3.4.4 Quantum error correlation (QEC) layer

A. Surface topological code

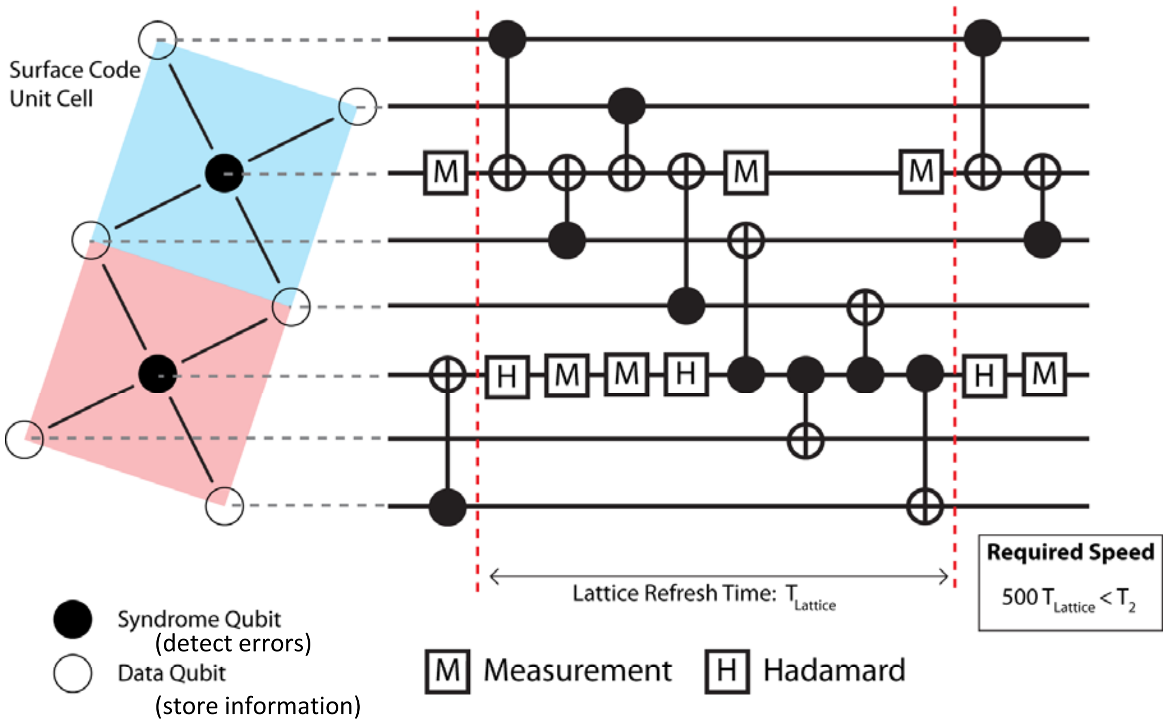
- Meaningful quantum computation requires an error rate of ~10^{-15} per gate operation, which is simply impossible on faulty hardware .

⇩

- Pump entropy (random errors) out in the form of an error syndrome and create a new set of computational resources – error-free logical qubits and gates –.
- Surface topological code satisfies a highest fault-tolerant threshold and requires only two-dimensional nearest-neighbor interaction.

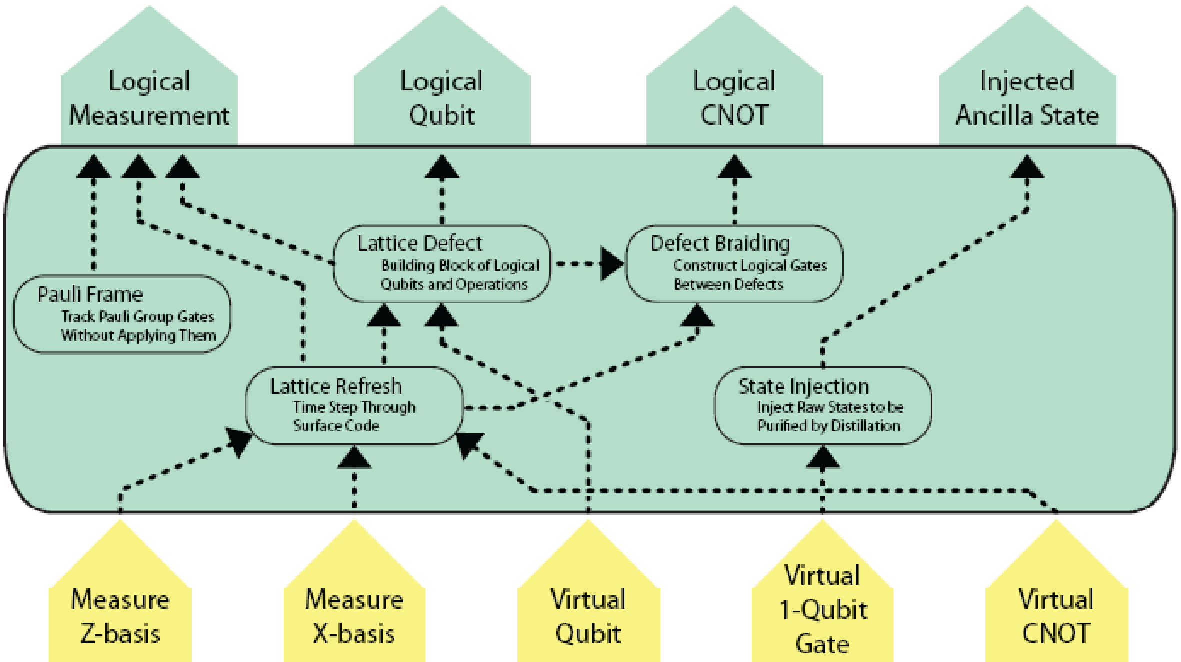
Two advantages:

- High threshold for gate errors $\lesssim$ 1.1%
- Requires only nearest-neighbor interaction



A lattice refresh cycle of the surface code can be performed in parallel across the entire 2D array of qubits.

A practical system must operate well (an order of magnitude) below the threshold, otherwise nearly infinite physical resources are required.



Maximum failure probability:

$$P_{\text{fail}} = 1 - (1 - \varepsilon_L)^{KQ} \simeq KQ\varepsilon_L \lesssim 10^{-2}$$ (less than 1%)

ε_L : error rate per logical gate

K : logical depth

Q : logical qubits

Surface code error rate:

$$\varepsilon_L \simeq C \left(\frac{\varepsilon_V}{\varepsilon_{th}}\right)^{\lfloor \frac{d+1}{2} \rfloor}$$

$$C \simeq 3 \times 10^{-2}$$

$$\varepsilon_{th} = 1.1 \times 10^{-2}$$ (threshold error gate)

$$\varepsilon_V \simeq 10^{-3}$$ (error rate per virtual gate)

d : surface code distance

Parameter	**Symbol**	**Value**
Threshold error per virtual gate [49]	$\varepsilon_{\text{thresh}}$	1.1×10^{-2}
Error per virtual gate	ε_{V}	1×10^{-3}
Logical circuit depth (in lattice refresh cycles)	K	3.6×10^{12}
Number of logical qubits ("Shor", Section 3.1)	Q	12288
Error per lattice refresh cycle	ε_{L}	2.3×10^{-19}
Surface code distance	d	35
Virtual qubits per logical qubit	VQ/LQ	8010

⇩

We need 8010 virtual qubits per logical qubit to successfully complete a 2048-bit integer factoring problem.

B. Pauli frames

The error syndrome reveals what Pauli errors occur and what Pauli gates (X, Y, or Z) should be applied for error correction.

⇩

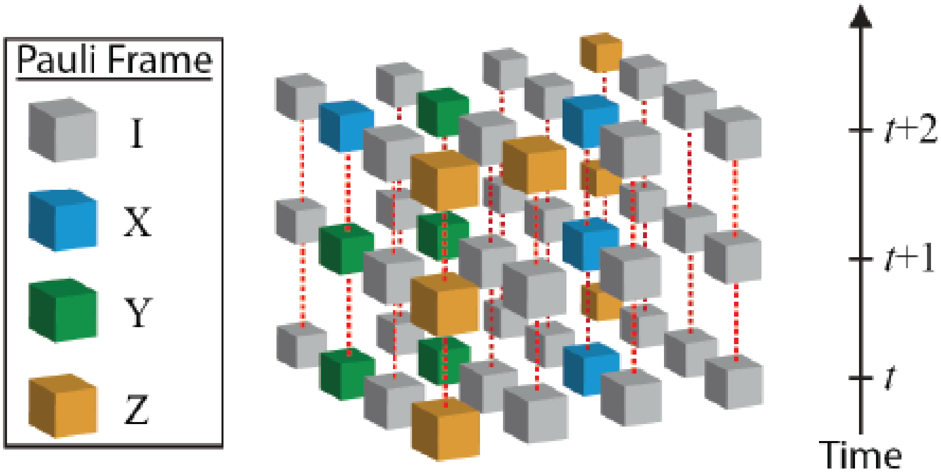
However, quantum gates are faulty, so that applying additional gates may introduce more errors into the system.

⇩

Rather than applying every correction operation, we can keep track of what Pauli correction would be applied and continue with computation. (Entropy is already pumped out by reading the syndrome!)

⇩

Final measurement results are modified based on the corresponding Pauli gates which should have been applied earlier. This stored Pauli gate is called the "Pauli frame", which changes the reference frame (interpretation) for the qubit.



3.4.5 Logical layer

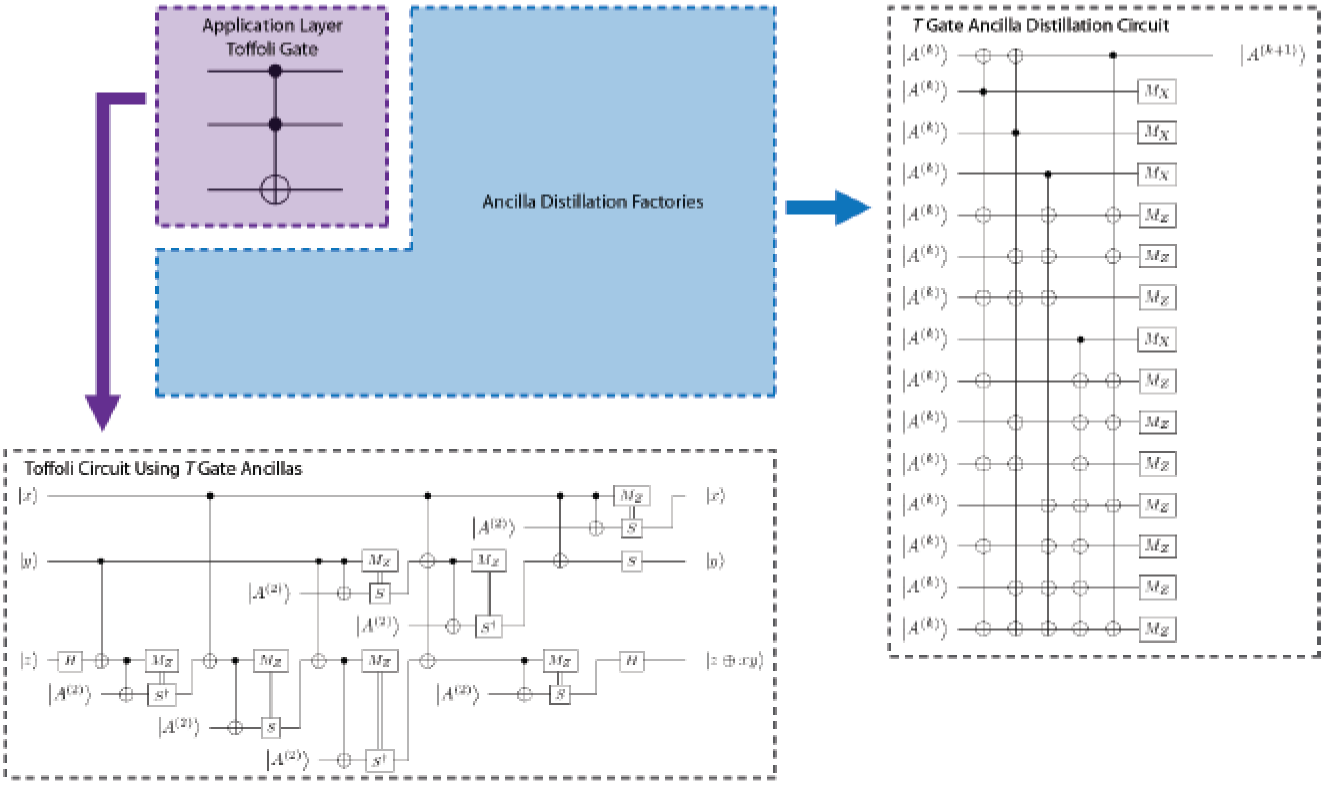
The logical layer creates any arbitrary quantum gate (Clifford group) needed for universal quantum computation.

- However, two ancilla states are needed for the surface code to produce a universal gate set:

$$|Y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \quad \Rightarrow \quad S = e^{i\frac{\pi}{4}z} \text{ gate}$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle\right) \quad \Rightarrow \quad T = e^{i\frac{\pi}{8}z} \text{ gate}$$

- Those ancillas must be produced by non-fault-tolerant methods in the virtual layer.
- Fortunately, a few "magic states" including $|Y\rangle$ and $|A\rangle$ can be distilled by consuming several low-fidelity ancillas and fundamental gates to produce one high-fidelity ancilla.

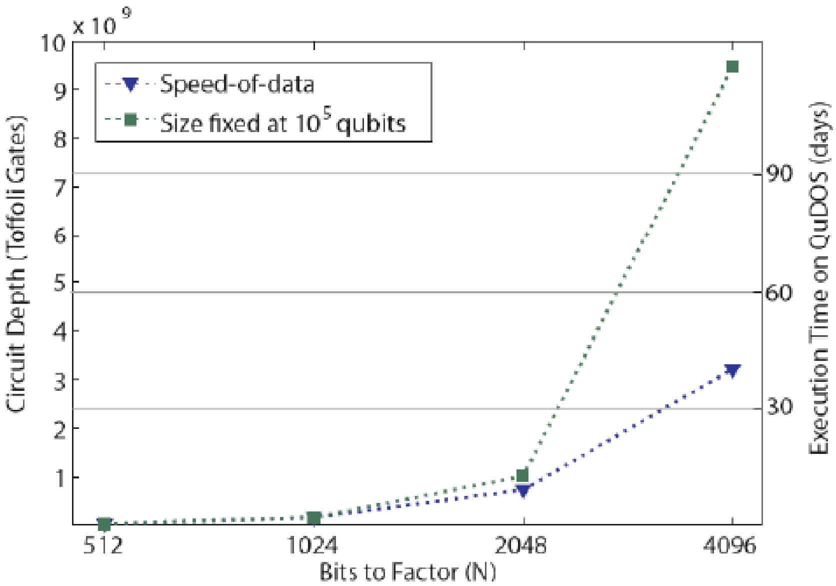


A Toffoli gate has only three application qubits, but substantially more logical qubits are needed for distillation circuits.

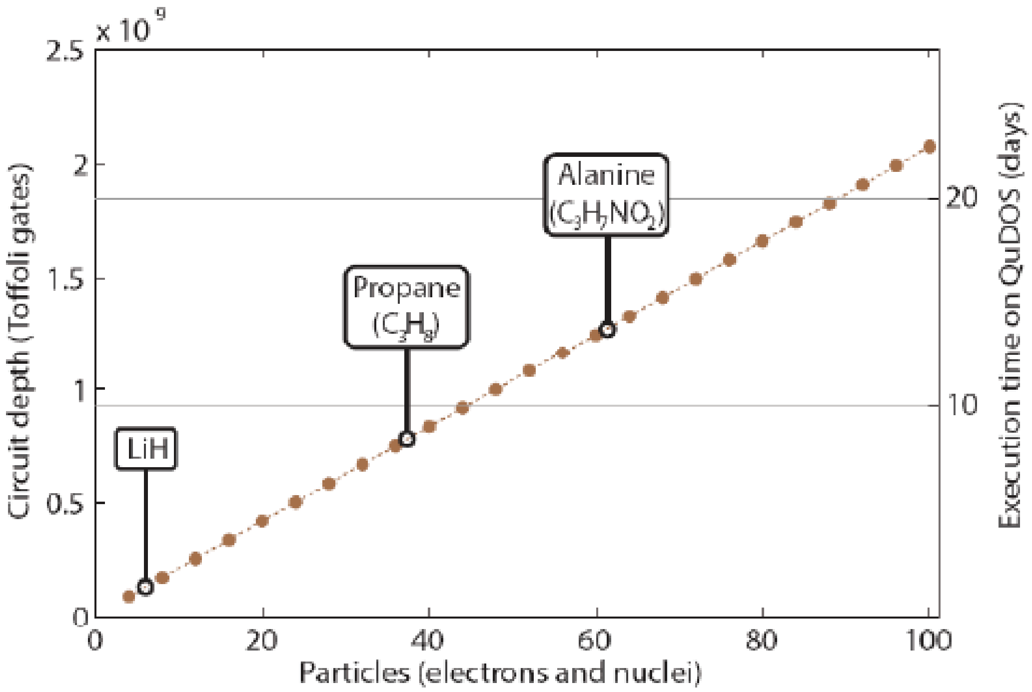
⇩

90% of the computing effort for a single Toffoli gate must be consumed for ancilla distillation circuits.

3.4.6 Sher's algorithm

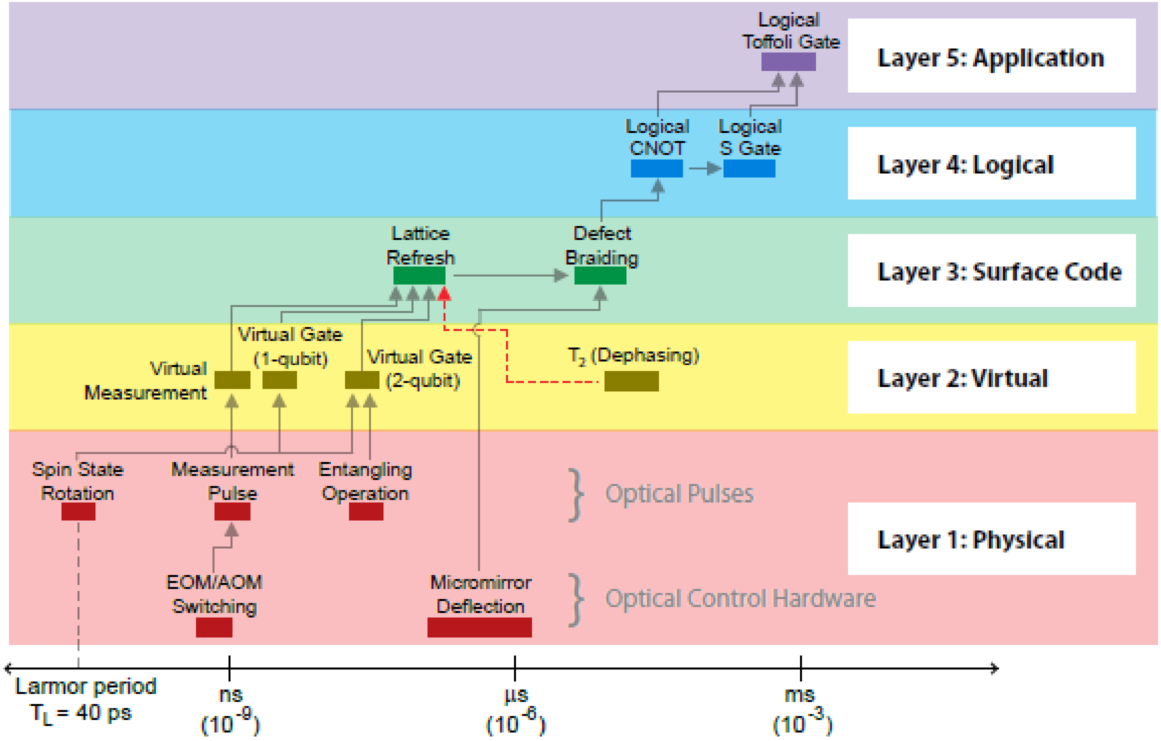


3.4.7 Quantum chemistry



Computing Resource	Shor's Algorithm (1024-bit)	Molecular Simulation (alanine)
(Layer 5) Application qubits	6144	6649
(Layer 5) Circuit depth (Toffoli)	1.68×10^8	1.27×10^9
(Layer 4) Log. distillation qubits	66564	15860
(Layer 4) Logical clock cycles	5.21×10^9	3.94×10^{10}
(Layer 3) Code distance	31	33
(Layer 3) Error per lattice cycle	6.53×10^{-19}	5.94×10^{-20}
(Layer 2) Virtual qubits	4.54×10^8	1.73×10^8
(Layer 2) Error per virtual gate	1.00×10^{-3}	1.00×10^{-3}
(Layer 1) Quantum dots (area on chip)	4.54×10^8 (4.54 cm^2)	1.73×10^8 (1.73 cm^2)
Execution time (est.)	**1.81 days**	**13.7 days**

3.4.8 Timing considaration



3.5 Simulation of quantum computers on classical computers

Gottesman-Knill theorem:

A quantum algorithm that initiates in the computational basis and employs only Hadamard (H), phase (S), Pauli (σx , σy, σz) and C-NOT gates, along with projective measurement in the computational basis can be efficiently simulated on a classical computer.

Is entanglement a sufficient resource for exponential speed-up in QC?

⇩

No. C-NOT gate can produce entanglement, but according to G-K theorem, C-NOT gate is not sufficient to construct quantum algorithm that cannot be simulated efficiently on a classical computer.

⇩

An essential part of quantum algorithm for exponential speed up is a “fractional phase rotation”.

- Deutsch-Jozsa algorithm
- Grover algorithm

} can be simulated efficiently on a CC.

- Shor algorithm ——— cannot be simulated efficiently on a CC.