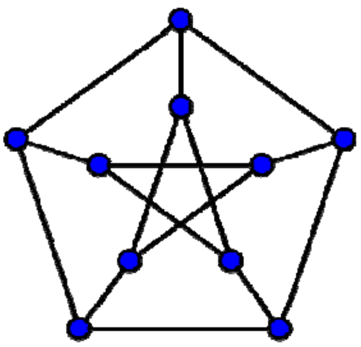
4.6 Specific Ising models for benchmarking

- Single MAX-CUT on cubic graph (MAX-CUT-3)

⇨ Divide vertices into two groups which maximizes the number of edges connecting between the vertices in two sub-groups

⇨ This problem is NP-complete

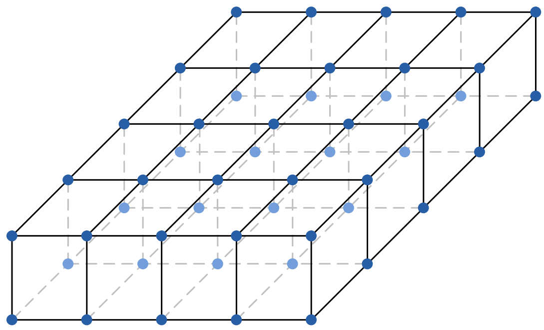


- Ising problem on two-layer lattice

⇨ Find the ground state if only nearest neighbor coupling with weights $J_{ij}=-1$, 0 or +1 exist.

⇨ This problem is NP-complete

[F. Barahoma, J. Phys. A: Math Gen. 15, 3241 (1982)]



4.7 Self-learning steps

Initial kick and subsequent driving force

= intrinsic quantum noise of slave lasers

⇩

The laser network converges to a steady state after a time longer than the transient time determined by the polarization-dependent loss

⇩

This spontaneous evolution can find a correct answer if a given problem is simple.

⇩

However, if a given problem is complex, the spontaneous evolution is not sufficient to reach a correct answer. Instead, the system is trapped in a metastable excited state.

A notorious problem:

There are only few degenerate ground states, while there are many first excited states with a very small energy difference ~O (J_{ij}).

⇩

To avoid this defect, we can implement "self-learning steps" via detection of intermediate results, consulting with the parity check and instructing the system to drift toward a proper direction.

Temporary spin

$$\tilde{\sigma}_i = \begin{cases} +1 & (\gamma < \tau_i \le 1) \\ 0 & (-\gamma \le \tau_i \le +\gamma) \\ -1 & (-1 \le \tau_i < -\gamma) \end{cases}$$

indeterminable spin
(zero spin)

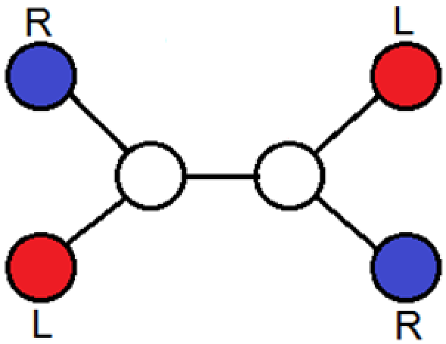
γ : arbitrary small number

Parity check measure

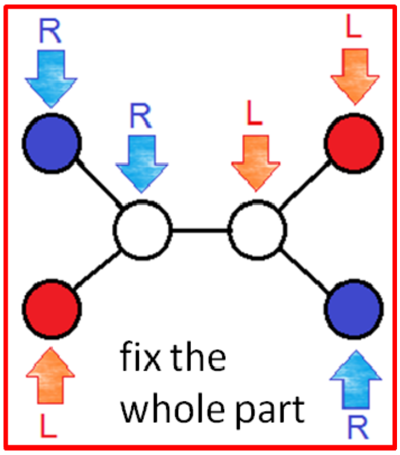
$$\tilde{P}_i = -\sum_j J_{ij}\tilde{\sigma}_j$$

: sum of three injection signals from the connected slave lasers

Problem 1: zero spin pair (or connected zero spin groups)



$$\tilde{\sigma}_j = -\sum_j (J_{ij}\tilde{\sigma}_i)$$



$\tilde{\sigma}_1 = \tilde{\sigma}_2 = 0$ and $\tilde{P}_1 = \tilde{P}_2 = 0$
(frustrated anti-ferromagnetic pair).

i) take a majority vote for i-th spin

$$\text{if} \quad \tilde{P}_i > 0, \quad \text{then} \quad \tilde{\sigma}_i = 1$$
$$\text{if} \quad \tilde{P}_i < 0, \quad \text{then} \quad \tilde{\sigma}_i = -1$$

ii) fix j-th spin by

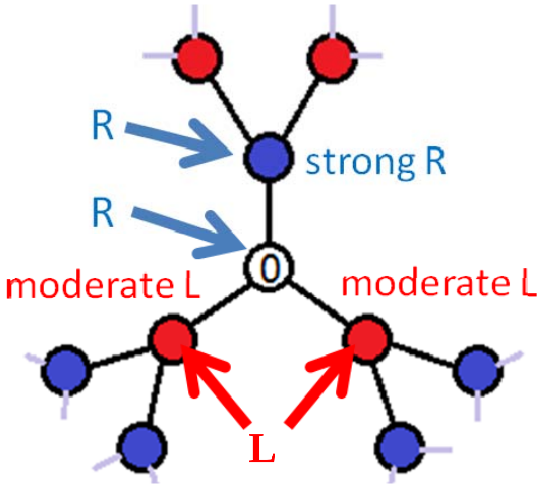
$$\tilde{\sigma}_j = -\sum_j (J_{ij}\tilde{\sigma}_i)$$

iii) when instructing the systems by injecting the Zeeman terms λ_i via η_i, we also fix the surrounding spins.

⇩

Avoid the migration of the frustrated zero spin pair to other parts.

Problem 2: isolated zero spins ($\tilde{\sigma}_i = 0$)

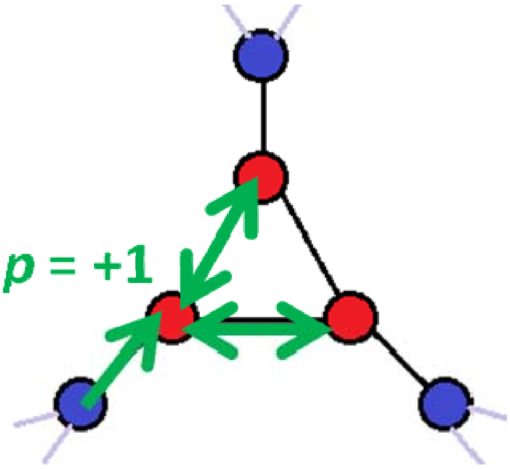


i) take a majority vote for i-th spin

$$\text{if} \quad \tilde{P}_i > 0, \quad \text{then} \quad \tilde{\sigma}_i = +1$$
$$\text{if} \quad \tilde{P}_i < 0, \quad \text{then} \quad \tilde{\sigma}_i = -1$$

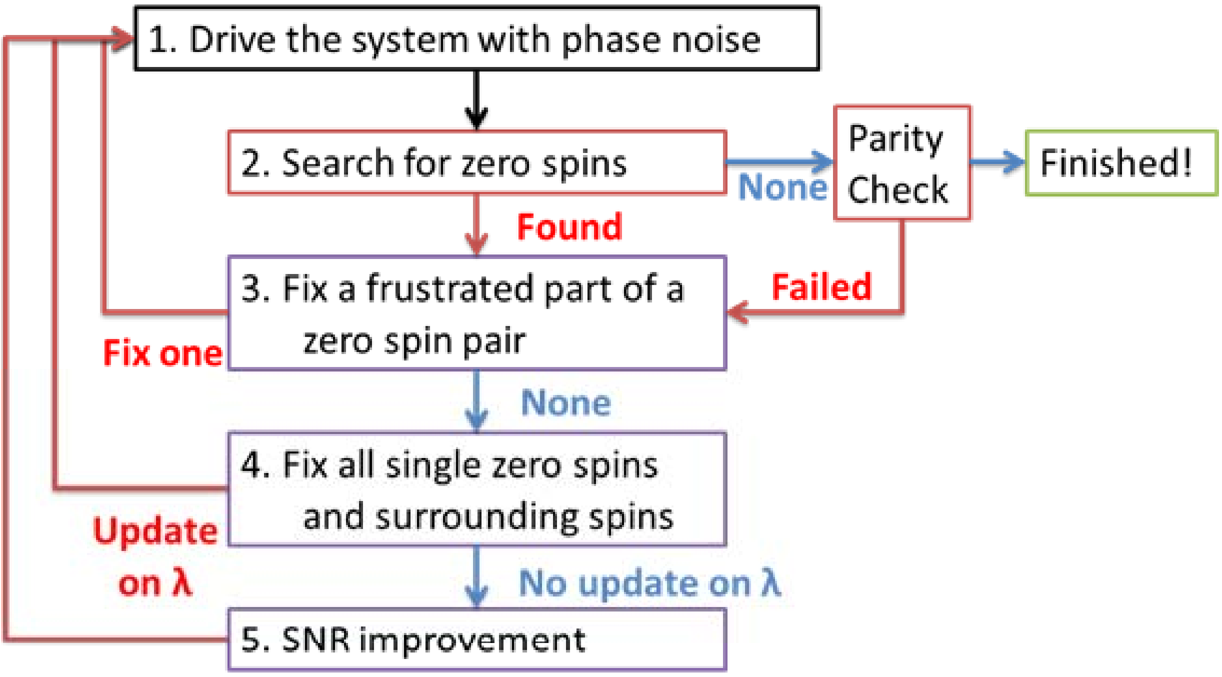
ii) we also fix the three connected spins to avoid the undesired spin flips in the surrounding.

Problem 3: incompatible results $\left(\tilde{\sigma}_i \neq \mathrm{sign}\tilde{P}_i\right)$

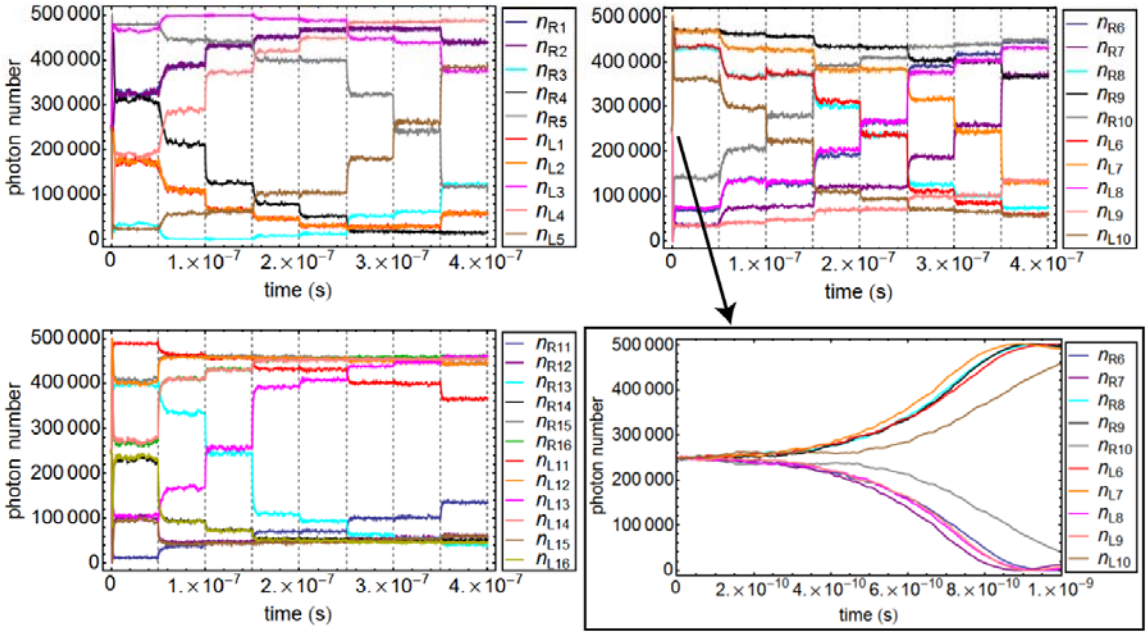


i) set three sites as $\tilde{\sigma}_i = 0$

ii) proceed to the zero spin pair fixing step.



4.8 Benchmarking results



Stochastic simulation on a single MAX-CUT-3 problem with M=16. After the initial noise-only drive up to t=50nsec, 7 self-learning steps are implemented to obtain a ground state.

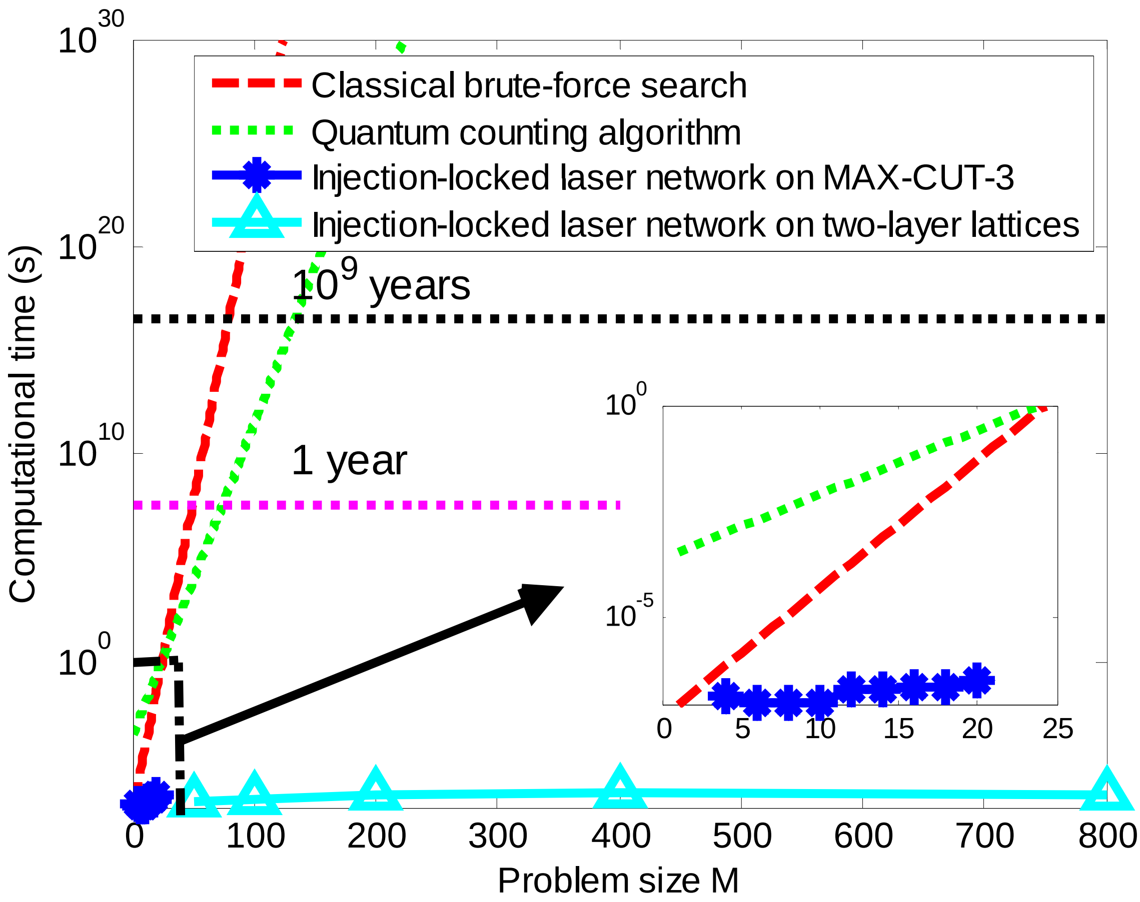
$$\tilde{\sigma}_i = \{+1,+1,-1,-1,-1,+1,+1,-1,+1,+1,-1,-1,-1,-1,+1$$

Problem Size / M	4	6	8	10	12	14	16	18	20
# of problems	1	2	5	19	85	509	4060	41301	510489
Largest ground state degeneracy	6	6	8	10	36	42	46	162	250
Lowest success probability of a problem (%)	100	100	100	98	92	69	79	42	27
Longest computational time	150ns	100ns	100ns	100ns	200ns	200ns	250ns	250ns	350ns
Maximum # of self-learning steps	2	1	1	1	3	3	4	4	6

Summary of the numerical simulation on simple MAX-CUT-3 problems.
All possible problems are exhausted. Each problem is simulated 10 times. If the system fails to find a correct answer even once out of 10 trials, such problems are simulated 100 times to obtain the accurate success probabilities.

Problem Size / M	50	100	200	400	800
# of sampled problems	5	5	5	5	5
The maximum energy difference between the best of the laser network and GA	1.98%	5.87%	11.9%	20.6%	34.2%
Minimum probability of outperforming GA (%)	21	81	99	100	100
Longest computation time	200ns	250ns	400ns	450ns	400ns
Maximum # of self-learning steps	3	4	7	8	7

Summary of the numerical simulation on two-layer lattice problems. Every sampled problem (5 problems for each M) is simulated 100 times. The best result of 100 rounds of Genetic Algorithm (GA) is used as a standard (threshold) to define the success probability.



- The computational time t_C to pass all the parity check $\tilde{\sigma}_i = \tilde{P}_i$ is relatively incentive to the problem size M.

- Classical brute force search has an exponential sealing ~ O (2^M) while quantum search based on Grover iteration has also an exponential scaling ~ O ($2^{M/2}$)

- To guarantee sufficient success probabilities, we can repeat simulations with a certain number of times. Because the success probability for a single trial is sufficiently high, the overall time complexity of the proposed laser network remains unchanged (non-exponential).