1.4 Basic concepts of quantum interference

1. Entanglement

If the two nonlinear crystals have identical parametric down-conversion efficiency, a “post-selected” state which has one signal and one idler photon is

$$|\psi_{12}\rangle_{post} = \frac{1}{\sqrt{2}}\left[|1,1,0,0\rangle + |0,0,1,1\rangle\right]$$

$|NL2\rangle_s |NL2\rangle_i$ $\quad$ $|NL1\rangle_s |NL1\rangle_i$

If the signal photon is emitted from NL2, then the idler photon is also from NL2, and vice versa. These two possibilities coexist.

⇓

entangled state

2. Which-path measurement

If the idler photon is lost into the reservoirs (no projective measurement), the information on which NL crystal fires is leaked into the reservoirs.

⇓

$$Tr_i\left[|\psi_{12}\rangle_{post\ post}\langle\psi_{12}|\right] = \frac{1}{2}\left[|NL1\rangle_{s\ s}\langle NL1| + |NL2\rangle_{s\ s}\langle NL2|\right]$$

which-path measurement

mixed state

This is why a single photon count rate is constant (no interference effect).

3. Quantum erasure

$$|\psi_{12}\rangle_{post} = \frac{1}{\sqrt{2}}\left[|NL2\rangle_s |NL2\rangle_i + |NL1\rangle_s |NL1\rangle_i\right]$$

$$= \frac{1}{2}\left[|D_B\rangle_i \left(|NL1\rangle_s + |NL2\rangle_s \right) + |D'_B\rangle_i \left(|NL1\rangle_s - |NL2\rangle_s \right) \right]$$

$$|D_B\rangle_i = \frac{1}{\sqrt{2}}\left[|NL1\rangle_i + |NL2\rangle_i \right] \Rightarrow$$ click at D_B

$$|D'_B\rangle_i = \frac{1}{\sqrt{2}}\left[|NL1\rangle_i - |NL2\rangle_i \right] \Rightarrow$$ no click at D_B

(an idler photon goes to undetected port)

$$U_{BS} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad |NL1\rangle_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |NL2\rangle_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U_{BS}|D_B\rangle_i = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow$$ detection

$$U_{BS}|D'_B\rangle_i = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow$$ no detection

The post-selected state, conditioned by the “click” at D_B:

$$|\psi_s\rangle_{post} = \sqrt{2}\,{}_i\langle D_B|\psi_{12}\rangle_{post} = \underbrace{\frac{1}{\sqrt{2}}\left[|NL1\rangle_s + |NL2\rangle_s \right]}_{\text{linear superposition state}}$$

Which path information is eliminated by the projective measurement performed on the idler photon. Note that a beam splitter is essential for the elimination of which-path information.

⇩

quantum erasure

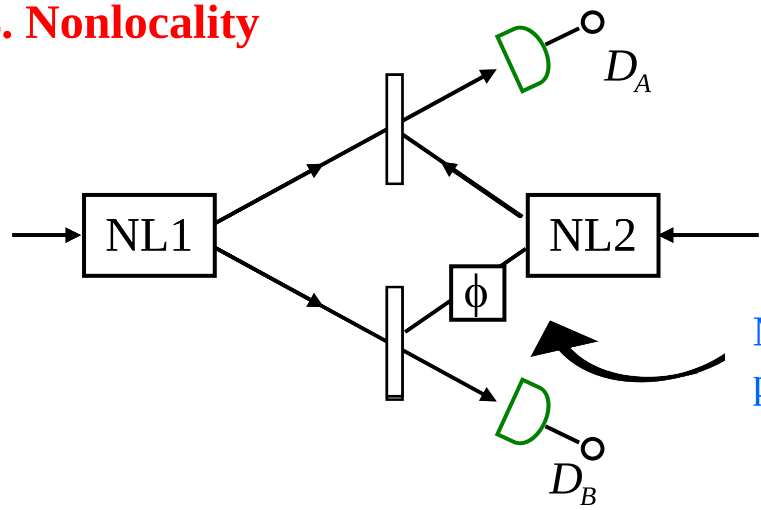
If we select $|D_B\rangle_i$, we have a cosine-type oscillation.

If we select $|D'_B\rangle_i$, we have a sine-type oscillation.

⇩

A simple sum (no selection)features no oscillation.

4. Nonlocality



Modulation of the optical path for the idler photon

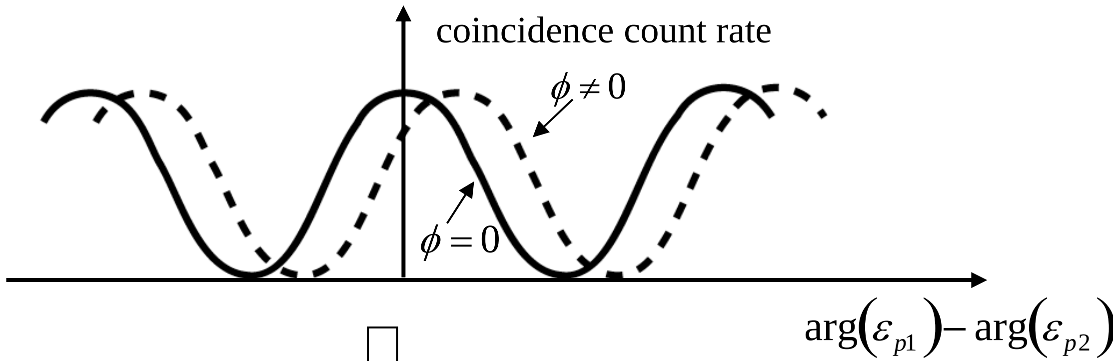
$$|\psi_{12}\rangle_{post} = \frac{1}{\sqrt{2}}\left[|NL1\rangle_s|NL1\rangle_i + e^{i\phi}|NL2\rangle_s|NL2\rangle_i\right]$$

Modulation imposed on the idler photon is shared by the signal photon which is spatially separated from the idler photon.

Post-selected state, conditioned by the "click" at D_B:

$$|\psi_s\rangle_{post} = \frac{1}{\sqrt{2}}\left[|NL1\rangle_s + e^{i\phi}|NL2\rangle_s\right]$$

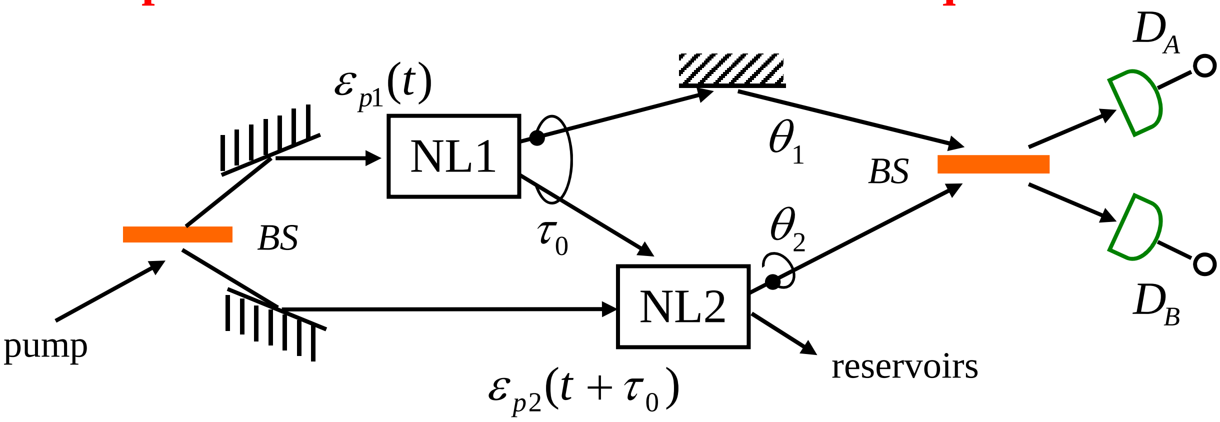
⇨ Equivalent to phase modulation of the signal arm.



⇩

nonlocality

1.5 Two photon interferometer with automatic quantum erasure



input states into beam splitter:

$$|\psi_{12}\rangle_{post} = c_{10}c_{20}|0\rangle_{s1}|0\rangle_{s2}|0\rangle_i + c_{11}c_{20}|1\rangle_{s1}|0\rangle_{s2}|1\rangle_i + c_{10}c_{21}e^{+i\omega_i\tau_0}|0\rangle_{s1}|1\rangle_{s2}|1\rangle_i$$

i_1 and i_2 are identical

field detected by D_A:

$$\hat{E}_A = \hat{E}_A^{(+)} + \hat{E}_A^{(-)}$$

$$E_A^{(+)} = \sqrt{\frac{\hbar\omega_s}{2\varepsilon_0 V}} \times \frac{1}{\sqrt{2}}\left(\hat{a}_{s1}e^{i\theta_1} + i\hat{a}_{s2}e^{i\theta_2}\right)e^{-i\omega_s t}$$

single photon count rate:

$$\omega(r_A, t) = S_A\langle\psi_{12}|\hat{E}_A^{(-)}(t)\hat{E}_A^{(+)}(t)|\psi_{12}\rangle$$

$$\hat{E}_A^{(+)}|\psi_{12}\rangle$$

$$= K_A e^{-i\omega_s t}\left(\hat{a}_{s1}e^{i\theta_1} + i\hat{a}_{s2}e^{i\theta_2}\right)\left[c_{10}c_{20}|0,0,0\rangle + c_{11}c_{20}|1,0,1\rangle + c_{10}c_{21}e^{+i\omega_i\tau_0}|0,1,1\rangle\right]$$

$$= K_A e^{-i\omega_s t}\left[c_{11}c_{20}e^{i\theta_1}|0,0,1\rangle + ic_{10}c_{21}e^{+i\omega_i\tau_0 + i\theta_2}|0,0,1\rangle\right]$$

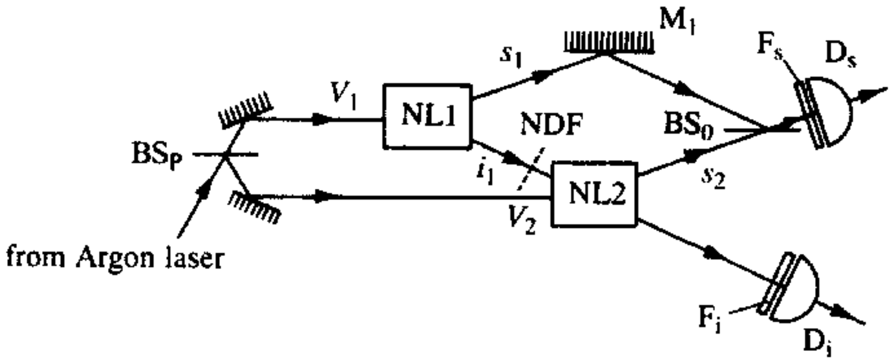
identical state

⇩

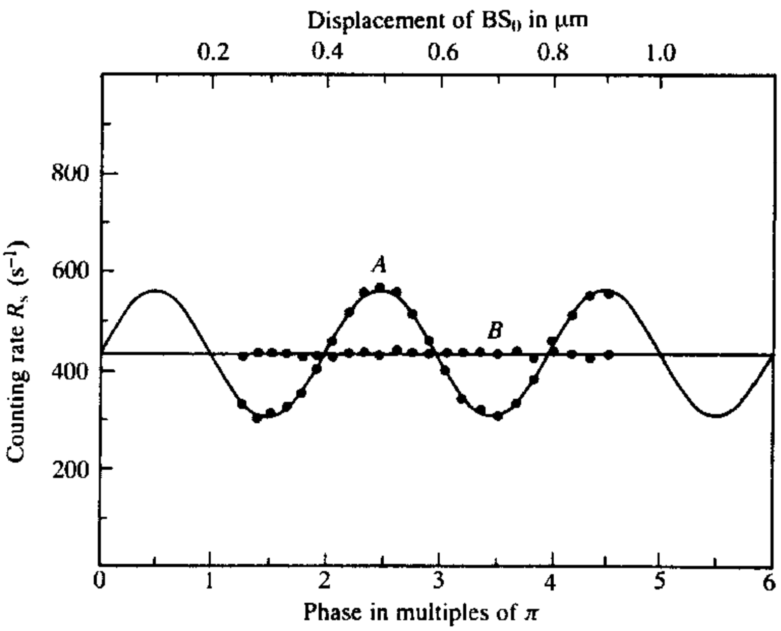
Two probability amplitudes interfere with each other.

$$\omega(r_A,t)=S_A K_A^2\left(|c_{11}c_{20}|^2+|c_{10}c_{21}|^2+2|c_{10}c_{20}c_{11}c_{21}|\cos\phi\right)$$

$$\phi=\omega_1\tau_0+\theta_2-\theta_1+const.$$



Outline of another interference experiment with two down-converters.



Results of the interference experiment giving the photon counting rate as a function of the displacement of BS_0 (a) with idlers i_1 and i_2 aligned, (b) with idler i_1 blocked.

- Quantum erasure, i.e. protection of which path information from leakage, is realized by optical alignment of i_1 mode and i_2 mode.

- If i_1 mode and i_2 mode are misaligned, the interference disappears. An actual measurement of i_1 mode or i_2 mode is not required. The “possibility” of a which path measurement is enough to destroy the interference.

- Modulation of an optical path $\omega_i \tau_0$ shifts the interference pattern.

  $\Rightarrow$ non-locality

1.6 Einstein locality and Bell's theorem

key words in this section

- (physical) reality

  If the outcome of a measurement for a physical quantity can be predicted with certainty, it is said there exists an element of reality corresponding to this physical quantity.

- Einstein - Podolsky – Rosen (EPR) paradox

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2\right]$$: polarization singlet state

$$|+45^\circ\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle + |V\rangle\right]$$

$$|-45^\circ\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle - |V\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[|+45^\circ\rangle_1|-45^\circ\rangle_2 - |-45^\circ\rangle_1|+45^\circ\rangle_2\right]$$

$$|R\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle + i|V\rangle\right]$$

$$|L\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle - i|V\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[|R\rangle_1|L\rangle_2 - |L\rangle_1|R\rangle_2\right]$$

If the polarization of a photon #1 is measured in the *(H-V)* basis and $|V\rangle_1$ is obtained, a photon #2 has a definite polarization $|H\rangle_2$ irrespective of whether it is actually measured or not and thus possesses an element of reality. If a photon #1 is measured in the (+45º, -45º) basis and $|-45^\circ\rangle$ is obtained, a photon #2 has a definite polarization $|+45^\circ\rangle$, and thus possesses an element of reality. However, the decision of which polarization basis is chosen can be made when the two photons are far apart (delayed choice). The two measurement setups cannot communicate in the available time, yet it influences the polarization state of the photon #2.

That is, if the photon #1 is measured in the ($+45^\circ$, -45°) basis, the photon #2 has a definite polarization in this basis. Now the *(H-V)* basis and ($+45^\circ$, -45°) basis are conjugate with each other according to QM so that they cannot have definite values simultaneously. This is in contradiction to Einstein's local realism, and they conclude QM is imcomplete.

- Einstein's local realism

  Science always tries to find an objective law of nature which governs and explains the world. Therefore, we tend to intuitively think that an objective reality must be associated with each system and its physical quantity.

- Copenhagen interpretation (N. Bohr)

  Before a measurement is performed and its result is read out, there exists only abstract information, called a vector or wavefunction. In a sense, an element of reality is created by a measurement. There is no "objective reality" in a quantum world.

- Local hidden variable theory (D. Bohm)

  An unmeasurable parameter determines the outcome of an experiment and can explain the observed non-local correlation.

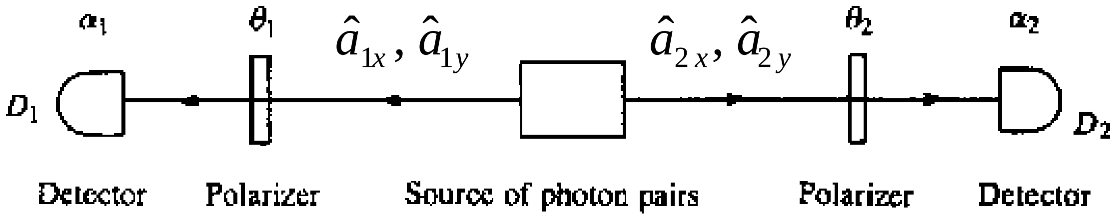
- Bell's inequality

  Local hidden variable theory based on Einstein's local realism imposes the upper limit to the correlation of distant events, which contradicts QM prediction.

  J.S. Bell, Physics 1, 195 (1964)

1.6.1. Quantum entanglement in photon-pair

Two-orthogonal polarization modes



EPR-Bell state: $$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left[|1\rangle_{1x}|0\rangle_{1y}|0\rangle_{2x}|1\rangle_{2y} - |0\rangle_{1x}|1\rangle_{1y}|1\rangle_{2x}|0\rangle_{2y}\right]$$

$|H\rangle_1|V\rangle_2$ $|V\rangle_1|H\rangle_2$

Projective property of a polarizer:

$$\hat{a}_1 = \hat{a}_{1x}\cos\theta + \hat{a}_{1y}\sin\theta$$

$$\Rightarrow \left[\hat{a}_1,\ \hat{a}_1^+\right] = \cos^2\theta\left[\hat{a}_{1x},\ \hat{a}_{1x}^+\right] + \sin^2\theta\left[\hat{a}_{1y},\ \hat{a}_{1y}^+\right] = 1$$

$= 1$ $= 1$

A commutator bracket is conserved

Single photon detection probability:

$$P_1(\theta) = \alpha_1\langle\psi_{12}|\hat{a}_1^+\hat{a}_1 \otimes \hat{I}_2|\psi_{12}\rangle$$

$$\hat{a}_1|\psi_{12}\rangle = \left(\hat{a}_{1x}\cos\theta_1 + \hat{a}_{1y}\sin\theta_1\right)\frac{1}{\sqrt{2}}\left[|1001\rangle - |0110\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\cos\theta_1|0001\rangle - \sin\theta_1|0010\rangle\right]$$

orthogonal $\Rightarrow$ which-path information is leaked into reservoirs

α_1 : quantum efficiency of a photodetector 1

$$P_1(\theta_1) = \frac{1}{2}\alpha_1$$

$$P_2(\theta_2) = \frac{1}{2}\alpha_2$$

$\Longrightarrow$ Each of the photon-pair is unpolarized if they are considered separately

two photon coincident detection probability:

$$\hat{a}_2\hat{a}_1|\psi_{12}\rangle = (\hat{a}_{2x}\cos\theta_2 + \hat{a}_{2y}\sin\theta_2)\frac{1}{\sqrt{2}}[\cos\theta_1|0001\rangle - \sin\theta_1|0010\rangle]$$

$$= \frac{1}{\sqrt{2}}[\cos\theta_1\sin\theta_2|0000\rangle - \sin\theta_1\cos\theta_2|0000\rangle]$$

$$P_{12}(\theta_1,\theta_2) = \alpha_1\alpha_2\langle\psi_{12}|\hat{a}_1^+\hat{a}_2^+\hat{a}_2\hat{a}_1|\psi_{12}\rangle$$

$$= \frac{1}{2}\alpha_1\alpha_2\sin^2(\theta_1 - \theta_2)$$

$\Longrightarrow$ quantum interference

If $\alpha_1 = \alpha_2 = 1$ (perfect photodetector), then

$$P(+,\theta_2;+,\theta_1) = \frac{1}{2}\sin^2(\theta_1 - \theta_2) \longleftarrow$$

A photon #1 is detected with a polarizer angle θ_1 and a photon #2 is detected with a polarizer angle θ_2.

$$P(-,\theta_2;-,\theta_1) = P\left(+,\theta_2 + \frac{\pi}{2};+,\theta_1 + \frac{\pi}{2}\right)$$

$$= \frac{1}{2}\sin^2(\theta_1 - \theta_2) \longleftarrow$$

A photon #1 is not detected with a polarizer angle θ_1 and a photon #2 is not detected with a polarizer angle θ_2.

$$P(+,\theta_2;-,\theta_1) = P\left(+,\theta_2;+,\theta_1 + \frac{\pi}{2}\right) = \frac{1}{2}\cos^2(\theta_1 - \theta_2)$$

$$P(-,\theta_2;+,\theta_1) = P\left(+,\theta_2 + \frac{\pi}{2};+,\theta_1\right) = \frac{1}{2}\cos^2(\theta_1 - \theta_2)$$

⇩

- Causality is preserved

Setting the angle of the polarizer *1* has no influence on the outcome of the measurement of the photon #2:

$$P(+,\theta_2/\theta_1)=P(+,\theta_2;+,\theta_1)+P(+,\theta_2;-,\theta_1)$$

$$=\frac{1}{2}$$ ⇨ independent of θ_1 and thus "superluminal communication" is not possible. The readout and transmission of the measurement result on photon #1 is required to transmit useful information.

- Non-locality exists

A conditional measurement result for the photon #2 is chosen at will by the orientatin of the polarizer angle θ_1.

1.6.2 Bell's inequality (Local hidden variable theory)

dichotomic observables $A(a)=\pm 1$ ← detection (+) and no detection (-) of the photon #1.

$B(b)=\pm 1$ ← detection (+) and no detection (-) of the photon #2.

↑

$a=\theta_1,\ b=\theta_2$ local parameters

ensemble average of many measurement results:

$$C(a,b)=\langle A(a)\,B(b)\rangle$$

$$=\int A(a,\lambda)\,B(b,\lambda)\rho(\lambda)d\lambda$$

Einstein locality (→ $A(a,\lambda)$, $B(b,\lambda)$): A depends only on a and B depends only on b.

local hidden variable (→ $\rho(\lambda)$): When a photon-pair is generated, the measurement results for A and B are promised.