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Recursive quantum repeater networks

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ABSTRACT
Internet-scale quantum repeater networks will be heterogeneous in physical technology, repeater functionality, and management. The classical control necessary to use the network will therefore face similar issues as Internet data transmission. Many scalability and management problems that arose during the development of the Internet might have been solved in a more uniform fashion, improving flexibility and reducing redundant engineering effort. Quantum repeater network development is currently at the stage where we risk similar duplication when separate systems are combined. We propose a unifying framework that can be used with all existing repeater designs. We introduce the notion of a Quantum Recursive Network Architecture, developed from the emerging classical concept of *recursive networks*, extending recursive mechanisms from a focus on data forwarding to a more general distributed computing request framework. Recursion abstracts independent transit networks as single relay nodes, unifies software layering, and virtualizes the addresses of resources to improve information hiding and resource management. Our architecture is useful for building arbitrary distributed states, including fundamental distributed states such Bell pairs and GHZ, W, and cluster states.

KEYWORDS
Quantum repeaters, distributed quantum computing, computer networks, recursive network architecture

1 Introduction

Communication is the sharing of data — bits, in modern digitial systems — between a pair of endpoints, with a certain probability of the shared data being the same at both ends. Shannon [1] addressed this problem for the case of a simple *channel*, where the endpoints are known *a priori*. In the sixty years since his seminal analysis, researchers have extended the concepts to apply to networks consisting of more than a single sender and receiver [2]. Except for the physical means of creating entanglement, quantum repeater networks require solutions to the same problems as all classical communications networks, allowing us to reuse many of the engineering principles developed for classical networks.

Quantum repeater networks support the sharing of quantum states. Distributed, entangled states of various kinds are used in distributed decision algorithms [3]–[5], the creation of shared, secret random numbers [6], [7], distributed arithmetic [8], secure, distributed function computation [9], quantum secret sharing [10], physical operations such as remote synchronization of clocks [11], and a range of other applications [12]–[15].

In this paper, we examine the problem of delivering distributed, entangled states across a large, topologically complex, technologically diverse internetwork,

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owned and operated by many different organizations and deployed over a period of years. These problems will be particularly evident as a communication request crosses the boundary between two networks based on different entanglement purification and forwarding schemes. We consider what lessons may be learned from the historical development of the Internet. In order to prevent similar problems from occurring in a quantum repeater Internet, we propose a solution based on classical recursive network architectures, extended to support requests for distributed computation. As our goal is planetary-scale networks composed of perhaps billions of nodes, scalability is a critical feature and motivates our work.

Researchers have proposed a “quantum Internet”, using the term “Internet” as a synonym for a shared, global network [16], [17]. The term “internetwork” encompasses a richer meaning, literally connecting multiple disparate networks. In any large network that grows and evolves over time, multiple different technologies and many different management domains will coexist, forming an internetwork or *internet*. This organic growth and the sheer scale of large networks pose several categories of problems: (1) ensuring interoperability among technologies that are heterogeneous (at both the physical and logical levels); (2) reconciling the competing needs and policies of independent organizations (including the desire to keep information about the network internals private); (3) choosing a technical approach for the routing, naming, and resource discovery problems that is robust in the face of this heterogeneity and federated operation; and (4) managing communication requests using incomplete, out-of-date information about the dynamic state of the network, including availability of resources and topological changes occurring as nodes join and leave, and network links go up and down.

The evolution of the Internet provides a guide to designing large-scale systems. It is a hierarchical system, with the communication functionality divided into a set of *protocol layers*, but not a truly recursive architecture (see Sec. 3). As the Internet grew from its predecessors, various routing protocols were devised, and a two-layer hierarchical structure created (using what are known as *interior gateway protocols* (IGPs) and *exterior gateway protocols* (EGPs)) to hide internal topologies and provide scalability and management autonomy and privacy. This hierarchy has other layers, but until recently the number and structure of layers has been fixed. Over time, it became desirable to build one network on top of another, or to translate addresses as data packets cross network boundaries. Virtual networks, tunnels, overlays, mobile IP, and network address translation (NAT) are used to achieve various technical and operational goals, but all interfere with the original, uniform scheme for addressing and routing, and have resulted in much duplicated engineering effort as functionality is re-implemented at various levels [2], [18]. Additional layers of this hierarchy have been added, using encapsulation-based subnets, such as LISP [19]. Recently, these layers have been understood as instances of a more general and flexible recursive architecture that supports arbitrary layering. Our goal is to apply that more general approach to quantum networking now, without restricting technological innovation or organizational choice, while supporting the quantum networking protocols already under development.

An emerging concept in classical networking is *recursive networks* [18], [20], in which a subset of a network can be represented as a single node at a different layer of that network. Recursive networking is used to unify multihop forwarding, embedded topologies and other forms of virtualization, and the software layering common in network protocol stacks.

Recursive networks allow individual networks to offer data forwarding services without requiring requesters to understand the detailed topology or technology of the network, and for that process to be repeated at multiple layers in the network more cleanly than the two-level EGP/IGP system. The networks that may form part of the path, but do not include nodes that are part of the requested state, are *transit networks*. To external requesters, a transit network appears as a single node. Internally, nodes within the network can in turn be networks, in recursive fashion. Fig. 1 shows an example network topology. The Internet and telephone network both exhibit a fixed set of layers, but recursive approaches are more general and scalable. Such approaches have only recently been applied in the Internet, in LISP [19] and Rbridges [21], but adoption of the concept earlier in their evolution may have alleviated some of the above scalability problems. Recursive networks will be described in more detail in Sec. 3.

Our engineering philosophy is to adapt classical solutions to the quantum domain where possible, an approach that has proven fruitful in other areas of quantum systems engineering. An excellent example is quantum error correction (QEC), where many of the ideas originate in classical error correcting codes (see Ref. [22] and Ref. [23] and references therein). Quantum arithmetic likewise has built on classical concepts (see e.g. Refs. [24]–[28]). Both fields are rich areas of study, resulting in both new ideas and rigorous, creative applications of old ones; we expect the same to be true as we engineer quantum networks.

We have chosen to adopt recursive networking as a framework for quantum networks. The principal differ-

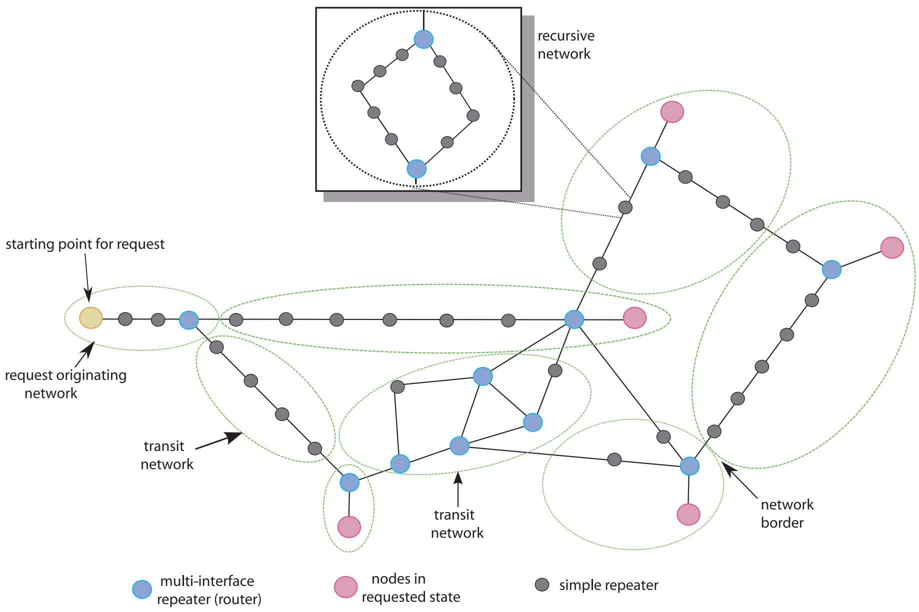

Fig. 1 Large-scale quantum repeater networks will consist of many nodes. Many long-line connections between routing-capable vertices ("routers") will consist of multiple-hop chains of simpler repeaters. In a recursive network, each node in the figure can actually represent a complete network itself that provides the services of a single node.

ence between classical and quantum networking lies in the semantics of the request. In a classical network, the arrival of a data packet represents an implicit request to forward that packet on toward its specified destination. Quantum networks (e.g., those based on teleportation) transmit not quantum data itself, but requests for the execution of operations which will *recreate* quantum states extinguished at other sites, or create new distributed, entangled states. Thus, *quantum requests are more complex than classical ones, and can no longer remain implicit*. Recursive quantum networks require significant extension of the classical concepts to offer distributed state creation services based only on a description of the desired state (or the computational steps necessary to create the desired state).

Our quantum recursive network architecture (QRNA) contributes to solving all four of the scaling problems, and will circumvent the evolutionary problems above. Ref. [29] defines a protocol stack for purify-and-swap repeaters (see Sec. 2) with separate, but stackable, protocol layers for entanglement swapping and one specific purification mechanism, building long-distance Bell pairs. QRNA generalizes and extends this to a more complete architecture for creating arbitrary distributed quantum states spanning heterogeneous, autonomous networks. In this paper, we describe the recursive framework, identify the contents of the necessary messages, show how they can be executed recursively, and enumerate the operational and architectural benefits.

After opening with networking background, both quantum (Sec. 2) and classical (Sec. 3), we describe the request structures that make the recursive architecture possible (Sec. 4), then show how this concept makes real-world deployment of truly large-scale, heterogeneous networks practical (Sec. 5).

2 Quantum networking

To date, most research on quantum networking has focused on systems for creating high-fidelity generic entangled states, such as end-to-end Bell pairs [30] or larger graph states on model networks [31], [32]. The resulting generic states are then used for the remote execution of quantum gates [33], teleportation of valuable application-level qubits [34]–[36], or the creation of shared classical random bits via measurement (e.g., for quantum key distribution (QKD) [6], [7]). The more general concept is the creation of arbitrary distributed, entangled states; thus, a quantum network is effectively a large-scale distributed quantum computing system.

The problems that must be solved to implement quantum networks include loss and fidelity degradation in optical channels, as well as network issues such as routing and resource management [37], [38]. The issue of an imperfect but abstract channel was the first attacked in the literature, so we will begin our discussion there.

Over a channel between sender and receiver, the goal has been to distribute entangled states without significant fidelity degradation. Even over short distances the fidelity will degrade, and some signals may also become lost; the fidelity and probability of successful entanglement vary depending on the physical mechanism and whether single photons or stronger signals are used [39]–[42].

The standard solution has been to establish multiple lower-fidelity Bell pairs between adjacent nodes, and then consume some of them in a *purification protocol* to leave the remaining links as close as possible to the desired Bell state [43]–[46]. Purification protocols can be run as many times as is required, consuming lower-fidelity Bell pairs at each step. Finally, a single high-fidelity Bell pair is produced end-to-end in the network, with the caveat that achievable fidelity remains capped by local gate, measurement and memory errors.

Standard purification requires two-way classical communication between the two ends of the Bell pair being purified, which can result in delays that harm both fidelity and performance, perhaps irreparably. There is also always a nontrivial possibility of failure of the purification: all the entangled links are consumed, but a high-fidelity Bell pair is not produced. This information must be shared between the end points. Purification can also be done using one-way communication and quantum error correction, with some loss in channel capacity [44], and can also be performed on graph states larger than two qubits [47], [48].

The solution to the problem of signal loss (exponential in distance for individual photons in optical fiber) has been to use *repeater nodes*, proposed to be placed at short intervals (generally of the order of tens of kilometers [39], [42], [49]). With either a chain of repeaters or a network comes the problem of moving data over multiple hops.

The obvious solution for moving quantum data is to simply forward a qubit from node to node using teleportation [34]. However, this hop-by-hop approach has long been considered unworkable in realistic environments because imperfect local gates and memories result in unacceptable degradation of fidelity [30], [50]. This limitation led to the proposed use of purification over multiple hops [30], [44]. To create the end-to-end entangled state, teleportation is often proposed to be used to perform *entanglement swapping*, often in a nested manner that doubles the span of entanglement at each step. With recent advances in the distributed use of error correction [51], [52] and operational tactics for repeaters [53], additional options are available. Perhaps the principal architectural choice facing the network architect is whether to attempt to revive the hop-by-hop approach, to operate in a distributed fashion (the purify-and-swap approach), or to take a radically different approach to moving data through the network, such as using the surface code [52], [54]–[56] or other quantum error correcting codes.

Recent development of such new approaches started with Jiang et al. [51], where error correction is used to replace some amount of purification, particularly in the context of long-distance QKD. Calderbank-Shor-Steane (CSS) [57], [58] encoding is used to replace long-range purification, although Bell pairs over single hops still require purification and the associated classical messaging, both between adjacent nodes and across the network in the case of failure events. More recently, Munro et al. have presented a scheme where purification is replaced entirely with quantum error correction and parallelization of low-fidelity link generation [53]. Instead of consuming multiple low-fidelity Bell pairs in a purification protocol, they show how a single high-fidelity Bell pair can be encoded in these multiple pairs, and give the explicit example of a repetition code. Only a single link-level classical message needs to propagate backwards from the receiver to the sender and, most importantly, the encoding is deterministic. There are therefore no end-to-end entanglement or purification failure messages that require propagation through the network.

In addition to using error correction codes to protect Bell pairs as they propagate, other recent work demonstrates that data qubits themselves can be transported in this way. Fowler et al. propose a scheme using the topological surface code on a regular cluster state superimposed on a repeater network [59]. Each node contains a block of surface code, with the block edges coupled through the entangled links between nodes. The data propagates through the network directly, rather than being held until a high-fidelity Bell pair is available and then teleported.

There are therefore many options available for managing and using a repeater network. Either standard purification or error correction can set up end-to-end Bell pairs. Error correction-based schemes can transmit application data in a hop-by-hop fashion, without end-to-end entanglement having ever been present at a given point in time. The hop-by-hop approach is attractive because of its similarity to classical networks, although a full accounting of resources used, including error correction to protect the transmitted states while they are operated on at the transit nodes, remains to be done. In the purify-and-swap case, activity must be coordinated with numerous waypoints or rendezvous points via classical messages, which, prior to the QRNA architecture proposed here, required explicit, static assignment of those points. In other architectures, the requirements vary from complete control of activity on an entire path to a send-it-and-forget-it approach similar to that of packet forwarding on the classical Internet. One of our goals is to create a request model that will support interoperation of all these approaches.

While the main focus of research into quantum repeaters has been for 2-party message transmission, it is also possible to use such a network for distributed measurement-based quantum communication protocols. These protocols use highly-entangled graph states to propagate information through the use of intermediate measurement, consuming the graph state resource in the process. Such schemes can propagate either application data or half of a Bell state to generate end-to-end entanglement. Managed properly, multiple data movement requests can also be satisfied from the same graph state, as in Fig. 2 [60]. As given, such schemes do not deal with realistic (impure) states, and will require adaption to an error correction/purification-based network. However, the key idea is already present in recent schemes, that a quantum network where data propagates directly is capable not only of hop-by-hop communication, but also distributed computation without end-to-end classical control messaging [61].

It is obvious from this discussion that repeater nodes in a quantum network differ significantly from classical signal repeaters. They are not classical signal amplifiers; they provide base-level entanglement with neighbors, data transmission services and the quantum computation and classical communication operations that are necessary for both purification and entanglement swapping. They are small, limited-functionality quantum computers in their own right, and fulfill the role of routers in the Internet. We will take advantage of these computational capabilities in QRNA, as will be described in Sec. 4. First, we preview some of the key ideas of QRNA while reviewing some of the principles of networking.

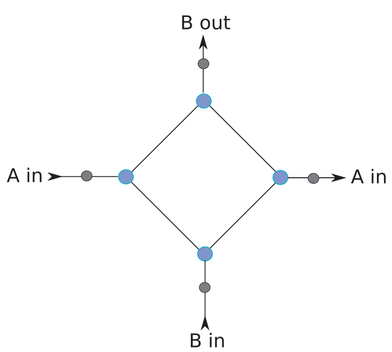

Fig. 2 Graph states are useful both at the application level and as communications channels; a diamond junction like this one can be used to send two qubits simultaneously from place to place in measurement-based quantum computing.

3 Computer networks

This section explains how quantum networking builds on concepts from classical networking, notably the emerging concept of *recursive networks*.

3.1 Network topology

A *network* is a group of interacting parties, where each of the members potentially wants to interact with any of the other members. A computer network consists of *nodes*, which are typically a form of computer, and *links*, which carry messages from node to node. A node uses a *physical interface* to connect to the link. Links may be bidirectional or unidirectional, and bidirectional links can be further divided into full- and half-duplex, depending on whether they support concurrent bidirectional transfer or must be time multiplexed.

In classical networks, links are also described as 2-party or multiparty. Two-party links involve two known, fixed nodes; multiparty links involve multiple nodes, in which the set of nodes may change over time (i.e., membership is not known, but rather discovered), and where a single message can be received by an individual node, or by all or some of the nodes (known as broadcast or multicast, respectively). The single-receiver case is equivalent to a bus, and numerous physical implementations of quantum systems support addressing individual qubits in such a fashion, generally through a shared waveguide or resonator (see, for example, Refs. [62]–[66]).

Classical broadcast or multicast copies data to all interested receivers. The direct equivalent for quantum communication would use GHZ states [67] for the FANOUT creation of GHZ-like states, giving each node a part of the state. In FANOUT [68], a single qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is expanded into an N-qubit GHZ-like state

$$|\psi'\rangle = \alpha|0\rangle^{\otimes N} + \beta|1\rangle^{\otimes N}, \tag{1}$$

where α and β conform to the usual normalization condition $|\alpha|^2 + |\beta|^2 = 1$ and the notation $|0\rangle^{\otimes N}$ indicates the N-qubit state $|0\rangle \otimes |0\rangle \ldots \otimes |0\rangle$. In distributed algorithms, this gives each node in the network access to the same quantum variable, which can be used in further quantum computations. This FANOUT of arbitrary quantum information allows for quantum computation to proceed in parallel (e.g., when used to distribute carry information in arithmetic [8]). Generic GHZ states are also used to make coordinated, distributed decisions [3].

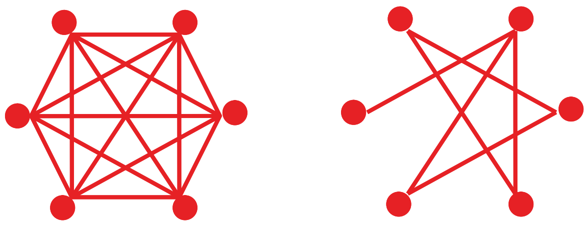

Fig. 3 Multiparty communication without forwarding requires N^2 links (left), but can use only N links (right) with forwarding.

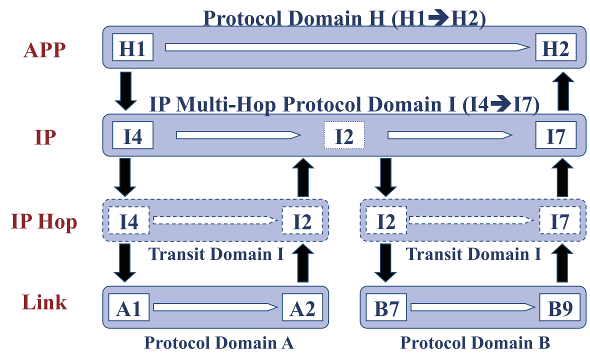

Fig. 4 Classical recursive multihop/multilayer architecture, including forwarding steps (Hop) as part of a multihop path (IP, here). APP is the application layer, IP is the Internet Protocol layer that connects different link networks, and the link layer provides a physical connection over a single hop (e.g., dial-up), or by a similar multihop network (not shown, e.g., Ethernet).

Other multi-party entangled states, such as W and graph states, will be used in different fashions in quantum algorithms. These entangled states may be created using some direct, multi-party physical interaction [69], [70], or more likely by local creation of the complex states that are propagated outwards using two-party distributed Bell pairs. This tradeoff is discussed further below.

As a result, quantum networks correspond most closely to classical networks with 2-party unidirectional links, with the primary remaining difference being the data being communicated (qubits *or* larger entangled states, and the long-distance gates necessary to execute them) and the means by which data is relayed.

3.2 Multihop communication

As noted above, quantum links are two-party, and the endpoints are known. This is the basis of Shannon's model of communication, but it requires N^2 links (given N nodes in a network) (see Fig. 3, left side). More commonly, such full connectivity is supported with a smaller set of links using multihop forwarding (see Fig. 3, right side). Forwarding is the fundamental concept that enables scalability in physical systems, both in distance and number of nodes: individual nodes have multiple physical interfaces, receive messages, and make decisions about how best to send the messages on to their respective destinations.

Given two nodes that want to communicate, a path must be found along the existing links, and communication along that path cascaded. In classical networks this is called *packet forwarding*, and is the basis of the Internet. There are other steps involved in classical networking, such as name resolution (finding the location of a named item), and routing (finding a network path to a location); we assume they function here as they would in any classical network, and address these issues further in Sec. 4.1.

In modern communication architectures, the functionality is divided into a set of *protocol layers*, as shown in Fig. 4. Each layer has a different role in supporting the end-to-end communication requested by the originating application. Each layer uses services provided by its lower layers to communicate with its peer layer at the remote node. In practice, the software implementing these layers may be integrated into a single module, but they are commonly described as if they were implemented separately.

Quantum networking is described well by the emerging concept of *recursive networks*. Recursive networking was developed in 2000 to describe multi-layer virtual networks that embed networks as nodes inside other networks [71], and has since evolved as a possible architecture for the future Internet [18], [20], [71]–[74]. It has been used to unify the layering of protocol software, message forwarding, and topology embedding. Classical message forwarding is explained by a kind of *tail recursion* [75], in which the last step performed in the operation is the recursion itself. Tail recursion is basically iteration accomplished using recursion, where instead of each step recursing (and thus pushing information on a stack) and the final step popping the entire stack, each step overwrites the top of the stack so that the last step can complete more efficiently. Quantum message forwarding is closer to topology embedding, as in the "recursive router" concept introduced in the X-Bone [71].

3.3 Recursive networking

A subset of a network can be embedded in the overall topology; such embeddings are useful in classical networks to hide complex subnet structure from being visible to the overall network (see Fig. 5). This subnetwork embedding is called *recursive networking*, and

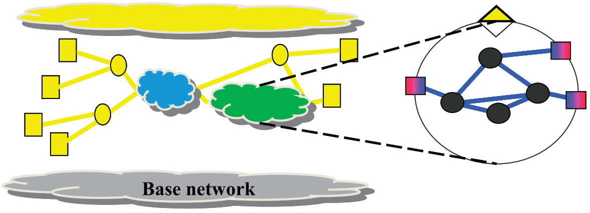

Fig. 5 Classical network recursion, showing a subnet (cloud) that acts as a router in the overlay (yellow), where the recursive router (inset circle) consists of ingress and egress virtual hosts (shaded squares on the inset) and interior routers (gray circles).

```
deliver(data, src, dst) {
  process(data) -> newdata
  WHILE (here != dst) {
    found = FALSE
    FOREACH (lowerlayer) {
      map(src,dst,lowerlayer) -> newsrc, newdst
      IF (deliver(newdata, newsrc, newdst)
          == TRUE) {
        found = TRUE
      }
    }
    IF (found == FALSE) {
      /* if you get here, you failed to deliver
         the data */
      FAIL
    }
  }
  /* if you get here, you're at the
     destination */
  RETURN TRUE
}
```

Fig. 6 The algorithm for recursive resolution and forwarding, adapted from Ref. [20]. This algorithm is executed at each node as it receives data to be delivered. `src` and `dst` are the source and destination addresses, and `lowerlayer` refers to e.g. the layer on the receiving end of a downward arrow in Fig. 4.

can be part of a broader approach to network architecture [72]–[74].

In classical networking, nodes acting as message sources or sinks are called *hosts*, and nodes acting as relays are called *routers*. A recursive network represents the embedded subnet as a router at the higher level, where the ingress and egress nodes of that subnet act as hosts inside the subnet (see Fig. 5, detail). Such embedding can happen many times, sometimes on top of existing embeddings, which is why this is called *recursive*. Note the similarity of Fig. 5 to Fig. 1. When a given layer in the protocol stack provides an interface to its clients that is identical to the interface of the services it uses (e.g., packet forwarding to nodes in a particular namespace), implementation of recursion is straightforward.

Classical networking uses recursion to represent topology hiding, but we can also consider the entire network architecture as recursive as well [18], [20]. As an architectural principle, recursive networking explains layering of protocols (and their modular software architecture), name resolution, routing, and forwarding as more than just artifacts of the current Internet [73].

In most forms of quantum networking, recursion as a request moves through the network is true recursion; we cannot transform it into a hop-by-hop iteration and optimize it as tail recursion. Fig. 2 of Ref. [29] shows how layers of quantum repeaters are used to compose a sequence of individual hops into a single, longer hop. This is the same composition process of tail recursion in classical recursive networks, except that the layering is left in place rather than collapsed as a simplification.

Consider the steps of classical recursive networking, shown in Fig. 6. When a packet is received by a node, the packet is implicitly requesting that the node forward the packet on toward its destination. This algorithm is executed by the node to perform the forwarding. The `process()` step may alter the packet, including turning a single packet into more than one. The `FOREACH` loop passes the packet(s) down the protocol stack, as in Fig. 4.

QRNA adopts a similar outline, with different semantics. In quantum networks, `data` contains a computation request using virtual identifiers for resources, and the `process()` step represents the local operations that are performed in a repeater toward fulfillment of that request, such as the entanglement swapping that happens when Bell pairs are spliced to form a longer pair [29], [30]. The output of `process()`, `newdata`, may be more than one request. The `map()` function may modify the addresses in a given request. In our architecture, the node identifiers don't change within a forwarding path, but requests may be retargeted from a network destination to a node destination, as we will describe in Sec. 5. The corresponding concept is shown in layered communication, as supported by recursive networking, in Fig. 4.

Beyond this basic structure for forwarding, in QRNA the requests themselves become recursive, and must be carried explicitly through the classical network. Next, we turn to the structure of these requests.

4 Recursive quantum requests

Recursion is a natural model for quantum repeater networks because purification, entanglement swapping, and Calderbank-Shor-Steane (CSS) [57], [58] or surface code error correction [52], [54]–[56] build on mixed, entangled states and produce other mixed, entangled states, working toward a common goal of a high-fidelity, wide area-distributed quantum state. The similarity of the interfaces on the top and bottom of a given protocol layer simplifies recursion, allowing more or less arbitrary composition of protocol stacks.

In a large network (millions to billions of nodes spread across many countries and organizations), direct management of the network as a single, synchronous, shared, centrally managed system is impractical, and even optimization of smaller portions of the network becomes a computationally intractable combinatoric problem. Applying recursion abstracts away much of this complexity and allows us to effectively manage the larger set of resources. Each protocol layer, node or network needs to recognize and be able to reach only a small subset of the entire network's resources, and hides much of the underlying complexity to allow its own clients to operate in a smaller subspace.

In order for recursion to be effective, we must have a well-defined request-response model that allows us to combine protocol layers. Before the requests and responses can be defined, we must be able to name the distributed entangled states themselves, and the resources comprising them. The next two subsections address these issues.

4.1 Naming a state

Over the course of the lifetime of a quantum network, many entangled quantum states will be created and consumed. Repeaters will make both independent and coordinated decisions about which states to purify, swap, error correct, forward, buffer, and discard, as they build states that satisfy users' requests. In order to communicate successfully about these states, nodes must be able to *name* states using a namespace that other repeaters will understand: "do operation U on this particular state we share." In order to construct such requests unambiguously, the qubits within the states must also be named.

The simplest naming scheme for a particular qubit is the tuple (N, A), where N is the node name and A is the physical qubit address within the node. However, there are three key problems with this scheme:

- each node is entitled to move the logical state of a qubit from one physical qubit to another;
- physical qubits are reused after being freed; and
- the node issuing the original request may need to refer to the qubit by name even before physical resources for it are actually allocated (e.g., a request for a gate to be executed may be issued at the same time as the initial entangling pulse).

These factors mean that physical address is a constraining and unreliable identifier for the quantum states that are our true subject of interest. All of these problems can be solved by allowing the original requesting node to assign a *virtual address* or other abstract identifier for the qubit; the node (or network) housing the physical resource is responsible for maintaining the mapping of virtual to physical resource. That mapping information is private to the node and need not be disclosed or coordinated with other nodes. In order to ensure that the virtual address assigned by the requester is unique, the full address tuple must include the requesting node and the actual request identifier.

The naming scheme must be prepared for names to shift as operations proceed. Multiple quantum states often merge to become a single state. Purification, entanglement swapping, and error correction all result in such mergers flowing up the protocol stack, and result in multiple requests moving down the protocol stack. Names for states and qubits may be remapped when crossing boundaries. Names for nodes, when visible, do not generally need to change, but requests moving from the outside of a network to the inside may become more specific at the boundary.

Each boundary in the system, whether a software boundary between modules or a hardware boundary between nodes or networks, represents a point at which resource names and requests may change. Logically, these boundaries represent points where these mappings and requests must be maintained, although in implementation this may vary.

4.2 Defining quantum requests

As classical distributed computation proceeds, applications running on several nodes request that the network subsystem send and receive messages or, using higher-level constructs, synchronize the state of distributed copies of shared data structures [76]. In the quantum world, a quantum request is for a specific state, spanning a named set of nodes.

The interface to the network subsystem must allow the requester to specify the desired state $|\psi_S\rangle$, while the network will actually return

$$\rho = \mathrm{Tr}_{AB}|\Psi\rangle\langle\Psi| \text{ where } |\Psi\rangle = |\psi'_S\rangle\widetilde{\otimes}|\psi_{A+B}\rangle \qquad (2)$$

where $|\psi'_S\rangle$ spans the set of state qubits, $|\psi_{A+B}\rangle$ is the set of ancillae (defined but unused qubits, for this state) plus the bath (the environment), and $\widetilde{\otimes}$ indicates that what we get in the real world is only an approximation of a separable state. The aim is to have

$$\rho \approx |\psi_S\rangle\langle\psi_S| \qquad (3)$$

within certain tolerances. The request must therefore also specify these tolerances on the state: a minimum fidelity and a maximum entanglement with the ancillae and bath. Thus, a density matrix should be viewed as *no-less-than* for the element(s) corresponding to the desired state, and *no-more-than* for the elements corresponding to undesired states.

Both the fidelity $F = \langle\psi_S|\rho|\psi_S\rangle$ and entropy $S = -\text{Tr}(\rho\log\rho)$ appear in the request to constrain the returned state ρ to be near the desired state. The fidelity is to ensure closeness to $|\psi_S\rangle$; the constraint on the entropy of ρ allows the system to filter out returned states that may be non-trivially entangled with other nodes in the system. In the limit of $F \to 1$, the entropy becomes unnecessary, but for fidelities bounded farther away from 1, the entropy becomes a useful tool. We assume that repeater nodes make repeated use of the same physical resources, and sometimes swap data qubits with ancillae, which if done imperfectly leaves behind some residual entanglement between qubits which should not be entangled. Further reuse of those ancillae can therefore further entangle data qubits in an undesired fashion. Because both the qubits on which $|\psi_S\rangle$ are defined and the ancilla qubits may be entangled with the environment (that is, in a mixed state), the state of ρ alone cannot determine if any node qubits are entangled with any of the ancillae. Limiting the entropy of ρ serves to limit the possible entanglement with ancilla qubits by limiting all external entanglement.

In addition to these properties, the requester must specify the desired logical or physical encoding of the quantum state. An application will request an absolute encoding, while each layer in the protocol stack provides a relative encoding (discussed further below), with the entire stack to provide the absolute encoding.

The tuple specifying a request for a state is

$$T = (ID, |\psi_S\rangle, F, S, ((N_i, A_i)), E_A), \tag{4}$$

where ID is the transaction identifier assigned by the requester, F is the minimum acceptable fidelity of ρ with $|\psi_S\rangle$, and S is the maximum acceptable entropy of ρ. $((N_i, A_i))$ is the set of nodes that are requested to comprise the state and the virtual addresses A_i that are to be used for the qubits, and E_A specifies the absolute quantum error correction encoding. $|\psi_S\rangle$ is the desired pure state; the exact encoding of the description of the requested state is beyond the scope of this paper, but can take numerous forms, including state vector, stabilizer, and circuit descriptions.

Requests may also be for *actions* to be executed on specific states, in which case the tuple is

$$T = (ID, C, F, S, ((N_i, A_i)), E_A), \tag{5}$$

where C is a circuit that may include both unitary and measurement operations.

The return value of a request is the tuple

$$R = (ID, \rho) \tag{6}$$

where ρ is the density matrix of the delivered state for request ID. The set of resources represented by ρ is specified by the basis $((N_i, A_i))$, the tuple of tuples including node (or network) identifiers N_i and the virtual addresses A_i included in the original request.

Benjamin et al. described a brokered approach to building large-scale graph states from smaller ones, tailored to a specific hardware implementation [77]. QRNA provides a framework for abstracting and generalizing this process, including support for cost functions that will allow intelligent decisions for constructing the sub-graphs.

5 Implementing recursion in quantum networks

5.1 Satisfying quantum requests

Requests naturally originate at applications running on specific nodes, and are processed through a series of software protocol modules that implement the layers of the protocol stack, with carefully defined interfaces between the layers. Each layer in the protocol stack has access to a set of resources it can use to satisfy requests: it knows about a certain set of network nodes (or, more scalably, how to *find out* relevant information about a set of network nodes), can ask for certain states (including entangled states) to be created on that set of nodes and for certain operations to be performed on those qubits, and can utilize its own internal capabilities. It has exclusive control of a certain set of resources, and may consult with the corresponding layer instances at remote nodes about the best way to satisfy requests. However, it should endeavor to make *independent but coordinated* decisions whenever possible, so that the latency penalty for explicit messaging can be avoided.

Each protocol instance has the ability to execute local quantum operations (unitary operations and measurements), as well as compute and communicate classically with other repeater nodes. This ability is often referred to as *LOCC*, local operations and classical communication. The instance has no access to distributed quantum states or operations beyond those it currently owns. If additional states are needed to complete an operation, they must be requested from protocol layers below or from peers.

Requests are not constrained to be 1 : 1; a single request from above may be mapped to multiple requests to the layer below. A protocol layer has the right to merge and split states and issue multiple requests to meet its obligations. The ability to *buffer* quantum states, to hold them while waiting for other resources to become available (e.g., other quantum states or answers to classical queries), is generally necessary when coordinating multiple requests.

Protocols that make decisions about how to get from place to place in the network must have access to a *cost function* for specific requests that can be used to make intelligent decisions, discussed next.

5.2 Finding rendezvous points

Purify-and-swap repeaters require the explicit use of named rendezvous points where the entanglement swapping occurs. On a modest-sized network, Dijkstra's algorithm [78] can be applied to select a path through the network, then the swapping points optimized on the chosen chain of repeaters [37]. The order of entanglement swapping can be either specified or left unspecified.

Store-and-forward repeaters need a similar path selection mechanism, but have no direct need for waypoints, although they do require an adequately scalable mechanism for calculating routes and choosing the correct next hop on a request-by-request basis. However, the Open Shortest Path First (OSPF) protocol [79], built on Dijkstra's shortest path first algorithm, is widely used on the Internet as an IGP in localized regions (called routing domains), but its use is impractical for more than a few thousand nodes. The Internet as a whole consists of tens of thousands of separate such domains interconnected by a separate routing protocol (BGP, used as an EGP), so that the hierarchy supports hundreds of thousands of interdomain connections, with the entire Internet consisting of perhaps tens of millions of routers and hundreds of millions of endnodes. Hierarchy is the principal means of solving such scalability problems, and the hierarchy and recursion allow the details of the routing mechanism at each level to remain irrelevant as long as the picture presented to the outside world is consistent.

5.3 Example

As an example, consider an application needing a three-qubit cluster state defined by the circuit in Fig. 8. The request originates at Node11, with the three qubits requested to be at Node11, Node55, and Node77 in the network in Fig. 7. The application begins by specifying the state it wants, then other ("system") software running at Node11 creates a global strategy for how to achieve the state, and sends requests to corresponding nodes or networks. The bulk of this work happens in the QRNA `process()` step in Fig. 6. The nodes that receive the requests will in turn craft their own strategies for the requests they receive. Although the two stages of creating a strategy and choosing where to send the application requests are intertwined, here we will describe them separately for clarity.

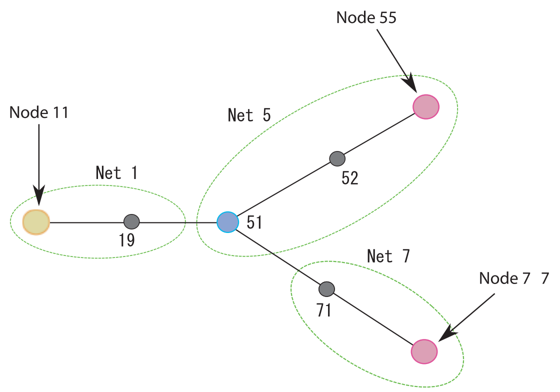

Fig. 7 Example of a small-scale internetwork composed of three networks. Our example request is initiated at Node11, and includes Node55 and Node77.

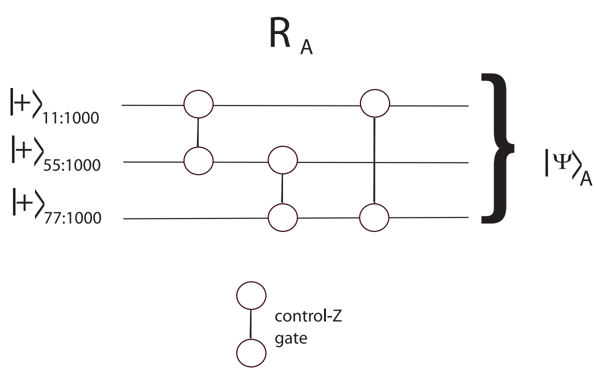

Fig. 8 Circuit for the three-qubit cluster state requested at Node11. 11 : 1000 and similar are the virtual addresses for the qubits, assigned by Node11.

The application running on Node11 creates a request of the form[1)]

$$
\begin{aligned}
R_A = & (1, |\psi_A\rangle, F \geq 0.99, S \leq 0.1, \\
& ((\text{Node11}, 1000), (\text{Node55}, 1000), \\
& (\text{Node77}, 1000)), \text{Raw}),
\end{aligned} \quad (7)
$$

where $|\psi_A\rangle$ is the cluster state created by the circuit in Fig. 8, 1000 is the virtual address chosen to be used for the qubit requested at each node, and Raw indicates that we are requesting an unencoded state.

To fulfill R_A, the first system software module to process the request (still at Node11) must create a global strategy. The principal decision is whether to create the state in one location and move the qubits via teleportation [34]–[36], or allocate the qubits in place and use

[1)] These example request values are taken from the output state of purification in a quantum network simulator [40].

teleported gates to execute the circuit in a remote fashion [33]. For our circuit, either approach results in three remote operations. The exact cost of a teleportation or a remote gate will depend on the network; we defer discussion of the detailed cost model and decision function to future work. We will consider the case where Node11 decides to ask for the state to be created in one location, then propagate the component qubits outward to the requested nodes.

With the global strategy chosen, the next step is selecting *where* each of the operations will take place. The routing table at Node11, shown in Table 1, contains information on how to get to all destinations on the network. To achieve scalability, the table has more precise information about nearby destinations, and vague information about more remote destinations, achieved using hierarchy and recursion, as in classical networks [2].

Table 1 The routing table at Node11 contains information on how to get to all destinations on the network. To achieve scalability, the table has more precise information about nearby destinations, and vague information about more remote destinations, achieved using hierarchy and recursion. Node55 resolves to Net5, and Node77 resolves to Net7, so that independent records are not needed for each node.

Destination	Route
Node19	(direct)
Net1	Local
Net5	Node19
Net7	Net5

The most important question is where to build the cluster state. Based on a cost function that uses the information in the routing table, Net5 is identified as being close to the "center" of this request. Thus, the strategy module chooses to ask Net5 to create the state, after which Net5 will teleport the qubits to Node11, Node55, and Node77.

As shown in Fig. 9, the original request (left side of the figure) is broken down into seven separate requests (right side of the figure): one for the state to be created local to Net5 (labeled R_{Net5}), three for Bell pairs to be used for teleportation (labeled $|\Psi\rangle_1$, etc.), and the teleportation operations themselves. In this case, R_{Net5} is the same circuit as in Fig. 8, with the resources specified as local to Net5 rather than distributed.

Each box in the figure lists the virtual addresses of the qubit resources to used for that request. The virtual addresses are created when the requests are created, but are not assigned to matching physical resources until the requests are processed at the receiving nodes. Each of these requests must also carry information about fidelity and entropy, with those values chosen to ensure that the delivered final state will meet the originally-requested constraints. Based on the routing table in Table 2, each of these requests is then sent via the classical network to each node involved; in this case,

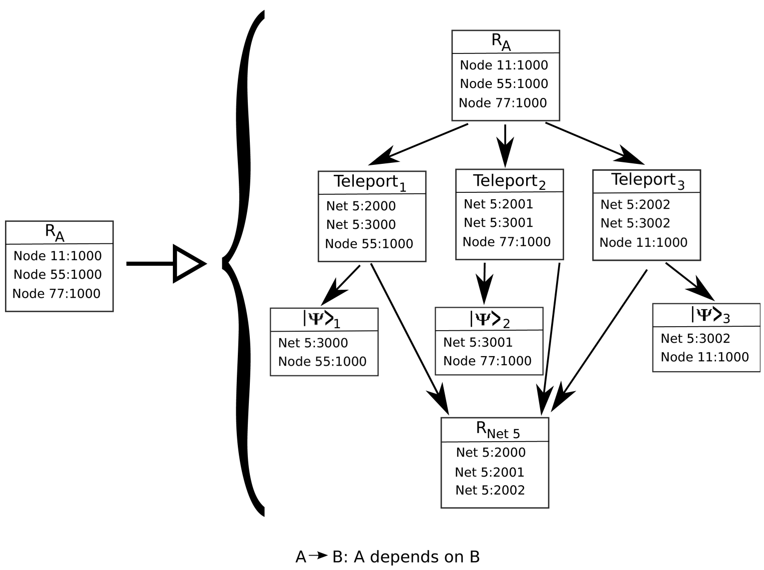

Fig. 9 The initial application request R_A is translated into a set of requests for sub-operations before leaving its origin, Node11. Each box lists not the full request tuple, but only the ID of a sub-request and the virtual addresses for the qubits assigned by the request creator.

Table 2 The routing table at Node51.

Destination	Route
Node52	(direct)
Node55	Node52
Net1	Node19
Net5	(process locally)
Net7	Node71

Node51, as the gateway to Net5, will receive most of the requests. Node51 will then forward the requests onward, or craft its own strategy, as appropriate. Requests can be executed once all dependencies (indicated with arrows in Fig. 9) are satisfied. The application's request is completed once all of the component requests finish.

Although this example shows only a single layer of recursion, the process may be repeated indefinitely for the physical nodes (as shown in Fig. 1), or for requests. To achieve adequately high fidelities, the node assigned to process each of these requests may in turn break the request down further into multiple requests for base-level entangled Bell pairs and purification operations. Likewise, for those operations spanning multiple hops, either entanglement swapping or hop-by-hop teleportation can be requested.

6 Conclusions

The fundamental difference between classical and quantum networks is the services they deliver. Classical networks move data from a source application to one or more destination applications over a distance. Quantum networks may likewise transport data from place to place, but in addition can produce distributed entangled quantum states, connecting two or more quantum applications. This difference requires a new form of interaction between network components. On the Internet, a received packet is implicitly a request: please forward this data toward the destination or destinations listed. In our Quantum Recursive Network Architecture (QRNA), rather than such an implicit request, the requester explicitly asks a node or network to participate actively in the creation of a larger state. Thus, rather than simply an information transfer system, a quantum network is a general-purpose distributed quantum computing system.

The problems of truly large-scale quantum repeater networks have much in common with the problems of classical distributed computing: naming and resource management are critical issues, and judicious use of the concepts of hierarchy and recursion provide the right abstraction to keep the systems efficient while the data structures that must be managed at each node remain tractable in size. Dynamic composition of the protocol stacks provides the needed flexibility, as well as isolation of responsibility.

All of these issues can be addressed through the use of recursive networking. QRNA abstracts subnetworks as individual nodes, allowing technology-independent requests for quantum state creation to be constructed with imperfect knowledge of the total network structure and state, and for those requests to be modified and processed in a recursive fashion as necessary to deliver the end-to-end quantum state required by applications running on quantum computers.

This paper has assumed that network nodes and repeaters are well-behaved and are not malicious, but in the real world those assumptions will not hold and the issues of robustness in the sometimes-hostile world will have to be addressed.

Long computations will naturally require not a single distributed state, but a sequence of them; reservations for such longer sequences, especially the real-time requirements, are beyond the scope of the current discussion.

Although we have focused in this paper on the creation of a core group of entangled states that are common building blocks for distributed algorithms, the mechanisms generalize quite easily to support direct distributed execution of any quantum algorithm.

We expect that adoption of QRNA will provide operational benefits and reduce redundant engineering effort as quantum repeater networks evolve and grow.

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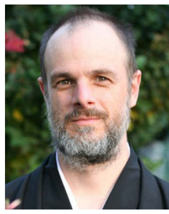

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