Chapter 1

Introduction

After Bose-Einstein condensation of dilute atomic gases was realized in 1995 [1, 2], the study of highly degenerate quantum gases has attracted the interest among theoretical and experimental scientists from various fields.

The quantum statistical property of a massless Bose particle was first studied by S.N. Bose in 1924, which is known today as photon statistics [3]. A. Einstein extended this work to a system of non-interacting massive Bose particles and presented the basic idea of Bose-Einstein condensation (BEC) in 1925 [4]. The BEC phase transition is achieved by the condensation of macroscopic number of particles into a lowest energy ground state at low enough temperatures and caused by the quantum statistics of Bose particles. However, for more than ten years, Einstein's prediction has been neglected as purely mathematical conclusion for a fictitious system of non-interacting ideal gas and with little relevance to real physics.

In 1938, P.L. Kapitza and independently J.F. Allen and A.D. Misener discovered the phenomenon of superfluidity, frictionless flow of fluids, in liquid ^{4}He [5, 6]. In the same year, F. London published his intuitive idea that superfluidity could be an experimental manifestation of BEC [7]. However, a superfluid ^{4}He is a strongly interacting system far from an ideal gas studied by A. Einstein, so that it was not straightforward at all to connect the two concepts of BEC and superfluidity. The first microscopic theory of interacting Bose gases in the context of BEC was formulated in 1947 by N.N. Bogoliubov [8]. In the mean time, L.D. Landau developed the phenomenological theory of superfluidity in terms of the excitation spectrum of ^{4}He liquid [9], which was later supported by the experimental confirmation for the postulated excitation spectrum and the microscopic theory developed by R.P. Feynman [10]. In spite of the successful development in understanding the superfluidity in early years, it is only after the realization of atomic BEC in 1995 that the theoretical concepts of BEC by A. Einstein [4] and N.N. Bogoliubov [8] are experimentally confirmed.

One of the significant features of BEC and the key to understand the connection between BEC and superfluidity is the concept of the off-diagonal long-range order. The subject has been extensively studied by many theorists, including Landau and Lifshits [11], Penrose [12], and Penrose and Onsager [13]. Another important aspect of BEC and superfluidity is the quantized vortices predicted by Onsager [14] and Feynman [15], which was first observed in superfluid ^{4}He and more recently in atomic BEC.

All interacting atomic systems, with the exception of spin-polarized hydrogen and helium atoms, undergo a phase transition to the solid phase at low enough temperatures, as shown in a typical pressure-temperature phase diagram (Fig. 1.1). In the figure the pressure-temperature $P(T)$ line characterizing the Bose-Einstein condensation (BEC) phase transition of the ideal Bose gas is indicated by a dashed line. Upper-left of this line, a dilute Bose gas would be in a BEC phase. However, the BEC phase is unstable since the true thermal equilibrium state under these conditions of pressure and temperature is the solid phase. At such low temperatures the BEC gas phase decay to form the stable crystal phase by the three-body recombination process. This discussion seems to rule out the possibility of realizing the BEC. However, the BEC phase can indeed exist as a "metastable" state if the following conditions are satisfied:

1. The density of the gas is so low that the three-body collisions are rare and thus it takes a long enough time for the system to enter a stable solid state from a gas phase.

2. The internal thermal equilibrium of the gas, nevertheless, can be established rather efficiently and quickly by the sizable two-body collisions and the established temperature of the gas is low enough so that quantum statistics play a role in the formation of the metastable gaseous state which is a BEC phase.

3. The time required to reach the BEC phase and the time required to enter the solid phase or the escape time of the Bose particles from the trap in a more practical situation are very different, so that we can experimentally probe the various properties of the BEC phase.

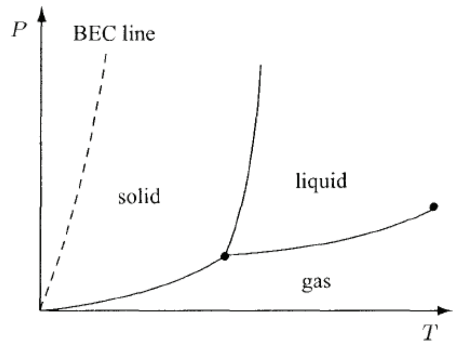

Figure 1.1: Typical pressure-temperature phase diagram. The BEC line lies in the region where the system, at equilibrium, is solid The Bose-Einstein condensed phase of the gas can consequently exist only in conditions of metastability.

The above conditions can be achieved by confining a very dilute, cold and spin-polarized atomic gas in a magnetic trap shown in Fig. 1.2(a) [1, 2] or optical dipole trap. The internal thermalization (energy relaxation) time due to the two-body collisions in such atomic BEC phases are significantly shorter than the lifetime of the sample, which is typically 1-10 sec in the presently available trap configurations. Therefore we can conclude a thermal equilibrium BEC phase can be formed in a dilute atomic gas. Figure 1.2(b) shows the BEC phase transition of cold rubidium atoms confirmed by the dramatic narrowing of the atomic distribution in momentum space[1].

(a) (b)

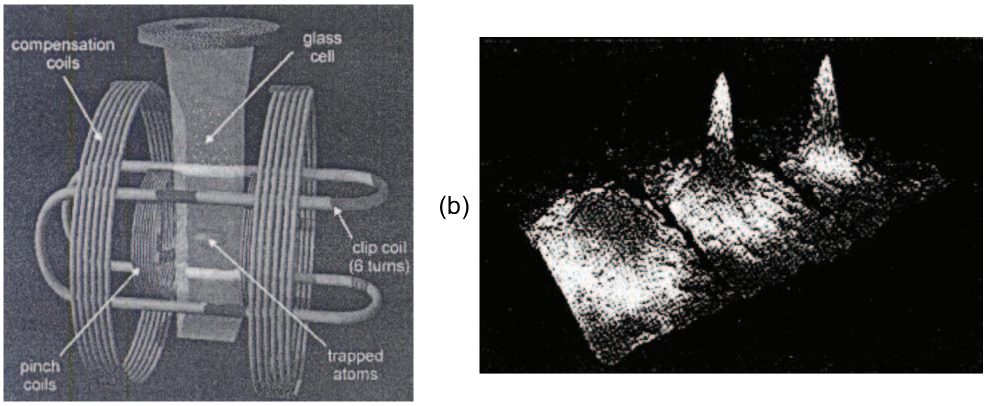

Figure 1.2: (a) A magnetic trap used for atomic BEC experiments. (b) Momentum distributions of rubidium atoms which feature a BEC phase transition [1].

Before the experimental studies on the atomic BEC was started in 1970s, the semiconductor physics community had investigated the possibility of BEC of quasi-particles, called excitons, already in 1960s [16, 17]. Excitons are elementary excitations in semiconductors but can be considered as compound bosonic particles consisting of two fermions: an electron at conduction band and a hole at valance band.

In spite of both experimental and theoretical efforts for more than four decades, the evidence of exciton BEC is still elusive. Main obstacles for exciton BEC experiments are two fold:

1. Excitons dissociate into an electron and hole plasma or non-radiatively decay by the Auger recombination process in the high density limit, so that it is very difficult to accumulate the exciton density up to a critical density of BEC at experimentally accessible temperatures.

2. Excitons are easily localized by crystal defects and suffer from relatively large inhomogeneous broadening. Thus, an ensemble of excitons in a real crystal cannot be considered as identical bosons, which is an indispensable prerequisite for BEC.

In 1996, a proposal was made with regard to the BEC of exciton-polaritons which are the hybrid quasi-particles consisting of quantum well (QW) excitons and microcavity (MC) photons in semiconductor planar microcavities [18]. By dressing QW excitons with MC photons or by the strong coupling between the two, a new quasi-particle with an extremely light effective mass is produced. The exciton-polaritons can overcome the localization and inhomogeneous broadening problem via their spatially extended wavefunctions. By inserting multiple QWs inside a microcavity, the exciton density per QW can be decreased with keeping the two-dimensional polariton density so that the other problem of the exciton density limit can be also circumvented. The exciton-polariton BEC has been indeed demonstrated very recently in several laboratories around the world [19, 20, 21, 22]. Figure 1.3(a) shows the semiconductor planar microcavity structure used in the exciton-polariton BEC experiment, while Figure 1.3(b) shows the energy-momentum dispersion relation, momentum space distribution and real space distribution of the exciton-polaritons across the BEC threshold.

The internal thermalization (relaxation) time of the exciton-polaritons trapped in semiconductor microcavities are comparable to or even longer than the lifetime of the exciton-polaritons, which is typically 1-10 psec in the presently available microcavities [19, 20, 21, 22]. Even though the macroscopic occupation is formed in a lowest energy ground state through the bosonic final state stimulation, the temperature of the exciton-polariton gas is not necessarily equal to the lattice temperature. There are three situations in such a system, determined by the ratio of the thermalization time to the lifetime of the ground state particles.

1. If the internal thermalization (energy relaxation) time is much longer than the lifetime of the exciton-polaritons, the thermal equilibrium state is not formed so that the temperature cannot be defined or the thermal equilibrium state is formed but the gas temperature is higher than the lattice temperature. Nevertheless, some of signatures of the BEC, such as the off-diagonal long-range order and the macroscopic population of the ground state, can still be realized. We refer to this non-equilibrium regime as "matter-wave lasers".

2. If the internal thermalization time is shorter than the lifetime of the exciton-polaritons, the gas temperature reaches the lattice temperature and the thermal equilibrium BEC phase is formed but only as a transient effect. Many signatures of the condensate can be experimentally probed by various stroboscopic measurement techniques. We call this transient regime as "dynamic condensation".

3. If the internal thermalization time is much shorter than the lifetime of the exciton-polaritons, the BEC phase of exciton-polaritons is indistinguishable from the atomic BEC or superfluide ^{4}He. We refer to this steady state regime as "thermal equilibrium BEC".

Table 1.1 summarizes the three distinct regimes of Bose-Einstein condensation, dynamic condensation and matter-wave lasers as a function of the ratio of the two relaxation time constants.

	Bose-Einstein condensation	Dynamic condensation	Matter-wave laser
τ_0/τ_{th}	$\gg 1$ (thermal equilibrium)	$\gtrsim 1$ (quasi-equilibrium)	$\ll 1$ (non-equilibrium)
Temperature	constant	varying	N.A.
Chemical potential	constant	varying	N.A.
Macroscopic population (ODLRO or Spatial coherence)	○	○	○
Linewidth (Temporal coherence)	N.A.	narrow	broad

Table 1.1: Three distinct regimes of cold bosonic gas systems. τ_0: lifetime of the sample (bose particles in a ground state), τ_{th}: internal thermalization (energy relaxation) time, N.A.: cannot be fined. ODLRO: off-diagonal long-range order

In this lecture we will start with the description of the conventional Bose-Einstein condensation at thermal equilibrium. After reviewing the basic properties of the BEC described by the nonlinear Schrödinger equation (or Gross-Pitaevskii equation) for a closed BEC system [23], we will move on the discussion of the dynamic condensation at quasi-equilibrium condition and matter-wave lasers at non-equilibrium condition. These two regimes must be analyzed by the quantum reservoir theory for an open-dissipative system [24].

The first part of this lecture deals with the conventional BEC in thermal equilibrium regime. Chapter 2 introduces several key concepts of BEC, such as an order parameter, spontaneous symmetry breaking, Nambu-Goldstone modes, off-diagonal long range order and higher-order coherence. A simple model of non-interacting Bose gas, first studied by A. Einstein, is presented in Chapter 3. In spite of its simplicity, the model still captures the basic properties of BEC, such as critical temperature/density and condensate fraction. Chapter 4 describes the Bogoliubov theory of a weakly interacting Bose gas. Again in spite of the simplicity of the theory, the model captures almost all key features of experimental BEC systems. We will derive the Gross-Pitaevskii equation, the Bogoliubov excitation spectrum, the sound velocity and the first-order coherence function. Superfluidity is a phenomenon closely related to BEC, and discussed in Chapter 5. We will present the Landau's criterion for superfluidity, the difference between BEC and superfluidity, the quantized vortices, vortex-pairs and vortex-lattice, and finally the Berezinskii-Kousterlitz-Thouless (BKT) theory of the superfluidity in uniform two-dimensional system. Chapter 6 introduces the superfluidity of Fermi particles interacting with attractive forces, which is a similar situation to the BCS phase transition of Cooper pairs of electrons in superconductors. A particular emphasis is given to the BEC-BCS crossover behavior when the Fermi particle density is increased from the dilute limit to the dense limit.

The second part of this lecture deals with the dynamic condensation in quasi-equilibrium regime and the matter-wave laser in non-equilibrium regime. Chapter 7 discusses the following questions to highlight the difference between the BEC and the matter-wave lasers: Why condensate fragmentation is suppressed and why a condensate prefers to acquire a stabilized phase. We will also discuss the dimensionality issue and the competing order to BEC in this chapter. The formal theory of a matter-wave laser is presented in Chapter 8. We will derive the Heisenberg-Langevin equation for the field operator by including a finite decay rate and gain rate. The modified excitation spectrum, amplitude and phase noise spectra and spectral linewidth (or temporal coherence function) are obtained. Chapter 9 presents the alternative theory of a matter-wave laser, which is based on the master equation for the density matrix and Fokker-Planck equation using the diagonal coherent state expansion. Finally, Chapter 10 concludes this lecture by mentioning the future outlook of the field, with a special emphasis on quantum information system applications.

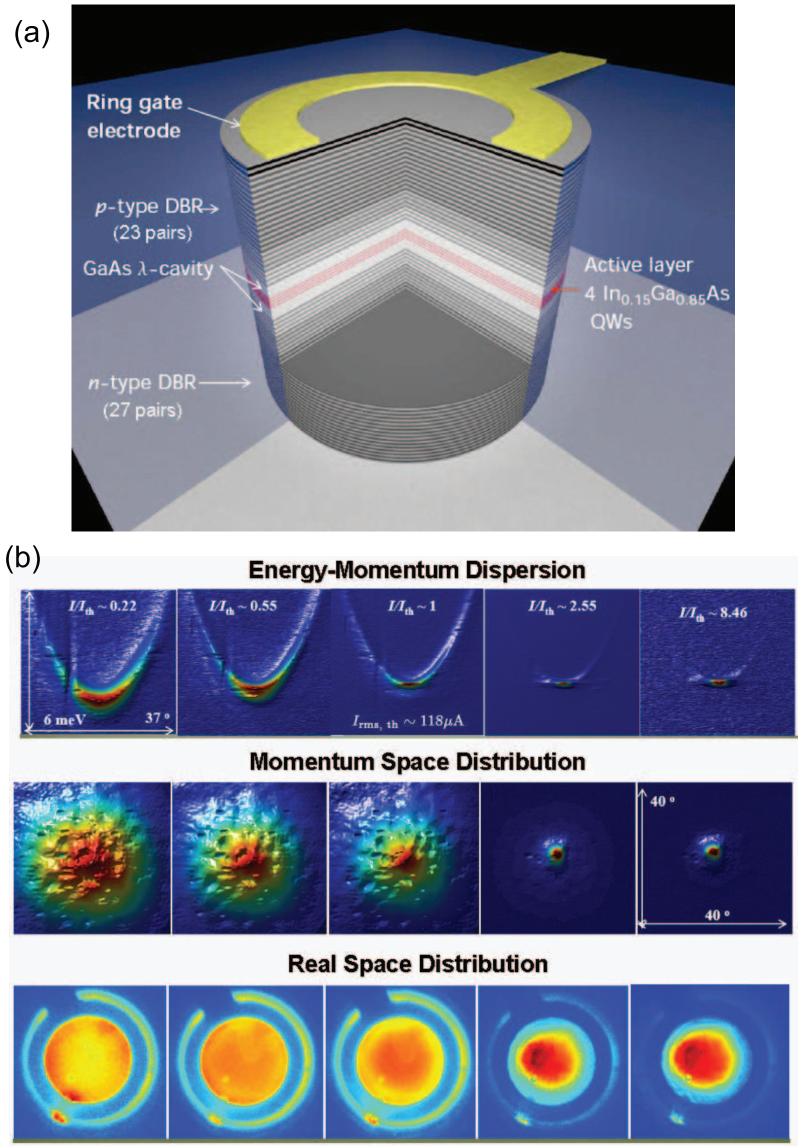

Figure 1.3: (a) A current injection type semiconductor planar microcavity with multiple quantum wells in a one-wavelength cavity, which is sandwiched by two $p-$type and $n-$type distributed Bragg reflectors. (b) Evolution of the energy-momentum dispersion relation, the momentum space distribution and the real space distribution of the exciton-polaritons across the exciton-polariton BEC phase transition.

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