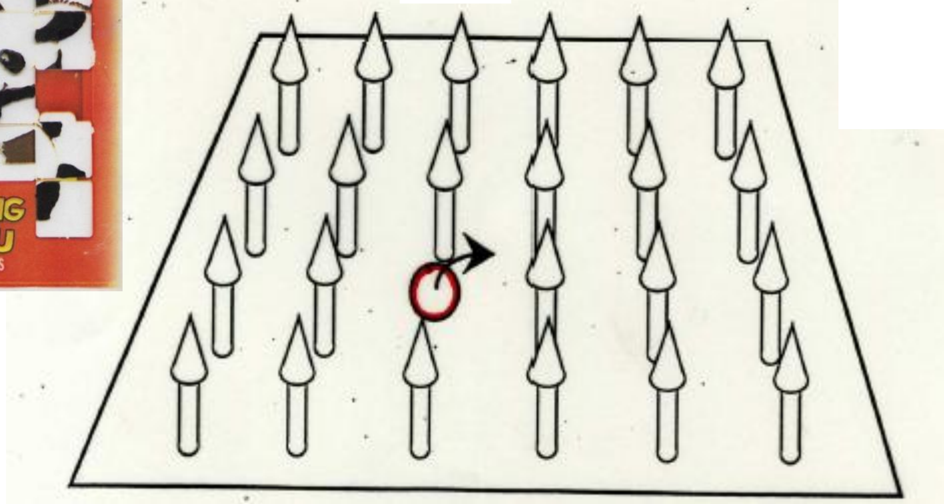
$D(E_F) \to \infty$ → Flat-band ferromagnetism (Lieb, Mielke, Tasaki)



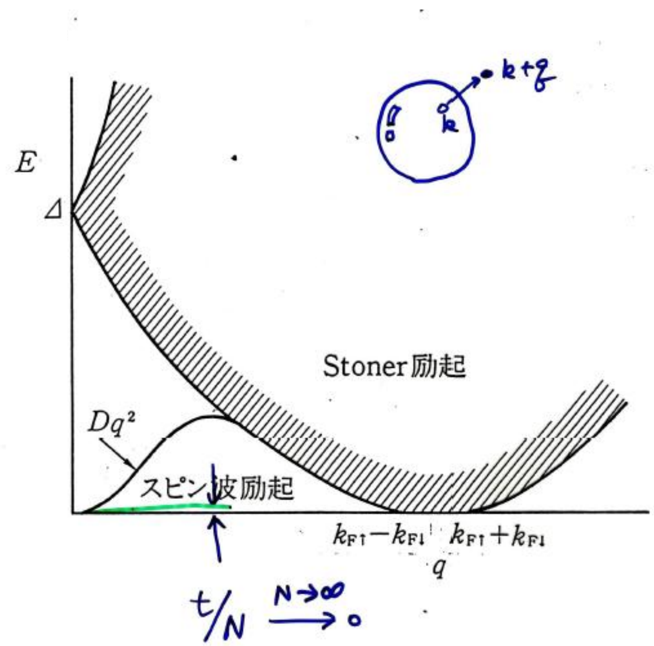


$U/t \to \infty$

AF

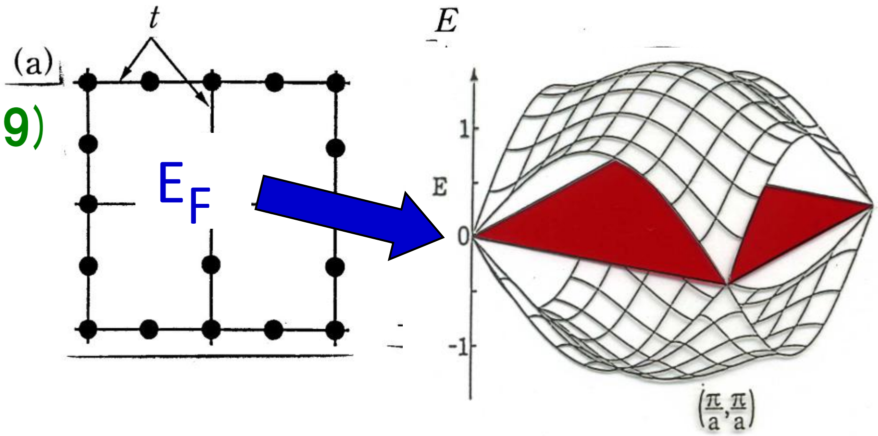


$U = \infty$ **F**

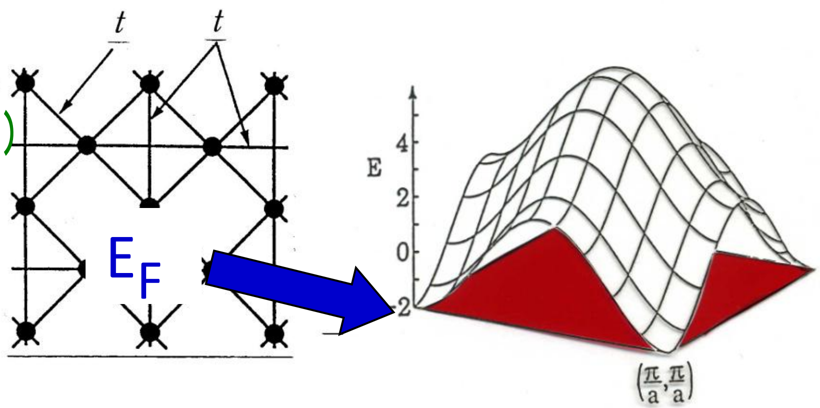


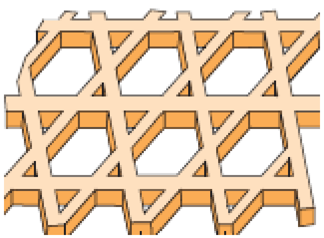
Hubbard model on flat-band systems

(Lieb, 1989)

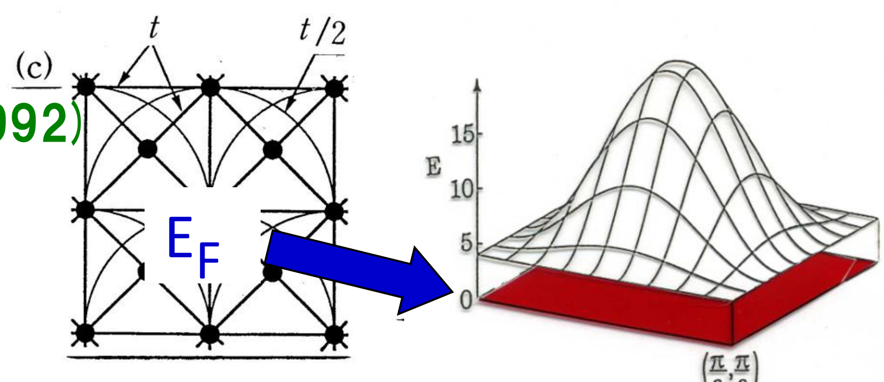


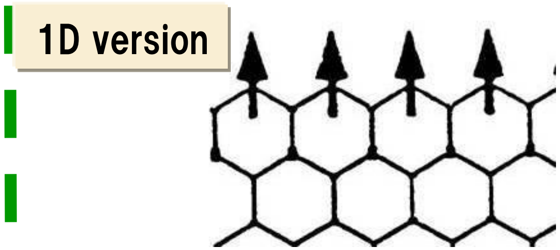
(Mielke, 1991)



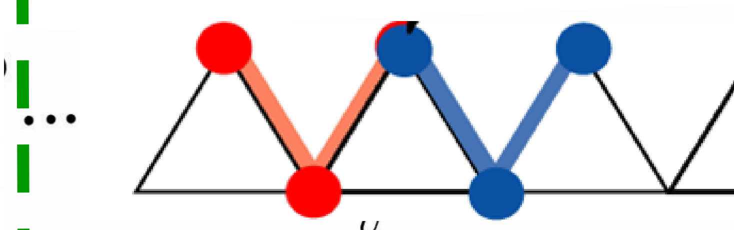


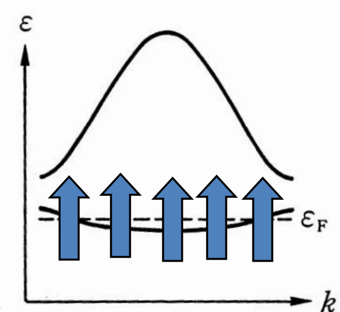
(Tasaki, 1992)



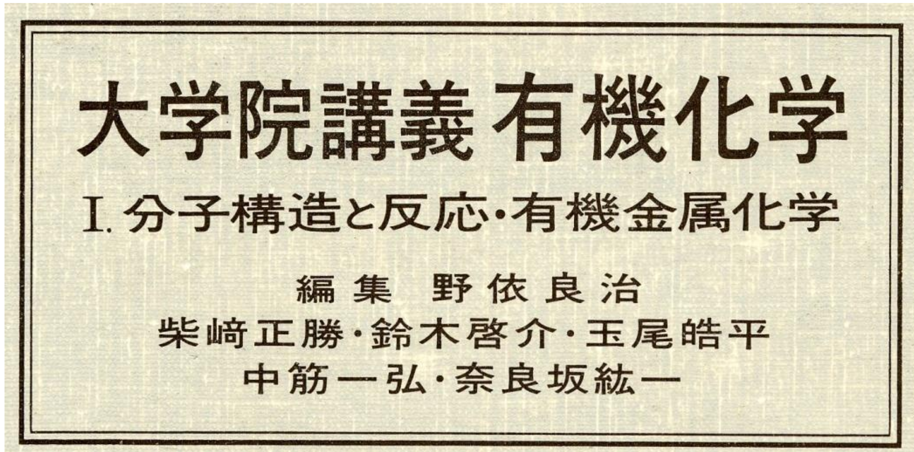


Connectivity condition
(``Wannier" orbits overlap)

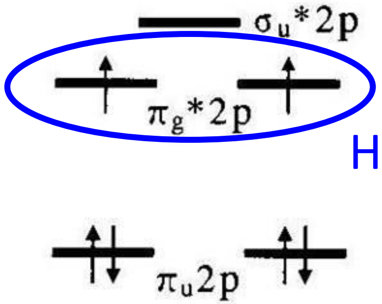




generalised Hund's coupling
(Kusakabe & Aoki, 1992)

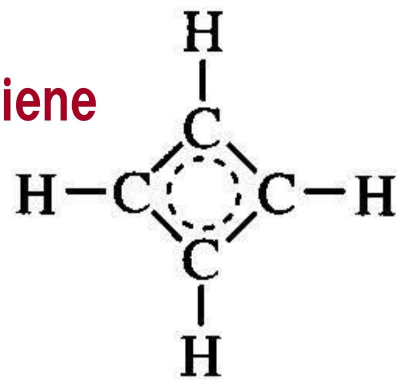


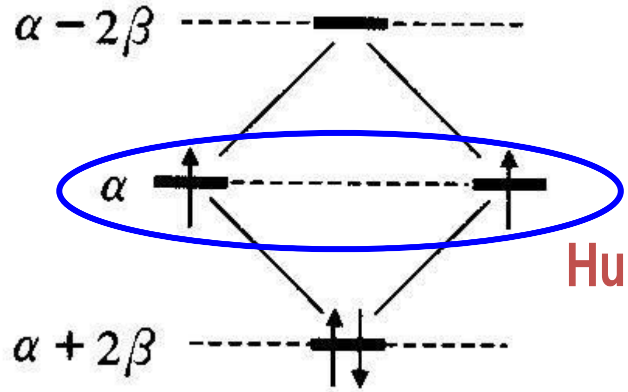
O_2



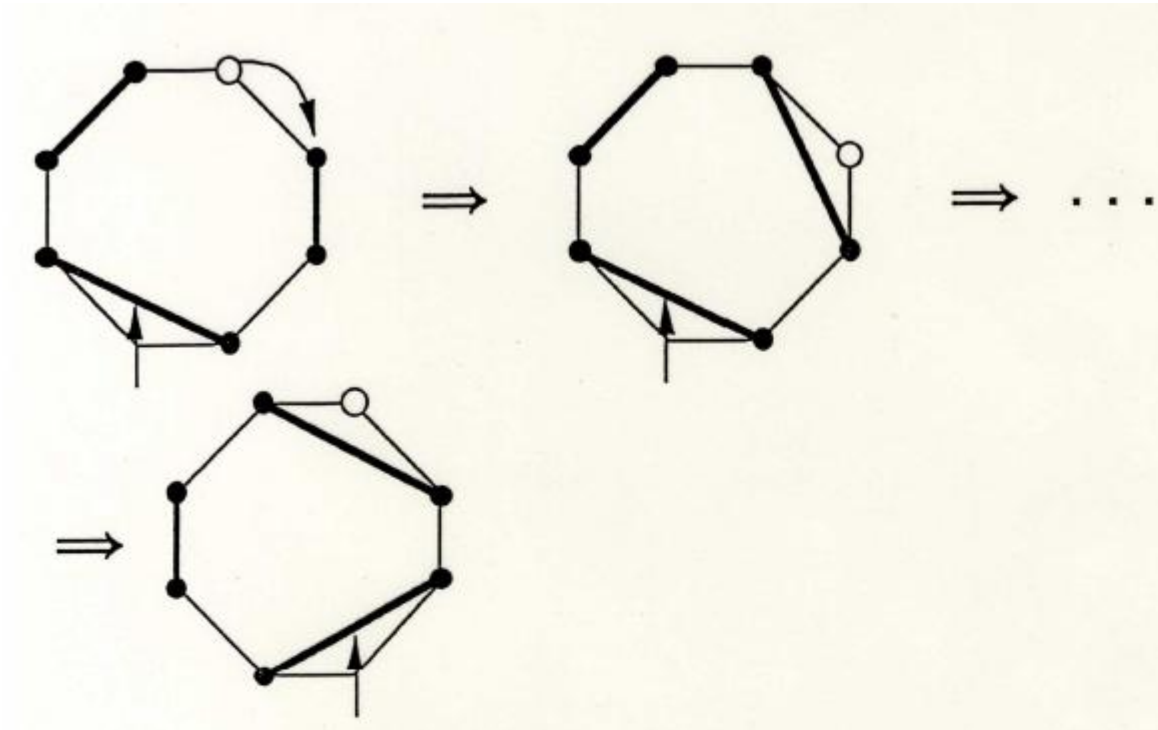
Hund

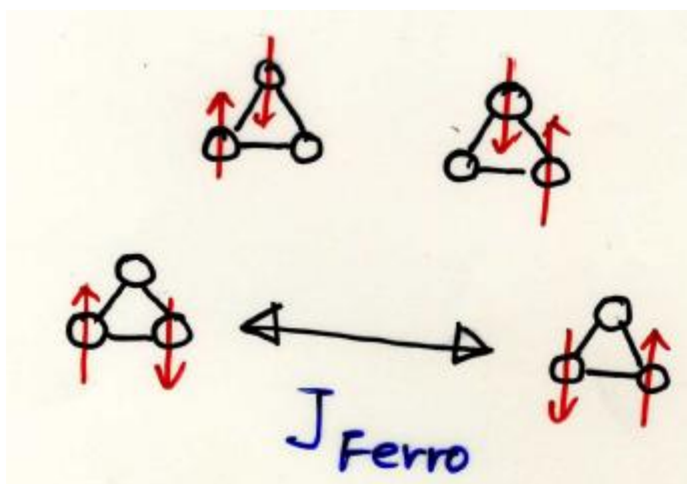
cyclobutadiene



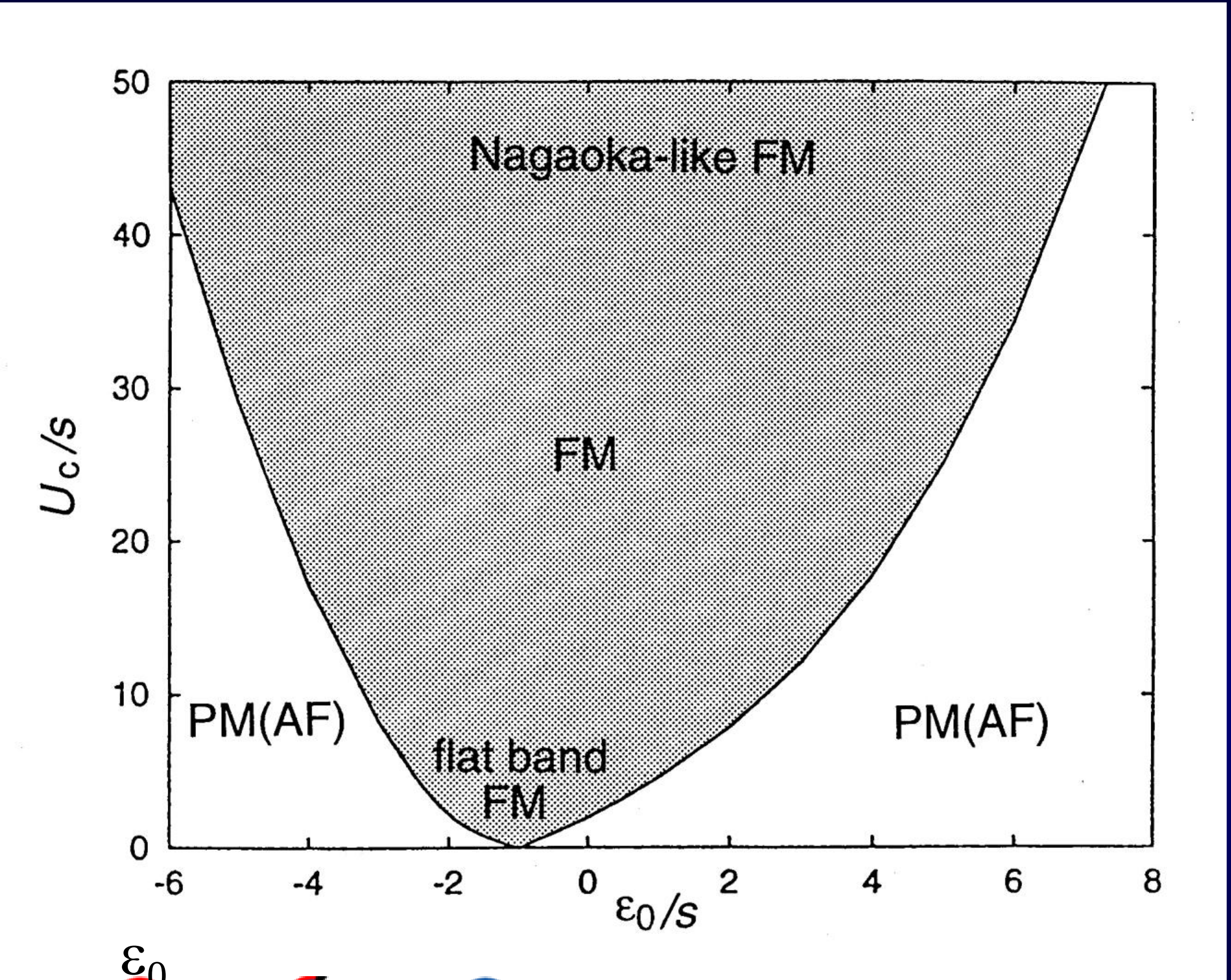


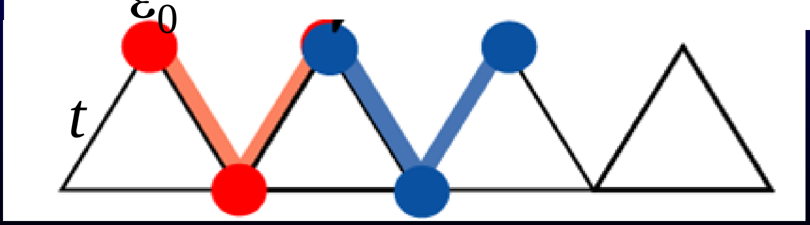
Hund



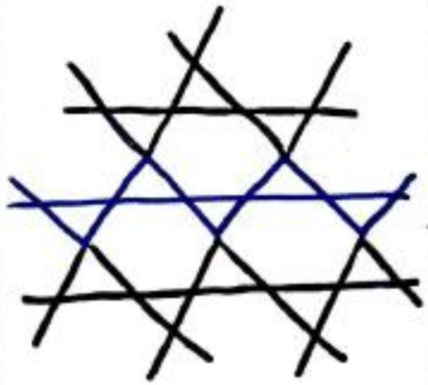
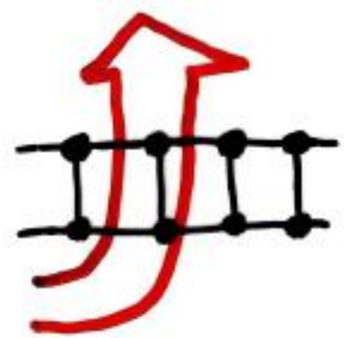

J Ferro

Flat-band F ←→ Nagaoka's F





(Penc et al, PRB 1996;
see also Vollhardt et al, PRL 2007)

	Hubbard model	
	Ferromagnetism	Superconductivity
2D		
1D		

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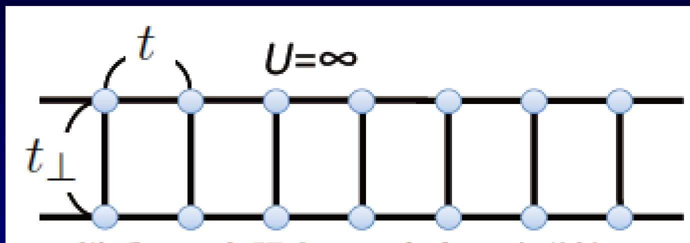
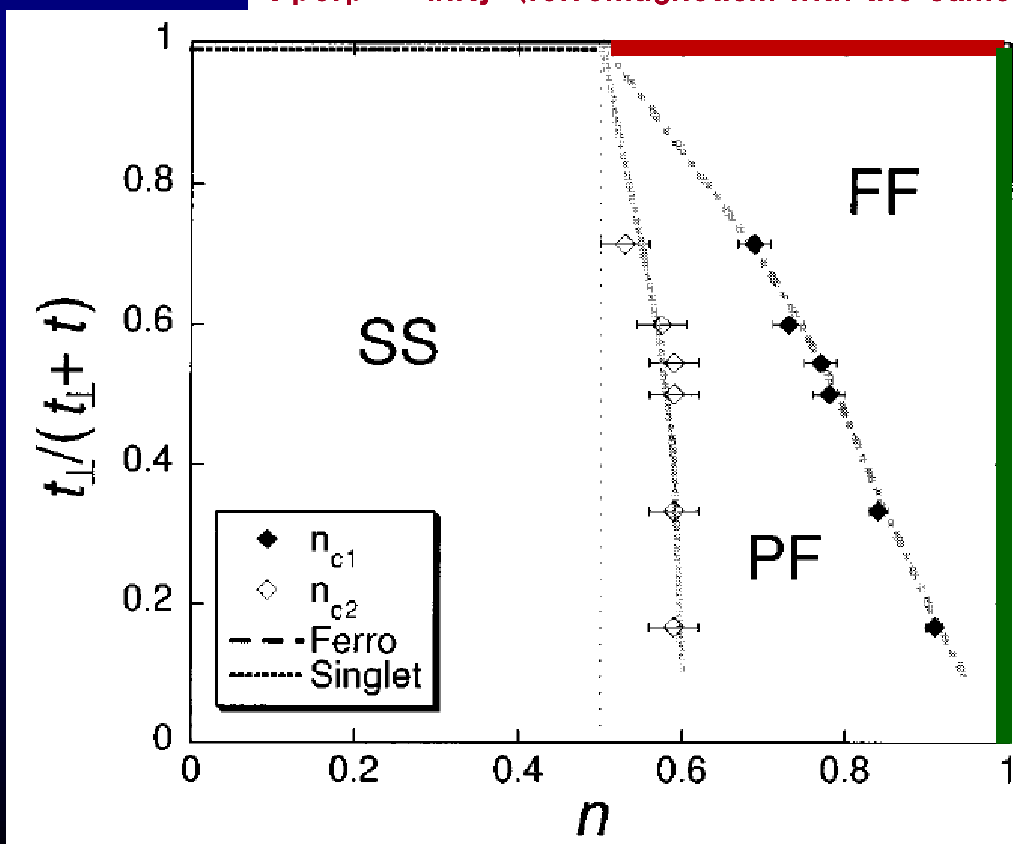
Why ladder ?



Itinerant (Stoner) F has been observed in cold atoms [Jo et al, Science 325, 1521 (2009)]

Ladder ← Nagaoka ferromagnetism

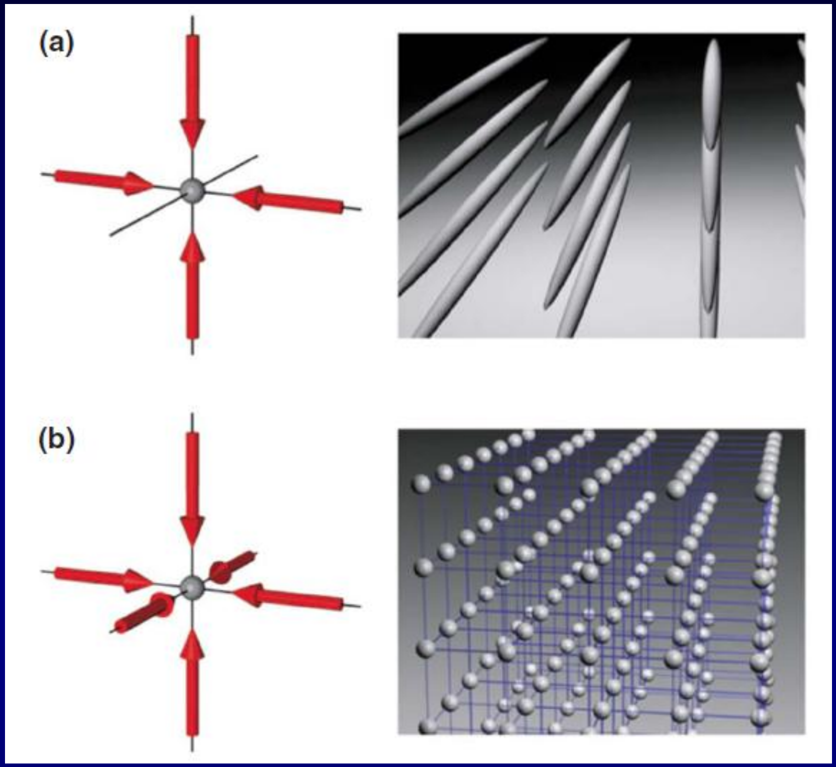
t perp → infty (ferromagnetism with the same proof as Nagaoka's)

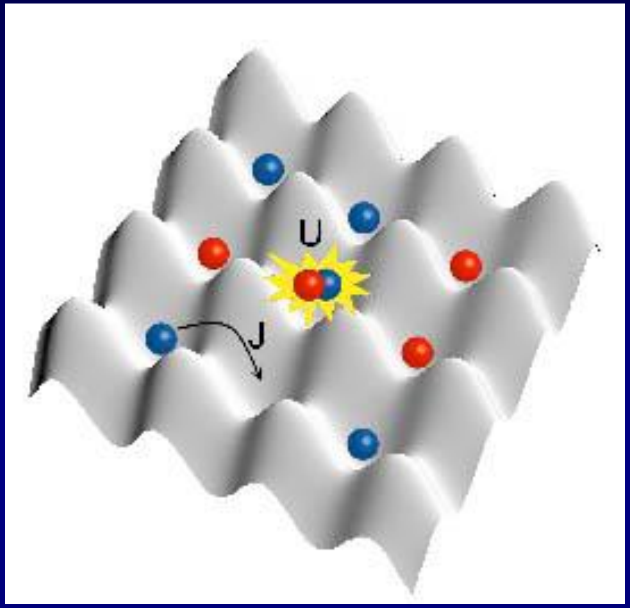


Nagaoka's ferromagnetism with one hole, U → infty

Kohno, PRB 56, 15015 (1997)

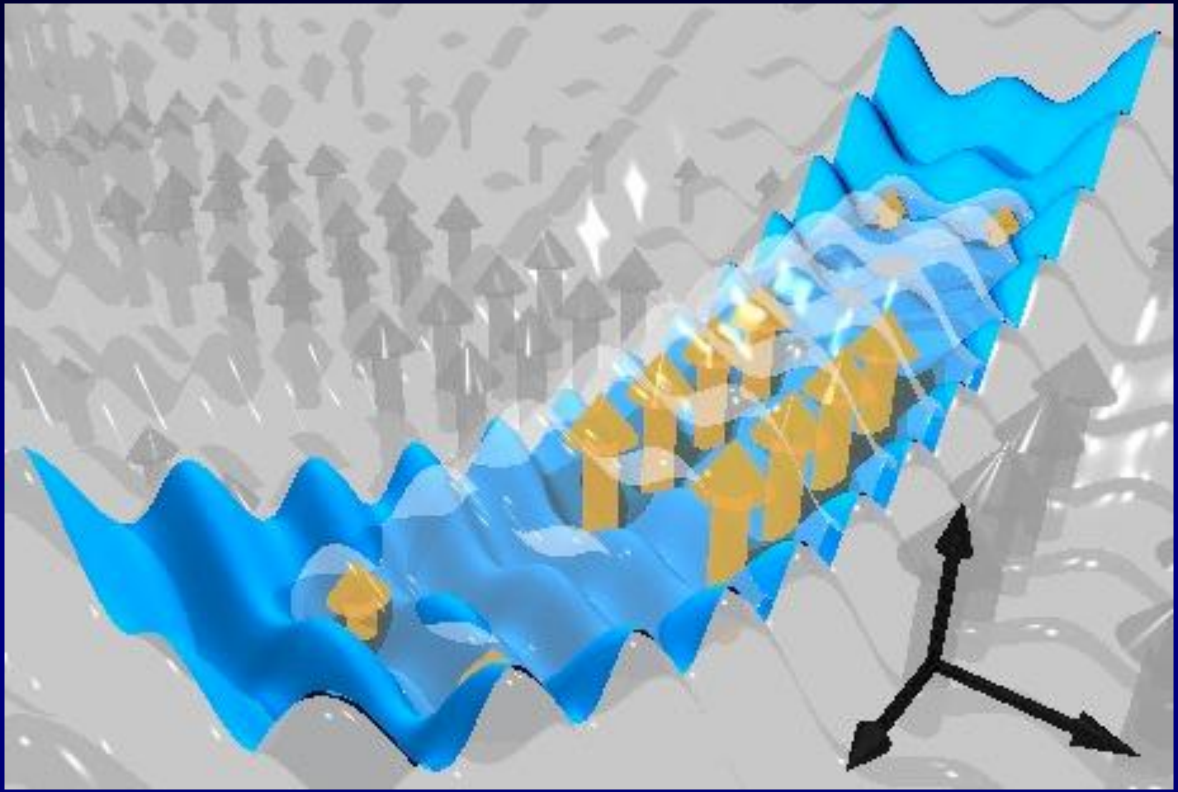
Optical lattice

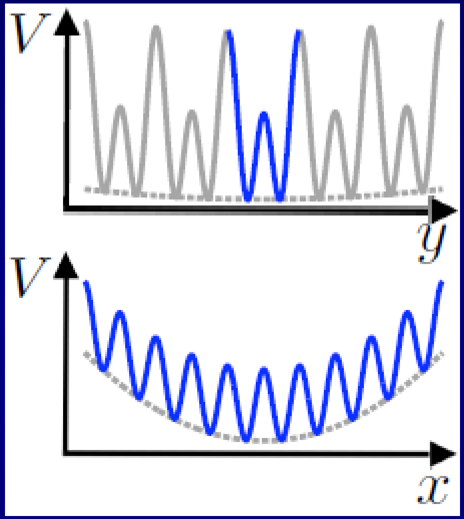




Bloch, et al, RMP **80**, 885 (2008)

Optical ladder in a trapping potential



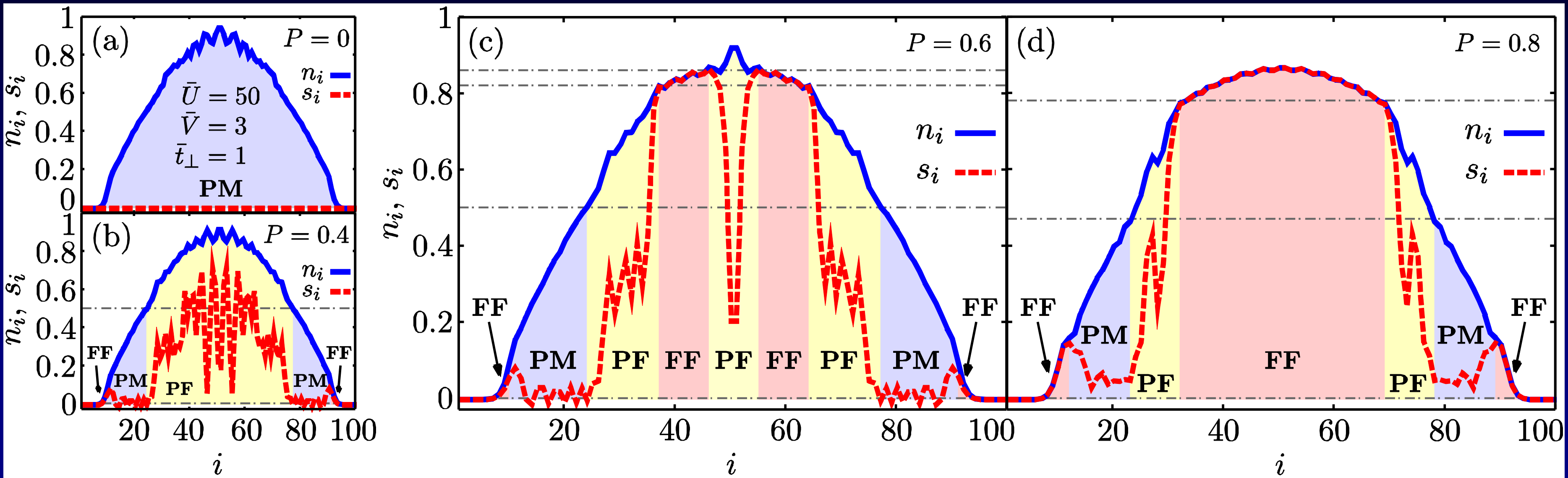


Ferromagnetism in ladders in a trapping potential --- not explored

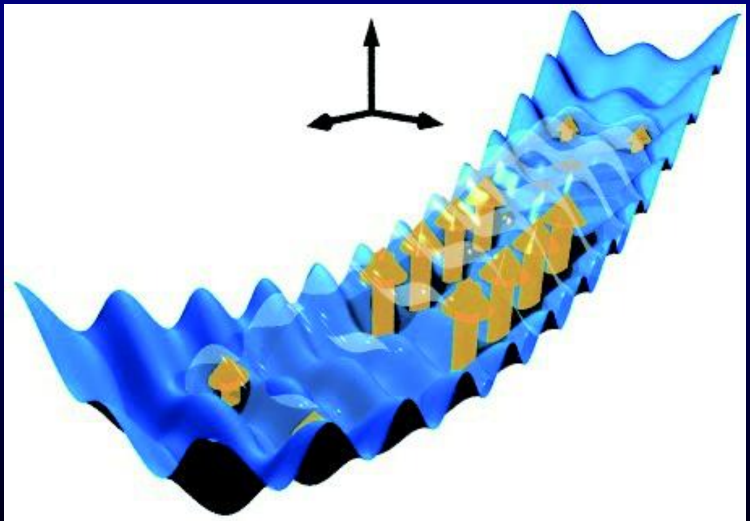
↑ we can fully exploit unique features in cold atoms in optical lattice
* interaction can be tuned
* spin imbalance manipulated
* trapping potential tuned

DMRG result for various spin imbalance: weak U = 1

$$P = (N_\uparrow - N_\downarrow)/N$$

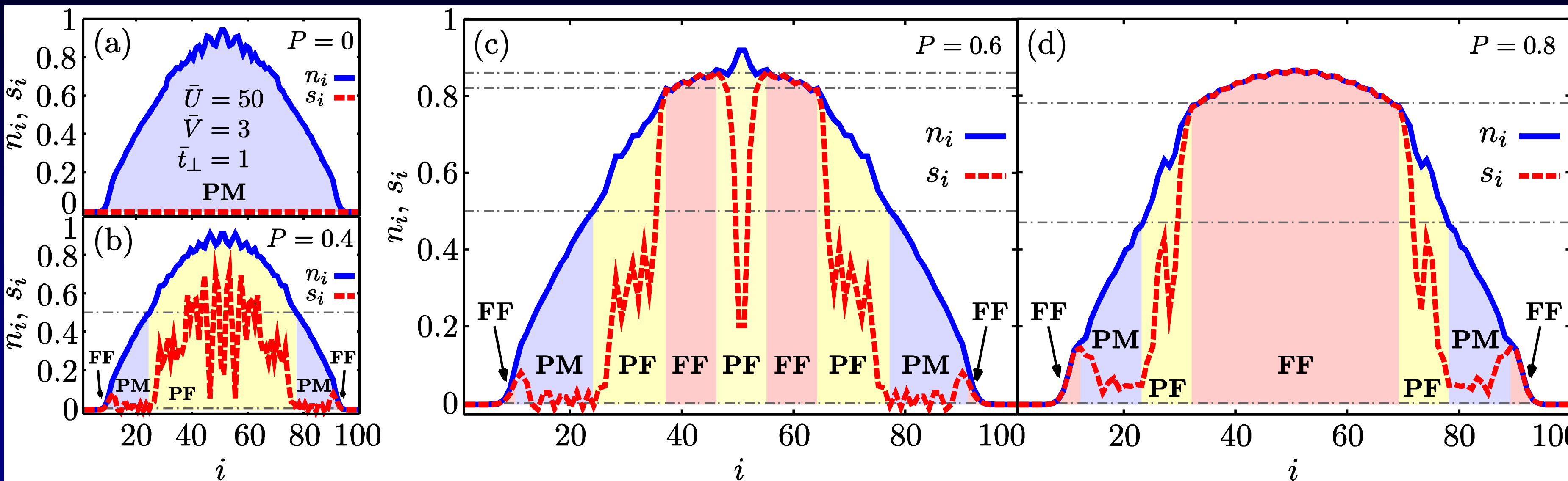


(Okumura et al, 2010)



Fully ferromagnetic domain emerges

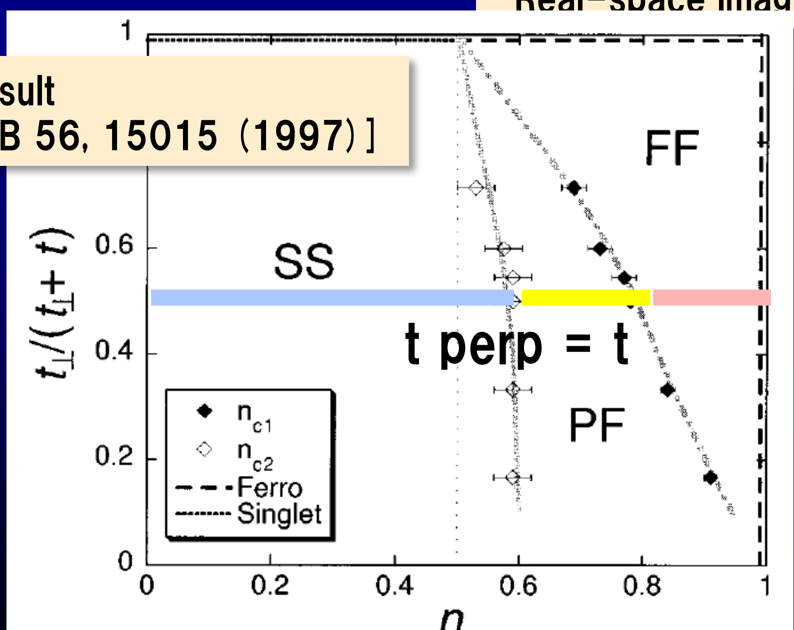
Result for various $P = (N_\uparrow - N_\downarrow)/N$: strong U = 50

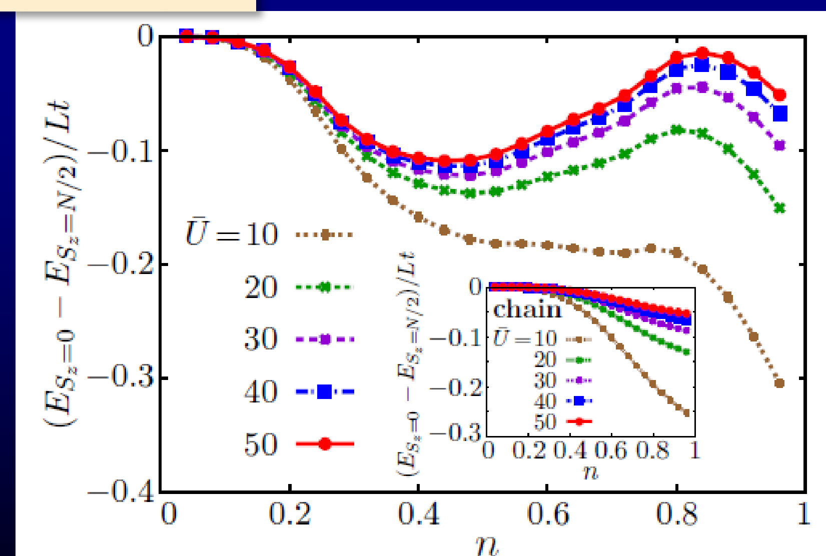


"Real-space imaging of the gap !

(Okumura et al, 2010)

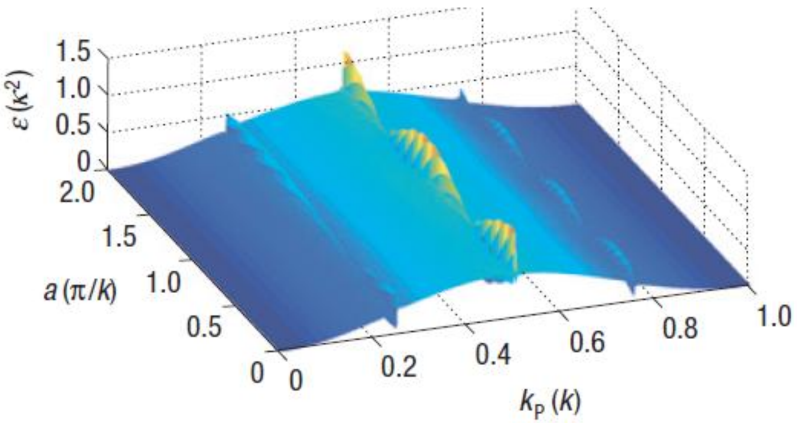
U = infty result
[Kohno, PRB 56, 15015 (1997)]





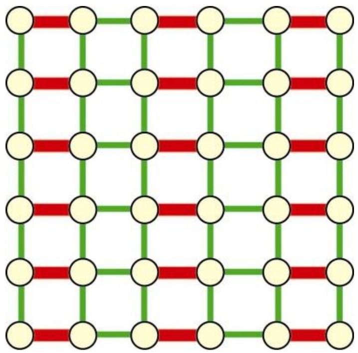
Spin-spin correlation

cf. Eckert et al: Quantum non-demolition detection of spatially-resolved AF states of a spin-1 optical lattice, nature phys 4, 50 (2008)



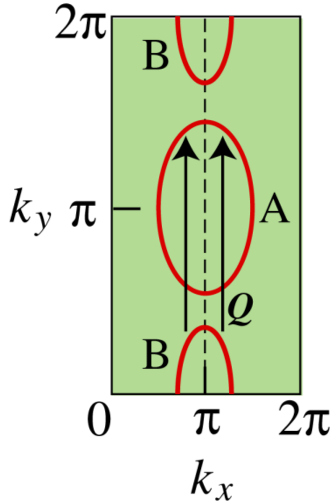
SC in modulated structures having disconnected FS's

$T_c \sim 0.1t$

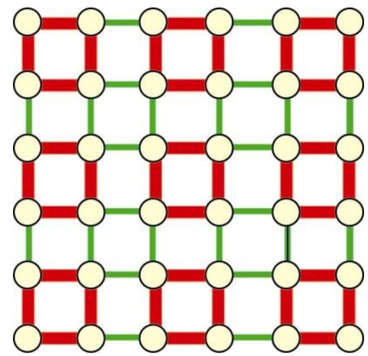


"two-leg ladder"

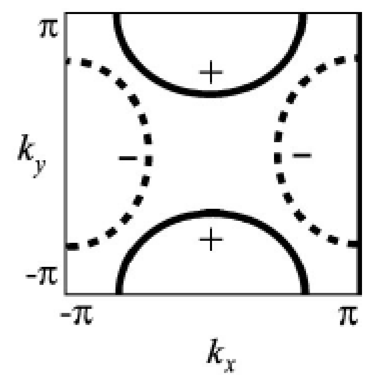
$t_{x2} > t_{x1}$



$T_c \sim 0.05t$



"plaquette"



(Kuroki, Arita & Kimura)

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    (Ota et al, arXiv:1008.3212)
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Collective (phase) modes in SC

Collective mode in one-band SC

Phase modes (Bogoliubov 1959, Anderson 1958, Nambu 1960)
= massless Nambu-Goldstone mode for neutral SC
→ massive for real (charged) SC (Anderson-Higgs mechanism 1963)
as observed in $NbSe_2$ (Raman: Sooryakumar & Klein 1980,
Littlewood & Varma 1981)

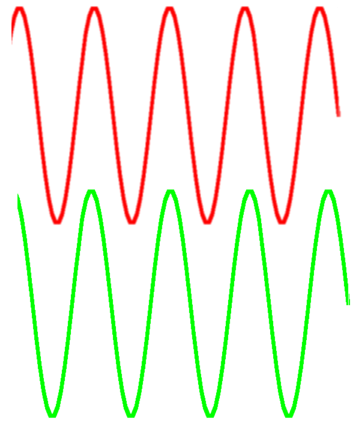
Collective modes in two-band SC

Out-of-phase (countersuperflow) mode (massive, Leggett 1966)

Two-band SC

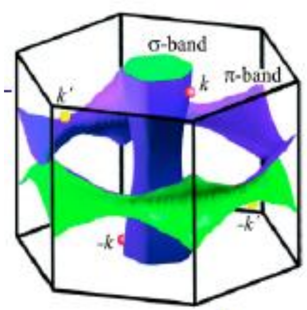
$\varphi^{(1)}$

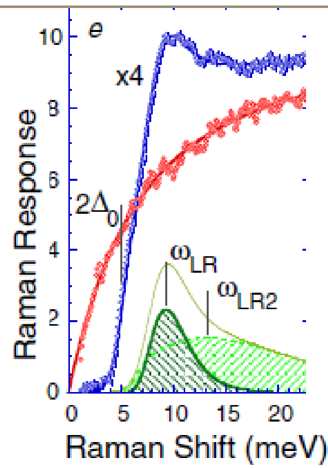
$\varphi^{(2)}$



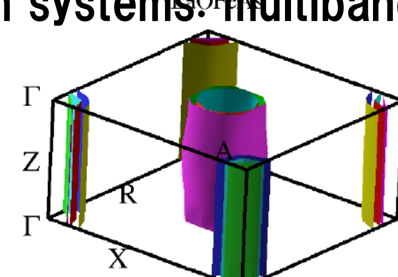
in phase

as observed in MgB_2 (Blumberg et al, PRL 2007)





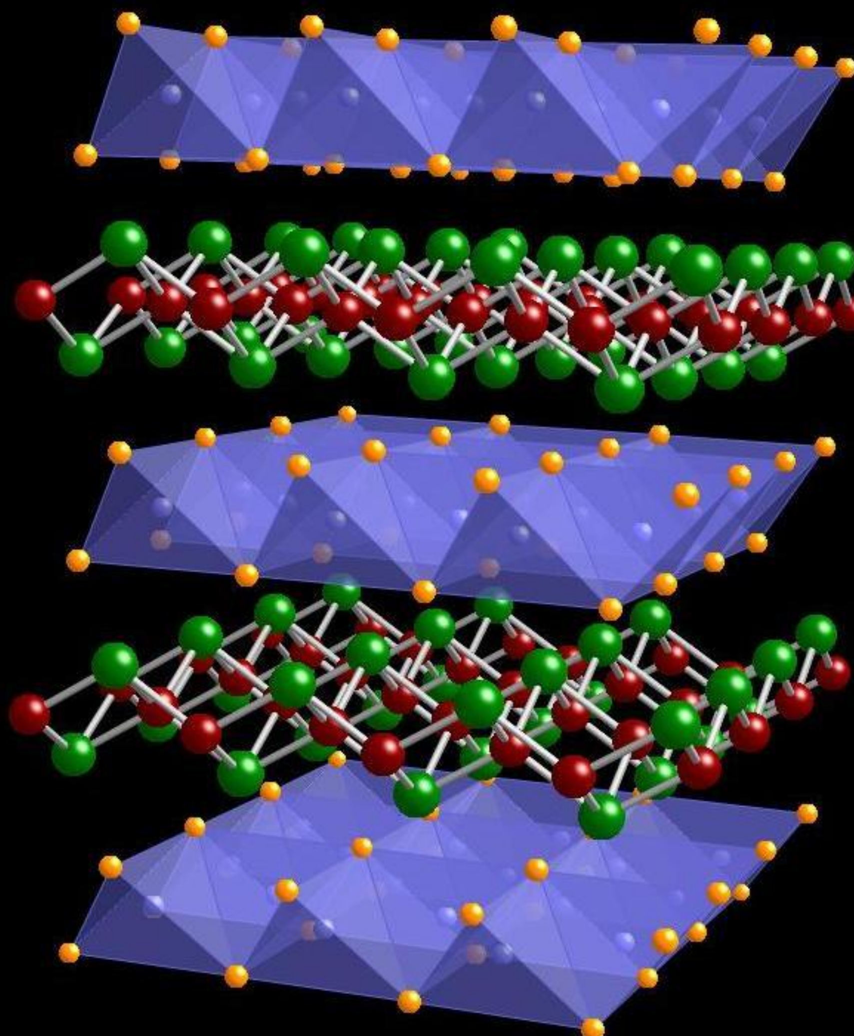
Electron systems: multiband SC



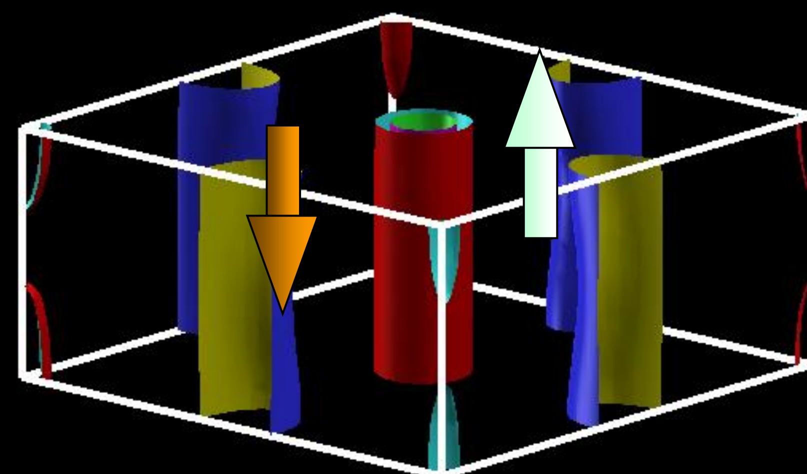
Cold atoms: multiple hyperfine states

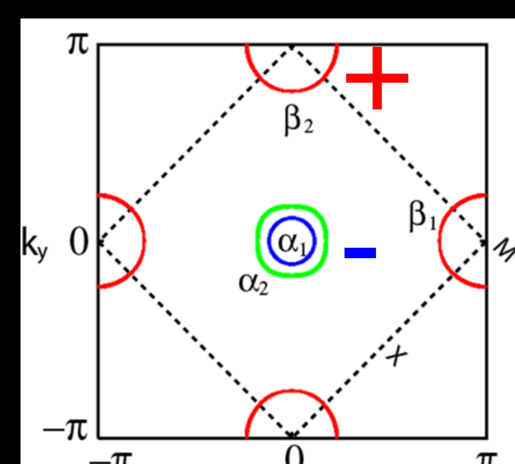
Fe-compound

Kamihara et al, JACS 130, 3296 (2008)



Kuroki et al, PRL 101, 087004 (2008)

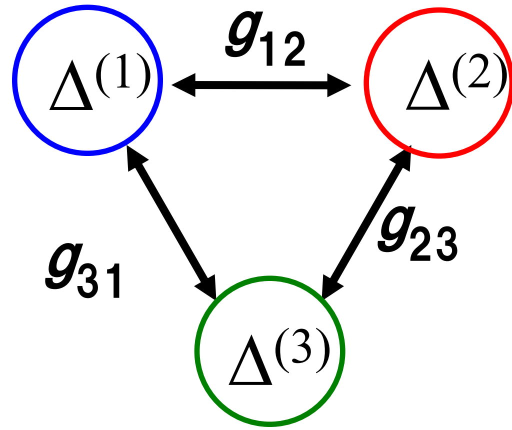




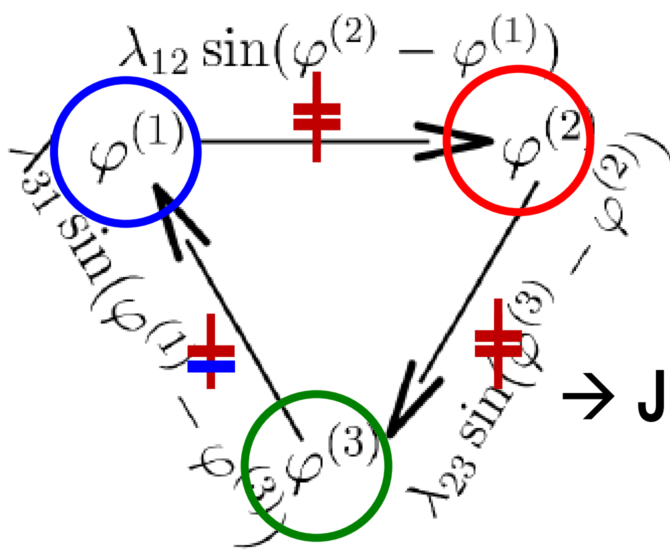
Question here: 3-band = 2-band in terms of the collective modes ?

(Ohta et al, 2010)

$$\hat{\mathcal{H}}_{\rm pair} = -\sum_{i,j} g_{ij} \underbrace{\hat{\psi}_{\uparrow}^{(i)\dagger}\hat{\psi}_{\downarrow}^{(i)\dagger}}_{i\text{th band}}\underbrace{\hat{\psi}_{\downarrow}^{(j)}\hat{\psi}_{\uparrow}^{(j)}}_{j\text{th band}}$$



- Ohta et al (in prep)



even / odd ← $(g^{-1})_{ij}$

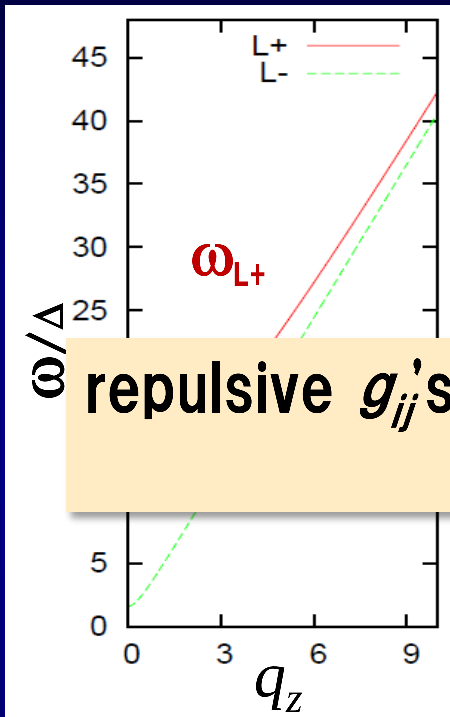
[class "even"]
→ internal Josephson c's add

[class "odd"]
→ Josephson c's subtract

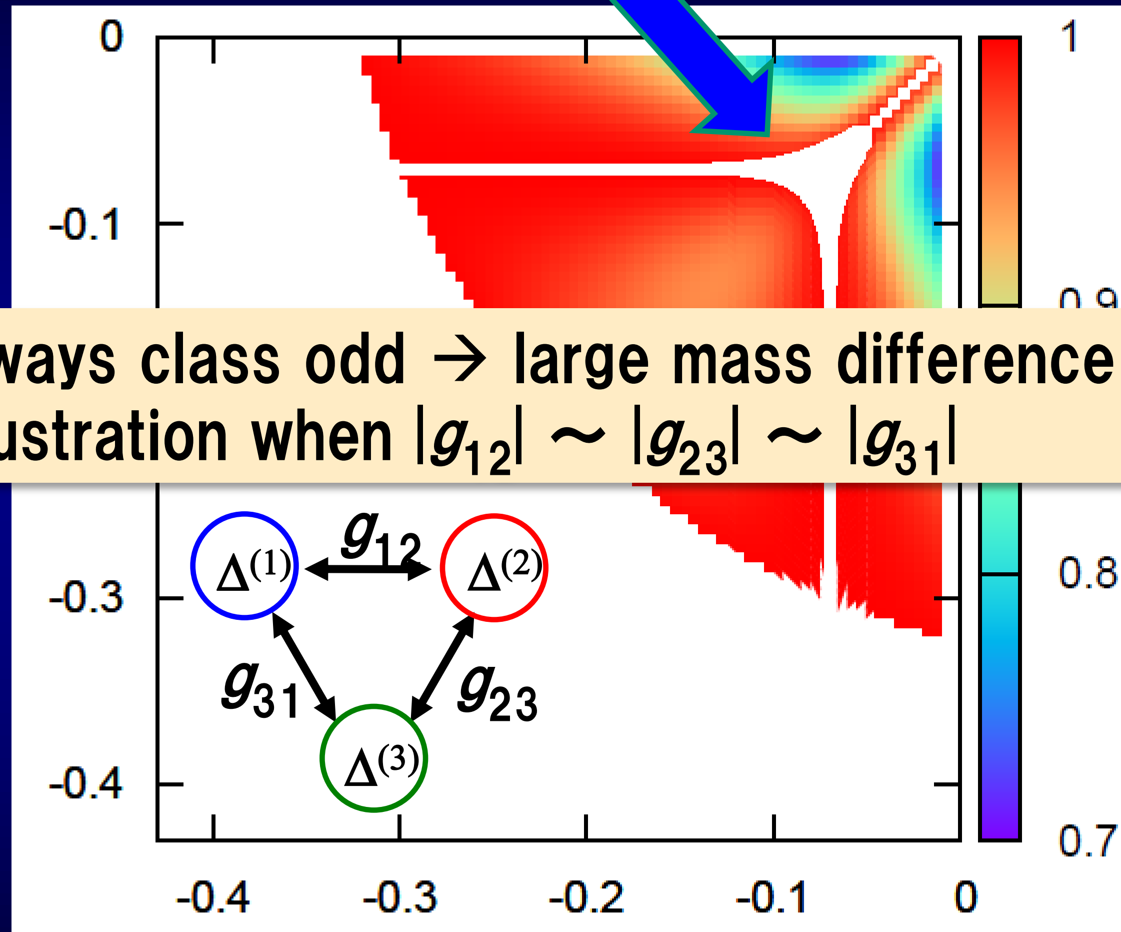
When the pairing interactions g_{ij} are varied

(Ohta et al, 2010)

multiple Leggett modes



mass difference



repulsive g_{ij}'s → always class odd → large mass difference

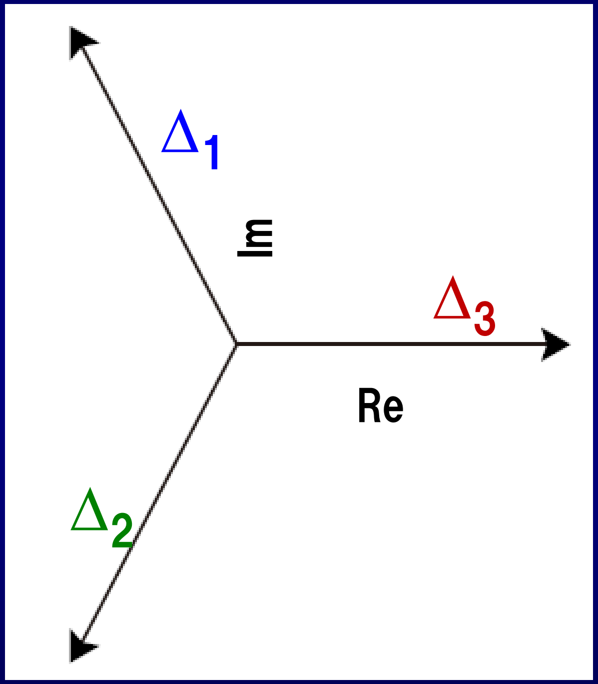
→ frustration when $|g_{12}| \sim |g_{23}| \sim |g_{31}|$

g_{12}

$g_{23}=-0.07$

Three-band SC can accommodate complex Δ i.e., spontaneously broken T

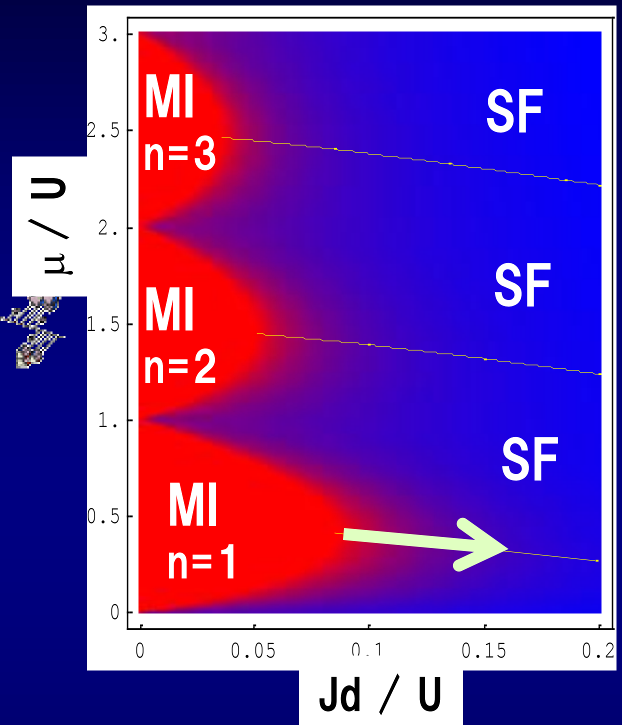
(Stanev & Tesanovic, PRB 2010)

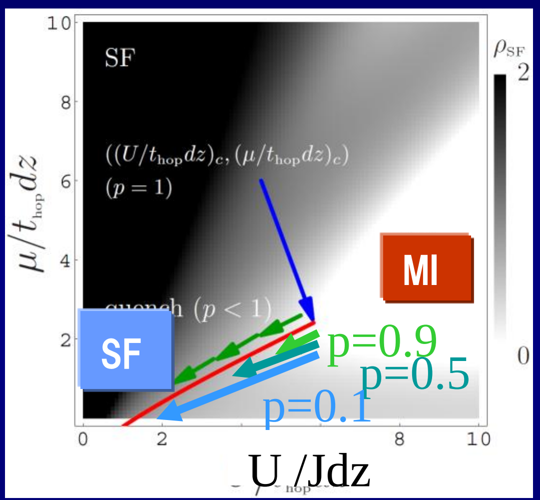


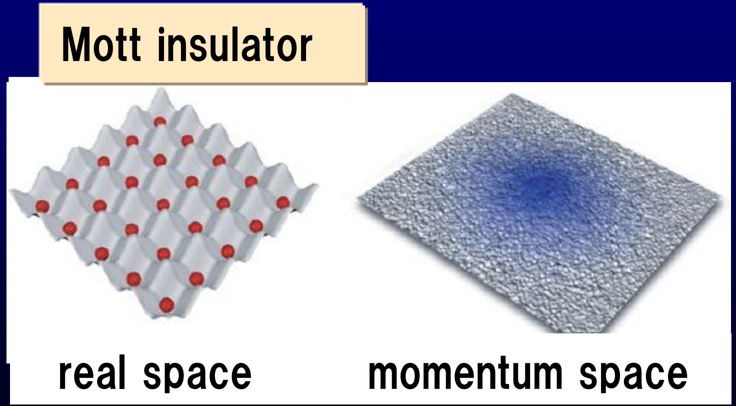
- 電子相関の物理
  - 磁性、超伝導
- 相関現象のplaygroundとしての光格子
  - 強磁性（梯子、籠目）
  - 超流動（多成分格子系での集団励起）
  - 超流動・モット絶縁体転移（Kibble-Zurek）

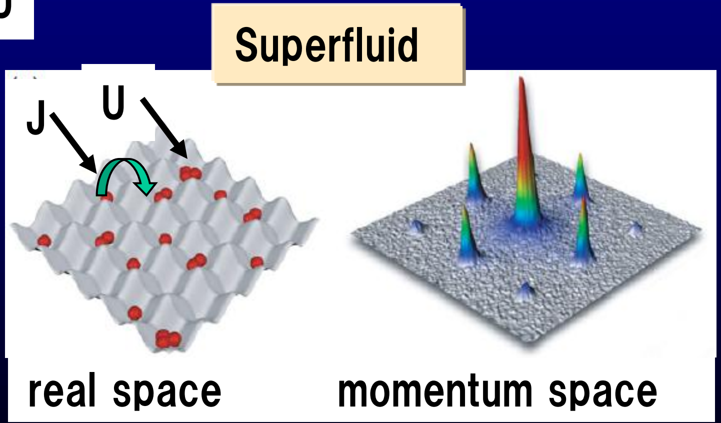
    [Horiguchi et al, J. Phys.: Conf. Series **150**, 032007 (2009)]
  - 格子系でフェルミオン間の相互作用を斥力⇔引力変換可？

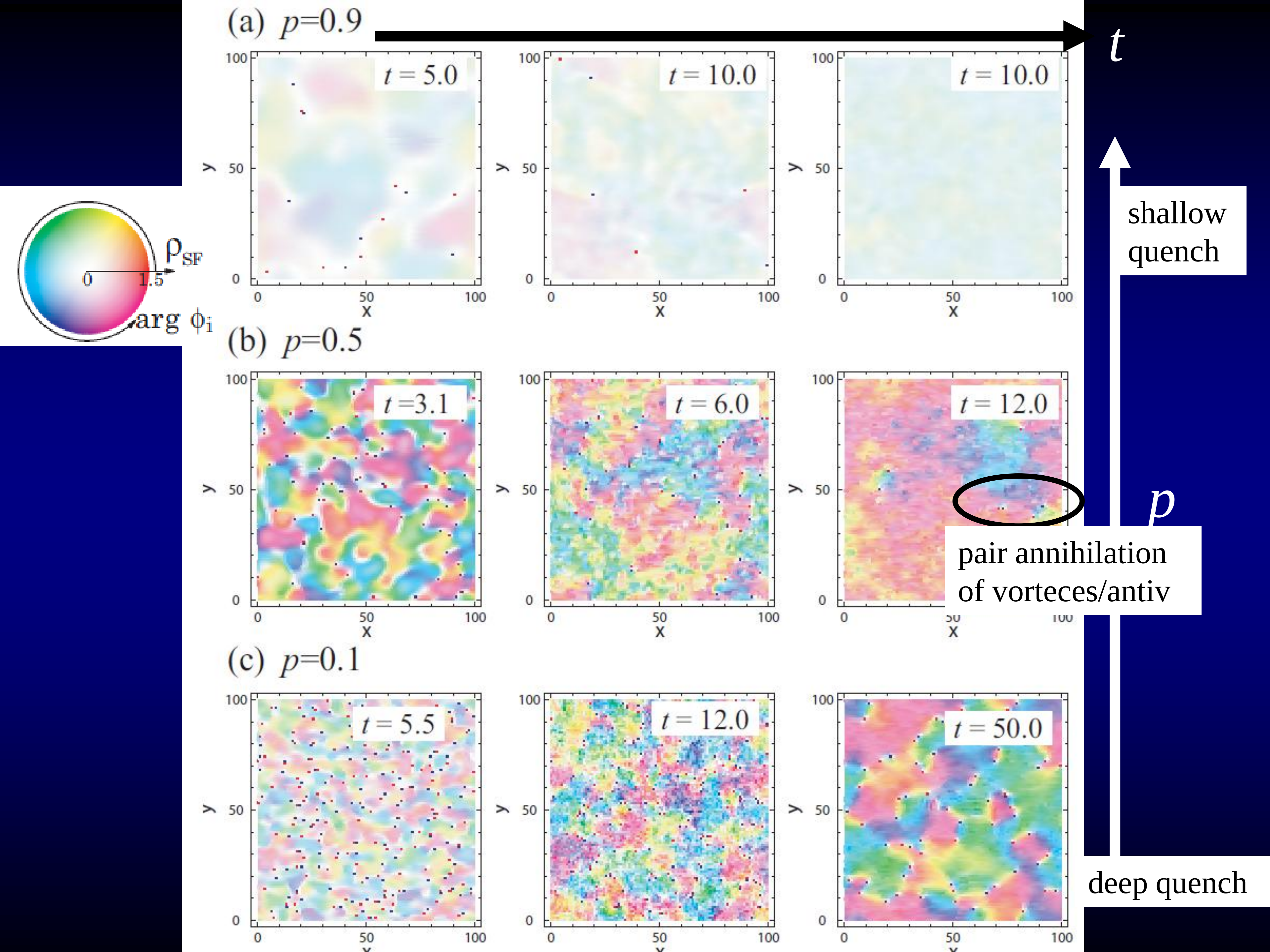
Bose-Hubbard for cold atoms in optical lattices – an ideal playground for nonequilibrium







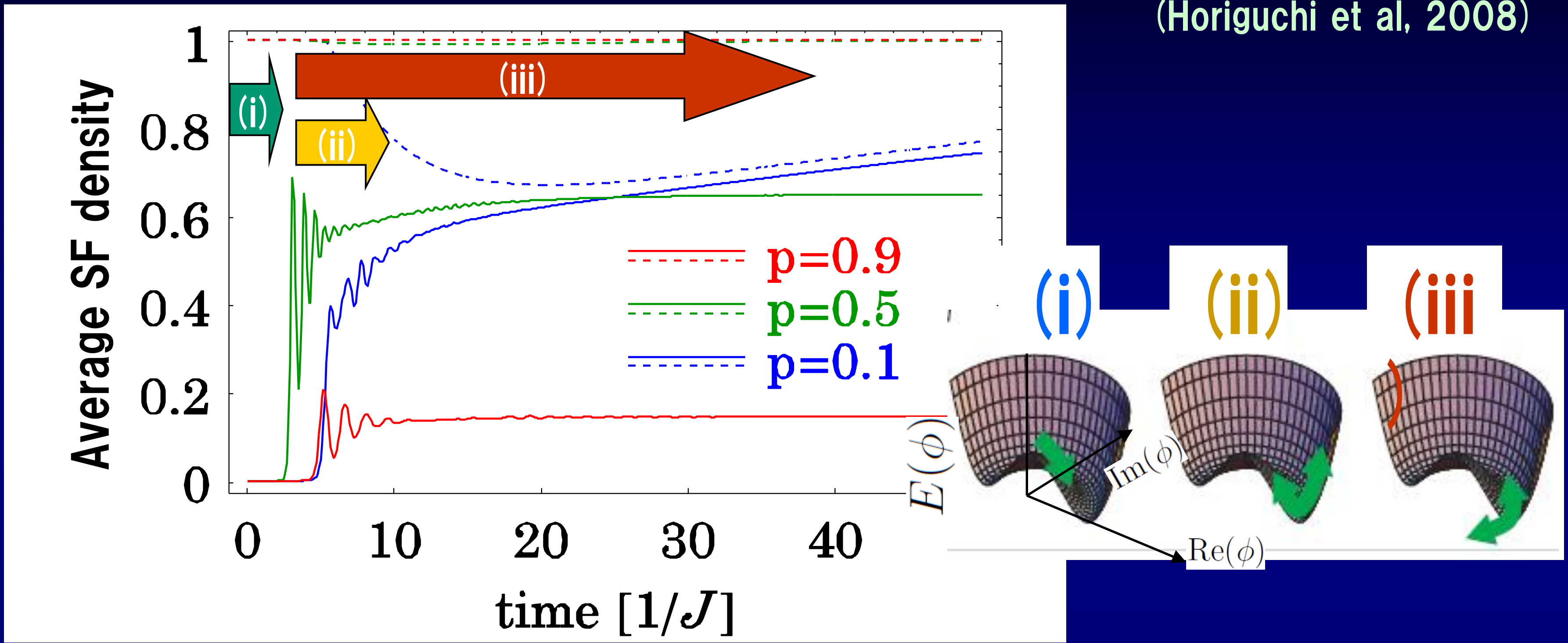




(a) p=0.9
t
t = 5.0
t = 10.0
t = 10.0
ρSF
0
1.5
arg φi
shallow quench
(b) p=0.5
t =3.1
t = 6.0
t = 12.0
p
pair annihilation of vorteces/antiv
(c) p=0.1
t = 5.5
t = 12.0
t = 50.0
deep quench

Kibble-Zurek for Mott→superfluid

(Horiguchi et al, 2008)

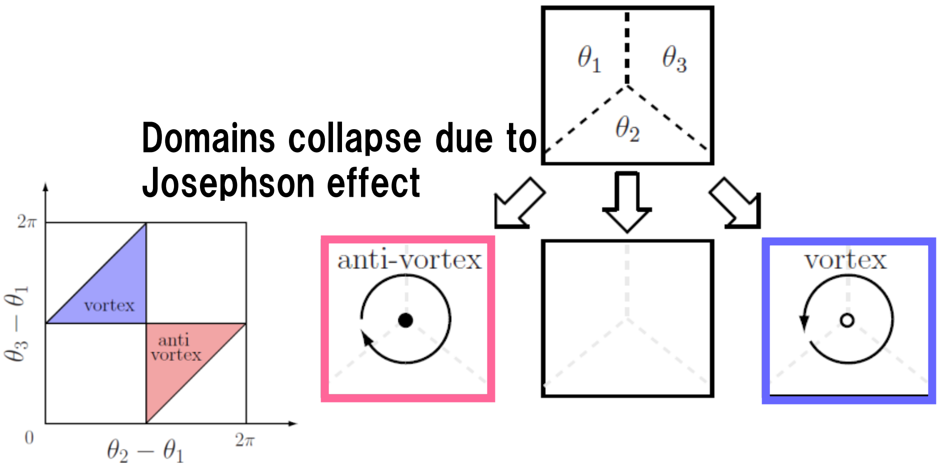


continuous symmetry broken in the condensate $\phi=|\phi|e^{i\theta}$
→ topological defects, Kibble-Zurek mechanism

Later process: Kibble-Zurek mechanism

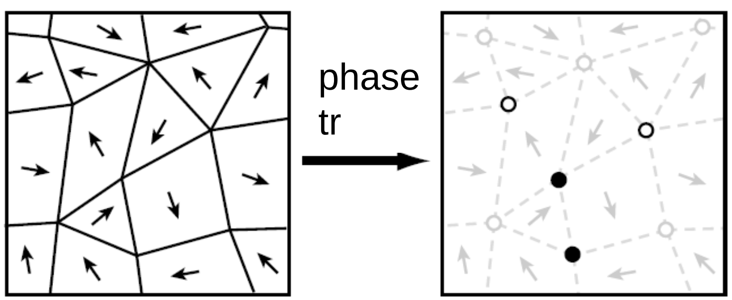
nodal point in a U(1) gauge domain structure

Domains collapse due to Josephson effect



Kibble, J. Phys. A **9**, 1387 (1977);
Zurek, Nature **317**, 505 (1985);
Zurek, Phys Rep **276**, 177 (1996):
generally applicable to 2nd order transition involving continuous phases

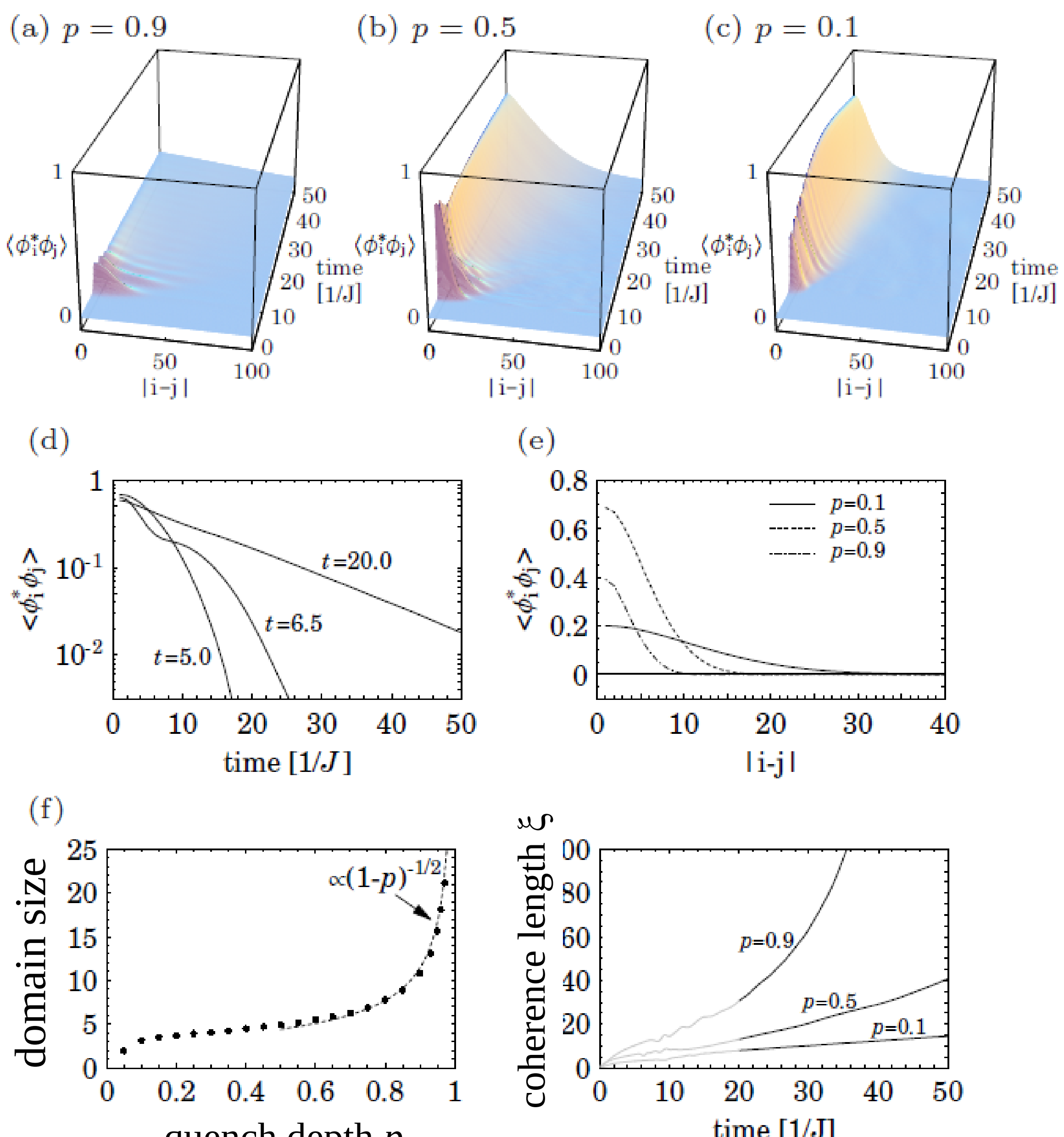
- Initial cosmic evolution (Kibble, 1977)
- Superfluid-normal in He (Zurek, 1985, ...)
- N-I transition in liq crystals (Chuang, 1991)
- Spinor BEC (Sadler, 2006)



MI SF

SF-MI transition
in cold atoms in optical lattices
(a strongly correlated system)

Correlation f's and domain size



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  - 超流動(多成分格子系での集団励起)
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  - 格子系でフェルミオン間の相互作用を斥力⇔引力変換可?

(Tsuji et al, arXiv:1008.2594)

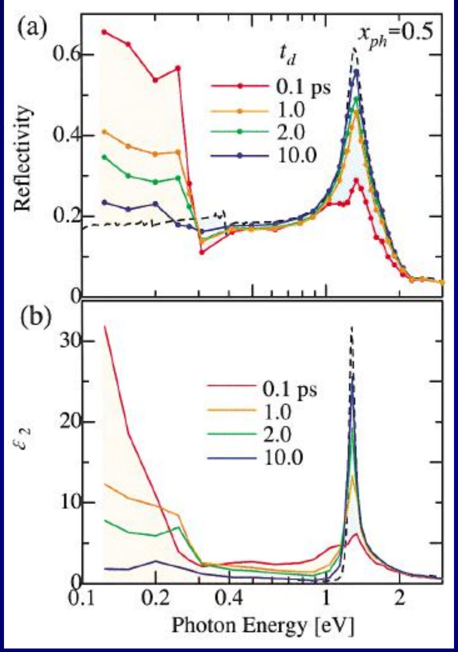
Correlated system ← intense laser: a wealth of phenomena

e.g., Photo-induced metallisation

$[Ni(chxn)_2Br]Br_2$:

1D CT insulator

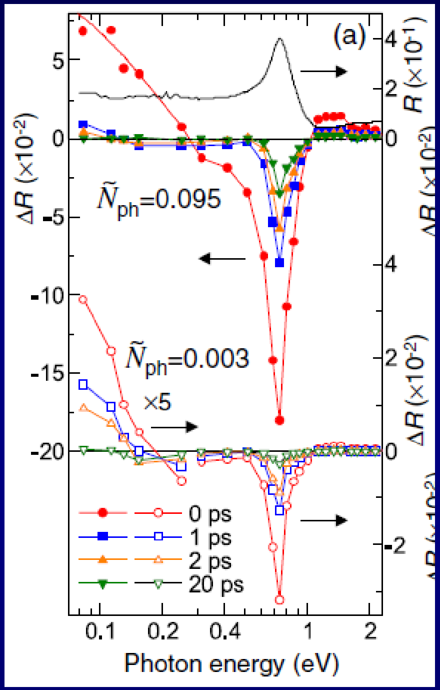
Iwai *et al*, PRL (2003)



$ET\text{-}F_2TCNQ$:

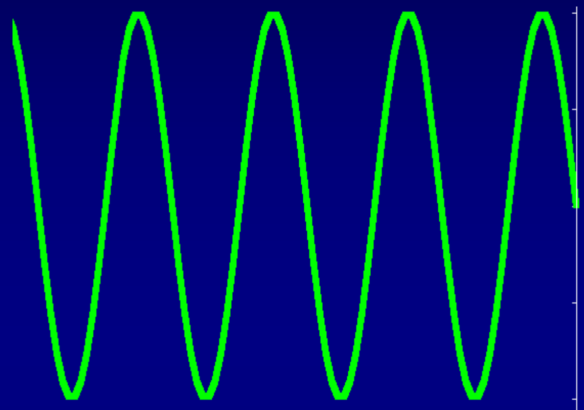
1D organic Mott insulator

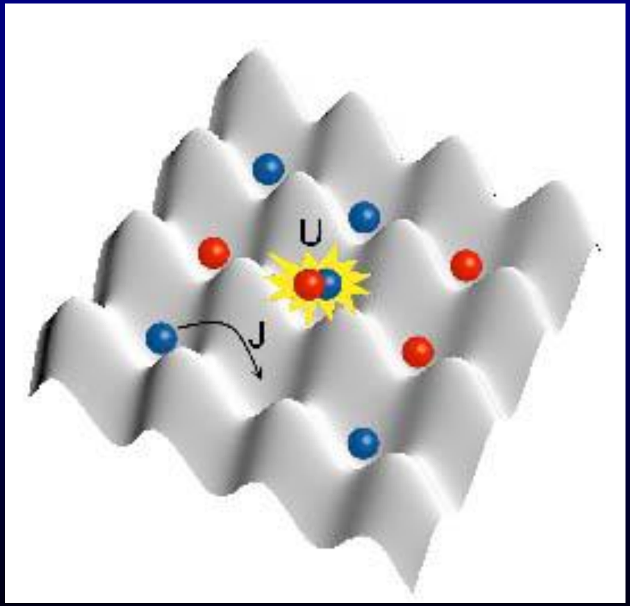
Okamoto *et al*, PRL (2007)



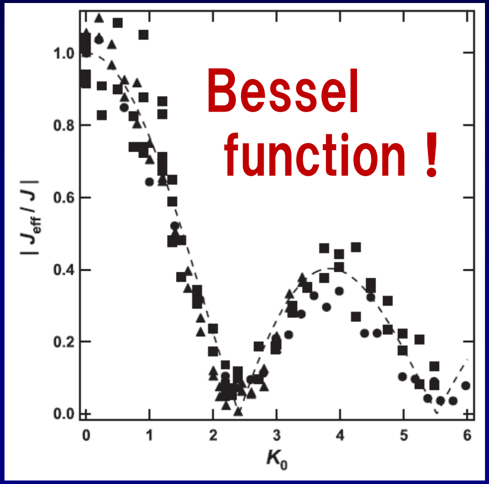
Non-equilibrium in AC fields

Cold atoms in an optical lattice + ac modulation
(Lignier et al, PRL 2007)





t eff / t



amplitude of ac / Ω

- Floquet theory
  gives a rigorous proof for Bessel F
  (Tsuji, Oka, Aoki, PRB, 2008)

Proposal / numerical finding

- Interacting lattice fermions driven by ac external fields:
  repulsive interaction → attraction !

- → ac-induced superconductivity / superfluidity

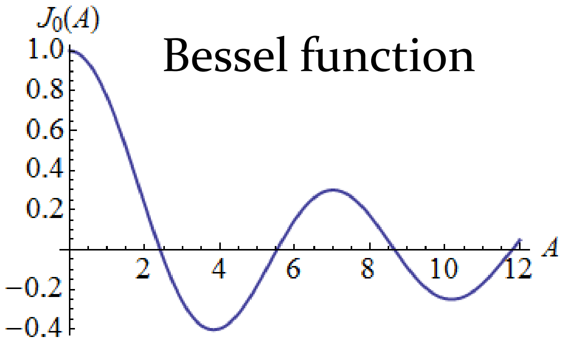
Bloch particles in ac field

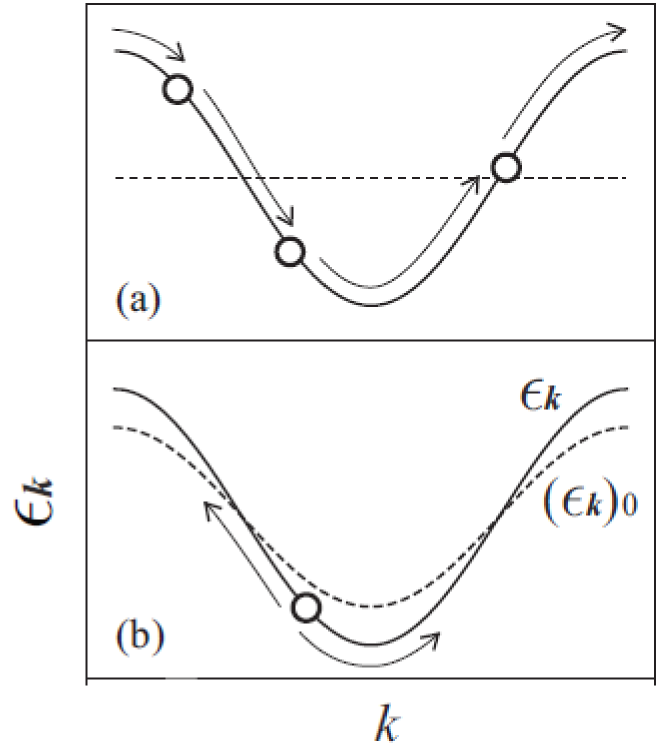
- Single-particle excitation energy for Bloch wave number k at time t

$$\epsilon_{\boldsymbol{k}-\boldsymbol{A}(t)} = -2J\cos(k - A\sin\Omega t)$$

- If an interband transition is absent, then

$$\epsilon_{\boldsymbol{k}-\boldsymbol{A}(t)} \to \langle \epsilon_{\boldsymbol{k}} \rangle = \frac{1}{\tau}\int_0^{\tau} dt\, \epsilon_{\boldsymbol{k}-\boldsymbol{A}(t)} = \mathcal{J}_0(A)\epsilon_{\boldsymbol{k}}$$





- Floquet theory gives a rigorous proof for the above statement. Tsuji, Oka, Aoki, PRB (2008)

particles in arbitrary ac fields --- Floquet's theorem

- 光 (振動電場) などの時間的に周期的な外場(強度は任意)に対し成り立つ.
- 空間的に周期的な系で成り立ったBlochの定理
  → 時間に周期的な系でのanalogue.

$$i\frac{d}{dt}\Psi(t) = H(t)\Psi(t) \qquad H(t+\tau) = H(t)$$

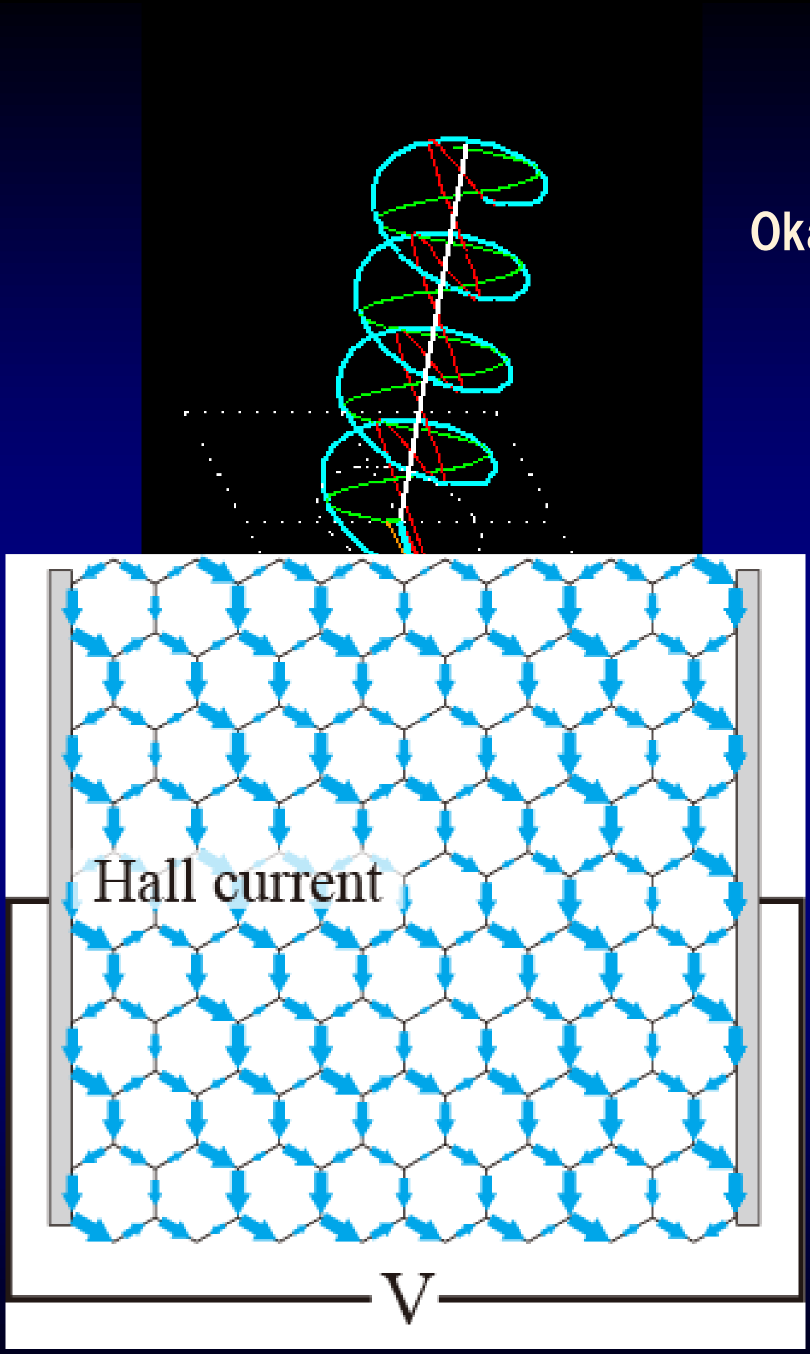
- Floquet state: $\Psi_\alpha(t) = e^{-i\varepsilon_\alpha t}u_\alpha(t), \qquad u_\alpha(t+\tau) = u_\alpha(t)$
- Fourier変換により、HamiltonianはFloquet 行列形式:

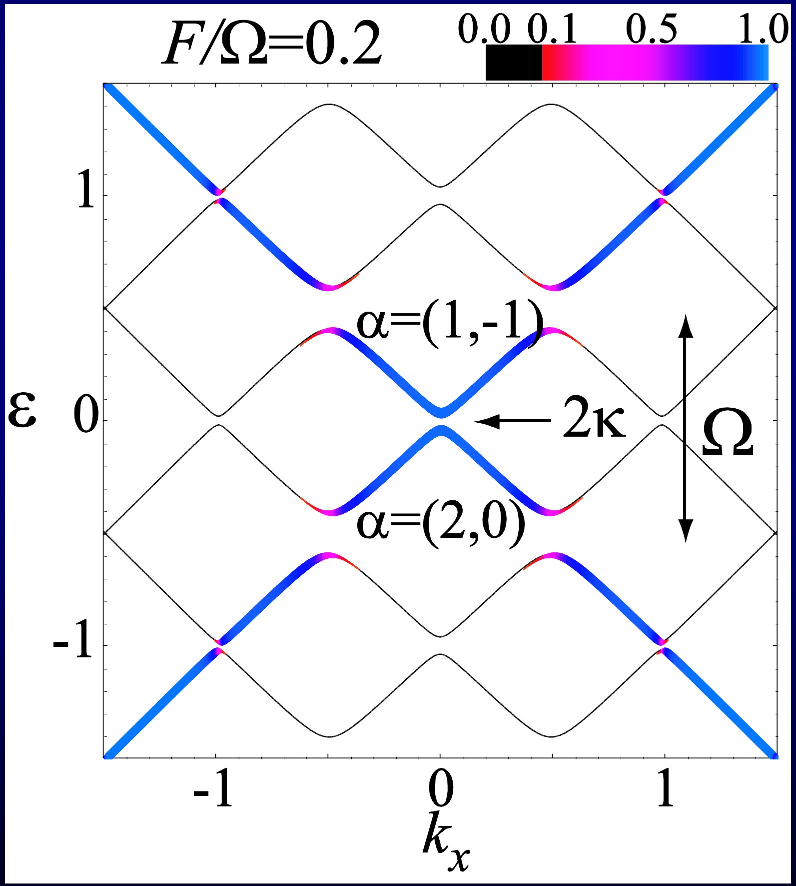
$$\sum_n H_{mn}\, u_\alpha^n = (\varepsilon_\alpha + m\Omega)\, u_\alpha^m$$

$$H_{mn} \equiv \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} dt e^{i(m-n)\Omega t} H(t)$$

- 結局、時間依存の問題 → 時間に依存しない問題に置き換わった
  (代償としてFloquet mode *n* という自由が加わった) .

Oka & Aoki, "Photovoltaic Hall effect in graphene"
(PRB **79**, 081406 (R) (2009); ibid 169901)

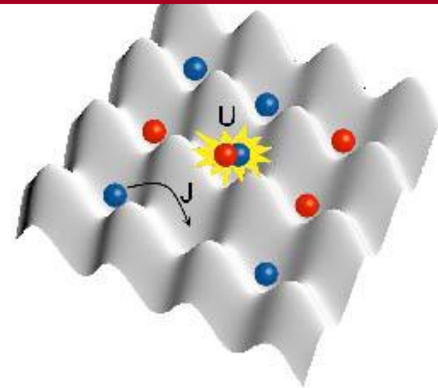




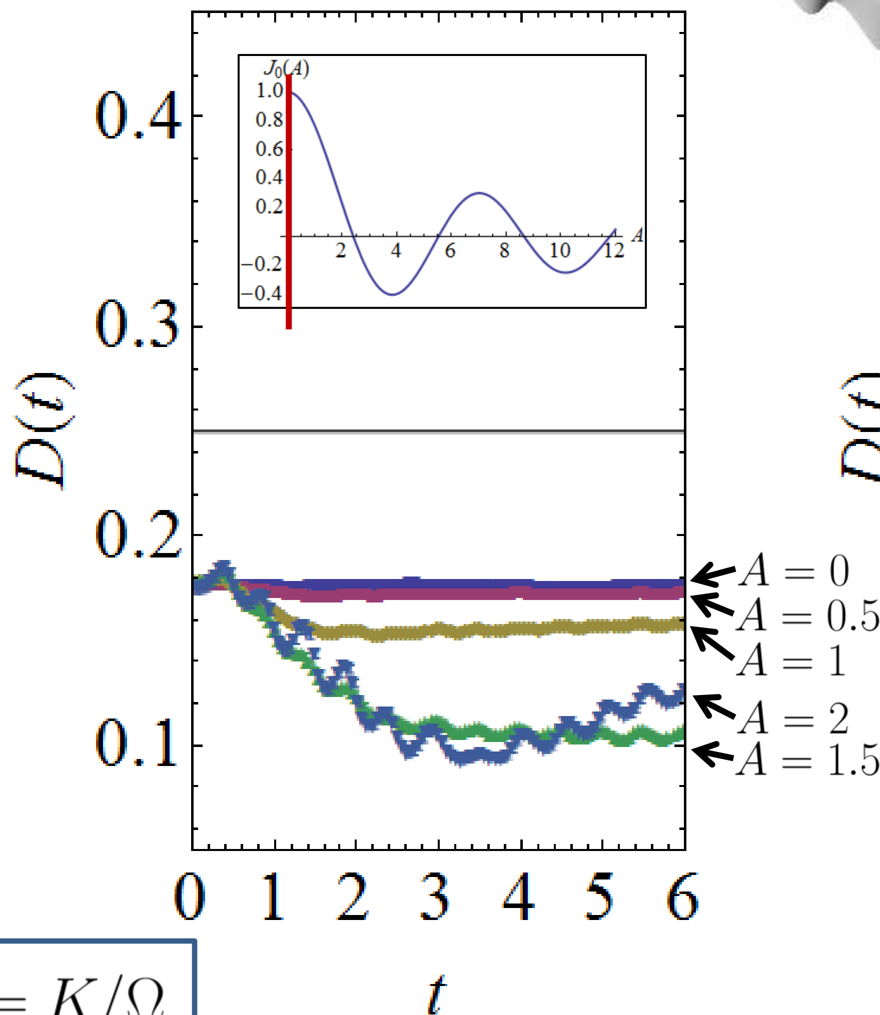
Numerical (DMFT) result for the double occupancy

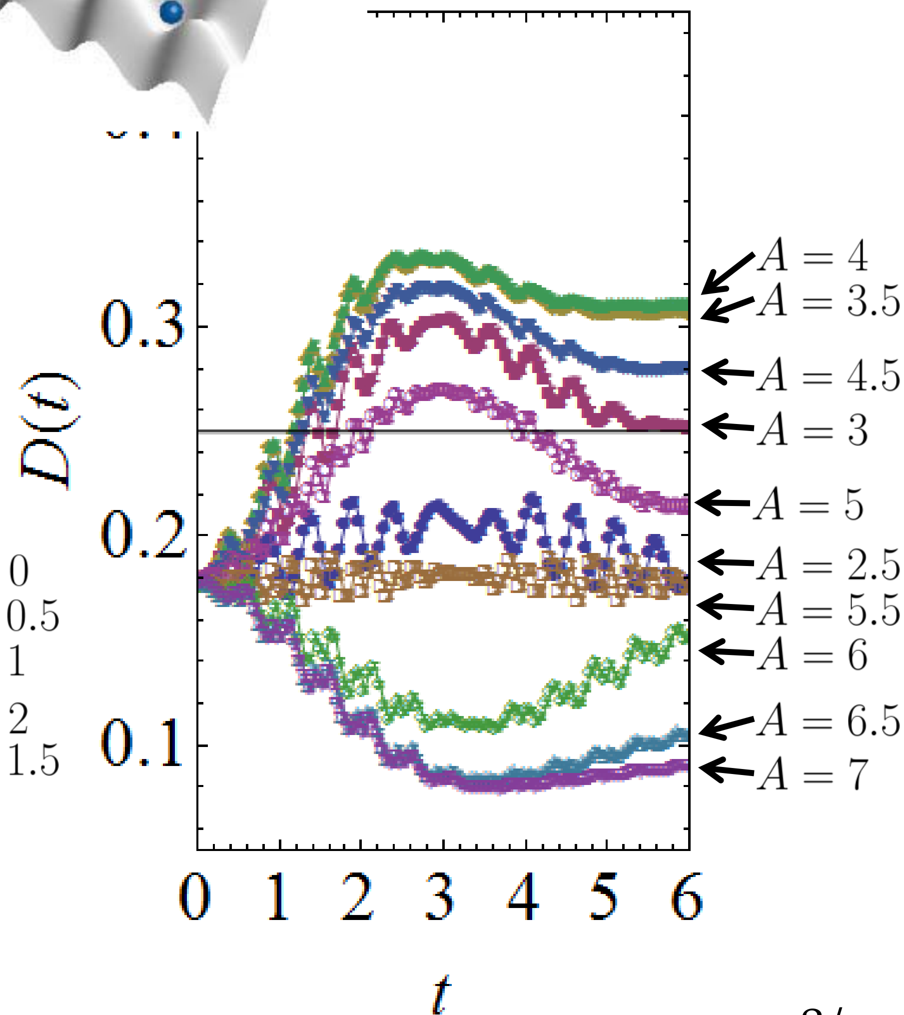
$$D(t) = \langle n_{i\uparrow}(t) n_{i\downarrow}(t) \rangle$$

free particle: $D = \langle n_\uparrow \rangle \langle n_\downarrow \rangle = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$



$U = 1, \Omega = 2\pi, \beta = 10$





$$A = K/\Omega$$

Positive or negative T ?

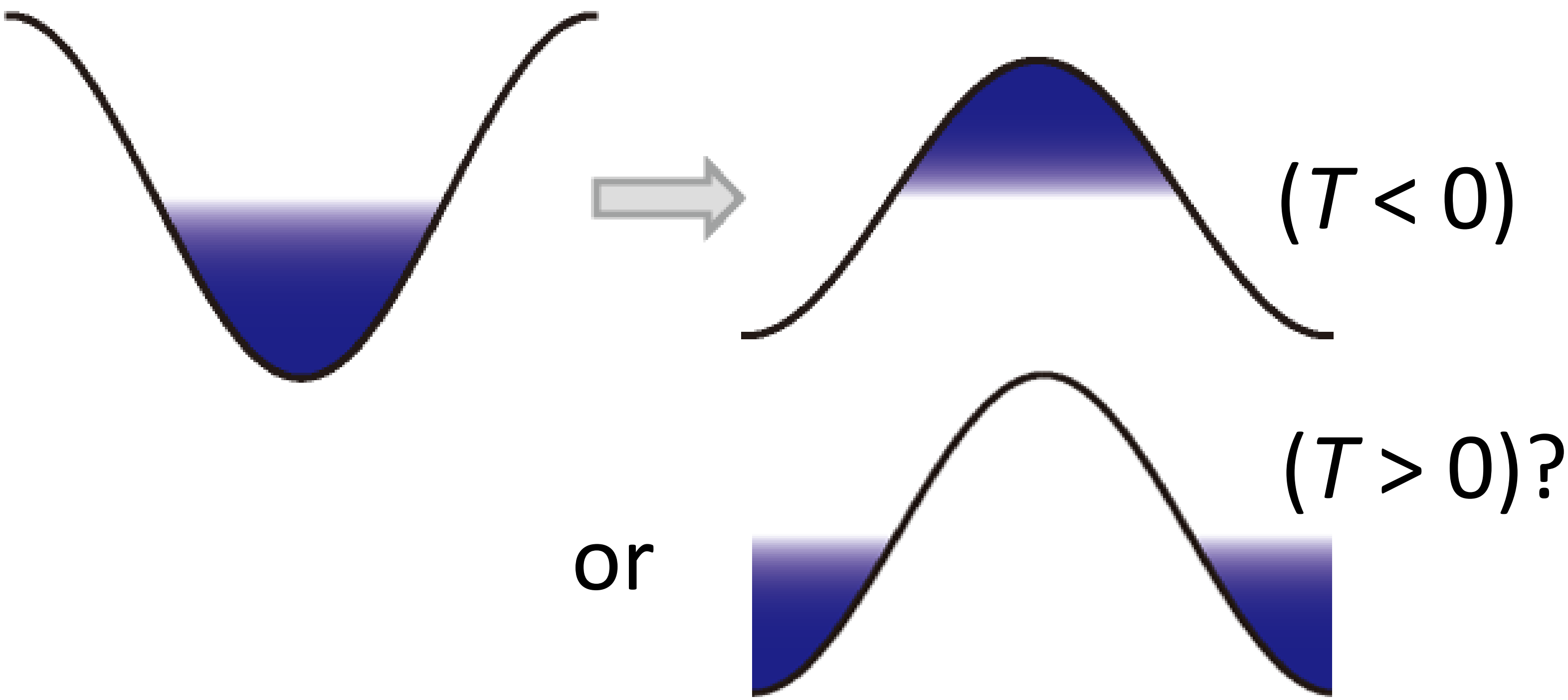
- Density matrix:

$$\rho(t) \xrightarrow{t\to\infty} \propto e^{-(-J_{\rm eff}\mathcal{H}_{\rm K}+U\mathcal{H}_{\rm I})/T^\star_{\rm eff}}$$

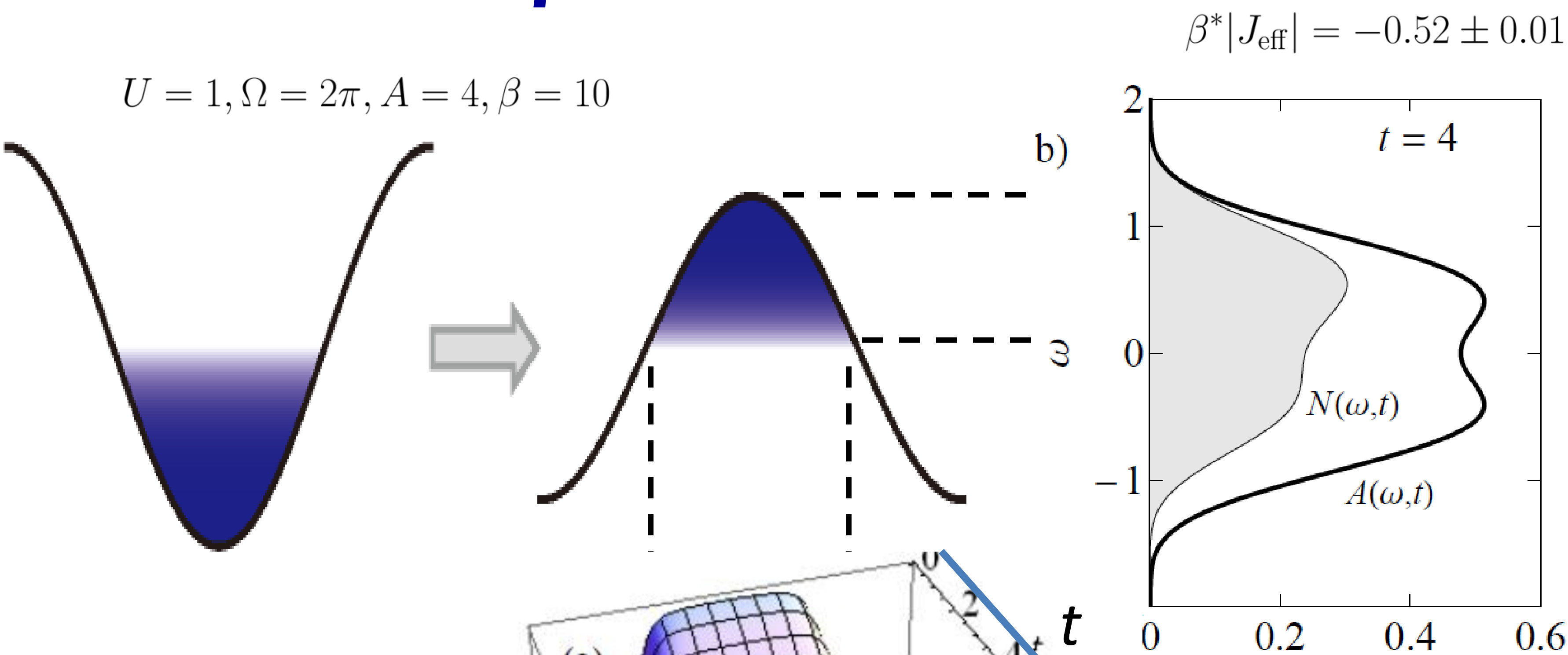
$$= e^{-(-J\mathcal{H}_{\rm K}+U_{\rm eff}\mathcal{H}_{\rm I})/[T^\star_{\rm eff}/\mathcal{J}_0(\mathcal{A})]}$$

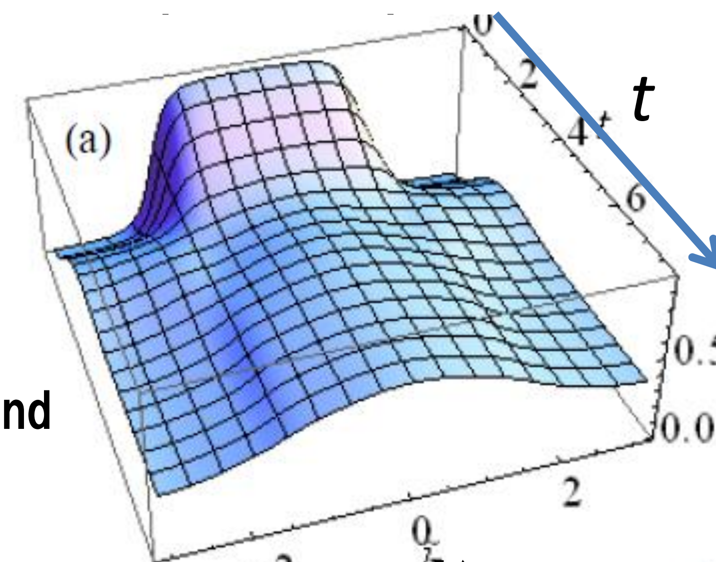
$$U \to U_{\rm eff} = U/\mathcal{J}_0(\mathcal{A})$$

- Positive U, negative T
- ←→ Negative U, positive T



Population inversion





Distribution sticks to the band
→ neg *T*!
(cf. cold atoms
[Lignier et al PRL 2007; Eckardt et al EL 2007])

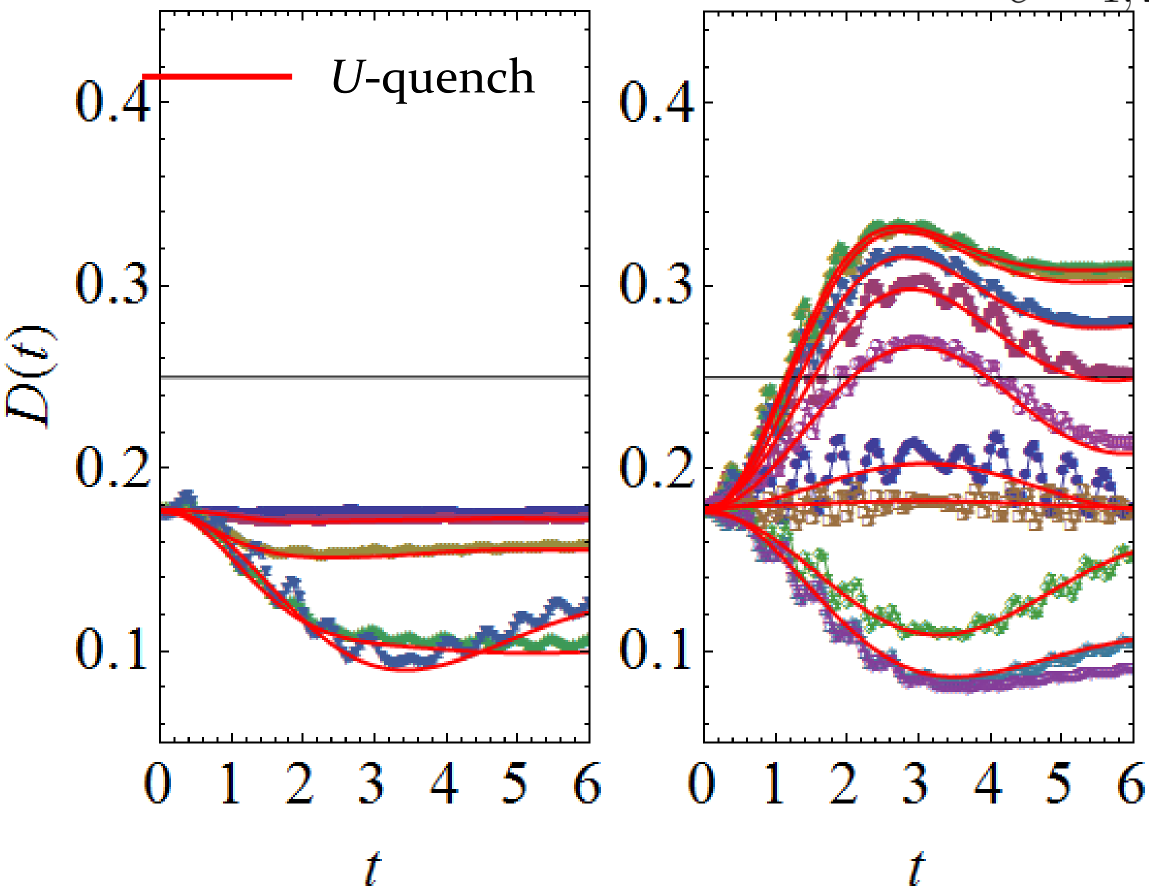
ac-quench vs U-quench

$$\rho(t) = e^{-it(-J_{\rm eff}\mathcal{H}_{\rm K}+U\mathcal{H}_{\rm I})}\rho(0)e^{it(-J_{\rm eff}\mathcal{H}_{\rm K}+U\mathcal{H}_{\rm I})} =$$
$$e^{\mp i\tilde{t}(-J\mathcal{H}_{\rm K}+U_{\rm eff}\mathcal{H}_{\rm I})}\rho(0)e^{\pm i\tilde{t}(-J\mathcal{H}_{\rm K}+U_{\rm eff}\mathcal{H}_{\rm I})} \equiv \rho^{\pm}_{\mathcal{A}}(t)$$

$\tilde{t} = |\mathcal{J}_0(A)|t$

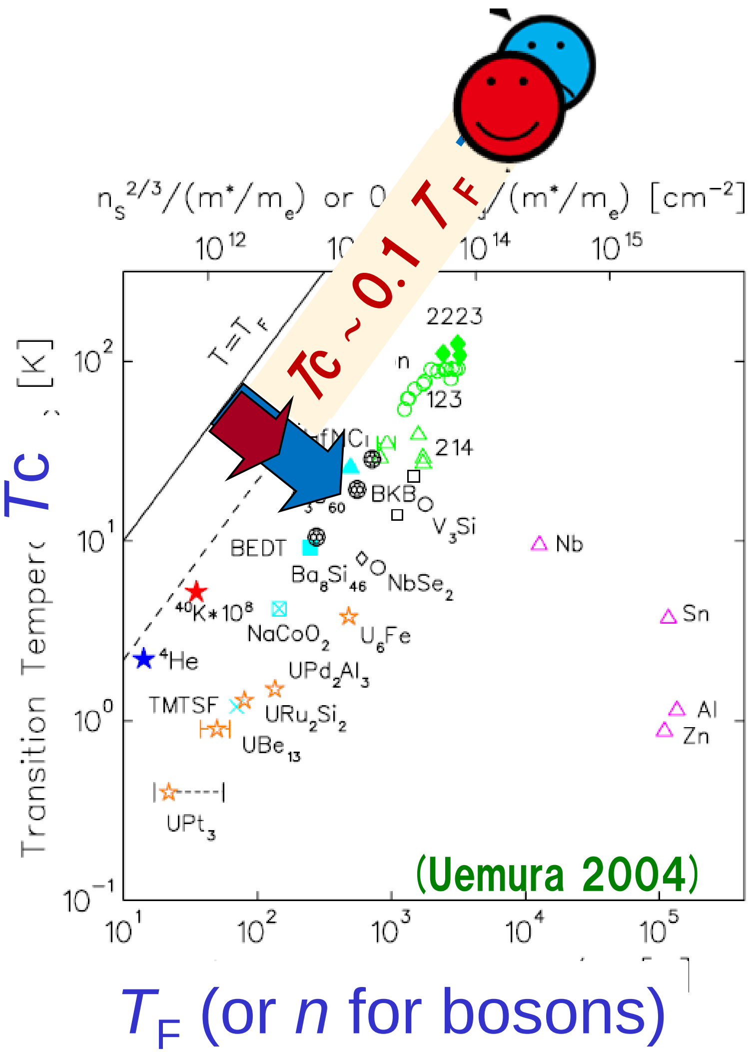
For J <0, system evolves with ρ-, with the inverted energy

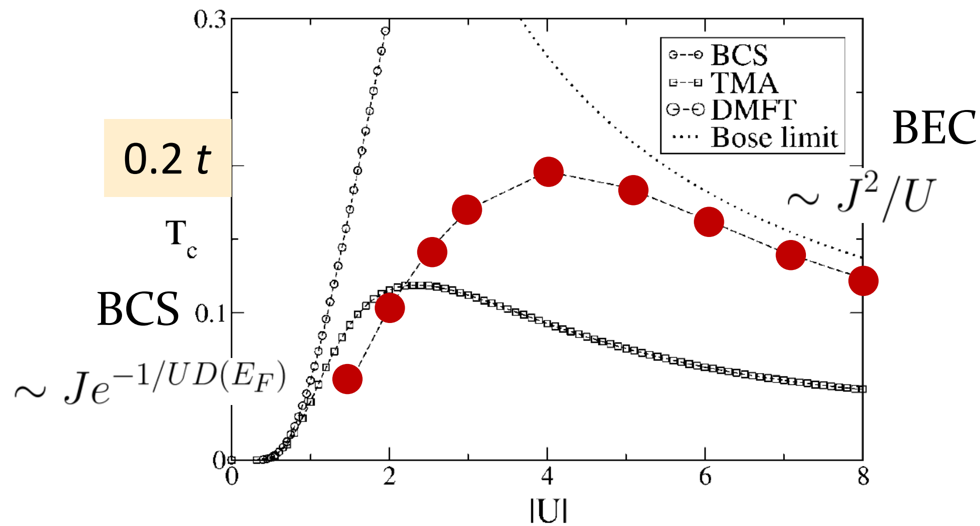
$U = 1, \Omega = 2\pi, \beta = 10$



Dynamical superconductivity

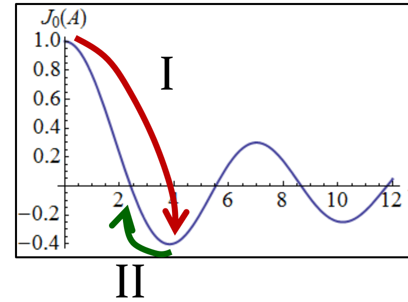
- Repulsive interaction (Tc ~ 0.01 J with anisotropic pairing)
  - → Attractive interaction (Tc ~ 0.1 J with s-pairing)

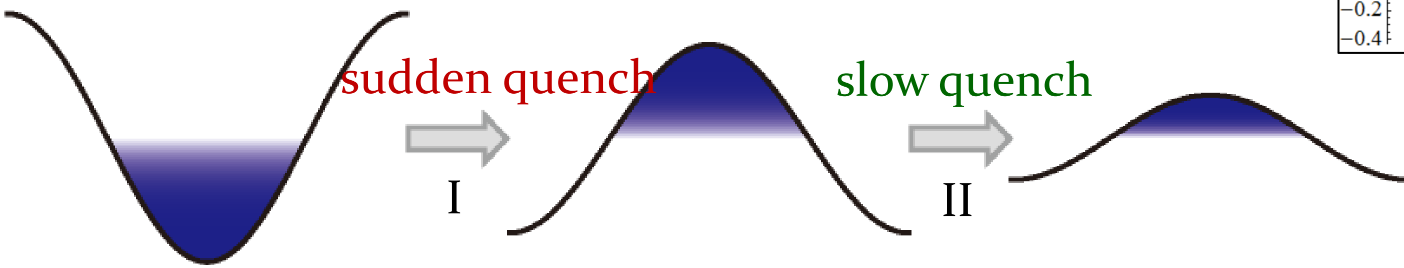


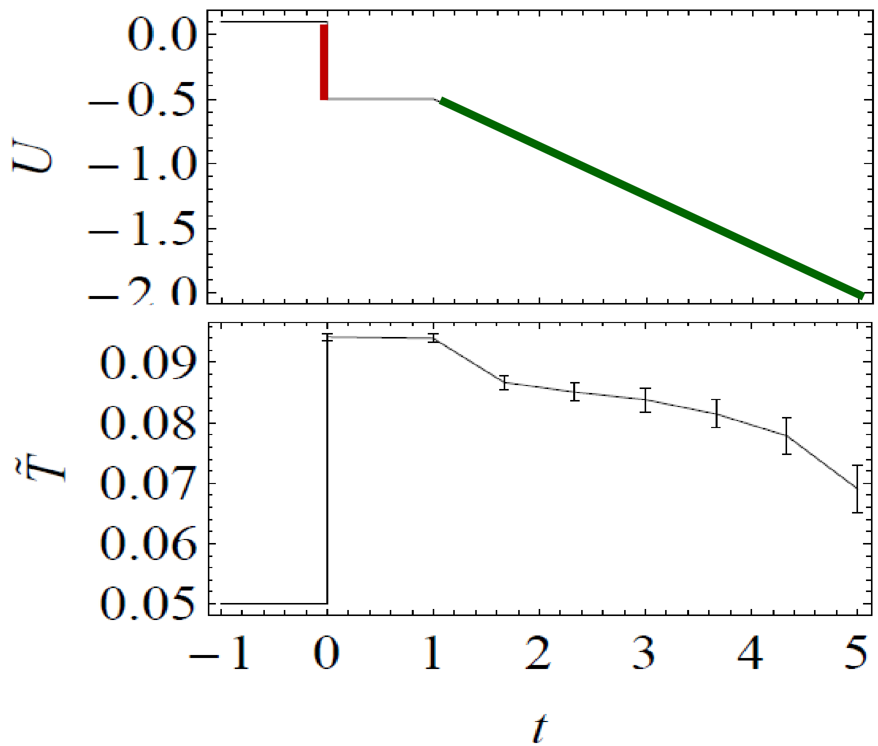


Keller et al, PRL (2001)

How to minimise the heating: Multi-step ramp







Summary

強相関系の物理 ⇔ 光格子

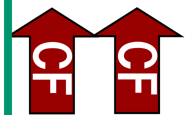
- 相関現象のplaygroundとしての光格子
  - 強磁性（梯子、籠目）
  - 超流動（多成分格子系での集団励起）
  - 超流動・モット絶縁体転移 (Kibble-Zurek)
  - 格子系でフェルミオン間の相互作用を斥力⇔引力変換可？

- 電子系⇔光格子： 共通の問題を追及できるだけでなく
  Cold atom systems: 制御性が大きいので、宝の山！

➡ 将来課題： より広範な現象・問題

FQHE state at $\nu = 5/2$

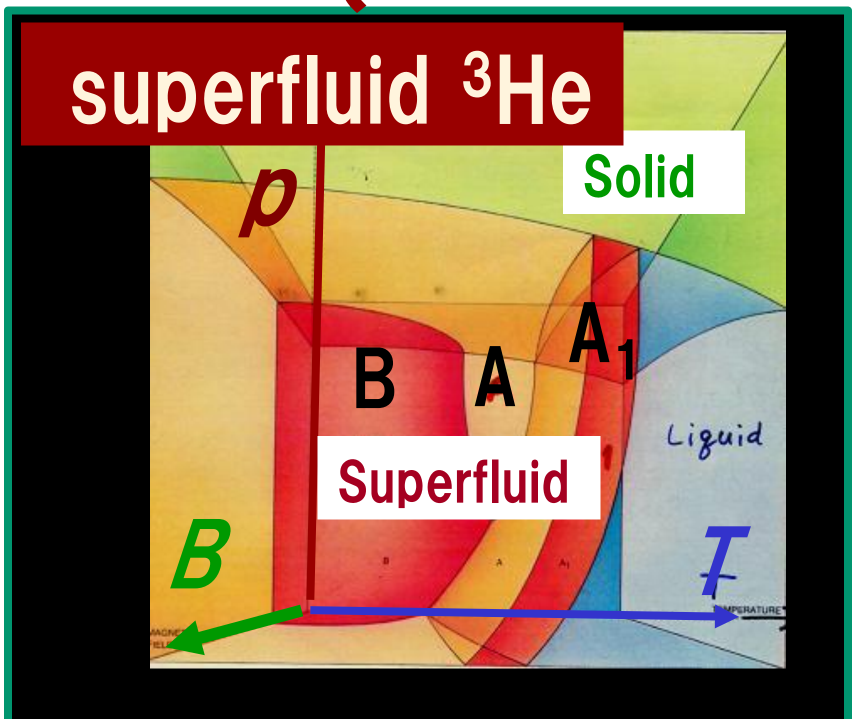
Triplet p-ip (Pfaffian state) → non-abelions

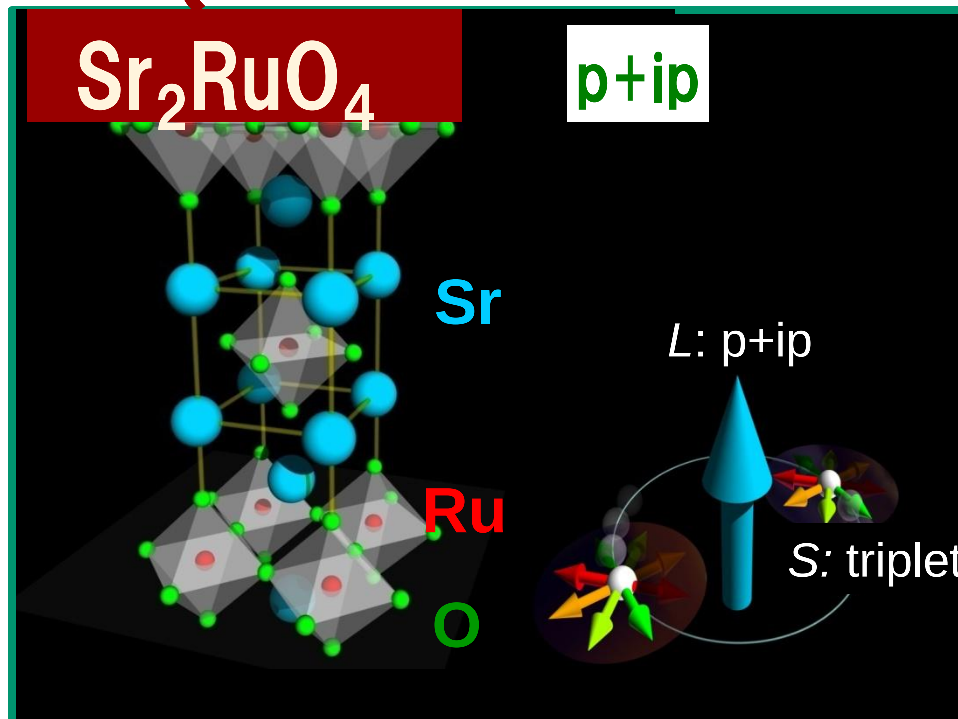


Trial wf: Moore-Read, Greiter-Wen-Wilczek 1991

Numerical: Morf 1998, Rezayi-Haldane 2000; Onoda-Mizusaki-Aoki 2003

Experiment: Willett-West-Pfeiffer 1998, 2002





新しい物質科学の創成
——超伝導100年

「科学」（岩波）2008年2月号特集

超伝導·超流動発見100年, 高温超伝導発見20年の今, ぜひ次のブレイクスルーがほしい. エキゾチック超伝導や多自由度系が注目される. ——青木秀夫

特集 新しい物質科学の創成

〈座談会〉
ブレイクスルーに向けて何が進んでいるか

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福山 寛 ふくやま ひろし（東京大学大学院理学系研究科, 低温物理学）

超伝導
超流動
BEC