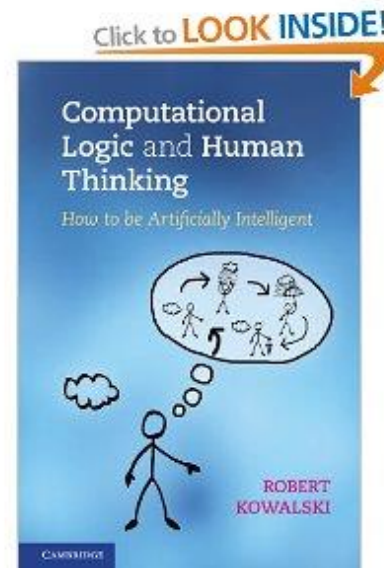


# Computational Logic and Human Thinking

Robert Kowalski  
Imperial College London



# Computational Logic (CL) and Human Thinking

- CL as the Language of Thought (LOT)
- CL as a connectionist model of the Mind
- Production systems as an alternative model of the Mind
- CL as a unifying framework

# CL as the Language of Thought (LOT)

In the philosophy of language, there are three schools of thought :

The LOT does not exist.

The LOT is a form of public, natural language.  
The natural languages that we speak  
influence the way we think.

The LOT is a private, language-like representation,  
which is independent of public, natural language.

In CL, clausal logic serves as an agent's private LOT,  
not dependent on any public natural language.

# How to investigate the LOT? Part 1 of 2

According to [relevance theory](#) [Sperber and Wilson, 1986], people understand natural language by attempting to extract the most information for the least effort.

It follows that:

If you want to find out whether there is a LOT, and what it is like, then study natural language texts that communicate useful information and are easy to understand.

Understanding the LOT can help us:

- communicate more effectively with other people
- develop better computer languages

# The Emergency Notice on the London underground

## Emergencies

Press the alarm signal button **to** alert the driver.

The driver will stop  
**if** any part of the train is in a station.

**If not**, the train will continue to the next station,  
where help can more easily be given.

There is a 50 pound penalty **for** improper use.

# The meaning (semantics) of the Emergency Notice

*the driver is alerted*

*if you press the alarm signal button.*

*the driver will stop the train in a station*

*if the driver is alerted*

*and any part of the train is in the station.*

*the driver will stop the train in the next station*

*if the driver is alerted*

*and not any part of the train is in a station.*

*help can more easily be given in an emergency*

*if the train is in a station.*

*You may be liable to a £50 penalty*

*if you use the alarm signal button improperly*

# The algorithmic behaviour intended by the writers of the Emergency Notice

To reason backwards using the beliefs:

*the driver is alerted*  
*if you press the alarm signal button.*

To reason forwards using the beliefs:

*the driver will stop the train in a station*  
*if the driver is alerted*  
*and any part of the train is in the station.*

*the driver will stop the train in the next station*  
*if the driver is alerted*  
*and not any part of the train is in a station.*

*help can more easily be given in an emergency*  
*if the train is in a station.*

*You may be liable to a £50 penalty*  
*if you use the alarm signal button improperly*



# British Nationality Act 1981

## 1981 CHAPTER 61

An Act to make fresh provision about citizenship and nationality, and to amend the Immigration Act 1971 as regards the right of abode in the United Kingdom.

[30th October 1981]

**B**E IT ENACTED by the Queen's most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows:—

### PART I

#### BRITISH CITIZENSHIP

##### *Acquisition after commencement*

**1.**—(1) A person born in the United Kingdom after commencement shall be a British citizen if at the time of the birth by birth or adoption, his father or mother is—

- (a) a British citizen ; or
- (b) settled in the United Kingdom.

(2) A new-born infant who, after commencement, is found abandoned in the United Kingdom shall, unless the contrary is shown, be deemed for the purposes of subsection (1)—

- (a) to have been born in the United Kingdom after commencement ; and
- (b) to have been born to a parent who at the time of the birth was a British citizen or settled in the United Kingdom.

1.-(1) **A person** born in the United Kingdom after commencement **shall be a British citizen** **if** at the time of the birth his father or mother is –

(a) a British citizen; **or**

(b) settled in the United Kingdom.

**The meaning of subsection 1.-(1)**

***A person shall be a British citizen by 1.-(1)***

***if*** *the person was born in the United Kingdom*

***and*** *the person was born after commencement*

***and*** *a parent of the person was a British citizen  
at the time of the person's birth* ***or***

*a parent of the person was settled in the United  
Kingdom at the time of the person's birth.*

# The syntax of logic programs

*Clauses* have the form:

*conclusion*  
*if condition*<sub>1</sub> *and condition*<sub>2</sub> .... *and condition*<sub>n</sub>

where *conclusion* is an atomic formula  
and *condition*<sub>*i*</sub> are atomic formulas or negations of atomic formulas,

If *n* = 0, then the clause is a “*fact*”

i.e. *conclusion if true*  
*conclusion*

If *conclusion* and all *condition*<sub>*i*</sub> are atomic formulas,  
then the clause is a *Horn clause*.

# What are forward and backward reasoning?

Given  $A \rightarrow C$  and the observation or assumption  $A$   
**forward reasoning** derives  $C$ .

Given  $A \wedge B \rightarrow C$  and the observations or assumptions  $A$  and  $B$   
**forward reasoning synthesizes** the new information  $C$ .

Given  $A \rightarrow C$  and the goal  $C$   
**backward reasoning** derives the subgoal  $A$ .

Given  $A \wedge B \rightarrow C$  and the goal  $C$   
**backward reasoning analyses**  $C$  into the components  $A$  and  $B$ .

**Backward reasoning** turns  $A \wedge B \rightarrow C$  into a **goal-reduction procedure**:  
Reduce the problem  $C$  to the subproblems  $A$  and  $B$ .

$A \wedge B \rightarrow C$  and  $C \leftarrow A \wedge B$  are equivalent (like  $1 < 2$  and  $2 > 1$ )

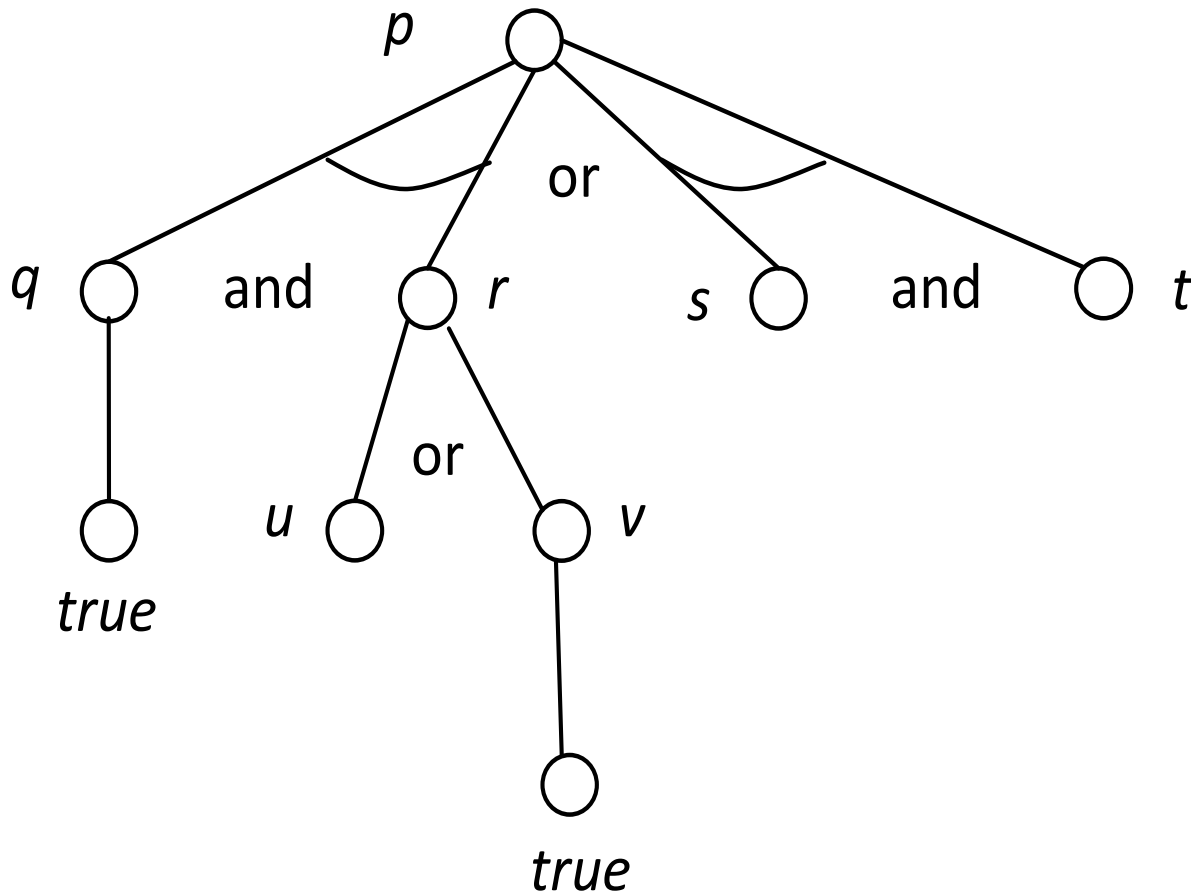
As Sherlock Holmes explained to Dr. Watson,  
*in A Study in Scarlet:*

“In solving a problem of this sort, the grand thing is to be able to **reason backward**. That is a very useful accomplishment, and a very easy one, but people do not practise it much. In the everyday affairs of life it is more useful to **reason forward**, and so the other comes to be neglected. There are fifty who can reason **synthetically** for one who can reason **analytically**.”

.....

“Most people, if you describe a train of events to them, will tell you what the result would be. They can put those events together in their minds, and argue from them that something will come to pass. There are few people, however, who, if you told them a result, would be able to evolve from their own inner consciousness what the steps were which led up to that result. This power is what I mean when I talk of **reasoning backward**, or **analytically**.”

# And-or trees can be represented by propositional Horn clause programs



$p \leftarrow q \wedge r$

$p \leftarrow s \wedge t$

$r \leftarrow u$

$r \leftarrow v$

$q$

$v$

# How to investigate the LOT? Part 2 of 2

According to **relevance theory**, people understand natural language by attempting to extract the **most information** for the **least effort**.

**It follows that:**

If you want to find out whether there is a LOT, and what it is like, then study advice about effective natural language communication.

Understanding the LOT can help us:

- to communicate more effectively with other people
- to develop better computer languages

INTERNATIONAL BESTSELLER

# THE PYRAMID PRINCIPLE

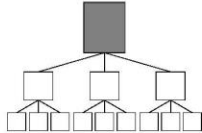
BARBARA MINTO

LOGICAL WRITING,  
THINKING AND PROBLEM SOLVING

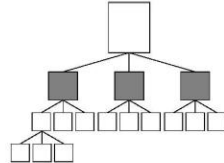
**FT** Prentice Hall  
FINANCIAL TIMES

# Ideas in writing should always form a pyramid

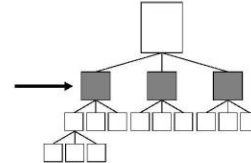
Only one answer on top level



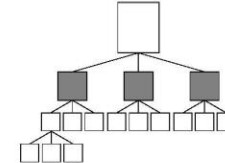
Ideas: relate horizontally (grouping or argument)



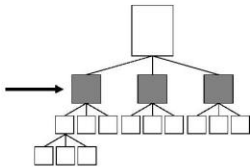
Each grouping: same kind of idea



Ideas: must be MECE

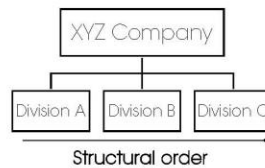


Groupings must be in logical order

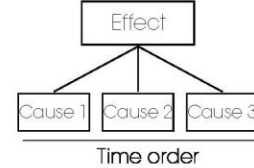


## The order dictated by the grouping

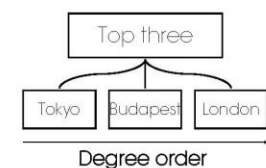
Divide a whole into its parts



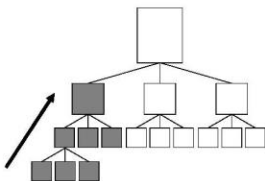
Determine the causes of an effect



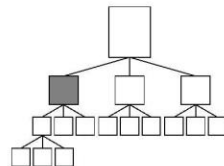
Classify like things



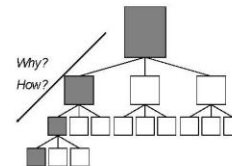
Ideas: summary of ideas grouped below



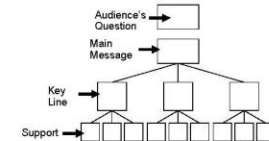
Ideas: Generate question in readers mind



Ideas: relate vertically  
A good argument forces reader into dialogue



Pyramid logic improves structure



Joseph M. Williams

# Style

Toward Clarity and Grace

*With two chapters coauthored by*  
Gregory G. Colomb

The University of Chicago Press  
Chicago and London

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# Clausal logic as a theory of the LOT can help people to communicate more effectively

By expressing communications:

**Clearly**            So that their meaning is unambiguous.

**Simply**            So that their meaning is close to their canonical form.

**Coherently**    So that it is easy to link new information to old information.

# To express yourself effectively in natural language

1. Avoid ambiguity. e.g.

John gave the book to Mary.

instead of:

He gave her the book.

clarity

2. Avoid unnecessary complexity. e.g.

Not: Our lack of knowledge of the topic of the talk prevented us from understanding it.

Better: Because we did not know the topic of the talk , we could not understand the talk.

simplicity

3. Connect related ideas together.

coherence

# Clausal logic is a simplified form of first-order logic (FOL)

In clausal logic, sentences have a simplified form, e.g.:

*has-feathers(X) ← bird(X).*  
*bird(john).*

In standard FOL, the same beliefs can be expressed in infinitely many, equivalent ways, including:

$\neg(\exists X((\neg \text{has-feathers}(X) \wedge \text{bird}(X)) \vee \neg \text{bird}(\text{john})))$   
 $\neg(\exists X((\neg \text{has-feathers}(X) \vee \neg \text{bird}(\text{john})) \wedge (\text{bird}(X) \vee \neg \text{bird}(\text{john}))))$

In clausal logic, reasoning is simpler than in standard FOL and can be reduced to forward or backward reasoning.

# Williams: Two Principles of Coherence

1. Put at the beginning of a sentence those ideas that you have already mentioned, referred to, or implied, or concepts that you can reasonably assume your reader is already familiar with, and will readily recognise.
2. Put at the end of your sentence the newest, the most surprising, the most significant information: information that you want to stress – perhaps the information that you will expand on in your next sentence.

# Coherence

Example: A.  
If A then B.  
If B then C.  
Therefore C.

Example: C?  
C if B.  
B if A.  
A.  
Therefore C.

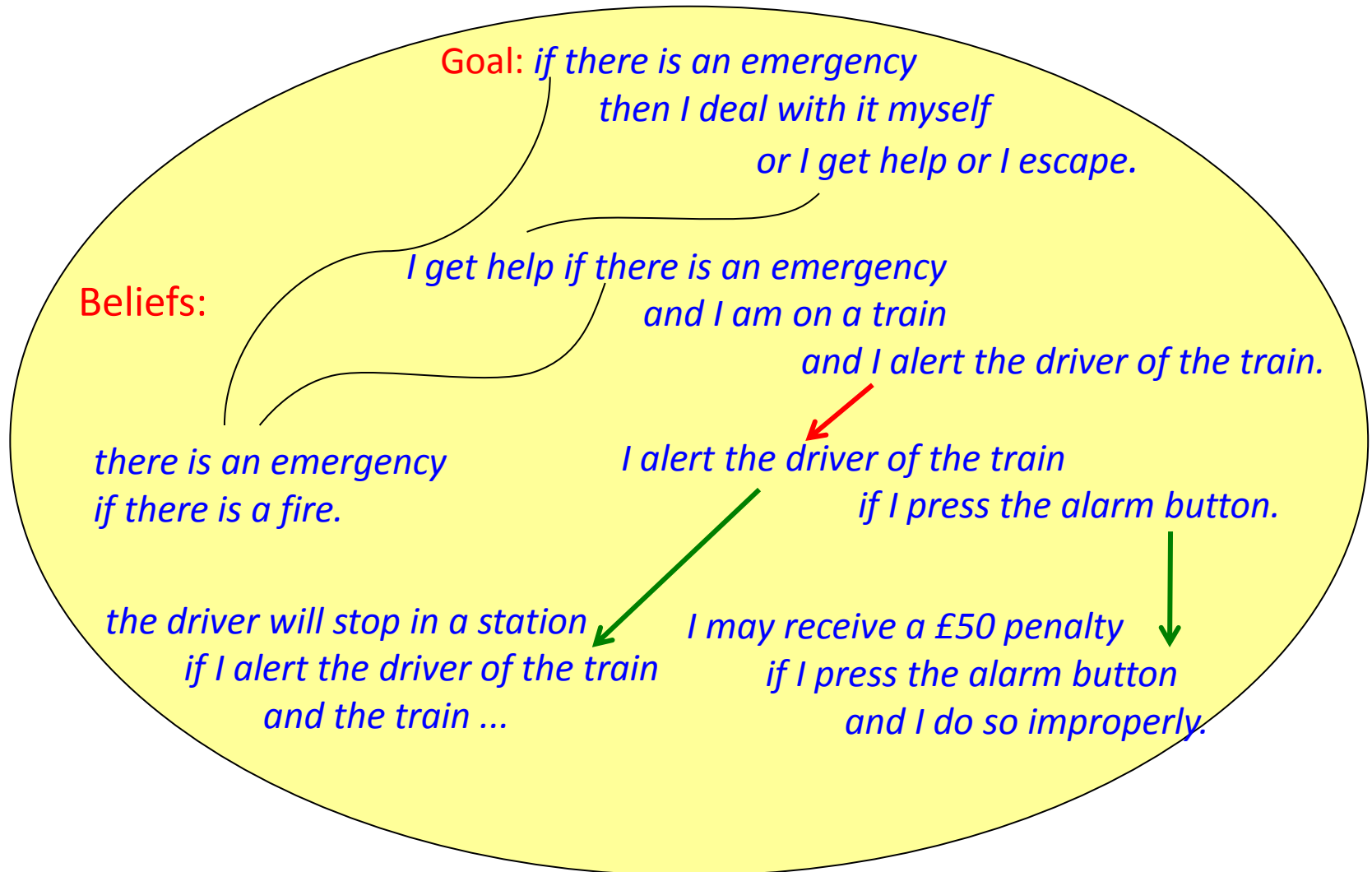
# Computational Logic and Human Thinking

- The Computational Logic (CL) as the Language of Thought (LOT)
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In Computational Logic, goals and beliefs are combined in a connectionist network



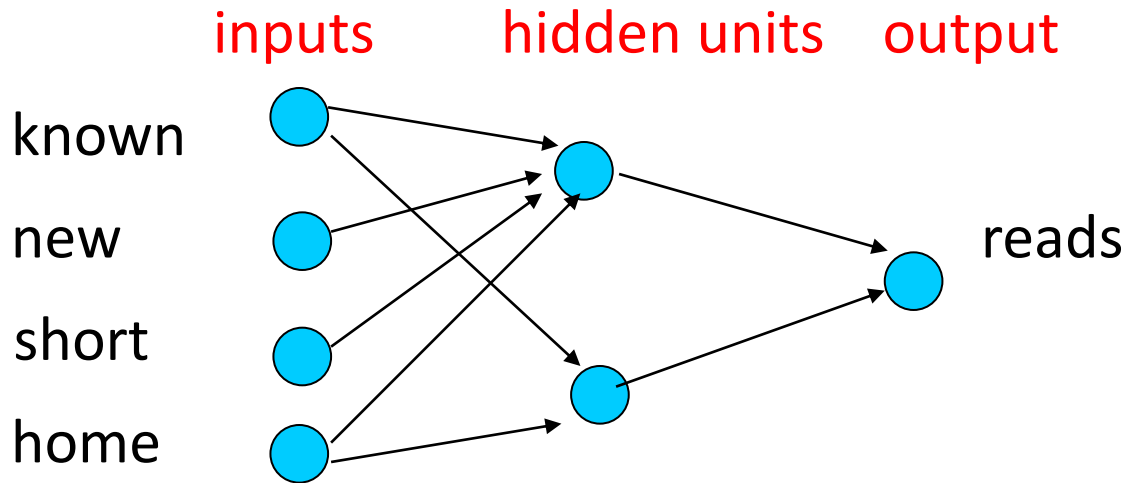
To understand a communication,  
translate it into clausal form and  
integrate it with your goals and beliefs



## The network of goals and beliefs can use information about previously useful connections

- Links can have **forward** or **backward** directions.
- Links can be **weighted** by statistics about how often they have been used successfully in the past.
- Input observations and goals can be assigned different **strengths** (or utilities).
- The strength of observations and goals can be **propagated** through the graph in proportion to the weights on the links.
- Activating links with the highest weighted strengths is like the **activation networks** of Patie Maes.

Feed-forward neural networks can be represented as logic programs  
(from Computational Intelligence, Poole, Mackworth, Goebel, 1998)



*reads with strength  $W$*

*if      arguably reads with strength  $W1$*

*and   arguably doesn't read with strength  $W2$*

*and    $W = f(2.98 + 6.88W1 - 2.1W2)$*

*arguably reads with strength W1*

*if known with strength W4*

*and new with strength W5*

*and short with strength W6*

*and home with strength W7*

*and  $W1 = f(-5.25 + 1.98W4 + 1.86W5 + 4.71W6 - .389W7)$*

*arguably doesn't read with strength W2*

*if known with strength W4*

*and new with strength W5*

*and short with strength W6*

*and home with strength W7*

*and  $W2 = f(.493 - 1.03W4 - 1.06W5 - .749W6 + .126W7)$*

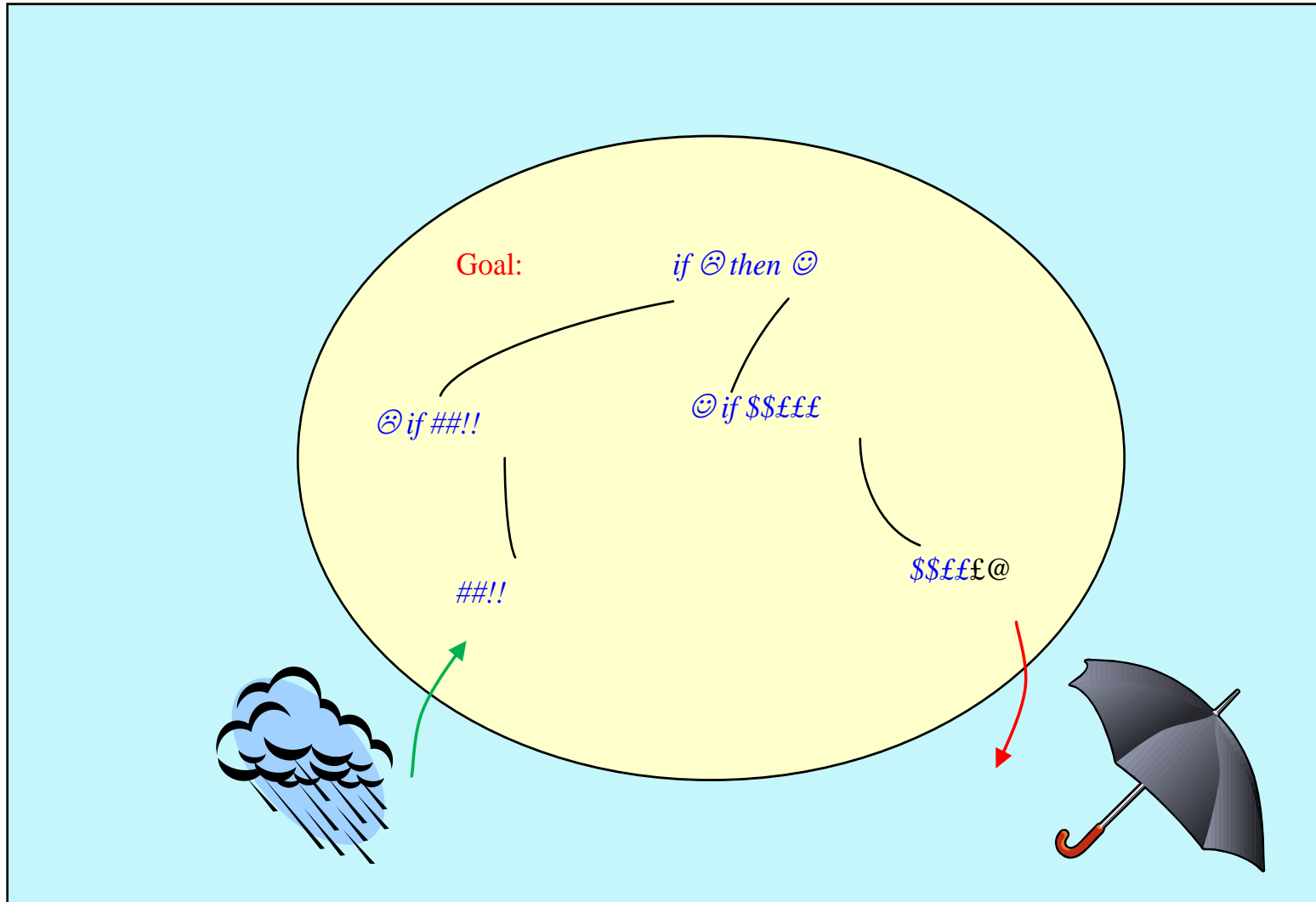
## In English

*A person will read a paper  
if there is strong reason to read the paper and  
there is no sufficiently strong reason not to read the paper.*

*There is a reason to read the paper  
if the author is known to the person, the topic is new,  
the paper is short and the person is at home.*

*There is a reason not to read the paper  
if the author is not known to the person, the topic is old,  
the paper is long and the person is not at home.*

# It can be difficult or impossible to put thoughts into words



# Computational Logic and Human Thinking

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- CL as a unifying framework

Complex thinking and decision-making can often be compiled into more efficient, lower-level maintenance goals, heuristics (or input-output associations)

For example:

*if there is a fire  
and I am on a train  
and I can not deal with the fire myself  
then I press the alarm button.*

# Heuristics are often represented as *condition-action* rules in production systems

Declarative “working memory” consisting of atomic sentences, and Procedures consisting of condition-action rules:

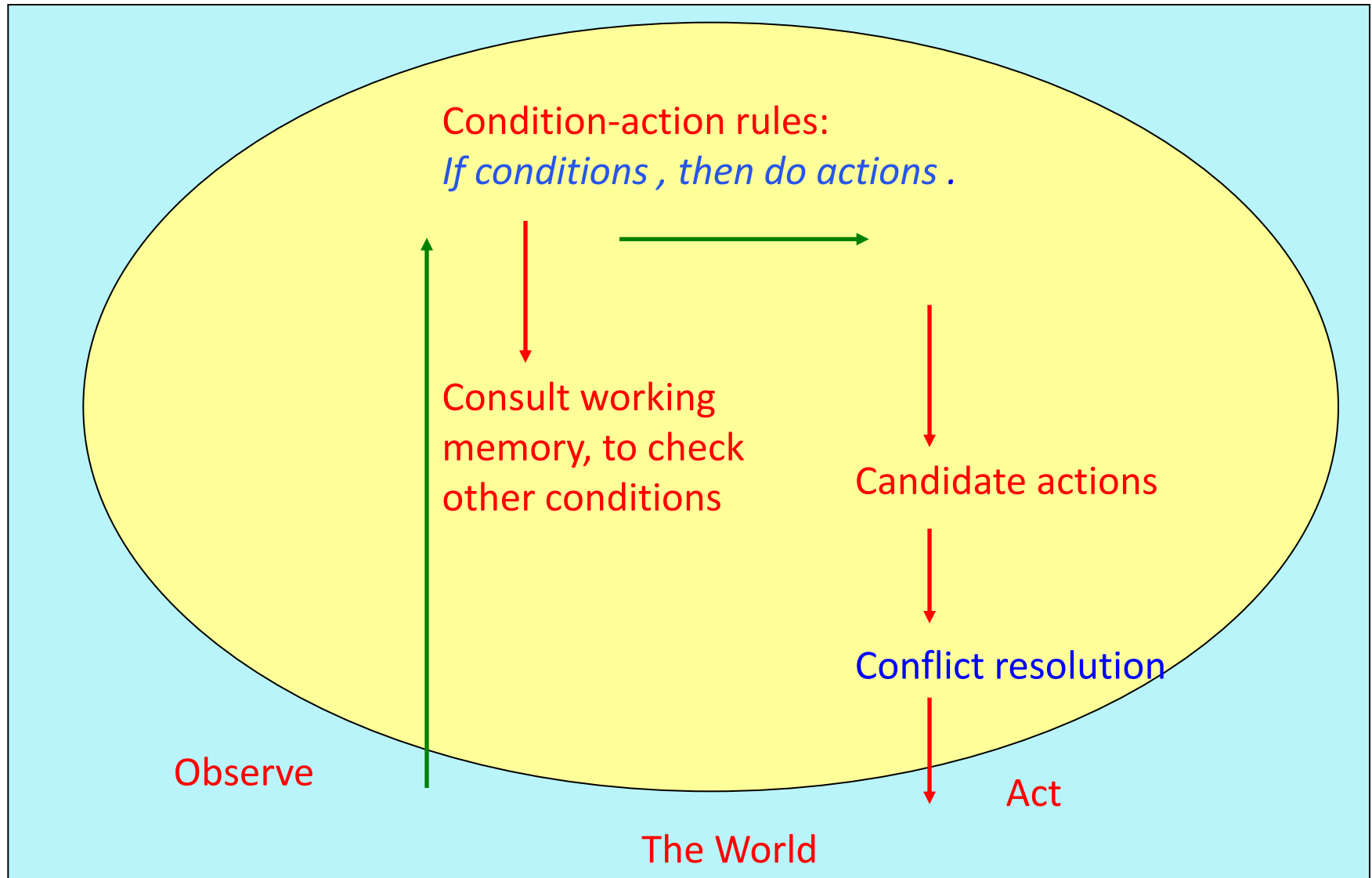
*If conditions C, then do actions A.*

Procedures look like logical conditionals, but do not have a logical semantics.

Production system cycle:

- observe a current input
- use *forward chaining* to match the input with a condition in C
- use *backward chaining* to verify the remaining conditions of C
- perform *conflict-resolution* to choose a single rule if the conditions C of more than one rule are satisfied, and
- execute the associated actions A.

# The production system cycle



# Conflict resolution

Several conflicting actions can be derived at the same time.

For example:

*If someone attacks me, then attack them back.*

*If someone attacks me, then get help.*

*If someone attacks me, then try to escape.*

The agent needs to use “conflict resolution” to *decide* what to do.

Production systems do not have a logical semantics

# Computational Logic and Human Thinking

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# A logical semantics for production rules

Maintenance goals in logical form:

*If a person attacks me,  
then I attack the person or I get help or I try to escape.*

Instead of production rules:

*If someone attacks me, then attack them back.  
then I attack the person and I get help and I try to escape.*

Given an observation or consequence of an observation:

*john attacks me*

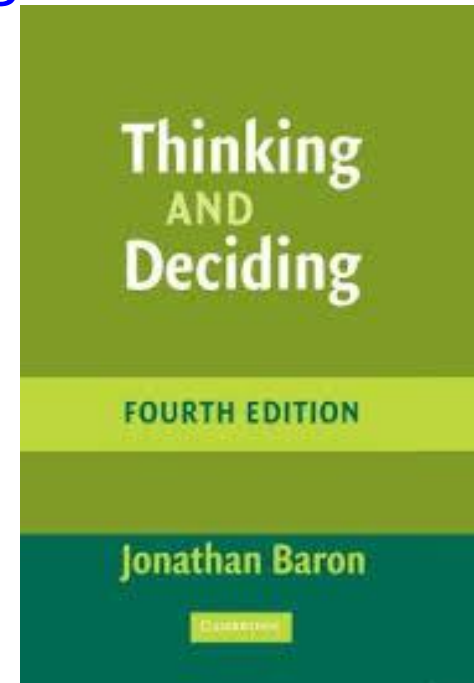
Reason forwards to derive the achievement goal:

*I attack john or I get help or I try to escape*

Decide between the different sets of actions:

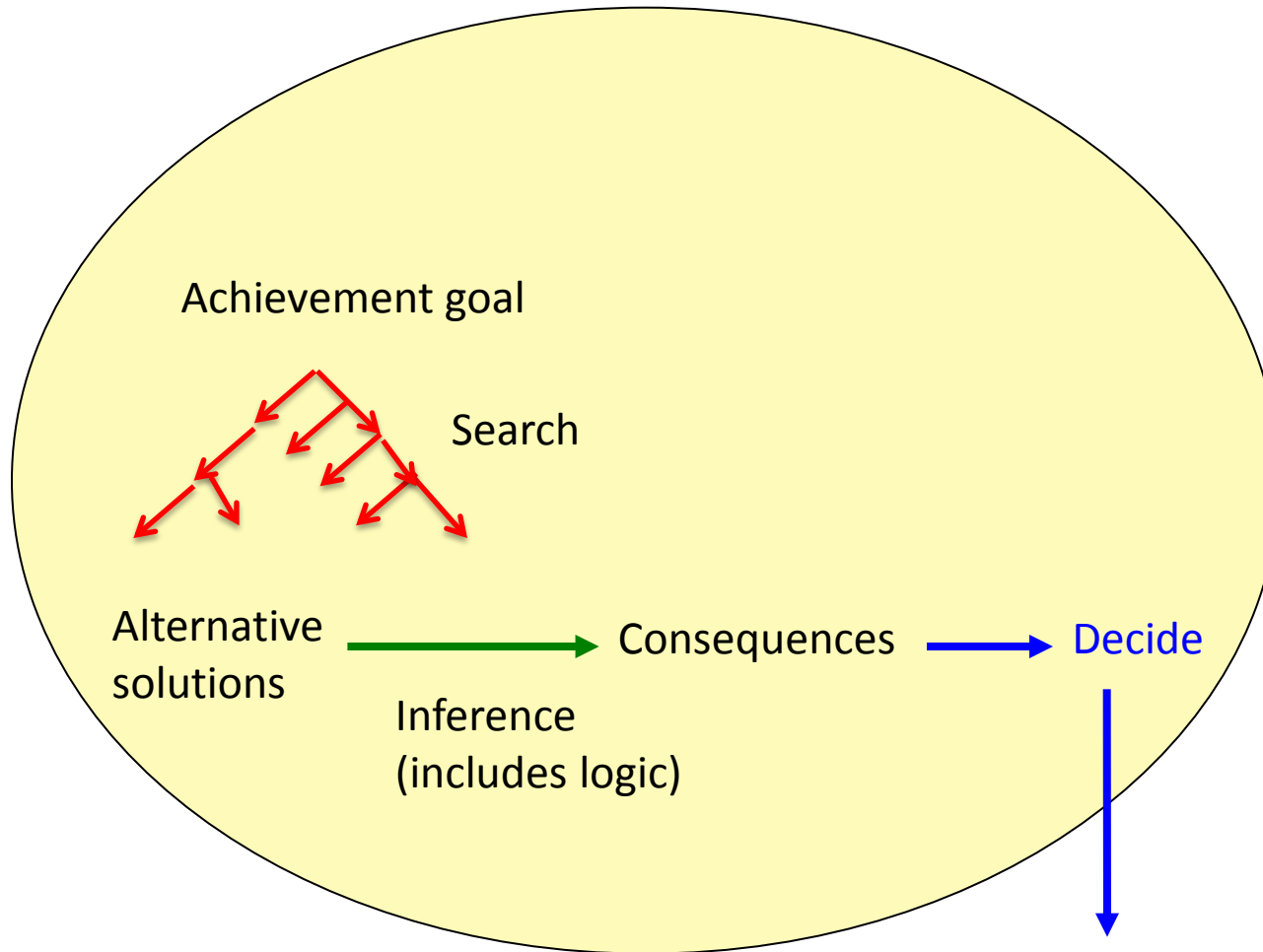
*I attack the person or I get help or I try to escape*

Jonathan Baron “Thinking and Deciding”  
(Fourth edition, 2008)



“*Thinking* about actions, beliefs and personal goals can all be described in terms of a common framework, which asserts that thinking consists of *search* and *inference*. We *search* for certain objects and then *make inferences* from and about the objects we have found.” (page 6)

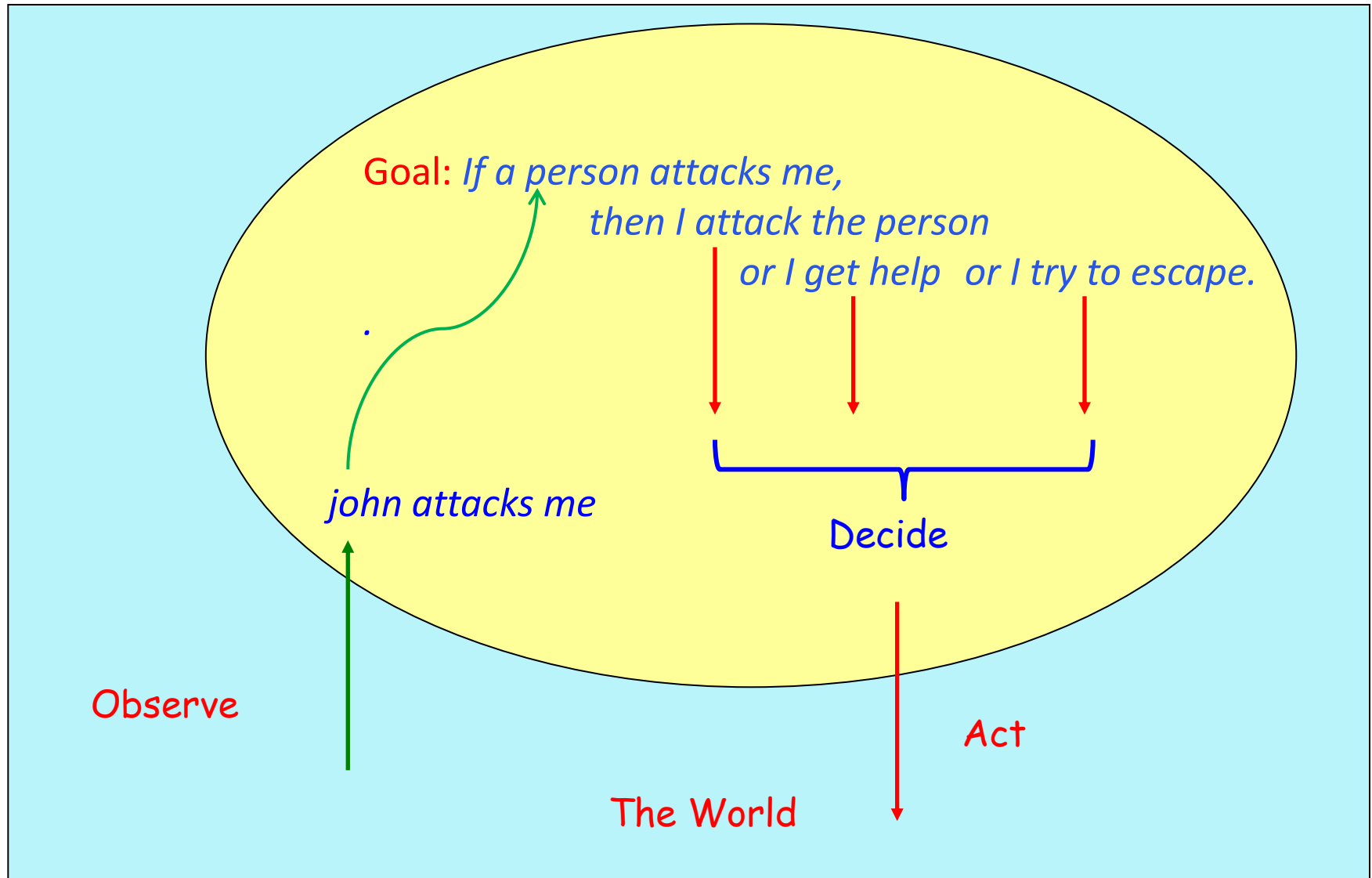
# Baron's view of search in relation to thinking and deciding



The world

A logical semantics for production rules:

Given observations, perform actions that make maintenance goals all true



# CL agents –semantics and pragmatics

**Beliefs**  $B$  describe the world as the agent sees it.

**Goals**  $G$  describe the world as the agent would like it to be.

Given **observations**  $O$ ,  
the agent's task is to find a set  $\Delta$   
of **actions** and **assumptions** such that:

} semantics

$G \cup O$  is *true* in the world determined by  $B \cup \Delta$ .

There can be many  $\Delta$  that solve the task.  
The challenge is to **choose the best**  $\Delta$   
within the resources available.

} pragmatics

# The syntax of maintenance goals = clausal form of first order logic

Goals: clauses of the form:

*If condition<sub>1</sub> and condition<sub>2</sub> .... and condition<sub>n</sub>  
then conclusion<sub>1</sub> or conclusion<sub>2</sub> .... or conclusion<sub>m</sub>*

If  $m = 0$ , then the goal is equivalent to a *denial* (or *constraint*):

*If condition<sub>1</sub> and condition<sub>2</sub> .... and condition<sub>n</sub> then false*

i.e. *it is not the case that condition<sub>1</sub> and condition<sub>2</sub> .... and condition<sub>n</sub>*

## Complex decisions can often be replaced by heuristic rules

Instead of the high-level maintenance goals:

*If a person attacks me,  
then I attack the person or I get help or I try to escape.*

and complex decision between the actions:

*I attack the person or  
I get help or  
I try to escape*

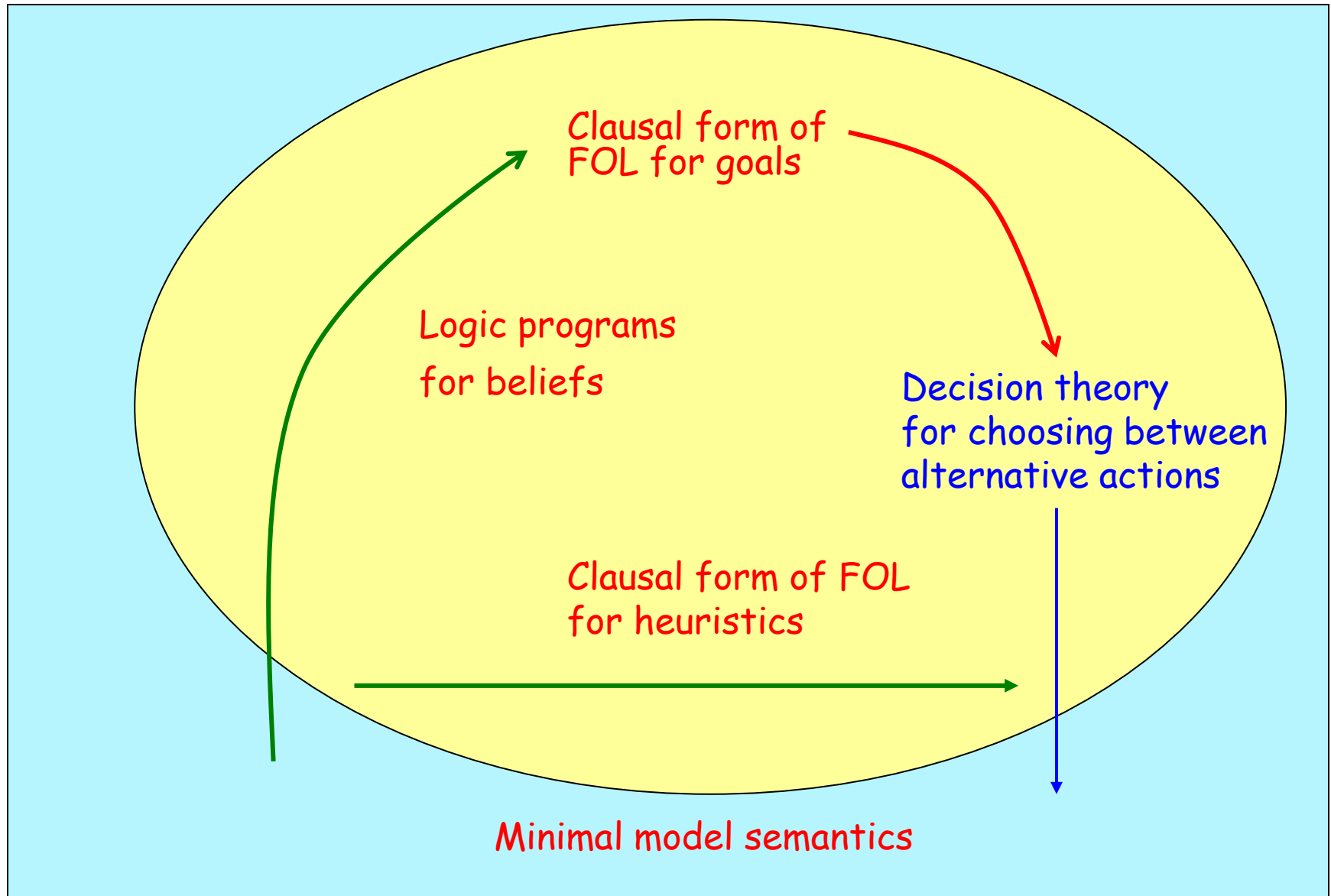
we can employ simpler, lower-level heuristic maintenance goals in logical form:

*If a person attacks me and I am stronger than the person,  
then I attack the person*

*If a person attacks me and I am weaker than the person,  
then I get help*

*If a person attacks me and I and my helpers are weaker than the person,  
then I try to escape*

# The CL Agent Model as a unifying framework



# The dual process model in Cognitive Psychology:

Lower-level heuristics and higher-level thinking and deciding are combined.

As Kahneman and Frederick (2002) put it:

the **intuitive, subconscious level** “quickly proposes intuitive answers to judgement problems as they arise”,

while the **deliberative, conscious level** “monitors the quality of these proposals, which it may endorse, correct, or override”.

# Conclusions

- The Computational Logic agent model combines
  - Logic
  - Connectionism
  - Production Systems
  - Decision Theory
- Computational Logic can be used to improve human and computer communication.