

Chapter 9

1/f Noise and Random Telegraph Signals

In practically all electronic and optical devices, the excess noise obeying the inverse frequency power law exists in addition to intrinsic thermal noise and quantum noise. An enormous amount of experimental data has been accumulated on 1/f noise in various materials and systems. However, a physical mechanism for 1/f noise has not been completely identified yet. We have one physical model that lead to the 1/f power law but it is not necessarily a unique physical mechanism for the 1/f noise.

In very small electronic devices the alternate capture and emission of carriers at an individual defect site generates discrete switching in the device resistance—referred to as a random telegraph signal (RTS). The study of RTS has demonstrated the possible microscopic origin of low-frequency (1/f) noise in these devices, and has provided new insight into the nature of defects at an interface.

As a consequence of recent advances in processing technology, it has now become possible to produce devices in which the active volume is so small that it contains only a small number of charge carriers. The examples are small-area silicon metal-oxide-semiconductor field-effect transistors (MOSFETs) and metal-insulator-metal (MIM) tunnel junctions. Figure 9.1 shows an example of the random telegraph signal (RTS) measured in the drain current of a MOSFET as a function of time; the times in the high- and low-current states correspond to carrier capture and emission times, respectively.

The bias-voltage dependence of the capture and emission times allows one to determine the location of the defects from the channel of conduction. In MOSFETs they are found to reside in the oxide up to a few nanometers from the interface and hence within tunnelling distance of the inversion layer. For the MIM tunnel junctions, the traps are also located in the insulator. Through the study of the temperature and bias-voltage dependence of these capture and emission times for a single defect, one can extract parameters such as capture cross-section, activation energy for capture and emission, and the temperature dependence of the trap energy level.

A principal theme of this chapter is the relationship between the RTS associated with these defects in small devices and the 1/f noise found in large devices. During the past two decades the origin of 1/f noise has been the subject of extensive investigation[1]-[6]. Despite this intensive effort, the subject of 1/f noise has been notorious for several

reasons: first, there has been a lack of data open to unambiguous interpretation; secondly, there has been a long-running and rather sterile debate over ‘mobility-fluctuation’ *versus* ‘number-fluctuation’ models. The basic reason why no consensus has emerged is that little detailed information comes from the conventional ensemble-averaged power spectrum. We shall discuss the recent results on the noise properties of microstructures in which the averaging process is incomplete and individual fluctuators can be resolved. In the case of MOSFETs and MIM diodes, it will be shown conclusively that the $1/f$ noise in large devices is caused by the summation of many RTSs due to the defects in the insulator. In addition, the distribution of physical characteristics measured for the defects accounts easily for the wide range of time constants necessary to generate $1/f$ noise.

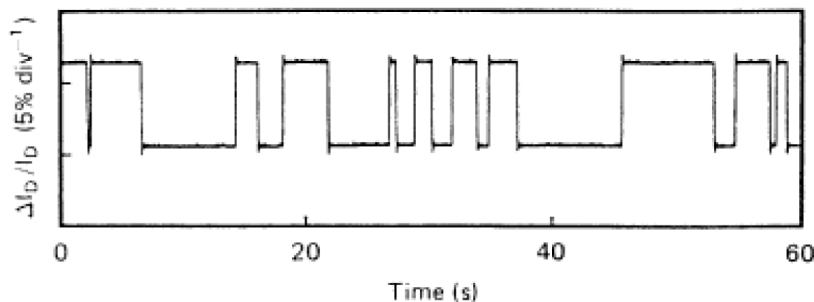


Figure 9.1: Random telegraph signal. Change in current against time. Active area of MOSFET is $0.4 \mu\text{m}^2$. $V_D = 10 \text{ mV}$, $V_G = 0.94 \text{ V}$, $I_D = 6.4 \text{ nA}$, $T = 293 \text{ K}$.

The electrical activity of defects at the Si/SiO₂ interface is normally studied using capacitance-voltage or conductance-voltage techniques. Recent experiments that have used the conductance technique show that there are two classes of interface defect: the first includes those defects normally seen, and which presumably reside at the interface, and are characterized by a single time constant; the second class incorporates defects residing in the oxide, which have a wide range of time constants and are responsible for the $1/f$ noise.

9.1 Characteristics of $1/f$ Noise

9.1.1 Scale invariance

A $1/f$ noise is characterized by a power spectral density function:

$$S_x(\omega) = C/\omega \quad , \quad (9.1)$$

where C is a constant. The integrated power in the spectrum between ω_1 and ω_2 is given by

$$P_x(\omega_1, \omega_2) = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} S_x(\omega) d\omega$$

$$= \frac{C}{2\pi} \ln \left(\frac{\omega_2}{\omega_1} \right) \quad , \quad (9.2)$$

This result shows that for a fixed frequency ratio ω_2/ω_1 , the integrated noise power is constant. Thus the total noise powers in between any decade of frequency, say 0.1 Hz to 1 Hz or 1 Hz to 10 Hz or 10 Hz to 100 Hz, are all identical. This property of 1/f noise is known as scale invariance.

9.1.2 Stationarity

Consider a 1/f noise, for which a noisy waveform $x(t)$ has the band-pass filtered power spectral density,

$$S_x(\omega) = \begin{cases} C/\omega & \text{for } \omega_1 \leq \omega \leq \omega_2 \\ 0 & \text{otherwise} \end{cases} \quad . \quad (9.3)$$

The autocorrelation function of $x(t)$ is obtained by using the Wiener-Khintchine theorem,

$$\begin{aligned} \phi_x(\tau) &= \frac{C}{2\pi} \int_{\omega_1}^{\omega_2} \frac{\cos \omega\tau}{\omega} d\omega \\ &= \frac{C}{2\pi} [C_i(\omega_2\tau) - C_i(\omega_1\tau)] \quad , \end{aligned} \quad (9.4)$$

where

$$C_i(z) = \int_{-\infty}^z \frac{\cos y}{y} dy \quad , \quad (9.5)$$

is the cosine integral. The series expansion of $C_i(z)$ is

$$C_i(z) = \gamma + \ln(z) + \sum_{k=1}^{\infty} \frac{(-1)^k z^{2k}}{(2k)!2k} \quad , \quad (9.6)$$

where $\gamma = 0.5772 \dots$ is Euler's constant. Thus, in the limit of $z \rightarrow 0$, the cosine integral reduces to $C_i(z) \simeq \ln z$. The mean-square of $x(t)$ is thus given by

$$\phi_x(\tau = 0) = \frac{C}{2\pi} \ln \left(\frac{\omega_2}{\omega_1} \right) \quad . \quad (9.7)$$

It is evident from the above argument that the band-pass filtered 1/f noise is statistically stationary because it has the second-order quantities depend only on the delay time τ and not on the absolute time at which the ensemble average is performed.

However, there is no experimental evidence for the existence of the low frequency limit ω_1 for 1/f noise. The reason is that an observation time T is always finite in practice and so a lower frequency region of the spectrum, $\omega \leq \frac{2\pi}{T}$, cannot be observed. The autocorrelation function and the mean square value that can be measured in actual experiments are thus given by replacing the low frequency limit ω_1 with $2\pi/T$ in Eqs. (9.4) and (9.7).

The Wiener-Levy process, discussed in Chapter 2, is a cumulative process of random walk. The power spectrum obeys $1/\omega^2$ law. By the very nature of the process, the Wiener-Levy process is statistically nonstationary and there is no possibility for the low-frequency limit ω_1 to exist. The corner (roll-off) frequency in the calculated spectrum (Chapter 2) is an artifact associated with the finite gate time T . In the case of 1/f noise, however, a low-frequency limit ω_1 may or may not exist. The stationarity of the process is still open to question.

9.2 Physical Model of 1/f Noise

9.2.1 Superposition of relaxation processes

If the noisy wave form $z(t)$ has an exponential relaxation process with a time constant τ_z , the power spectral density has the general form:

$$S_z(\omega) = \frac{g(\tau_z)}{1 + \omega^2\tau_z^2} \quad , \quad (9.8)$$

where $g(\tau_z)$ depends on the generation mechanism of the random pulse trains. Suppose an overall noisy waveform $x(t)$ is constructed from linear superposition of such relaxation processes, whose decay constants are distributed between upper and lower limits τ_1 and τ_2 with a probability density $p(\tau_z)$. The overall power spectral density is then

$$S_x(\omega) = \int_{\tau_1}^{\tau_2} S_z(\omega)p(\tau_z)d\tau_z = \int_{\tau_1}^{\tau_2} \frac{p(\tau_z)g(\tau_z)}{(1 + \omega^2\tau_z^2)} d\tau_z \quad . \quad (9.9)$$

If the numerator $p(\tau_z)g(\tau_z)$ is independent of τ_z and equal to a constant P , the above integral reduces to

$$S_x(\omega) = P \left[\tan^{-1}(\omega\tau_2) - \tan^{-1}(\omega\tau_1) \right] / \omega \quad . \quad (9.10)$$

When the two time constants τ_2 and τ_1 satisfy $\omega\tau_2 \gg 1$ and $0 \leq \omega\tau_1 \ll 1$, respectively, the two terms in the numerator are approximately equal to $\pi/2$ and 0. Thus, a superposition of exponential relaxation processes can give rise to a spectrum

$$S_x(\omega) = \left(\frac{\pi P}{2} \right) / \omega \quad . \quad (9.11)$$

If the product $p(\tau_z)g(\tau_z)$ is proportional to $\tau_z^{\alpha-1}$, the power spectral density has a more general form of $\omega^{-\alpha}$.

9.2.2 Distributed trapping model

One physical model of such 1/f noise is the trapping of charged particles with a wide spread of time constants. If a free carrier in a conducting channel is immobilized by falling into a trap, it is no longer available for current transport. The modulation of carrier numbers has the form of random telegraph signal with a Poisson point process as shown in Fig.9.2. The probability of observing m telegraphic signals in the time interval T is given by

$$p(m, T) = \frac{(\nu T)^m}{m!} e^{-\nu T} \quad , \quad (9.12)$$

where ν is the mean rate of transitions per second. If τ_+ and τ_- are the average times spent in the upper and lower states, respectively, the probability distributions of the upper state time t_+ and lower state time t_- are

$$p(t_{\pm}) = \tau_{\pm}^{-1} \exp\left(-\frac{t_{\pm}}{\tau_{\pm}}\right) \quad . \quad (9.13)$$

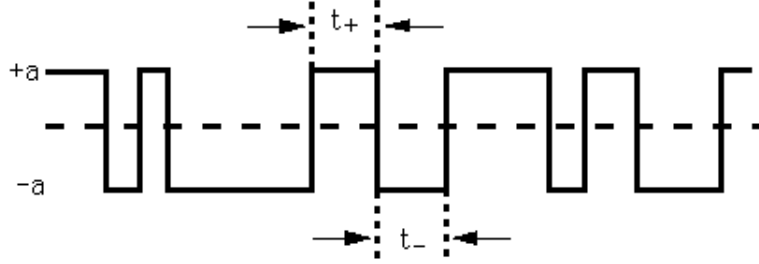


Figure 9.2: A random telegraph signal produced by a carrier trap.

The product $x(t)x(t + \tau)$ is equal to $+a^2$ if an even number of transitions occur in the interval $(t, t + \tau)$ and to $-a^2$ if an odd number of transitions occur in the same interval. Therefore, the autocorrelation function is

$$\begin{aligned}
 \phi_\tau(x) &= a^2 [p(0, \tau) + p(2, \tau) + \dots] \\
 &\quad - a^2 [p(1, \tau) + p(3, \tau) + \dots] \\
 &= a^2 e^{-\nu\tau} \left[1 - \nu\tau + \frac{(\nu\tau)^2}{2!} - \frac{(\nu\tau)^3}{3!} + \dots \right] \\
 &= a^2 e^{-2\nu\tau} \quad .
 \end{aligned} \tag{9.14}$$

The power spectrum is thus calculated by the Wiener-Khintchine theorem,

$$\begin{aligned}
 S_x(\omega) &= 4 \int_0^\infty \phi_x(\tau) \cos(\omega\tau) d\tau \\
 &= \frac{2a^2/\nu}{(1 + \omega^2/4\nu^2)} \\
 &= a^2 \frac{4\tau_z}{(1 + \omega^2\tau_z^2)} \quad .
 \end{aligned} \tag{9.15}$$

Here $\tau_z = 1/2\nu$ is the time constant of the trap. If τ_z is distributed according to the function $p(\tau_z)$, the power spectral density of the total carrier number fluctuation is

$$S_n(\omega) = 4\phi_n(\tau = 0) \int_0^\infty \frac{\tau_z p(\tau_z)}{(1 + \omega^2\tau_z^2)} d\tau_z \quad . \tag{9.16}$$

Here it is assumed $\int_0^\infty p(\tau_z) dz = 1$.

Suppose the carrier trap occurs by the tunneling of charged carriers from a conducting layer to traps inside the oxide layer at depth w , the time constant obeys

$$\tau_z = \tau_0 \exp(\gamma w) \quad , \tag{9.17}$$

where τ_0 and γ are constants. If the traps are uniformly distributed between the depth w_1 and w_2 , corresponding to the time constants τ_1 and τ_2 , we obtain

$$p(\tau_z) d\tau_z = \begin{cases} \frac{d\tau_z/\tau_z}{\ln(\tau_2/\tau_1)} & (\tau_1 \leq \tau_z \leq \tau_2) \\ 0 & (\text{otherwise}) \end{cases} \quad . \tag{9.18}$$

Using Eq. (9.18) in Eq. (9.16), the power spectral density of the total number fluctuation is given by

$$\begin{aligned} S_n(\omega) &= \frac{4\phi_n(0)}{\ln(\tau_2/\tau_1)} \int_{\tau_1}^{\tau_2} \frac{d\tau_z}{(1 + \omega^2\tau_z^2)} \\ &= \frac{4\phi_n(0)}{\ln(\tau_2/\tau_1)} \times \frac{\tan^{-1}(\omega\tau_2) - \tan^{-1}(\omega\tau_1)}{\omega} . \end{aligned} \quad (9.19)$$

As we discussed before, Eq. (9.19) shows 1/f power law in the frequency range of $\omega\tau_2 \gg 1$ and $0 \leq \omega\tau_1 \ll 1$.

The above argument applies also for the intrinsic bulk transport property of the hopping conduction. The essential requirement to obtain the 1/f power law is the Poissonian telegraphic event with a distributed time constant which obeys $1/\tau_z$ distribution function, as shown in Eq.(9.18).

9.3 Random Telegraph Signals

The main purpose of the following subsections is to provide a detailed analysis of random telegraph signals (RTSs) and the capture and emission kinetics of individual defects.

9.3.1 Probability distribution of RTS

Referring back to Fig. 9.1, we shall take the high-current state of the RTS to be state 1 and the low-current state to be state 0. We shall assume that the probability (per unit time) of a transition from state 1 to state 0 (i.e. from up to down) is given by $1/\bar{\tau}_1$, with $1/\bar{\tau}_0$ being the corresponding probability from 0 to 1 (i.e. from down to up). We also assume the transitions are instantaneous. We now intend to show that these assumptions imply that the times in states 0 and 1 are exponentially distributed, that is, the switching is a Poisson process.

Let $p_1(t)dt$ be the probability that state 1 will not make a transition between times o and t , then will make a transition between times t and $t + dt$. Thus

$$p_1(t) = A(t)/\bar{\tau}_1 \quad , \quad (9.20)$$

where $A(t)$ is the probability that after time t state 1 will not have made a transition and $1/\bar{\tau}_1$ is the probability (per unit time) of making a transition to state 0 at time t . However,

$$A(t + dt) = A(t)(1 - dt/\bar{\tau}_1); \quad (9.21)$$

that is, the probability of not making a transition between times o and $t + dt$ is equal to the product of the probability of not having made a transition between times o and t and the probability of not making a transition during the interval from t to $t + dt$. We can rearrange Eq. (9.21) to give

$$\frac{dA(t)}{dt} = -\frac{A(t)}{\bar{\tau}_1} \quad . \quad (9.22)$$

Integrating both sides of Eq. (9.22), we find

$$A(t) = \exp(-t/\bar{\tau}_1) \quad , \quad (9.23)$$

such that $A(0) = 1$. Thus

$$p_1(t) = \frac{1}{\bar{\tau}_1} \exp\left(-\frac{t}{\bar{\tau}_1}\right) . \quad (9.24)$$

$p_1(t)$ is correctly normalized such that

$$\int_0^\infty p_1(t) dt = 1 .$$

The corresponding expression for $p_0(t)$ is

$$p_0(t) = \frac{1}{\bar{\tau}_0} \exp\left(-\frac{t}{\bar{\tau}_0}\right) . \quad (9.25)$$

Hence, on the assumption that the up and down times are characterized by single attempt rates, we expect the times to be exponentially distributed. The mean time spent in state 1 is given by

$$\int_0^\infty t p_1(t) dt = \bar{\tau}_1 , \quad (9.26)$$

and the standard deviation is

$$\left[\int_0^\infty t^2 p_1(t) dt - \bar{\tau}_1^2 \right]^{1/2} = \bar{\tau}_1 . \quad (9.27)$$

Equivalent expressions hold for the down state. Thus the standard deviation is equal to the mean time spent in the state. Equation (9.27) can be used as a simple test for exponential behavior.

9.3.2 Power spectrum of RTS: Lorentzian spectrum

Here we shall outline the derivation of the power spectrum of an asymmetric RTS. Initially, we need to evaluate the autocorrelation function of the RTS. It is convenient to choose the origin of the coordinate system such that state 0 has amplitude $x_0 = 0$, and state 1 has amplitude $x_1 = \Delta I$. In addition, all statistical properties will be taken to be independent of the time origin. The probability that at any given time the RTS is in state 1 is $\bar{\tau}_1/(\bar{\tau}_0 + \bar{\tau}_1)$, and similarly for state 0 it is $\bar{\tau}_0/(\bar{\tau}_0 + \bar{\tau}_1)$. Then we have

$$\begin{aligned} c(t) &= \sum_i \sum_j x_i x_j \times \{\text{Prob. that } x(s) = x_i\} \\ &\quad \times \{\text{Prob. that } x(s+t) = x_j, \text{ given } x(s) = x_i\} . \end{aligned} \quad (9.28)$$

Since $x_0 = 0$ and $x_1 = \Delta I$, we obtain

$$\begin{aligned} c(t) &= (\Delta I)^2 \frac{\bar{\tau}_1}{\bar{\tau}_0 + \bar{\tau}_1} P_{11}(t) \\ &= (\Delta I)^2 \times \{\text{Prob. that } x(s) = \Delta I\} \\ &\quad \times \{\text{Prob. of even no. of transitions in time } t, \text{ starting in state 1}\} . \end{aligned} \quad (9.29)$$

If we define $P_{10}(t)$ as the probability of an odd number of transitions in time t , starting in state 1 then we have

$$P_{11}(t) + P_{10}(t) = 1 . \quad (9.30)$$

In addition,

$$P_{11}(t + dt) = P_{10}(t) \frac{dt}{\bar{\tau}_0} + P_{11}(t) \left(1 - \frac{dt}{\bar{\tau}_1}\right); \quad (9.31)$$

that is, the probability of an even number of transitions in time $t + dt$ is given by the sum of two mutually exclusive events: first, the probability of an odd number of transitions in time t and one transition in time dt ; and secondly, the probability of an even number of transitions in time t and no transitions in time dt . We can make dt small enough that the probability of more than one transition is vanishingly small. Substituting from Eq. (9.30) into Eq. (9.31), we obtain the following differential equation for $P_{11}(t)$:

$$\frac{dP_{11}(t)}{dt} + P_{11}(t) \left(\frac{1}{\bar{\tau}_0} + \frac{1}{\bar{\tau}_1}\right) = \frac{1}{\bar{\tau}_0} \quad . \quad (9.32)$$

This equation can be solved by using $\exp [f(1/\bar{\tau}_0 + 1/\bar{\tau}_1)dt]$ as an integrating factor:

$$P_{11}(t) = \frac{\bar{\tau}_1}{\bar{\tau}_0 + \bar{\tau}_1} + \frac{\bar{\tau}_0}{\bar{\tau}_0 + \bar{\tau}_1} \exp \left[- \left(\frac{1}{\bar{\tau}_0} + \frac{1}{\bar{\tau}_1} \right) t \right] \quad , \quad (9.33)$$

where $P_{11}(t)$ has been normalized such that $P_{11}(0) = 1$. Equations (9.33) and (9.29) can now be used to evaluate the power spectral density $S(f)$:

$$S(f) = 4 \int_0^\infty c(t) \cos(2\pi f\tau) d\tau = \frac{4(\Delta I)^2}{(\bar{\tau}_0 + \bar{\tau}_1)[(1/\bar{\tau}_0 + 1/\bar{\tau}_1)^2 + (2\pi f)^2]} \quad . \quad (9.34)$$

Here we have used the Wiener-Khintchine theorem. (The d.c. term, which contributes a delta function at $f = 0$, has been ignored.) For the case of a symmetric RTS, that is, $\bar{\tau}_0 = \bar{\tau}_1 = \bar{\tau}$ for example, this equation simplifies to

$$S(f) = \frac{2(\Delta I)^2 \bar{\tau}}{4 + (2\pi f \bar{\tau})^2} \quad . \quad (9.35)$$

The total power P in the RTS can be obtained by integrating Eq. (9.34) over all frequencies:

$$P = \frac{(\Delta I)^2}{(\bar{\tau}_0 + \bar{\tau}_1)(1/\bar{\tau}_0 + 1/\bar{\tau}_1)} \quad . \quad (9.36)$$

As one would expect, $P = (\frac{1}{2}\Delta I)^2$ when $\bar{\tau}_0 = \bar{\tau}_1$; P is a maximum under these conditions.

9.3.3 Occupancy levels and grand partition function

In order to be precise in our meaning, we shall introduce the nomenclature ‘occupancy level’ $E(n + 1/n)$ to describe the energy level of a defect: $E(n + 1/n)$ marks the Fermi level E_F at which the defect’s occupancy changes from n electrons to $n + 1$ electrons. We can determine the occupancy of the defect using the grand partition function, Z_G . This is written as

$$Z_G = \sum_{ASN} \exp \left(- \frac{E_S - NE_F}{kT} \right) \quad , \quad (9.37)$$

where ASN implies that the summation is to be carried out over all states S of the system for all numbers of particles N . We have adopted the convention of semiconductor

physics and set E_F to be equivalent to the temperature-dependent chemical potential. The absolute probability that the system will be found in a state (N_1, E_1) is given by

$$p(N_1, E_1) = \frac{\gamma \exp[-(E_1 - N_1 E_F)/kT]}{Z_G} \quad , \quad (9.38)$$

where the state is orbitally (and perhaps also spin) degenerate with degeneracy γ .

Consider a defect system that has only two states of charge, n and $n + 1$, available. Let the energy zero of the system correspond to the defect occupied by n electrons. Then

$$Z_G = \gamma(n) \exp\left(\frac{nE_F}{kT}\right) + \gamma(n + 1) \exp\left[-\frac{E(n + 1/n) - (n + 1)E_F}{kT}\right] \quad , \quad (9.39)$$

where $\gamma(n)$ and $\gamma(n + 1)$ are the degeneracies of the n - and $(n + 1)$ -electron states. Then the probability of finding the defect in the $(n + 1)$ -electron state is

$$f = p(n + 1) = \left\{ 1 + g \exp\left[\frac{E(n + 1/n) - E_F}{kT}\right] \right\}^{-1} \quad , \quad (9.40)$$

where

$$g = \gamma(n)/\gamma(n + 1) \quad . \quad (9.41)$$

This looks like a Fermi-Dirac distribution with a degeneracy factor g . In addition, we can write

$$\frac{p(n + 1)}{p(n)} = \frac{\gamma(n + 1)}{\gamma(n)} \exp\left[-\frac{E(n + 1/n) - E_F}{kT}\right] \quad . \quad (9.42)$$

That is, when the Fermi level crosses the level $E(n + 1/n)$, the $(n + 1)$ -electron state dominates over the n -electron state.

For an individual RTS generated by a trap with occupancy level $E(n + 1/n)$ and with mean capture and emission times $\bar{\tau}_c$ and $\bar{\tau}_e$, we have

$$f = \frac{\bar{\tau}_e}{\bar{\tau}_c + \bar{\tau}_e} = \left\{ 1 + g \exp\left[\frac{E(n + 1/n) - E_F}{kT}\right] \right\}^{-1} \quad , \quad (9.43)$$

$$\bar{\tau}_e = \frac{\bar{\tau}_c}{g} \exp\left[-\frac{E(n + 1/n) - E_F}{kT}\right] \quad , \quad (9.44)$$

where g is given by Eq. (9.41).

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