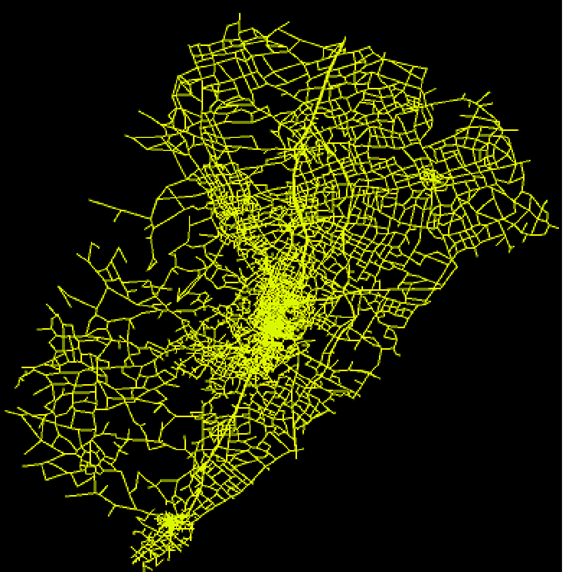
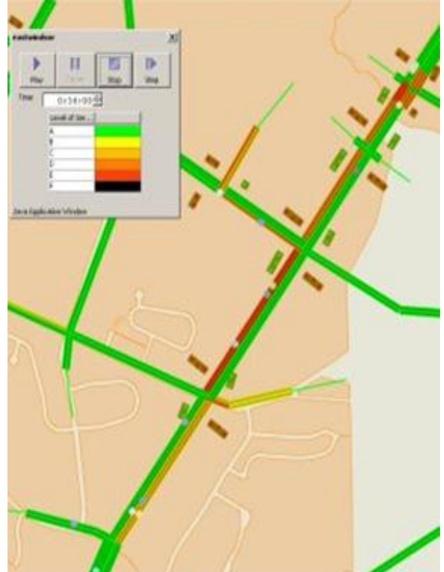


Innovative Methodologies for Large-scale Stochastic Dynamic Transportation Network Systems

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NICTA

UNSW THE UNIVERSITY OF NEW SOUTH WALES | Transportation

RCITI Research Centre for Integrated Transport Innovation

Presentation Goals

- Core concepts of transport network modelling
  - Applicability and typical level of rigor
- Very brief evolution and history of transport network models
- Advanced current model examples
  - **Dynamics**
    - Integrated demand/network model
    - Deployed large-scale network tool
  - **Information**
    - Novel emerging network models

Transport Planning/Modelling

- In essence, mathematically model individual travel **choice** and resulting system **impacts**
  - Trip/activity Destination Departure-time
  - Mode choice Toll Usage Route
  - Lane Acceleration

  - Congestion Emissions Safety
  - Energy Use Reliability Accessibility

  - **And the list continues to grow**

Modeling is universal …… “common sense” **IS** a model

Evolution of Network Models

- Questions regarding transport systems continually grow more complex.
- As a result, the modeling tools match (and often exceed) this complexity.
- Domain-specific network issues
  - Physics of traffic/transit
  - Individual operational behaviour (e.g., reaction time, distraction ,stress)
  - Individual strategic behaviour (e.g., route/mode/toll/trip choice)

Recent advances include **dynamics** and **information**

My recent and ongoing relevant efforts

- Past and current centers established (as founding director)
  - **rCITI at UNSW**
  - **Network Modeling Center – DTA (UT-Austin)**
  - **NSF Center for EVs (UT-Austin)**

- Over 200 papers and 40+ funded projects for:
- ARC, NICTA, NSF, FHWA, SHRP, Texas/Illinois/Ohio/NJ DOTs, NCTCOG, Chicago RTA, MAE Center, SWUTC, USDOT, Port Authority of NY and NJ, Cities of Austin and San Francisco,
- Parson Brinkerhoff, Research Systems Group, Booze Allen, Evans & Peck
- TSS, PTV (transport software companies) and GoGet (car-share company)
- On applications including:
- Traffic/transit network optimization, routing algorithms, integrated financial analysis/PPPs, ITS, V2V, disrupted behavior, sustainability, EVs, health, environmental justice, etc.

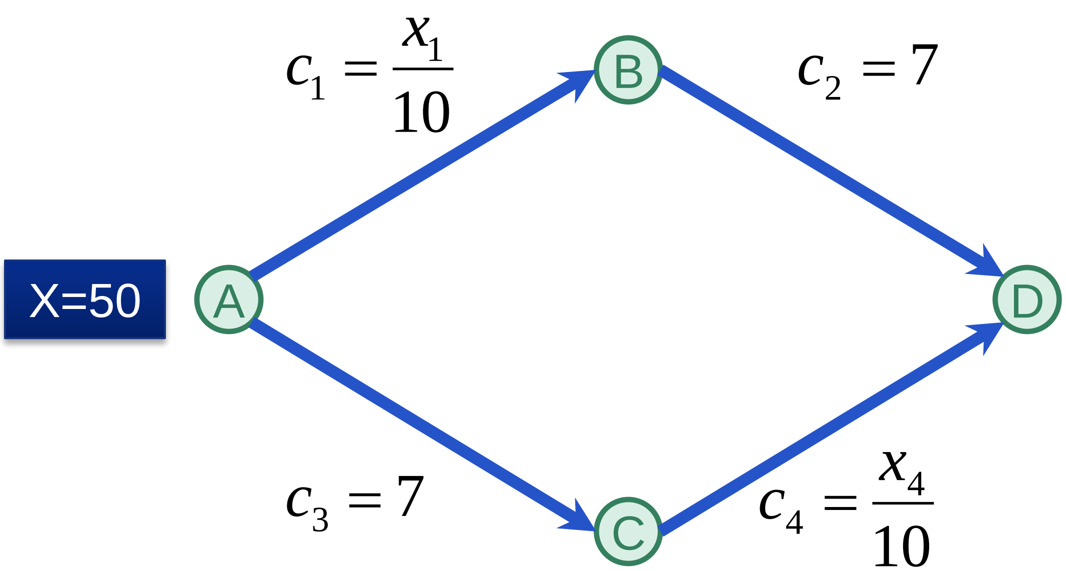
Dynamic Network Assignment

- Representation of traveler behavior within a large-scale network context
  - Primarily addressed through equilibrium models combined with simulation-based optimization approaches

- Sponsors to-date:
  - North Central Texas Council of Gov.
  - Federal Highway Administration
  - Chicago Regional Transportation Authority
  - Strategic Highway Research Program
  - Capitol Area Metropolitan Planning Organization
  - City of Austin
  - National Science Foundation
  - Texas DOT

Let’s start with traditional “static” approaches to the problem

Simplified Static Equilibrium Model
Braess's Paradox (simplified example)



2 Paths

- P_1= A-B-D
- P_2 = A-C-D

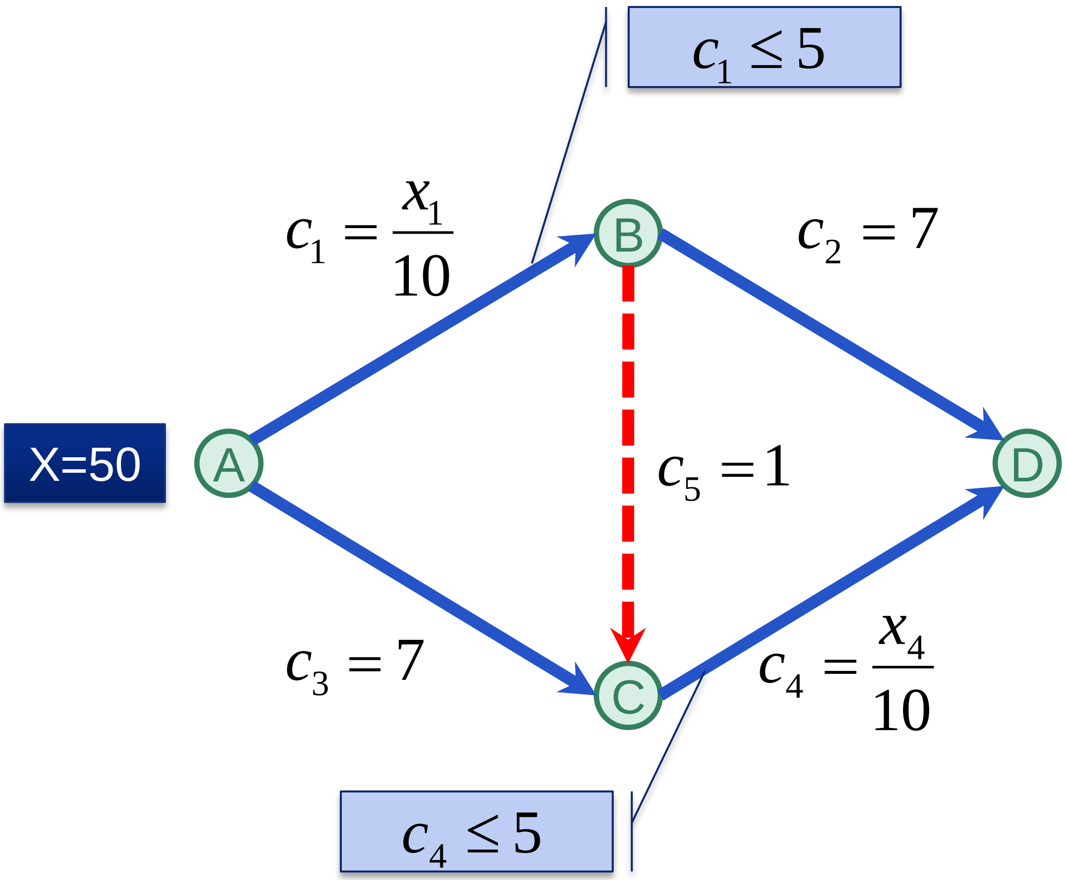
Equilibrium flows

- $P_1 = P_2 = 25$

$c_1+c_2 = c_3+c_4 =$ **9.5**

Total cost = 475

Braess's Paradox Example



3 Paths

- P_1 = A-B-D
- P_2 = A-C-D
- P_3 = A-B-C-D

Equilibrium flows

$P_3 = 50$, $P_1 = 0$, $P_2 = 0$

$C_1 + C_5 + C_4 = \mathbf{11}$

Total cost = 550

“Static” Traffic Assignment

- Formulation (Beckman, 1956)

$$\min \sum_a \int_0^{x_a} c_a(\omega)\,d\omega$$

$$s.t.$$

$$\sum_k h_k^{rs} = q_{rs} \qquad \forall r, s$$

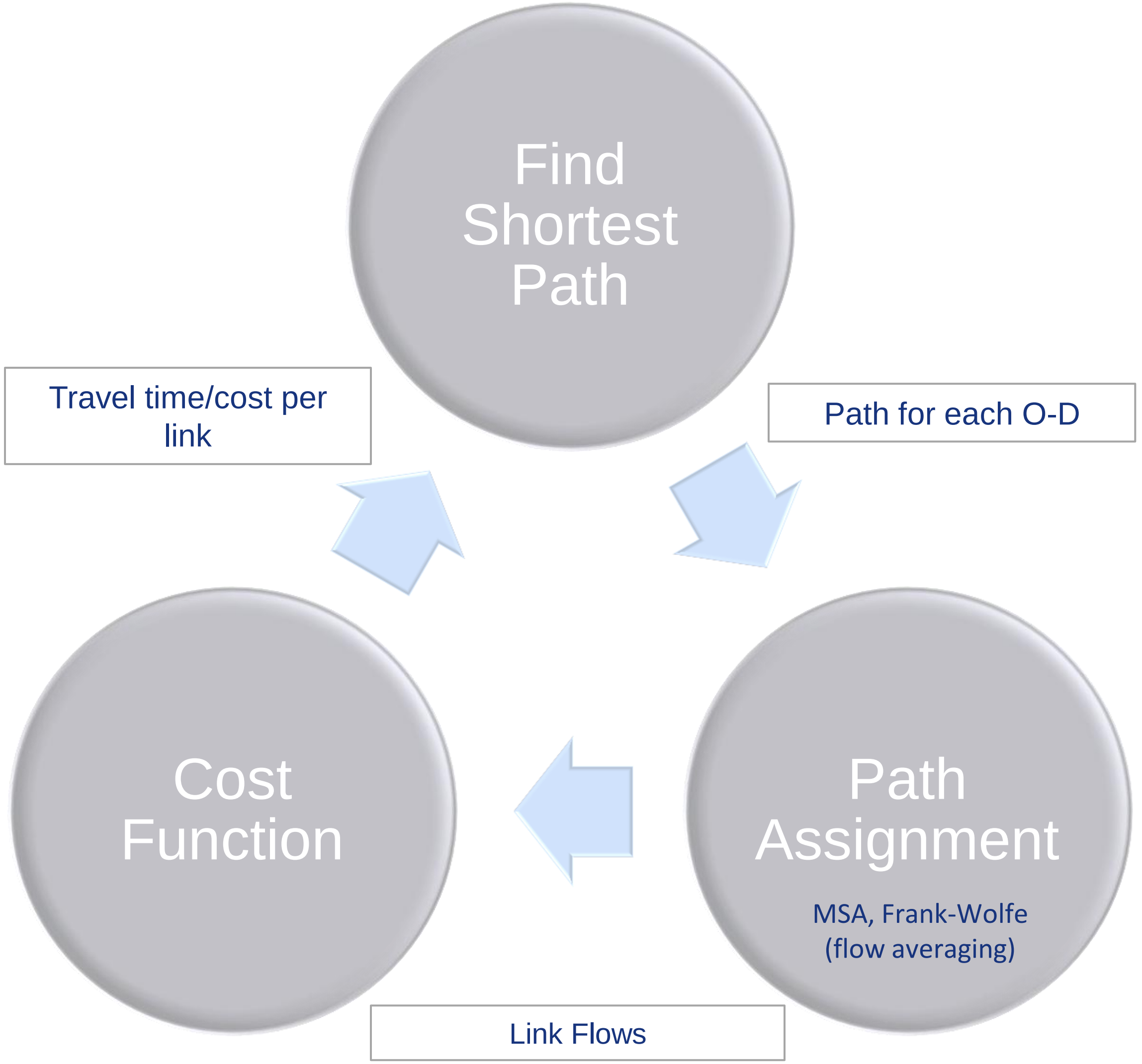
$$h_k^{rs} \geq 0 \qquad \forall k, r, s$$

$$x_a = \sum_r \sum_s \sum_k h_k^{rs} \delta_{a,k}^{rs} \qquad \forall a$$

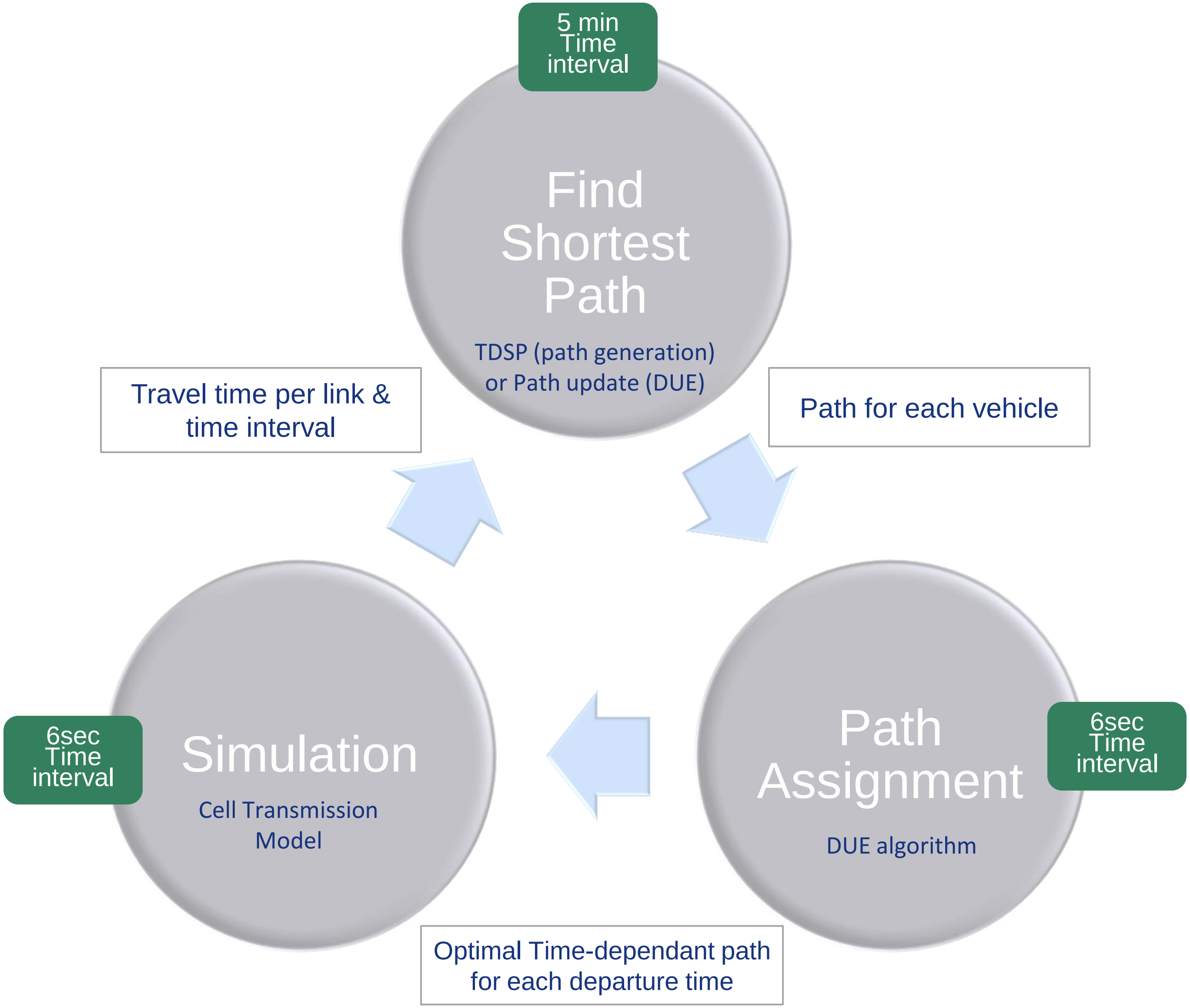
Advances in Network Realities

- Numerous advances over the past 60 years
  - Stochasticity
  - **Dynamics**
  - Multiple classes of travel behaviour
  - Pricing
  - Network design
  - Signal design
  - **Information**
  - Demand/Supply integration
  - Many others

"Static" Network Assignment Solution Approach



DTA Solution Approach



DTA and Travel Demand Formulation

$$DTA: \Psi(\Xi^*)^T(\Xi - \Xi^*) \geq 0 \qquad \forall \Xi \in D$$

$$DEMAND: \Psi(\Xi^*) = S(P(Z(\Psi(\Xi^*))))$$

Ξ = Any feasible DTA solution(vector)

Ξ^* = Optimal DTA solution(vector)

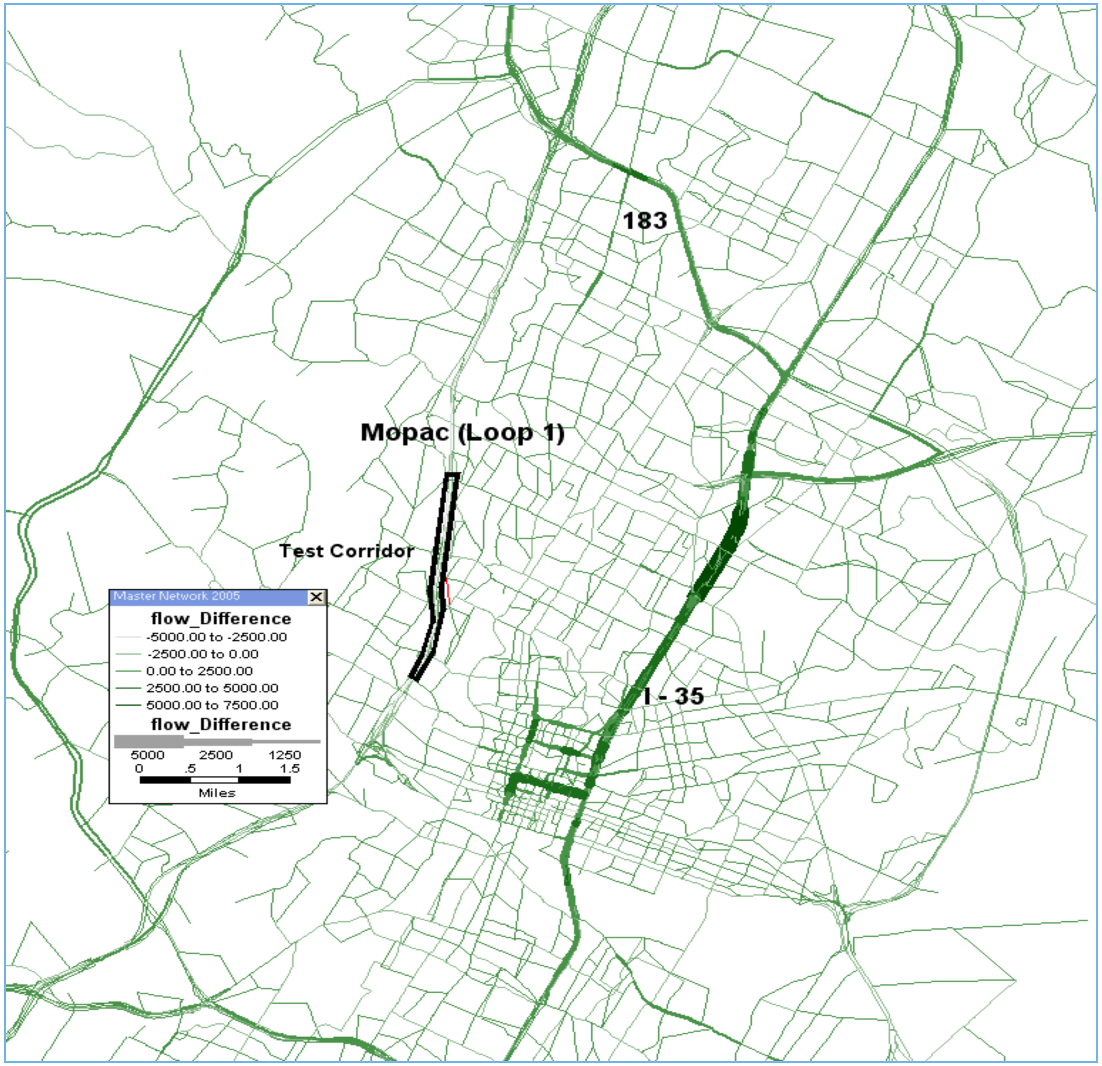
$\Psi(\Xi)$ = Path cost vector resulting from DTA Ξ

$Z(\Psi(\Xi))$ = Dynamic trip table resulting from path cost vector $\Psi(\Xi)$

$P(Z(\Psi(\Xi)))$ = User paths vector from assigning trip table $Z(\Psi(\Xi))$

$S(P(Z(\Psi(\Xi))))$ = Path cost vector obtained from simulating user paths $P(Z(\Psi(\Xi)))$

Corridor-level to Network-level Effects



- Incorporate ITS or pricing approach into embedded simulation
- DTA evaluates changes in route-choice and network-wide impacts
- **Example from a project evaluating ATM on Mopac corridor**

DTA Model Status

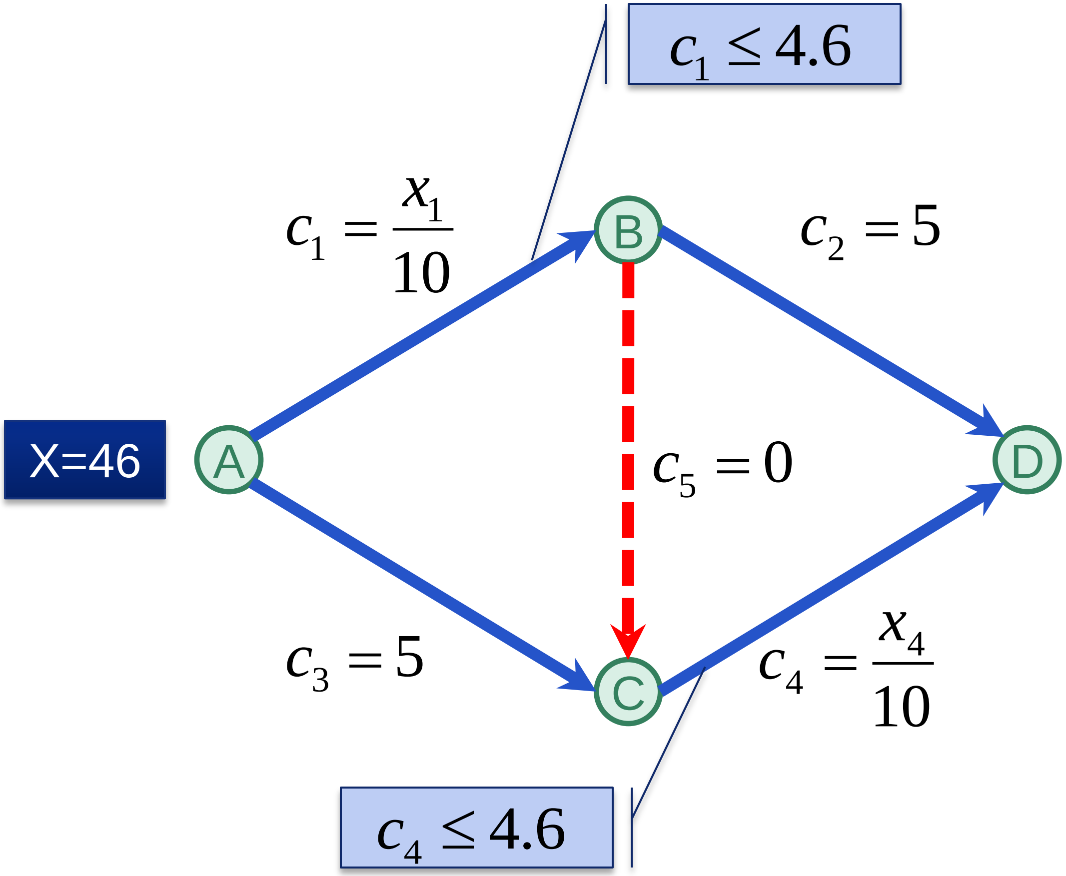
- Existing model being employed by the Central Texas region for long-term planning investments
- Numerous new capabilities and requests are in development
- Additional work is underway elsewhere in the state and nation

Recognized outcome: **the region has gained the ability to answer specific planning questions quantitatively that they previous could not**

Evolving Stochastic Dynamic Network Research (examples)

- Past NSF Awards
  - **“CAREER Accounting for Information and Recourse in the Robust Design and Optimization of Stochastic Transportation Networks”**
  - **“Multi-stage Optimization of Stochastic Dynamic Transportation Networks”**
- Ongoing NSF Grants
  - **“Predicting Disrupted Network Behavior”**
    - Collaborative with Psychology (Prof. Brad Love)
  - **“Center for Transportation and Electricity Convergence”**

Recall: Braess's Paradox Example



3 Paths

- A-B-D (y_1)
- A-C-D (y_2)
- A-B-C-D (y_3)

$y'_1 = y'_2 = 0$

$y'_3 = 46$

$c'1 + c'5 + c'4 = 9.2$

$Z' = 423.2$

$Z' - Z = 87.4$

New Model for Our Problem

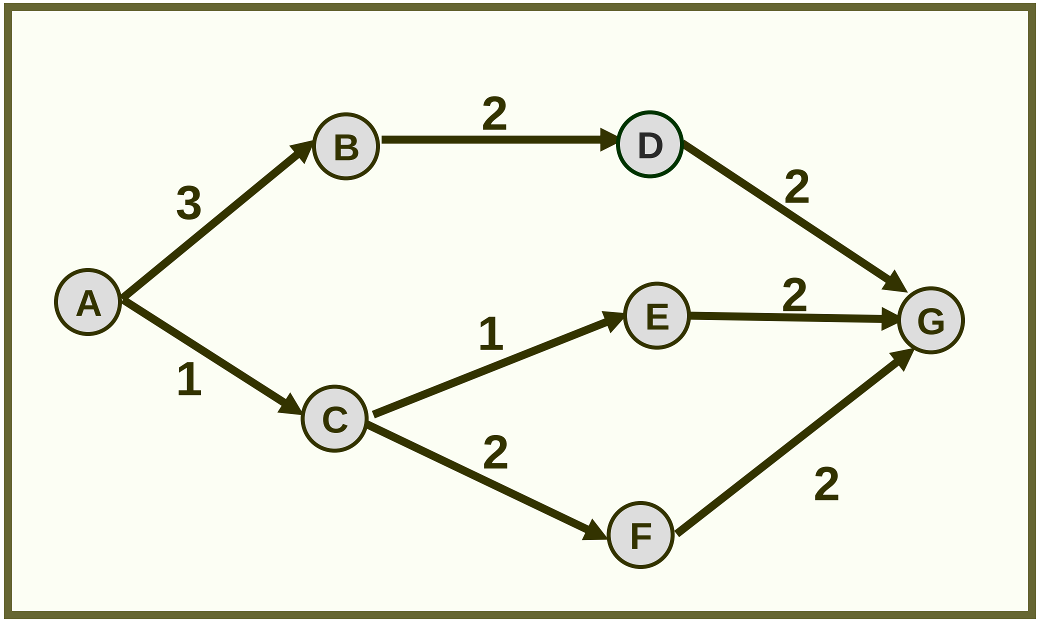
- We need similar models for **information** and **uncertainty** evaluation

- True impact of real-time ITS?
  - Fundamental **behavior**, including **anticipation**, will change

- We will begin with an examination of *individual routing under information*

Deterministic Costs:
Example Network

Path Costs
ABDG: 7
ACEG: 4
ACFG: 5

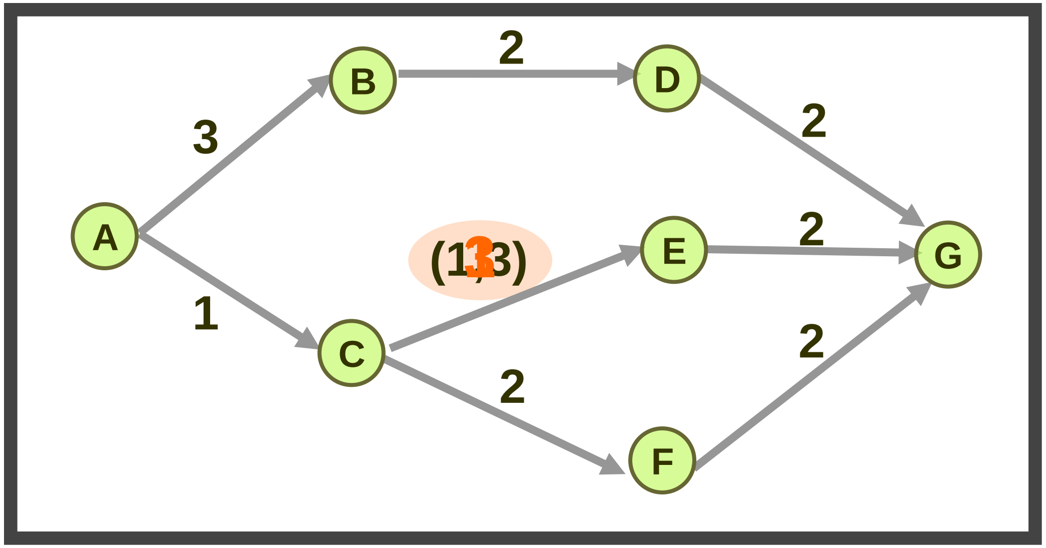


A user travel from A to G

Costs do not change with flow

Three elementary paths

Stochastic Costs:
Arc States & Hyper-paths



2 states

State 1 with cost 1

State 2 with cost 3

Both states have equal probability

Online Routing: Users learn the state of CE when they reach C

Recourse : Users change their paths en-route depending on the information received

State 1: ACEG

State 2: ACFG

Solution : Model assigns users to **hyperpaths**

AC/1-EG

AC/2-FG

Online Shortest Path (OSP)

- Numerous issues exist for even simple OSPs

- A couple quick examples and solution properties

Notation

o = origin node d = destination node

$S_{a,b}$ = Set of possible states for arc (a,b)

E[b|a,s]= expected cost to d from b, given that arc (a,b) is traversed at state s

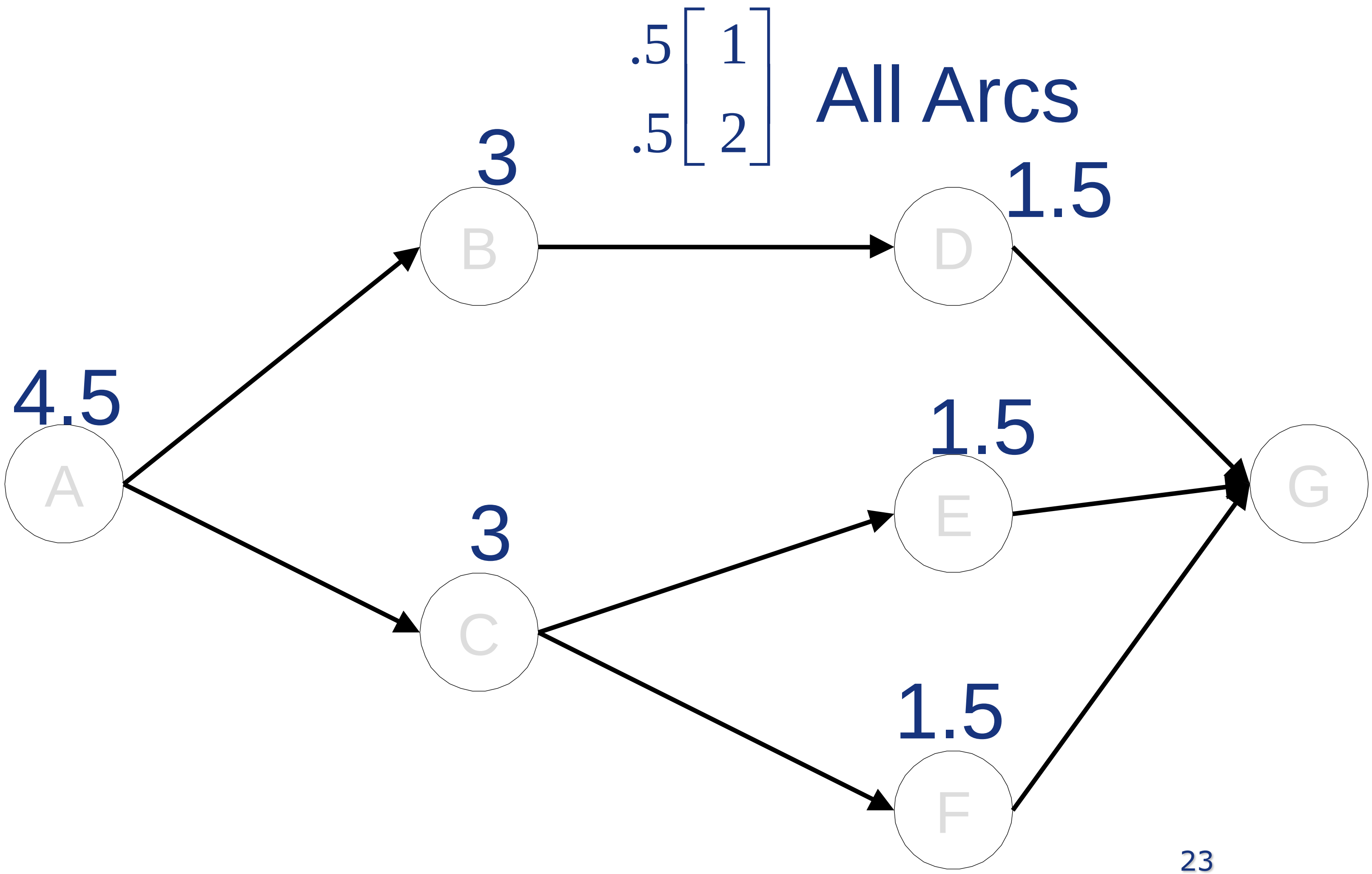
$$p_{s,k}^{a,b,c} = \text{probability that arc } (b,c) \text{ is in state } k, \text{ given that arc } (a,b) \text{ was in state } s$$

SE = scan eligible list

$\Gamma^{-1}(a)$ = set of all predecessor nodes of a

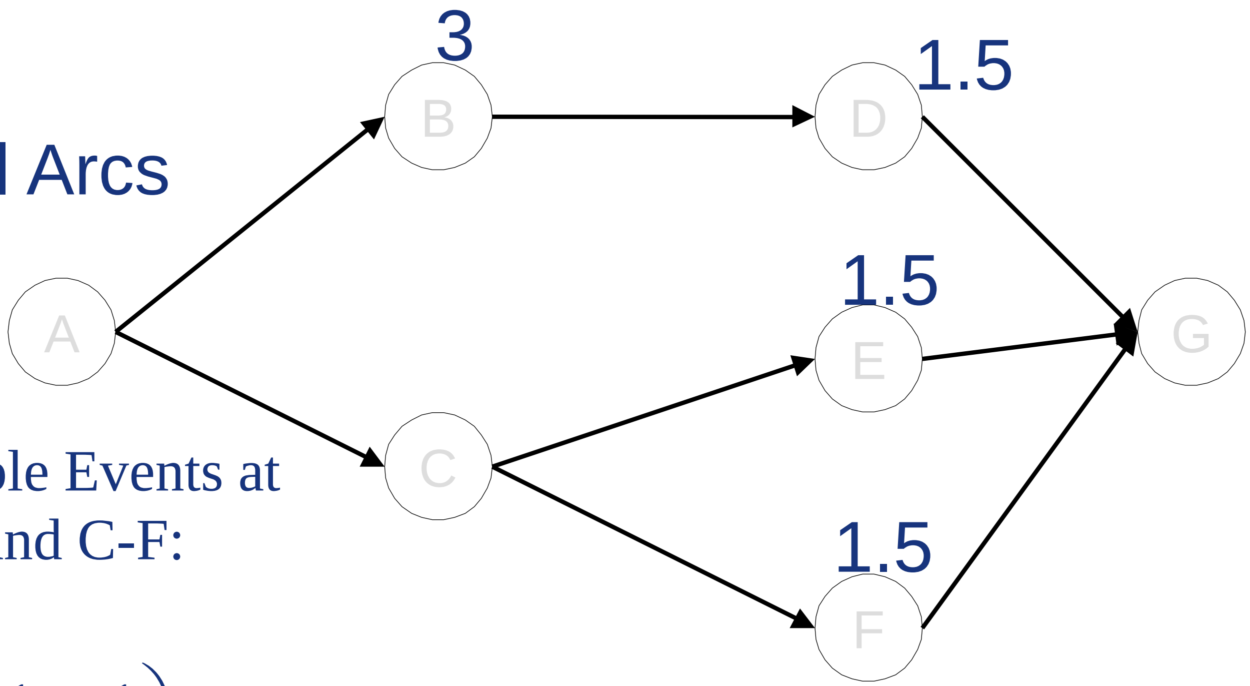
$\Gamma(a)$ = set of all sucessor nodes of a

A Priori (offline) Example



On-line Example

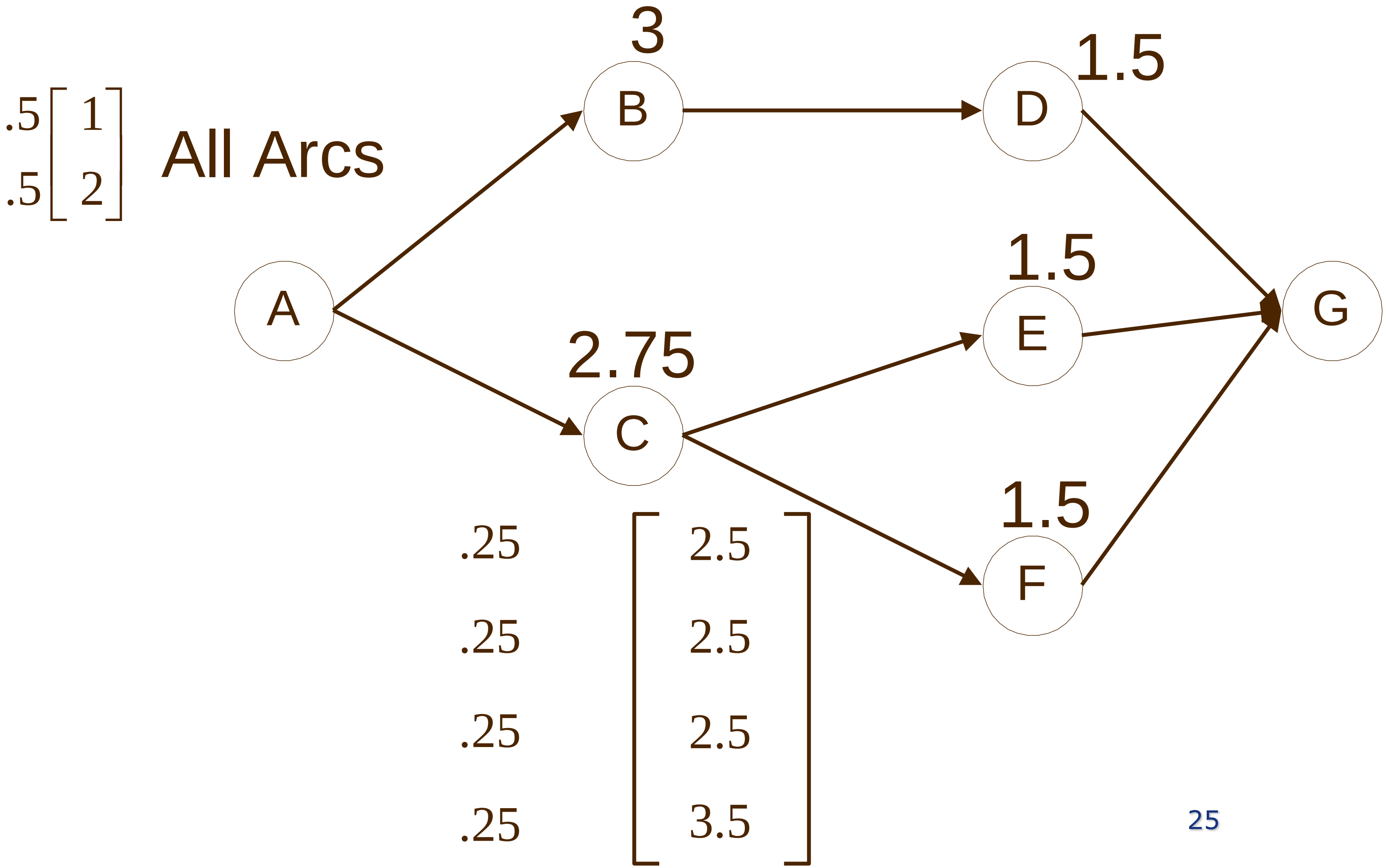
$$\begin{matrix} .5 \\ .5 \end{matrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$ All Arcs



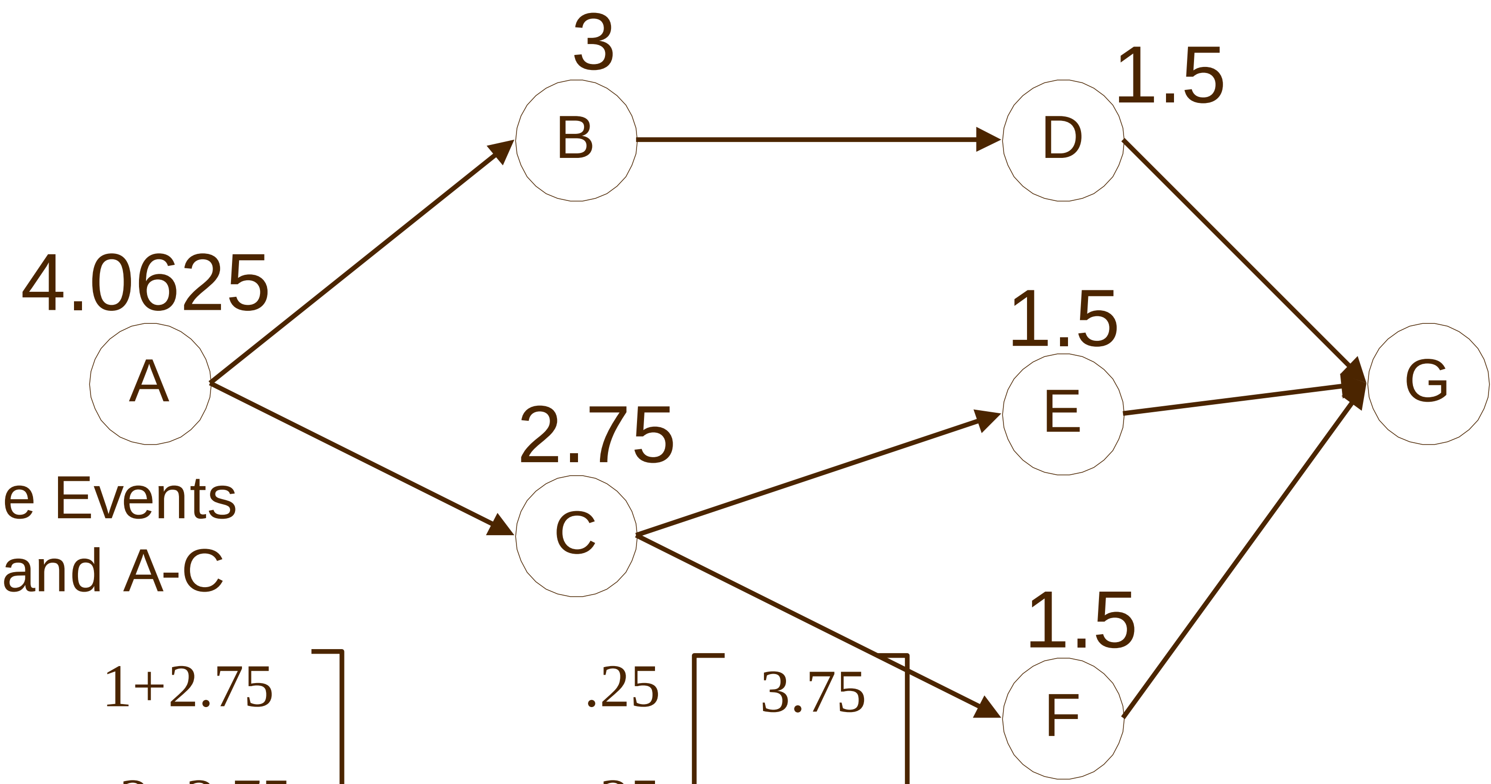
Possible Events at C-E and C-F:

$$\begin{matrix} .25 \\ .25 \\ .25 \\ .25 \end{matrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{pmatrix}$$

On-line Example



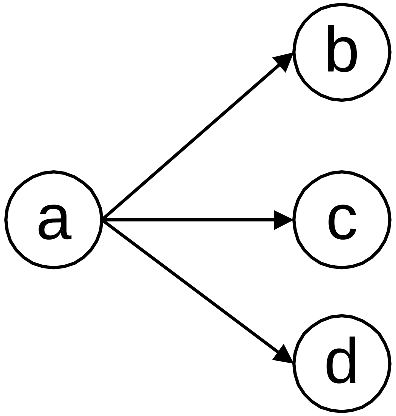
On-Line Example



Possible Events
at A-B and A-C

$$\begin{matrix} .25 \\ .25 \\ .25 \\ .25 \end{matrix} \begin{bmatrix} 1+3 & 1+2.75 \\ 1+3 & 2+2.75 \\ 2+3 & 1+2.75 \\ 2+3 & 2+2.75 \end{bmatrix} \Rightarrow \begin{matrix} .25 \\ .25 \\ .25 \\ .25 \end{matrix} \begin{bmatrix} 3.75 \\ 4 \\ 3.75 \\ 4.75 \end{bmatrix}$$

Simple Combined Probability Matrix



$$P^{a,b} = \begin{matrix} .333 \\ .333 \\ .333 \end{matrix}\begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}, P^{a,c} = \begin{matrix} .5 \\ .5 \end{matrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix},$$

$$P^{a,d} = \begin{matrix} .5 \\ .5 \end{matrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$E = \begin{matrix} .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \\ .0833 \end{matrix}\begin{bmatrix} \min(1+E[b], 2+E[c], 3+E[d]) \\ \min(1+E[b], 6+E[c], 3+E[d]) \\ \min(1+E[b], 2+E[c], 5+E[d]) \\ \min(1+E[b], 6+E[c], 5+E[d]) \\ \min(4+E[b], 2+E[c], 3+E[d]) \\ \min(4+E[b], 6+E[c], 3+E[d]) \\ \min(4+E[b], 2+E[c], 5+E[d]) \\ \min(4+E[b], 2+E[c], 5+E[d]) \\ \min(8+E[b], 2+E[c], 3+E[d]) \\ \min(8+E[b], 6+E[c], 3+E[d]) \\ \min(8+E[b], 2+E[c], 5+E[d]) \\ \min(8+E[b], 6+E[c], 5+E[d]) \end{bmatrix}$$

Pair-Wise Combination

- Combine first two arcs:

$$\begin{bmatrix} .166 & \min(1+E[b], 2+E[c]) \\ .166 & \min(1+E[b], 6+E[c]) \\ .166 & \min(4+E[b], 2+E[c]) \\ .166 & \min(4+E[b], 6+E[c]) \\ .166 & \min(8+E[b], 2+E[c]) \\ .166 & \min(8+E[b], 6+E[c]) \end{bmatrix}$$

- There can be at most 5 unique states in this matrix.
- Therefore, this matrix can be reduced and then combined with another arc.

Matrix Reduction

- 1)Create an empty dynamic Linked List (LL)
- 2)Remove row (a), consisting of a state cost and probability, from the original matrix
- 3)Perform a Binary Search on LL for the state of (a)
- 4)If it exists, add the probability from (a) to element in LL
- 5)If it does not exist, insert (a) into LL at the place pointed to by the binary search

Complexity of Reduction

- Take S to be the maximum number of States on any arc.
- This procedure must be carried out until the original combined matrix is empty, at most S^2 times.
- Each steps takes O(1) except 3.
- The maximum size of a reduced matrix is nS.
- Step 3 can be completed in log(nS).
- Reduction takes S^2 log(nS). For each pair-wise combination

Probability Bounds, Positive Costs

- C = Minimum Arc Cost, M = Maximum Arc Cost
- N = Number of Nodes,   E=Expected # of Arcs
- p(i) = Probability of exactly i cycles
- F = Cumulative distribution for # of Arcs

- C * E[# of Arcs] $\leq$ NM

$$E = \sum_{i=0}^{\infty} i * p(i)$$

Probability Bounds

- $C * E \le NM$

$$E = \sum_{i=0}^{\infty} i * p(i)$$

- Take $\varepsilon(j)$ as a lower bound on E:
- $$\varepsilon(j) = \sum_{i=j}^{\infty} j * p(i)$$ where $j \ge 0$ integer
- $\varepsilon(j) = j*(1-F(j))$
- Since $\varepsilon(j) \le E \le NM/C$
- $\Rightarrow 1-F(j) \le NM/(Cj)$

Properties and Complexity

- Cumulative probability F() that the optimal solution will contain j arcs is bounded:
  - $1-F(j) \leq nM/(Cj)$

- State space matrices can be iteratively bounded and reduced

- Yields algorithm complexity, given error ε
  - $O(n^2 m S^2 M(nM-C) / (C^2 \varepsilon))$

Online Algorithm

Step 1.

$E[d|i,s]=0 \quad \forall\ i\in \Gamma^{-1}(d),\ s\in S_{i,t}$

$E[n|i,s]=\infty \quad \forall\ n\in N/d, \quad i\in\Gamma^{-1}(n),\ s\in S_{i,n}$

SE:= d

Step 2.

while SE$\neq \varnothing$

Remove an element, n, from the SE

for each $i\in\Gamma^{-1}(n)$, $s\in S_{i,n}$, $j\in\Gamma(n)$

$$\pi[n\,|\,i,s]=\sum_{k\in S_{n,j}} p_{s,k}^{i,n,j}(c_k^{n,j}+E[j\,|\,n,k])$$

If $\pi[n|i,s]< E[n|i,s]$, then $E[n|i,s]:=\pi[n|i,s]$

SE:=SE $\cup\{j\in\Gamma^{-1}(i)\}$

UER Network Assignment Model

Equilibrium Formulation

- Accounts for **congestion effect**
- Costs are a function of flow & network state

Model Assumptions

- Cost **functional form varies** according to the network state
- Travelers **learn the cost** functional form of an arc when they reach **upstream node**

Network Equilibrium with Recourse

Develop **analytical formulation** for traffic network assignment problem under online information provision

- User Equilibrium
- System Optimal

Develop a Frank-Wolfe based **solution algorithm** for solving the problem

- Static network assignment
- Limited one-step information

UER Model Definitions & Assumptions

Arc states follow a **discrete probability distribution**

When a **traveler reaches node *i*** they learn the cost functional form for **ALL arcs *(i,j)***

Special case: travelers learn the **capacity** on each arc

$C_{ijs}(\ )$ is the state-dependent cost function

$s \in S_{ij}$

S_{ij} is the set of possible states for arc *(i,j)*

Model A: All users see the same node state

Model B: Users see different node states

Model A : Expected Hyperpath Cost

Node State: combination of emanating link state realizations

System State: combination of node state realizations

Hyperpath Flow: H^k (for hyperpath ***k***)

Link/Hyperpath incidence: $\gamma^k_{i-j/u}\begin{cases}\mathbf{1} & \text{if hyperpath } \boldsymbol{k} \text{ uses arc } \boldsymbol{(ij)} \text{ under state } \boldsymbol{u}\\ \mathbf{0} & \text{otherwise}\end{cases}$

Hyperarc Flow: $f_{i-j/u} = \sum_k \gamma^k_{i-j/u} H^k$ (given system state ***u***)

Hyperarc Flow Vector: $F = \Delta H$

Hyperpath flow vector

Node-hyperpath accessibility matrix

Hyperpath-Hyperarc Accessibility Matrix: $P_{l,k} = p_u \gamma^k_{i-j/u}$

Probability of system state u

Expected Hyperpath Cost Vector: $P^T C[\Delta H]$

Model A: Formulation & Solution Algorithm

CONVEX FORMULATION

$$Min \;\; Z[F(H)] = \sum_{iju} \int_{x=0}^{f_{i-j/u}} p_u.C_{i-j/u}(x)dx$$

Subject to $\quad F = \Delta H \quad t = BH \quad H \geq 0$

SOLUTION ALGORITHM : FRANK-WOLFE

Step 1: At iteration n, fix the costs on the arcs $C_{i-j/u}(f^n_{i-j/u})$

Step 2: Determine the optimal hyperpath H

Step 3: Conduct all-or-nothing assignment on H

Step 4: Determine the auxiliary link flows $y^{n+1}_{i-j/u}$

Step 5: Determine $f^{n+1}_{i-j/u}$ by a linear combination of $y^{n+1}_{i-j/u}, f^n_{i-j/u}$

Step 6: Test for convergence. If no set $n=n+1$, go to Step 1

Model A: Equilibrium Condition

Property: *A traffic network is in UER if each user follows a route that guarantees the minimum hyperpath (strategy) available and no user can unilaterally change his/her route to improve their travel time*

EQUILIBRIUM CONDITION

$$H^T[P^T C[\Delta H] - B^T u] = 0$$

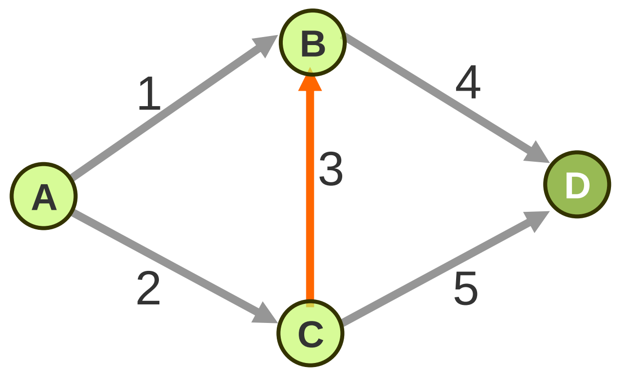
$$P^T C[\Delta H] - B^T u \geq 0$$

$$H \geq 0$$

INSIGHTS

- All used hyperpaths will have equal (and minimum) expected cost.
- This implies that those network users who follow a UER solution without options, still receive precisely the same benefit as those users who actually experience the options.

Without information



- ***Arc CB*** has ***2 STATES***:
  - ***State 1:*** $C_3(x)=1000$ (wp 0.2)
  - ***State 2:*** $C_3(x)=1$ (wp 0.8)
- Other arcs: ***single states***
  - $C_1(x)=5$, $C_2(x)=x/10$ (wp 1)
  - $C_4(x)=X/10$, $C_5(x)=5$ (wp 1)

PATHS

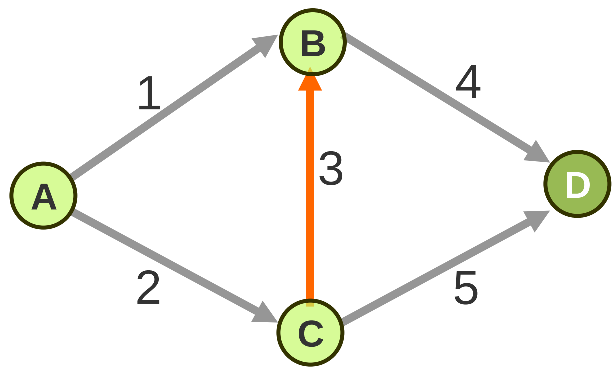
P1: A-B-D

P2: A-C-D

P3: A-C-B-D

- **DEMAND:** ***40 users*** want to travel from A to D
- **Solution:** all users split over paths P1 and P2 (*P3 too risky*)
- P1 = P2 = 20
- **User Cost = 7**

UER Example



- ***Arc CB*** has ***2 STATES***:
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  - ***State 2:*** $C_3(x)=1$ (wp 0.8)
- Other arcs: ***single states***
  - $C_1(x)=5$, $C_2(x)=x/10$ (wp 1)
  - $C_4(x)=X/10$, $C_5(x)=5$ (wp 1)

HYPERPATHS

H1: A-B-D

H2: A-C/1-B-D & A-C/2-B-D

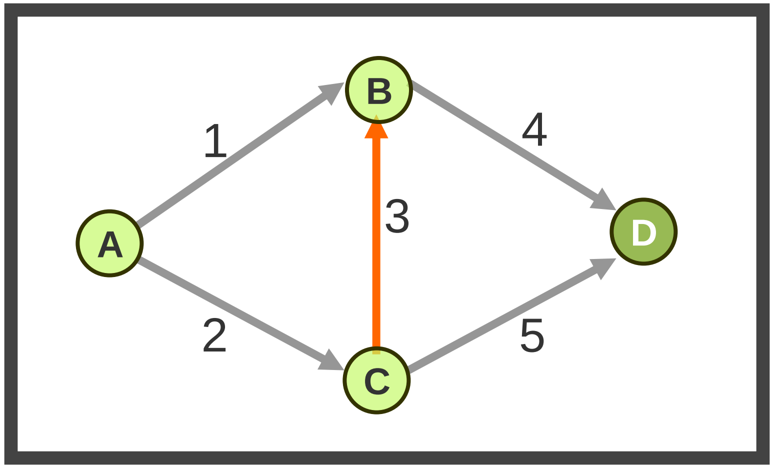
H3: A-C/1-B-D & A-C/2-D

H4: A-C/1-D & A-C/2-D

H5: A-C/1-D & A-C/2-B-D

- **DEMAND:** ***40 users*** want to travel from A to D
- Users assigned to ***HYPERPATHS***

UER Example



HYPERPATH	*FLOW*	*EXP COST*
H1	8.33	8.1666
H2	0	207.1333
H3	0	208.3333
H4	2.5	8.1666
H5	29.166	8.1666

All used hyperpaths have equal and minimal expected costs

*Flow on **BD** depends on **state of C**. Even though states are not correlated, the flow induces dependency*

UER vs UE Without Information: Braess Paradox

Expected User Cost
UER : 8.1666

Expected User Cost
No Information: 7

If everybody has access to the network state information, system performance may be worse than under a no-Information scenario

Fundamental implications when **planning for information provision** through **ITS devices**

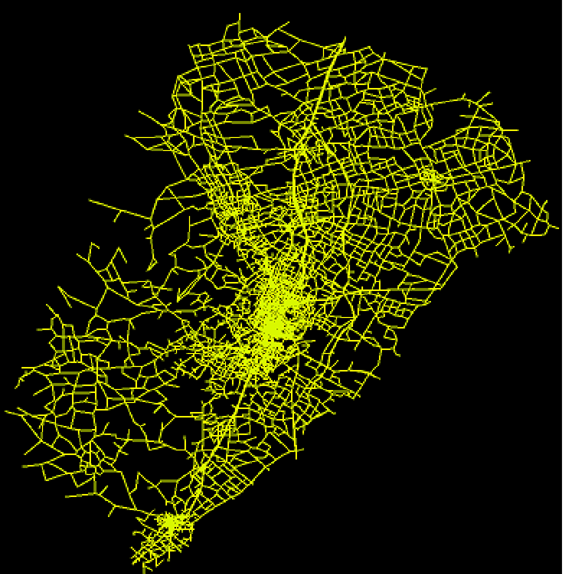
These analytical models form the next generation of deployable practical models

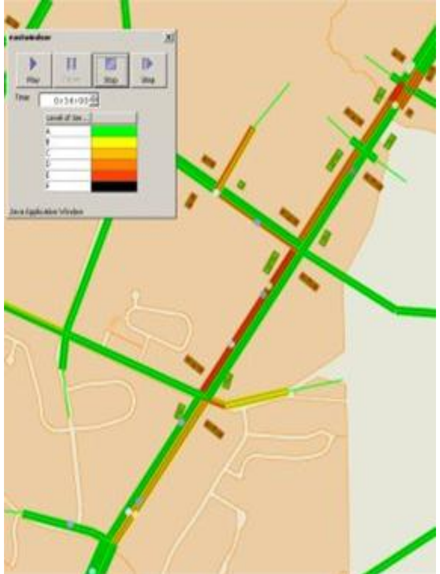
We need additional algorithmic computational improvement

Summary

- Overview of network equilibrium and DTA
- New algorithms for online shortest path
- New models for user equilibrium with recourse

These models form one specific piece of the overall integrated system.





Questions?

Consideration: Possible Special Issue

- **Computer-Aided Civil and Infrastructure Engineering (CACAIE)**
  - Publisher: Wiley
  - **Impact Factor: 3.382**
  - ISI Journal Citation Report
    - 1/56 (Construction & Building Technology)
    - 2/28 (Transportation Science & Technology)
    - 3/118 (Engineering Civil)
    - 9/99 (Computer Science Interdisciplinary Applications)