Computational approaches to analyze complex dynamic systems: model-checking and its applications.

Part 2: Model-checking of timed transitions systems

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NII - *Inoue Laboratory*
École Centrale de Nantes - IRCCyN - *MeForBio team*

Lecture Series - Lecture 2 / NII - 2013/04/03

1. Timed models
   - Timed, Hybrid and Linear Hybrid Automata
   - Time Petri nets
   - Other timed models
   - State space abstractions
2. Formalizing specification through timed modal logics
   - Reminders about linear and branching-time logics
   - Timed extensions of linear logics
   - Timed extensions of branching-time logics
3. Biological application
4. An introduction to control of timed systems
   - Control of discrete-events systems
   - Control of timed systems

Motivations

Objective: formal verification of properties

- Model the system S :
  - Discrete models: finite state automata, Petri nets, ... ⇒ Lecture 1
  - Timed models:
    - timed extensions of finite state automata: timed/hybrid automata ⇒ **Lecture 2**
    - timed extensions of Petri nets: time/stopwatch Petri nets ⇒ Lecture 3
- Formalize the specification φ :
  - Observers
  - Temporal logics: LTL, CTL, ... ⇒ Lecture 1
  - Timed extensions of temporals logics: TCTL, ... ⇒ **Lectures 2** & 3
- Does $S \models \varphi$?

Model-checking algorithms

⇒ State space exploration

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Model-checking algorithms

$\Rightarrow$ State space exploration

Some major issues

Need for modeling tasks with suspending/resuming features

Expressivity/Decidability compromise to discuss ⇒ **Lectures 2** & 3

State space combinatorial explosion

- Need for symbolic approaches ⇒ **Lectures 2** & 3
- Need for new models and abstracted algorithms ⇒ Lecture 4

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Today's and next week's issue

Tricky question

Coming from France, why do I need an average 3-4 days period not to be jet-lagged anymore in Tōkyō?

Observation

- Discrete models do not encompass sufficient information to get a thorough description of the gene regulation network behind the circadian clock **w.r.t. time**
- Some related issues:
  - Is it possible to determine the lower limit of the day/night period cycle during which the circadian clock continues to stabilize?
  - Why does the body better support backward phase delay than advance phase delay?
- → On-going modeling project with **biologists** and **computer scientists** (CNRS PEPII funded project CirClock)

Contribution

Scientific challenge

How can we get information about the **production and degradation rates** of a protein in a biological regulatory network?

Objectives of this talk and the forthcoming one

- From discrete model to timed model $\rightarrow$ emphasize on the **progressive enrichment** of model and its drawbacks
- Focus on the introduction of **quantitative** timing information
- Discuss the most appropriate **time semantics** adapted to the model
- Apply the general methodology to practical examples coming from biology

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Discrete-event systems vs timed systems

Discrete-event systems

Focus on the sequence of *observable* events (**chronology**):

t_1 t_2 t_3 t_2 t_1 t_1 t_1 ...

Timed systems

Focus on dated *observable* events (**chronometry**):

(t_1, d_1) (t_2, d_2) (t_3, d_3) (t_2, d_4) (t_1, d_5) (t_1, d_6) (t_1, d_7) ...

with:

- d_1: date at which the first t_1 occurs
- d_2: date at which the first t_2 occurs, ...

Remark: events are **asynchronous**, but dates d_i are authorized to be equal to 0

Semantics of timed systems

Discrete-time semantics vs dense-time semantics

- **Discrete-time** semantics: events occur at integer dates only
- **Dense-time** semantics: events occur at any time

$\Rightarrow$ We will discuss the precise links between dense-time, discretization and discrete-time in Lecture 3.

Timed Automata

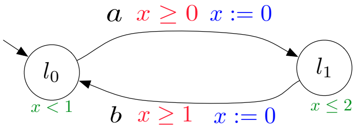

Figure: A **Timed Automaton** (from [CR08])

- State of a TA = (Location, clock valuations)
- The timed language $\mathcal{L}(\mathcal{A})$ of a TA $\mathcal{A}$ is the set of all words (traces) accepted by $\mathcal{A}$.
- The behavioral semantics of a TA $\mathcal{A}$ is a timed transition system $S_{\mathcal{A}}$

Timed Automata

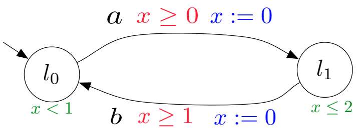

Figure: A **Timed Automaton** (from [CR08])

A path: $(\ell_0, 0) \xrightarrow{0.78} (\ell_0, 0.78) \xrightarrow{a} (\ell_1, 0) \xrightarrow{1.5} (\ell_1, 1.5) \xrightarrow{b} (\ell_0, 0) \cdots$

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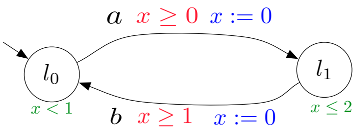

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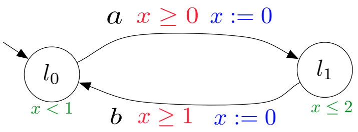

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Timed Automata [AD91]

Definition

- A finite set of **locations** l
- A finite set of **clocks** v (over $\mathbb{R}$ or $\mathbb{N}$)
- An **invariant function**, mapping each location with a predicate over v
- A finite set of **transitions**
- A *labelling* function
- An **initial location**

Timed Automata [AD91]

About transition

A transition is composed of

- a unique **source** location
- a unique **target** location
- a **guard**, *i.e.* an enabling condition ($g := x \sim c | g \wedge g$, where $\sim \in \{<, \leq, =, \geq, >\}$
- a **label** (that can be used for **synchronization**)
- a subset (potentially empty) of clocks to be **reset**

Timed Automata [AD91]

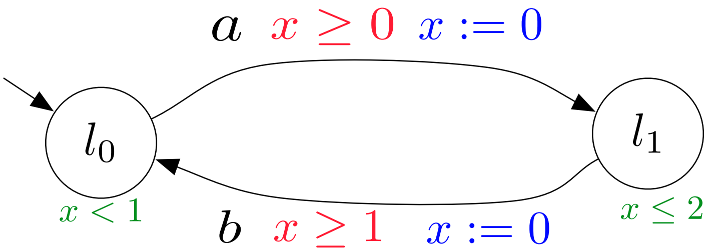

Figure: A Timed Automaton with its invariants, guards and clocks to reset

Semantics of a timed automaton

Definition as a **timed transition system**

- An **action** transition: $(l, v) \xrightarrow{a} (l', v')$ if there exists an a-labelled transition from l to l' such that:
  - v satisfies the guard of the transition
  - $v' = v[r \leftarrow 0]$, with r the set of clocks to be reset
- A **delay** transition: $(l, v) \xrightarrow{\delta(d)} (l, v + d)$, where (l, v) is a state of the timed automaton, and d belongs to the time domain in (l, v)

Hybrid automata [ACH+95]

Key idea

Every location is mapped with a set of **ordinary differential equations** defining the evolution of the variables

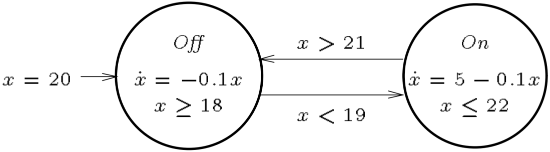

Figure: **Hybrid Automaton** describing a thermostat (from [ACH+95])

Hybrid automata

Definition

- A finite set of **locations** l
- A finite set of **variables** v over $\mathbb{R}$
- A finite set of **initial states** (couples (l, v))
- A finite set of **transitions**
- A **flow function**, mapping each location with with a predicate over v and $\dot{v}$
- An **invariant function**, mapping each location with a predicate over v
- A **jump condition function**, mapping each transition with a predicate over v
- An **initialization condition**, mapping the initial state with a predicate
- *A finite set of synchronization labels*

Linear Hybrid Automata [Hen96]

Key ideas

- The invariant, flow and jump conditions are **boolean combinations of linear equalities**.
- Every location is mapped with **a set of ordinary differential equations** $\sum \dot{x} \leq k$, with $k \in \mathbb{R}$, defining the evolution of the variables.

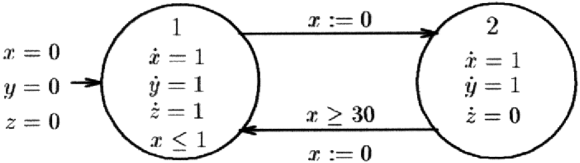

Figure: **Linear Hybrid Automaton** describing a leak in a gas-heating process (from [Hen96])

Petri net - Reminder

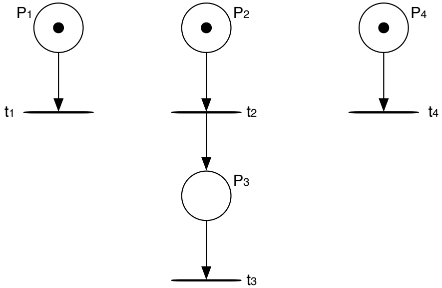

Figure: A Petri net

$$\{P_1, P_2, P_4\}$$

Petri net - Reminder

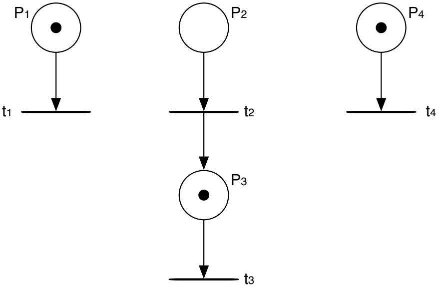

Figure: A Petri net

$$\{P_1, P_2, P_4\} \xrightarrow{t_2} \{P_1, P_3, P_4\} \xrightarrow{t_1} \ldots$$

Time Petri nets - Introduction

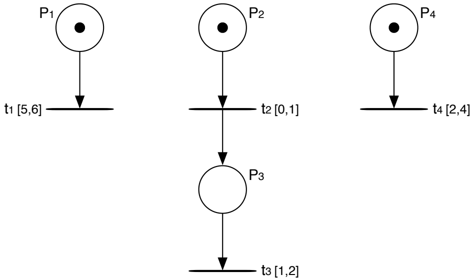

Figure: A time Petri net

$$\begin{array}{l}\{P_1, P_2, P_4\}\\ \theta(t_1) = 0\\ \theta(t_2) = 0\\ \theta(t_4) = 0\end{array} \xrightarrow{0.2} \begin{array}{l}\{P_1, P_2, P_4\}\\ \theta(t_1) = 0.2\\ \theta(t_2) = 0.2\\ \theta(t_4) = 0.2\end{array}$$

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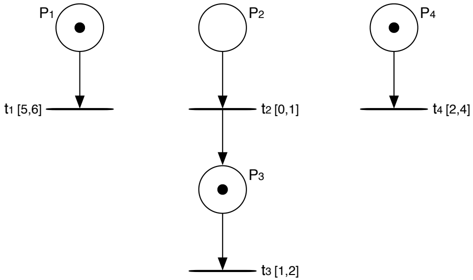

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Time Petri nets: Definition [Mer74]

A *Time Petri Net* (TPN) is a tuple $\mathcal{T} = (P, T, {}^\bullet(), ()^\bullet, M_0, a, b)$ where :

- $P = \{p_1, p_2, \ldots, p_m\}$ is a non-empty finite set of *places*;
- $T = \{t_1, t_2, \ldots, t_n\}$ is a non-empty finite set of *transitions* $(T \cap P = \emptyset)$;
- ${}^\bullet() \in (\mathbb{N}^P)^T$ is the *backward incidence function*; $()^\bullet \in (\mathbb{N}^P)^T$ is the *forward incidence function*;
- $M_0 \in \mathbb{N}^P$ is the *initial marking* of the net;
- $a \in (\mathbb{Q}^+)^T$ and $b \in (\mathbb{Q}^+ \cup \{\infty\})^T$ are functions giving for each transition respectively its *earliest* and *latest* firing times $(a \leq b)$.

(Un)decidability results

Problem [JLL77]

Reachability, liveness and boundedness problems are **undecidable** for time Petri nets.

Berthomieu et al. proved [BM83]:

Theorem

Reachability and liveness problems are decidable for **bounded** *time Petri nets.*

(Un)decidability results

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About newly enabled transitions

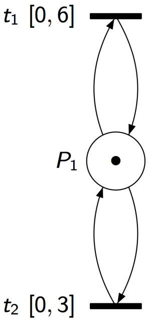


Figure: A time Petri net

We fire t_1

About newly enabled transitions

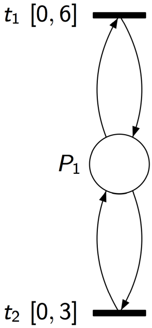


Figure: A time Petri net

We fire t_1

t_1 and t_2 are not enabled by $M - {}^{\bullet}t_1$ (M represents the marking of the net)

About newly enabled transitions

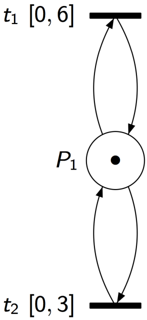


Figure: A time Petri net

We fire t_1

t_1 and t_2 are not enabled by $M - {}^\bullet t_1$

t_1 and t_2 are newly enabled

About newly enabled transitions

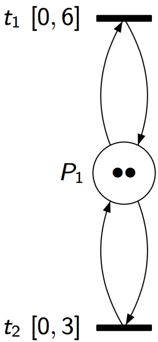


Figure: A time Petri net

We fire t_1

About newly enabled transitions

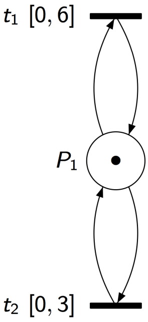


Figure: A time Petri net

We fire t_1

t_1 and t_2 are enabled by $M - {}^\bullet t_1$ but t_1 is the fired transition

About newly enabled transitions

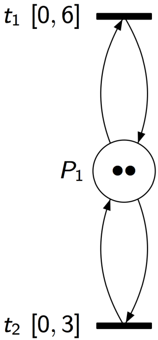


Figure: A time Petri net

We fire t_1

t_1 and t_2 are enabled by $M - {}^\bullet t_1$ but t_1 is the fired transition

t_2 remains enabled, t_1 is newly enabled

Other timed models

A large family of models

- On the thin red **line between decidability and undecidability**
- Variants of **timed automata**:
  - Stopwatch automata: clocks can be stopped in some locations
  - Updatable timed automata: not only clock resets, but also clock updates $x := c$ or $x := y + c$
  - Priced Timed Automata
- Variants of **time Petri nets**:
  - TPNs with self modification
  - Different semantics w.r.t.:
    - time elapsing: strong, weak
    - transition firing: intermediate, atomic

Need for abstractions for timed models

Problem

The state space of a timed transition system is infinite (in general)

$\Rightarrow$ Group states into equivalence classes (abstraction)

Major challenge

What is a relevant abstraction for the model, that **preserves desired properties**?

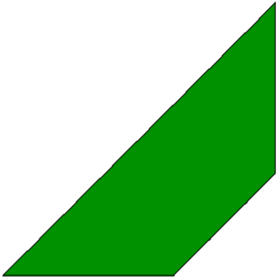
$\Rightarrow$ We will illustrate this abstraction-based approach on one example targeting TPNs.

Abstractions for TPNs

- Infinite state-space $\Rightarrow$ Abstractions
- TPNs: Zone-based simulation graph [GRR06]
- TPNs: State class graph [BD91]

State Class

$$C = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \right.$$



$$\left. \right\}$$

TPNs: Zone (encoded by a Difference Bound Matrix (DBM) $[d_{ij}]_{i,j \in [0..n]}$):

$$\begin{cases} -d_{0i} \leq \theta_i - 0 \leq d_{i0}, \\ \theta_i - \theta_j \leq d_{ij} \end{cases}$$

Basic Algorithm for state space computation

begin
  $Passed = \emptyset$
  $Waiting = \{C_0\}$
  while $Waiting \neq \emptyset$
    $C = \text{pop}(Waiting)$
    $Passed = Passed \cup C$
    for t firable from C
      $C' = \text{AbstractSuccessor}(C, t)$
      if $C' \notin Passed$
        $Waiting = Waiting \cup C'$
      end if
    end for
  end while
end

Computing the state class graph

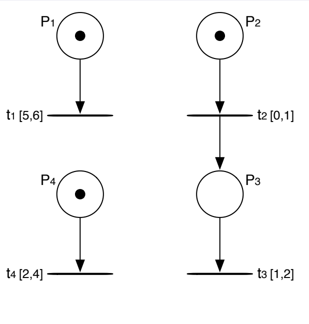
Let $C = (M, D)$ and $D = (A.\Theta \leq B)$. We fire t_f.

- $M' = M - {}^\bullet t_f + t_f{}^\bullet$
- D' is computed by:
  - for all enabled transitions t_i, constrain by $\theta_f \leq \theta_i$
  - for all enabled transitions t_i, $\theta_i' = \theta_i - \theta_f$
  - eliminate variables for disabled transitions (e.g. using Fourier-Motzkin method)
  - add new variables for newly enabled transitions t_i:

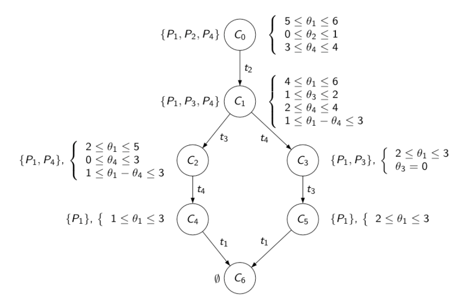
$$\alpha(t_i) \leq \theta_i \leq \beta(t_i)$$

State class graph computation: an example

TPN



State class graph



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Computation paths vs computation tree - Reminder

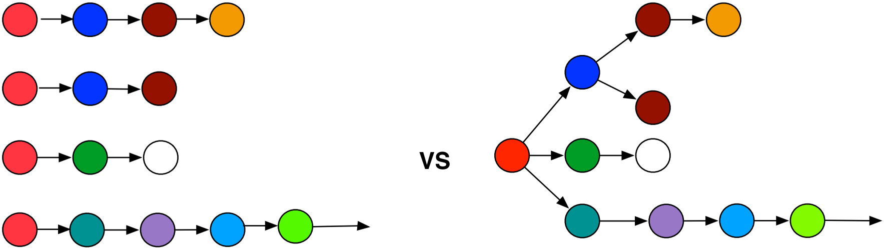

Figure: Execution can be seen as a set of execution paths or as an execution tree

Model-checking formal properties - Reminder

Qualitative properties

- LTL (linear-time properties): *on a given* **path**, $X\varphi$, $\varphi U\psi$ + $G\varphi$, $F\varphi$
- CTL (branching-time properties): *in a given* **state**,
  - $EX\varphi$, $E\varphi U\psi$ + $EG\varphi$, $EF\varphi$
  - $AX\varphi$; $A\varphi U\psi$ + $AG\varphi$, $AF\varphi$
- CTL* (superset including, but not equal, to the union of LTL and CTL)

Model-checking of LTL properties - Reminder

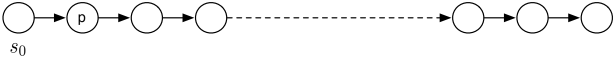

Figure: $s_0 \models Xp$

Model-checking of LTL properties - Reminder

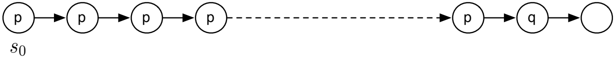

Figure: $s_0 \models pUq$

Model-checking of LTL properties - Reminder

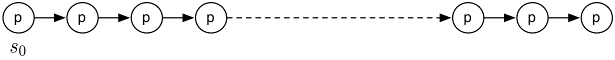

Figure: $s_0 \models Gp$

Model-checking of LTL properties - Reminder

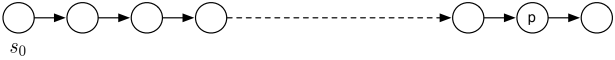

Figure: $s_0 \models Fp$

Model-checking of CTL properties - Reminder

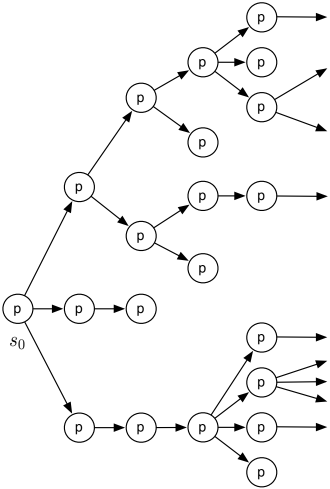

Figure: $s_0 \models AGp$

Model-checking of CTL properties - Reminder

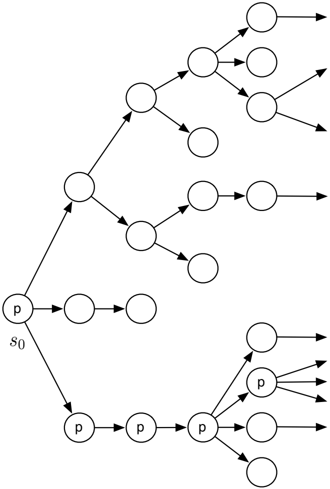

Figure: $s_0 \models EG\mathrm{p}$

Model-checking of CTL properties - Reminder

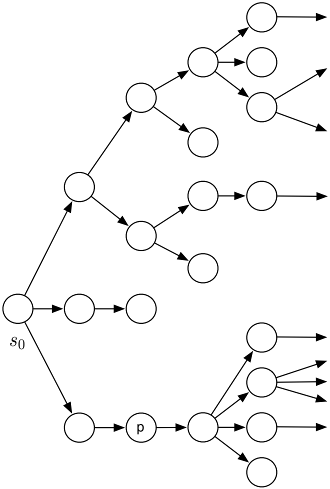

Figure: $s_0 \models EF\,p$

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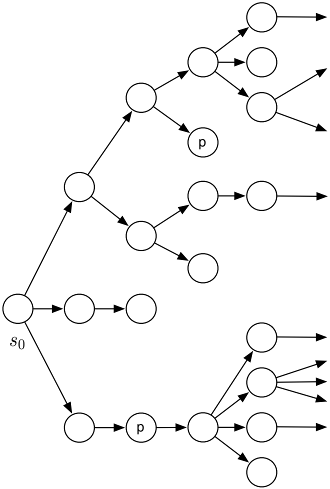

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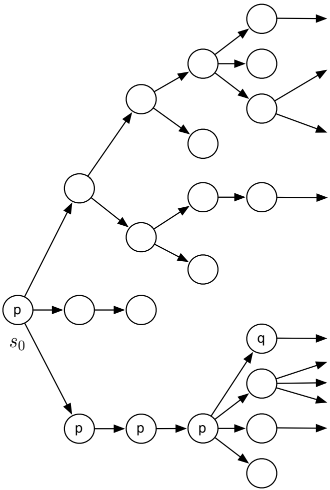


Figure: $s_0 \models$ pEUq

Model-checking of CTL properties - Reminder

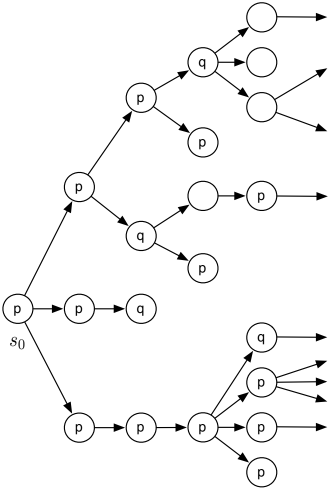


Figure: $s_0 \models pAUq$

Need for timed extensions of modal logics

Quantitative timing properties

How can we formalize a sentence like: "any problem is followed by an alarm **in at most 5 time units**"?

Enrich temporal logics

- "Any problem is followed by an alarm": $AG(\text{problem} \rightarrow AF\text{alarm})$
- Extend temporal logics:
  - Add subscripts to temporal operators, e.g. $AG(\text{problem} \rightarrow AF_{\leq 5}\text{alarm})$
  - Use real clocks to assert timed constraints, e.g. $AG(\text{problem} \rightarrow x \in (x \leq 5 \wedge AF\text{alarm}))$

$\Rightarrow$ **Timed** temporal logics

Timed temporal logics: From a **path** point of view

Extensions of Linear Temporal Logics

- **Metric Temporal Logic** (MTL) [Koy90]
  - Add **subscripts** to temporal operators
  - Example: $G(\text{problem} \rightarrow F_{\leq 5}\text{alarm})$
- **Timed Propositional Temporal Logic** (TPTL) [AH94]
  - Add **real clocks** to formulae
  - Example: $G(\text{problem} \rightarrow x.F \in (x \leq 5 \wedge \text{alarm}))$, where $x.\varphi$ means that clock x is reset at the current position (*i.e.* before evaluating φ).

Remark: next (X) operator from LTL is **removed** (no meaning in dense-time semantics)

Model-checking of MTL properties: An example

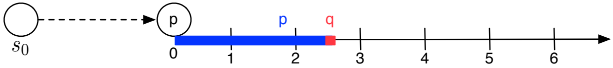


Figure: $s_0 \models p U_{[2;4]} q$

Timed temporal logics: From a **branching-time** point of view [ACD93]

Extensions of CTL*

- TCTL with **subscripts**, e.g. $AG(\text{problem} \rightarrow AF_{\leq 5}\text{alarm})$
- TCTL with **explicit clocks** added to formulae, e.g. $AG(\text{problem} \rightarrow x \in (x \leq 5 \wedge AF\text{alarm}))$

Remark: next (X) operator from CTL* is **removed** (no meaning in dense-time semantics)

Model-checking of TCTL properties: An example

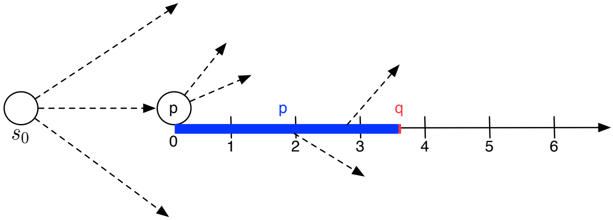


Figure: $s_0 \models E(\mathrm{p} U_{[2;4]} \mathrm{q})$

Timed temporal logics: Expressiveness results [BCM05]

Subscripts vs explicit clocks

- TPTL has been proven to be **strictly more expressive** than MTL (e.g. $x.F(a \wedge x \leq 1 \wedge G(x \leq 1 \Rightarrow \neg b)))$
- TCTL with explicit clocks has been proven to be **strictly more expressive** than TCTL with subscripts.

Timed temporal logics

Quantitative timing properties

A TCTL formula:

$$\varphi := ap \mid \neg ap \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid A\varphi U_I \varphi \mid E\varphi U_I \varphi$$

with:

- *ap* an atomic assertion
- I an interval from $\mathbb{R}^+$ with **integer bounds** s.t. $[n, m]$, $[n, m[$, $]n, m]$, $]n, m[$, or $[m, \infty[$, $n, m \in \mathbb{N}$

Some additional TCTL examples

Bounded liveness/response [DT98]

- "Whenever a property p becomes true, q must be true within n seconds" ($n \in \mathbb{N}$)
- $AG(p \Rightarrow AF_{[0,n]} q)$
- Denoted $p \rightarrow_{[0,n]} q$ in most model-checkers

Decidability results w.r.t. model-checking [Alu99]

Following problems are **undecidable**

- Model-checking of timed automata for MTL properties
- Model-checking of TPNs for TCTL properties
- Satisfaction problem for TCTL (TA/TPN)

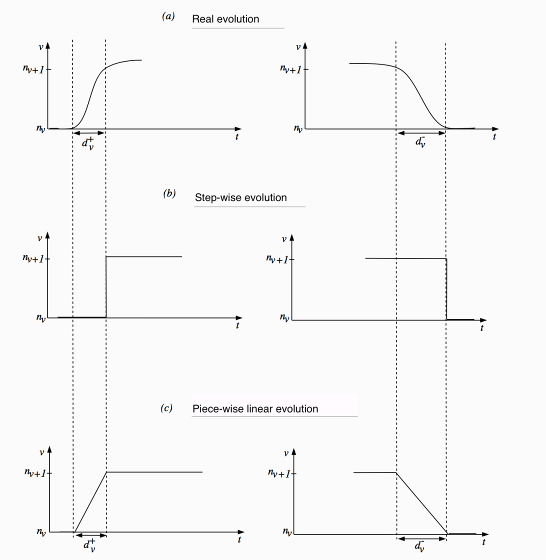
Following problems are **decidable**

- Model-checking of timed automata for TCTL properties
- Model-checking of **bounded** TPNs for a subset (no nesting) of TCTL with subscripts

Overview

1. Timed models
   - Timed, Hybrid and Linear Hybrid Automata
   - Time Petri nets
   - Other timed models
   - State space abstractions
2. Formalizing specification through timed modal logics
   - Reminders about linear and branching-time logics
   - Timed extensions of linear logics
   - Timed extensions of branching-time logics
3. Biological application
4. An introduction to control of timed systems
   - Control of discrete-events systems
   - Control of timed systems

Biological application (from [AR10])



Biological application (from [AR10])

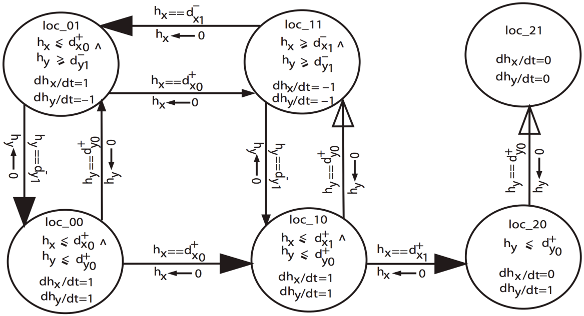


Figure: Linear Hybrid Automaton modeling Pseudomonas Aeruginas

Biological application (from [AR10])

Aim

Identify **cycles** and **attractors**

Methodogy

- Use a **model-checker** on hybrid automata (e.g. HyTech, PHAVer, ...)
- Interpret results thanks to a **parameterized polyhedra library** (e.g. PolyLib)

Overview

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The **control** problem

Real-life system

- Uncontrollable events
- Controllable events
- To be discussed: **Observability** $\Rightarrow$ **full** observability vs **partial** observability

The control problem

Control problem

Does there exist a controller C that guarantees the given properties φ such that $S \parallel C \models \varphi$?

Controller synthesis problem

Can we build a controller C that guarantees the given properties $\varphi \Rightarrow \exists C,\ S \parallel C \models \varphi$?

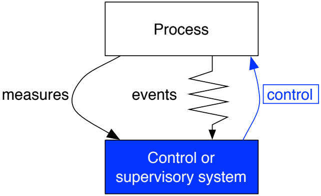


Figure: The control problem

A first approach to **control** problem

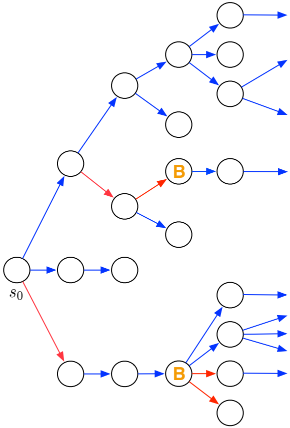


Figure: Branching execution of a model: blue actions stand for controllable actions; red actions stand for uncontrollable ones; B stands for bad states that should be avoided

A first approach to **control** problem

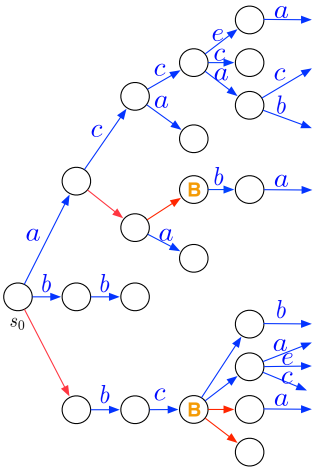
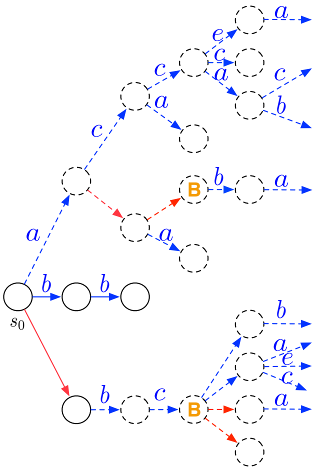


Figure: Blue actions = controllable ones; red actions = uncontrollable ones; B = bad states

A first approach to **control** problem



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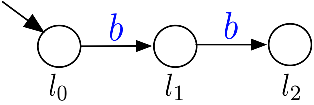
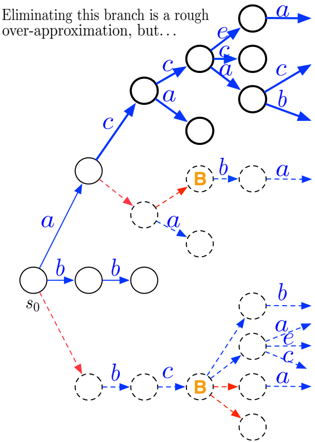


Figure: Supervisor automaton to avoid that the system reach bad states

A first approach to **control** problem



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A first approach to **control** problem

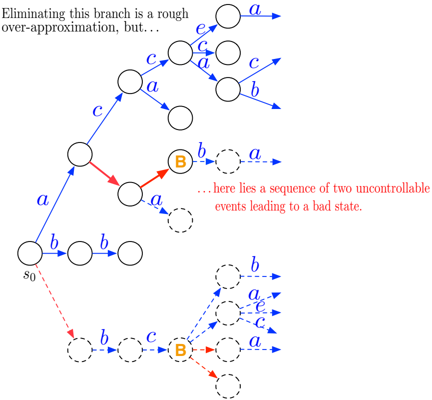


Figure: Blue actions = controllable ones; red actions = uncontrollable ones; B = bad states

Supervisory control theory

Ramadge-Wonham framework [RW89]

- Discrete-events system, modeled as a **finite** automaton with:
  - Uncontrollable events
  - Controllable events
- **Specification**
  - E.g.: Avoid any sequences leading to a state where the property bad is satisfied
  - $\Rightarrow$ specifications as a **language**
- Principle: **Supervisor**, described as a synchronous automaton, observes the events generated by the system and might prevent it from generating a subset of the controllable events

Solving a **control** problem

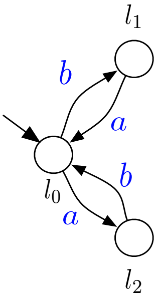


Figure: System S (both a and b are controllable). We would like that only one execution $a.b$ can occur (specification φ). Does there exist a controller C such that $S \parallel C \models \varphi$?

Solving a **control** problem

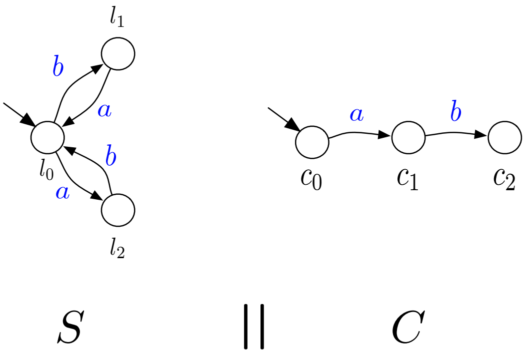


Figure: System S with its supervisor C so that only one execution $a.b$ can occur.

Solving a **control** problem: key idea

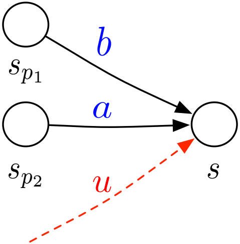


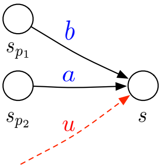
Figure: Basic idea behind the notion of controllable predecessors: I_{p_1} and I_{p_2} might be in the set of controllable predecessors of I

Solving a **control** problem: key idea

Controllable predecessors technique

Let:

- S be the "safe" states, *i.e.* the ones meeting the specification φ
- $\pi(X)$ is the set of **controllable predecessors** of a given state X [MPS95]: $\pi(X)$ is computed as the **greatest fix-point** of $\pi(X) = \pi(X) \cap S$



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Control

If the initial state of the automaton belongs to $\pi(S)$, then there exists a supervisor satisfying the specification φ.

Solving a **control** problem: controllable predecessors

Theorems

- For finite automata, the semi-algorithm that computes the set of controllable predecessors **terminates** (because of the finite number of discrete states)
- For Petri nets, the semi-algorithm that computes the set of controllable predecessors **may not terminate**.

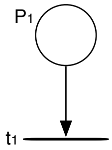

Figure: Example of Petri net for which the computation of the set of **controllable predecessors** will not terminate.

Control as a game (from [CM07])

Definition of the problem

- Open-system = game with two players:
  - Environment plays uncontrollable events
  - Controller plays controllable events
- **Control** objective = **Winning** condition (e.g. avoid bad states)
- Control problem: find a **strategy** (a controller) to win the game

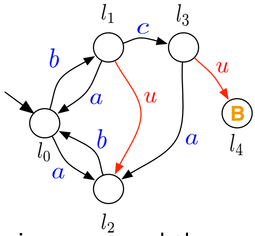

Figure: Game between the environment and the controller: bad state must be avoided (blue actions are controllable; red ones are uncontrollable)

Control as a game (from [CM07])

Related concepts

- **Strategy**: gives, for each finite run, the controllable action to perform
- **Winning strategy**: strategy which generates only runs that leads to a set of states S meeting the specification φ
- **Winning states**: set of states s in which there exists a winning strategy from s (*i.e.* $\pi(S)$)

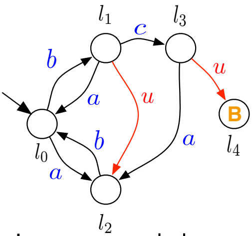

Figure: Game between the environment and the controller: bad state must be avoided (blue actions are controllable; red ones are uncontrollable)

Key issues w.r.t. control as a game problem

Criteria needed to the correct definition of the problem

- **Observability**, again: **full** observability vs **partial** observability
- Type of games:
  - Concurrent games: each opponent can play at any turn
  - Turn-based games: each opponent plays alternatively

Introduction to timed control

Control for timed systems

- Natural extension of the control of discrete-events systems
- A *run* = a succession of **discrete** and **time elapsing** steps
- Extension of the controllable predecessors algorithm

Application to the control problem for timed automata

Control is viewed as a **Timed Game Automaton** [AMPS98]

Control of timed automata

Principle

- **Full observability**: the controller observes both discrete and time-elapsing steps
- **Two options** for the controller:
  - Delay action
  - Perform a controllable action (among the possible ones)
- Define a **strategy**
  - "Wait as long as the system permits"
  - Build the most permissive controller (i.e. the one that restricts the behavior of the environment as little as possible)
  - Towards **optimal** control
- Extension of the controllable predecessors algorithm

Remark: the controller can prevent time to elapse by taking only controllable moves ⇒ **zeno-controllers** (which are usually excluded)

Control of timed automata

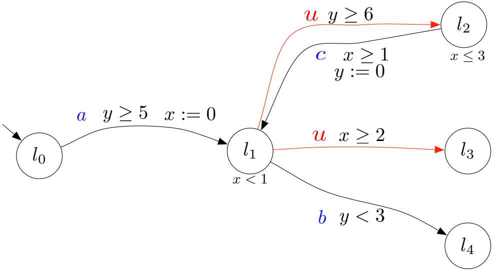

Figure: Timed automaton with controllable and uncontrollable actions

Extension of the controllable predecessors algorithm

Key ideas

- A state s_p is a time controllable predecessor of state s iff, on the time elapsing path between s_p and s, there is no uncontrollable discrete step leading to a bad state s_b
- A **symbolic version** of $\pi(X)$, the set of **controllable predecessors** of a given state X, can be defined [AMPS98]

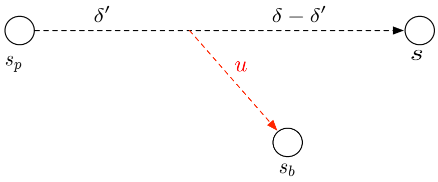

Figure: Time controllable predecessor(s)

Verification vs Optimization

Verification

- **Checks** logical properties
- Implementation: consider the whole state-space of the model

Optimization

- Find **optimal** solutions w.r.t. a set of criteria
- Implementation: cut techniques to avoid non-optimal parts of the state space

Introduction to optimal control

Given a logical property, does there exist an **optimal controller** that guarantees the property, *i.e.* a controller that guarantees the property and optimizes a set of criteria?

Verification vs Optimization

Verification

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Introduction to Optimal Timed Games [BCFL04]

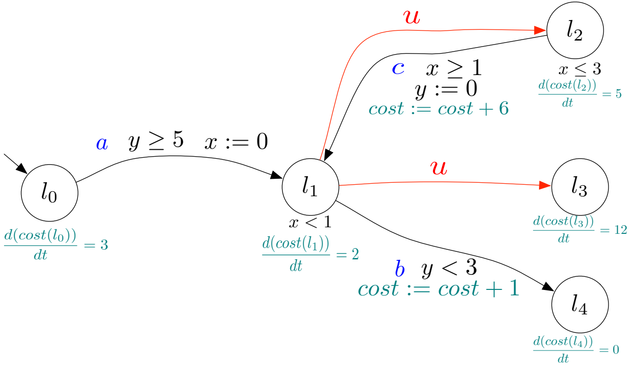

Figure: Game between the environment and the controller: blue actions are controllable; red ones are uncontrollable

Introduction to Optimal Timed Games [BCFL04]

Principle of a reachability timed game

- Does a **best cost** *whatever the environment does* exist? If yes, what is its value?
- Is there a **strategy** to achieve this optimal cost?
- Is this strategy **computable**?

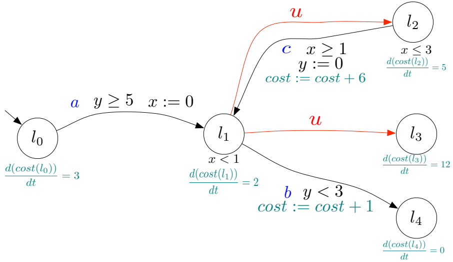

Figure: **Priced timed game automaton** between the environment and the controller: blue actions are controllable; red ones are uncontrollable

Optimal Timed Games [BCFL04]

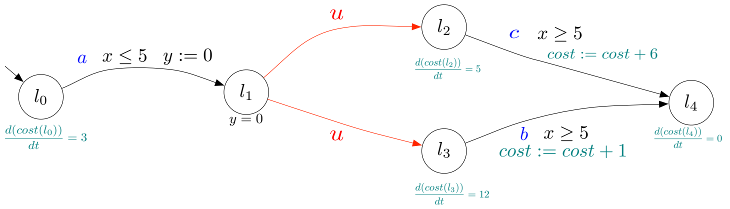

Figure: **Priced timed game automaton** between the environment and the controller: blue actions are controllable; red ones are uncontrollable

Basic illustration of a reachability timed game

- **Best cost** *to reach l_4 whatever the environment does*: $\inf_{0 \leq t \leq 5} max(3t + 5(5-t) + 6; 3t + 12(5-t) + 1) = \frac{11}{9}$, where t represents the time to remain in l_0
- **Strategy** to achieve this optimal cost: wait in l_0 till $t = \frac{11}{9}$, then fire a

Optimal Timed Games [BCFL04]

Problem

- **Priced Timed Game Automaton** (PTGA) = Timed Automaton + **cost function** which associates to each location a cost rate and to each discrete transition a cost
- Usual assumptions on PTGA:
  - **Deterministic w.r.t. controllable actions**
  - **Time-deterministic**: let s, s_1 and s_2 be three states of a timed transition system and $d \in \mathbb{R}$. If $s \xrightarrow{d} s_1$ and $s \xrightarrow{d} s_2$, then $s_1 = s_2$
- Link between **optimal control** for a PTGA and **reachability control** for a Linear Hybrid Game Automaton

Application to scheduling [BLR04]

- Aircraft landing
- Job shop scheduling

Adding timed informations to models

Key factors

- Expressivity: **clocks** vs **stopwatches** vs **variables with more complex dynamics**
- **Asynchronous** events vs **synchronous** events
- **Zenoness**

Timed and hybrid models

Summary

- A wide range of models
- Gaining **expressively** often leads to undecidability
- But **undecidability** is not always incompatible with practical problems

Further work

- Discuss the quantitative **time semantics**
- Discuss the respective **expressivity** of models (timed extensions of automata vs timed extensions of Petri nets)
- Application to practical biological problems

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