QIS 385: Bose-Einstein Condensation and Matter-Wave Lasers

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After Bose-Einstein condensation of dilute atomic gases was realized in 1995, the study of highly degenerate quantum gases has attracted the interest among theoretical and experimental scientists from various fields. More recently an elementary excitation in semiconductor cavity QED systems, called an exciton polariton, has been investigated extensively from the viewpoints of dynamic condensation and matter-wave lasers. In this lecture we will start with the conventional argument of Bose-Einstein condensation (BEC) at thermal equilibrium. After reviewing the basic properties of the BEC described by the conservative Gross-Pitaevskii equation, we will move on the discussion of the dynamic condensation at quasi-equilibrium and matter-wave lasers at non-equilibrium, described by the open dissipative Gross-Pitaevskii (Heisenberg-Langevin) equation or maser equation. Topics covered in the course include the coherence properties of BEC, BEC of non-interacting particles, Bogoliubov theory of interacting bosons, superfluidity and quantized vortices, BCS phase transition, several nontrivial issues of BEC, quantum reservoir theory of matter-wave lasers and potential applications to future quantum information processing.

Course Outline

Chapter 1: Introduction

Chapter 2: Order parameter, symmetry breaking and long-range order

2.1 Order parameter and symmetry breaking
2.2 Off-diagonal long range order and coherence functions
2.3 Nambu-Goldstone bosons

Chapter 3: Bose-Einstein condensation of an ideal gas

3.1 The ideal Bose gas in the canonical and grand canonical ensemble
3.2 BEC critical temperature
3.3 Density of states and BEC in a uniform system
3.4 Thermodynamic functions
3.5 Spatial coherence function

Chapter 4: Bogoliubov theory of the weakly interacting Bose gas

4.1 General features of dilute and cold gases
4.2 Hamiltonian of the system and the lowest-order approximation
4.3 Higher-order approximation
4.4 Excitation spectrum