

Inner-iteration preconditioned block GMRES

どんな研究？

Algorithms for solving least squares problems.

Mainly least squares problems with many right hand sides. (Solving many problems at the same time.)

Precondition the problems to speed up CPU time.

状況設定

$$\min \|AX - C\|_F, \quad A \in \mathbb{R}^{m \times n}, \quad m > n, \quad C \in \mathbb{R}^{m \times p}.$$

Algorithm NR-SOR inner-iteration block BA-GMRES

- 1: Choose $X_0 \in \mathbb{R}^{n \times p}$, $R_0 = C - AX_0$,
- 2: apply l steps SOR to $A^T A w = A^T R_0$ to obtain $W_0 = P^l A^T R_0$, (NR-SOR),
- 3: $[V_1, R] = qr(W_0)$,
- 4: **for** $i = 1, 2, \dots, k$ **do**
- 5: $U_i = AV_i$,
- 6: apply l steps SOR to $A^T A W_i = A^T U_i$ to obtain $W_i = P^l A^T U_i$, (NR-SOR)
- 7: **for** $j = 1, 2, \dots, i$ **do**
- 8: $H_{i,j} = V_j^T W_i$, $W_i = W_i - V_j H_{i,j}$,
- 9: **end for**
- 10: $[V_{i+1}, H_{i+1,i}] = qr(W_i)$,
- 11: Compute $Y_i \in \mathbb{R}^{i \times p}$ which minimizes $\|R_i\|_F = \|R - H_{(i+1)p, ip} Y_i\|_F$,
- 12: $X_i = X_0 + [V_1, V_2, \dots, V_i] Y_i$, $R_i = C - AX_i$.
- 13: **if** $\|A^T R_i\|_F < \epsilon \|A^T R_0\|_F$ **then**
- 14: **stop**
- 15: **end if**
- 16: **end for**

何がわかる？

Solving numerical models to obtain the pressure distributions, parameters in biological models, price of options, the structure of the underground and etc.

TABLE : CPU time of block BA-GMRES and IP Block BA-GMRES

p	Iter.(B=A ^T)	CPU (s)	Iter.(IP)	CPU (s)
1	547	0.4075	201	0.2535
2	285	0.6229	126	0.2025
3	195	0.3908	92	0.1707
4	149	0.3268	76	0.1751
5	121	0.2722	65	0.1728
6	102	0.2344	58	0.1728
7	88	0.2307	53	0.1842

研究内容

Algorithm Block NR-SOR

- 1: Let X^0 be the initial solution and $R = C - AX^0$, $0 < \omega < 2$.
- 2: **for** $k = 1, 2, \dots, l$ **do**
- 3: **for** $i = 1, 2, \dots, n$ **do**
- 4: $\Delta_i^T = (\omega / \|a_i\|_2^2) R^T a_i$,
- 5: $X_i^{k+1T} = X_i^{kT} + \Delta_i^T$,
- 6: $R = R - a_i \Delta_i^T$ (rank-1 update).
- 7: **end for**
- 8: **end for**